3. WGS 84 ELLIPSOID

3.1 General

In geodetic applications, three different surfaces or earth figures are normally involved. In addition to the earth's natural or physical surface, these include a geometric or mathematical reference surface, the ellipsoid, and an equipotential surface called the geoid (Chapter 6). In determining the WGS 84 Ellipsoid and associated parameters, the WGS 84 Development Committee, in keeping with DMA guidance, decided quite early to closely adhere to the thoughts and approach used by the International Union of Geodesy and Geophysics (IUGG) when the latter established and adopted Geodetic Reference System 1980 (GRS 80) [3.1]. Accordingly, a geocentric equipotential ellipsoid of revolution was taken as the form for the WGS 84 Ellipsoid. The parameters selected to define the WGS 84 Ellipsoid are the semimajor axis (a), the earth's gravitational constant (GM), the normalized second degree zonal gravitational coefficient ($\bar{C}_{2,0}$), and the angular velocity (ω) of the earth (Table 3.1). These parameters are identical to those for the GRS 80 Ellipsoid with one minor exception. The coefficient form used for the second degree zonal is that of the WGS 84 Earth Gravitational Model rather than the notation J_2 used with GRS 80. Accuracy estimates (one sigma) are also given in Table 3.1 for the defining parameters. Cited references treating GRS 80 have been borrowed from freely in preparing this Chapter.

3.2 The Equipotential Ellipsoid

An equipotential ellipsoid, or level ellipsoid, is an ellipsoid defined to be an equipotential surface. If an ellipsoid of revolution (with semimajor axis a and semiminor axis b) is given, then it can be made an equipotential surface

$$U = U_0 = \text{Constant}$$

of a certain potential function U, called the normal (theoretical) gravity potential. This function is uniquely determined by means of the ellipsoid surface (semiaxes a, b), the enclosed mass (M), and the angular velocity (ω), according to a theorem of Stokes-Poincare, quite independently of the internal density distribution. Instead of the four constants, a, b, M, and ω , any other system of four independent parameters may be used as defining constants. However, as noted in Section 3.1, above, the defining parameters of the WGS 84 Ellipsoid are the semimajor axis (a), the earth's gravitational constant (GM), the earth's angular velocity (ω), and the normalized second degree zonal harmonic coefficient ($\bar{C}_{2,0}$) of the gravitational potential. With this choice of parameters, the equipotential ellipsoid furnishes a simple, consistent, and uniform reference system for all purposes of geodesy - the ellipsoid as a reference surface for geometric use (mapping, charting, etc.) , and a normal (theoretical) gravity field at the earth's surface and in space, defined in terms of closed formulas, as a reference for gravimetry and satellite geodesy.

The standard theory of the equipotential ellipsoid regards the normal (theoretical) gravitational potential as a harmonic function outside the ellipsoid, which implies the absence of an atmosphere. Thus, the computations are based on the theory of an equipotential ellipsoid without an atmosphere. The reference ellipsoid is defined to enclose the whole mass of the earth, including the atmosphere. As a visualization, the atmosphere can be considered to be condensed as a surface layer on the ellipsoid. The normal (theoretical) gravity field at the earth's surface and in space can therefore be computed without any need for considering the variation of atmospheric density.

If atmospheric effects must be considered, this can be done by applying corrections to the measured values. This is the standard procedure in the case of the effect of atmospheric refraction on angle and electronic distance measurements. A similar procedure is used for gravity data, where atmospheric corrections are applied to measured values of gravity. A table of atmospheric corrections for gravity measurements was included as an integral part of the report on GRS 80 [3.1]. A similar table pertaining to WGS 84 is included in Chapter 4. Due to the importance of this application, additional details are also given in [3.2].

It is important to note that defining the reference ellipsoid (the WGS 84 Ellipsoid) to enclose the mass of the earth, including the atmosphere, differs from the definition adopted for previous WGS Ellipsoids. (The WGS 72 and earlier WGS Ellipsoids did not include the mass of the atmosphere.) This change in definition affects theoretical gravity and must be considered in the calculation of gravity anomalies. More discussion on this topic will occur later in this Chapter, as well as in Chapter 4.

3.3 Defining Parameters

3.3.1 Semimajor Axis (a)

The value adopted for the semimajor axis (a) of the WGS 84 Ellipsoid, a defining parameter, and its one-sigma accuracy estimate are:

$$a = (6378137 \pm 2) \text{ meters.}$$

This a-value, which is the same as that for the GRS 80 Ellipsoid, is two meters (m) larger than the value of 6378135 m adopted for the WGS 72 Ellipsoid [3.3]. As stated in [3.4], the GRS 80 (and thus the WGS 84) a-value is based on estimates from the 1976-1979 time period, determined using laser, Doppler, radar altimeter, laser plus radar altimeter, and Doppler plus radar altimeter data/techniques. These efforts yielded values from 6378134.5 m to 6378140 m. The best estimate was considered to lie between 6378135 m and 6378140 m, with a representative value being

$$a = (6378137 \pm 2) \text{ m.}$$

For comparison purposes, Table 3.2 contains the semimajor axis and ellipticity (flattening) of the WGS 84 Ellipsoid and other well-known reference ellipsoids.

Since 1979, estimates of the semimajor axis have been made using satellite radar altimeter data, satellite laser ranging data, and Doppler satellite data. Results from these determinations, summarized in [3.5], range from 6378134.9 to 6378137.0 meters. Therefore, a = 6378137 meters and $\sigma_a = \pm 2$ meters are good choices for the semimajor axis of the WGS 84 Ellipsoid and its accuracy.

Two viewpoints exist with respect to the accuracy of the semimajor axis of the WGS 84 Ellipsoid (or any ellipsoid). From one perspective, the WGS 84 Ellipsoid is a mathematical figure adopted for the earth, and since it is an adopted figure having a selected (adopted) a-value, the assignment of an accuracy to "a" is not required and should not be done. However, when attempting to adopt a geocentric ellipsoid that best fits the figure (mean sea level surface) of the earth, an accuracy value for the semimajor axis has merit since it is a measure of how well such a fit has been accomplished. It is in this context that $\sigma_a = \pm 2$ meters is stated for the semimajor axis of the WGS 84 Ellipsoid, and that is its primary, if not sole, use. For example, σ_a should not be used in any error analysis treating the accuracy of WGS 84 geodetic latitude and geodetic longitude values determined from WGS 84 rectangular (X,Y,Z) coordinates.

3.3.2 Earth's Gravitational Constant (GM)

3.3.2.1 GM (With Earth's Atmosphere Included)

The value of the earth's gravitational constant adopted as one of the four defining parameters of the WGS 84 Ellipsoid and its one-sigma accuracy estimate are:

$$GM = (3986005 \pm 0.6) \times 10^8 m^3 s^{-2} . \qquad (3\text{-}1)$$

This value includes the mass of the atmosphere and is based on several types of space measurements. These measurement types and the associated estimates for GM are [3.1]:

Spacecraft radio tracking...............$(3986005.0 \pm 0.5) \times 10^8 m^3 s^{-2}$
Lunar laser data analysis...............$(3986004.6 \pm 0.3) \times 10^8 m^3 s^{-2}$
Satellite laser range measurements......$(3986004.4 \pm 0.2) \times 10^8 m^3 s^{-2}$

From these results, a representative value for GM is

$$GM = (3986004.7 \pm 0.3) \times 10^8 m^3 s^{-2}$$

or, when rounded

$$GM = (3986005 \pm 0.6) \times 10^8 m^3 s^{-2} .$$

Although more recent estimates of GM [3.6] provide a representative value of

$$GM = 3986004.4 \times 10^8 m^3 s^{-2}$$

it is not sufficiently different from the internationally adopted GRS 80 value for GM to warrant its adoption for use with WGS 84.

3.3.2.2 GM_A of the Earth's Atmosphere

3.3.2.2.1 Calculated as a Product of G and M_A

For some applications, it is necessary to either have a GM value for the earth which does not include the mass of the earth's atmosphere, or have a GM value for the earth's atmosphere itself. For this, it is necessary to know both the mass of the earth's atmosphere, M_A , and the universal gravitational constant, G.

The values of G and M_A adopted for WGS 72, as well as recent International Association of Geodesy (IAG) and International Astronomical Union (IAU) recommended values for these parameters, are listed in Table 3.3. Using the value recommended for G [3.7] by the IAG, and the more recent value for M_A [3.8], the product GM_A to two significant digits is

$$GM_A = (3.5 \pm 0.03) \times 10^8 m^3 s^{-2}$$

which is the value currently recommended by the IAG for this product [3.7].

This product is necessary to obtain a value for GM that excludes the mass of the earth's atmosphere, given a GM value that includes it (Section 3.3.2.1). This value of GM_A, with a more conservative accuracy value assigned, was adopted for use with WGS 84; i.e.:

$$GM_A = (3.5 \pm 0.1) \times 10^8 m^3 s^{-2} \qquad (3\text{-}2)$$

3.3.2.2.2 Implied by Atmospheric Correction to Gravity Values

As alluded to earlier, an atmospheric correction to observed gravity (g) is required for gravity anomaly (Δg) determination when theoretical gravity (γ) in the equation

$$\Delta g = g - \gamma + \text{gravity reduction terms} \qquad (3\text{-}3)$$

is for an ellipsoid that includes the mass of the earth's atmosphere, as is the case with the WGS 84 Ellipsoid. A value for GM_A is implied by the corresponding atmospheric correction that must be applied to observed gravity in this situation.

A method for determining the GM_A value implied by a given set of atmospheric corrections to gravity is to compare theoretical gravity values determined from a GM that includes the mass of the earth's atmosphere with theoretical gravity values determined from a GM that excludes the mass of the atmosphere. This was done for the three quantities $\bar{\gamma}$, γ_e, γ_p, where

$\bar{\gamma}$ = average value of theoretical gravity

$$\bar{\gamma} = \gamma_e\left(1 + \frac{1}{6}e^2 + \frac{1}{3}k + \frac{59}{360}e^4 + \frac{5}{18}e^2k + \frac{2371}{15120}e^6 + \frac{259}{1080}e^4k + \frac{270229}{1814400}e^8 + \frac{9623}{45360}e^6k\right) \quad (3\text{-}4)$$

γ_e = theoretical gravity at the equator

$$\gamma_e = \frac{GM}{ab}\left(1 - m - \frac{m}{6}\frac{e'q_o'}{q_o}\right) \quad (3\text{-}5)$$

γ_p = theoretical gravity at the poles

$$\gamma_p = \frac{GM}{a^2}\left(1 + \frac{m}{3}\frac{e'q_o'}{q_o}\right) \quad (3\text{-}6)$$

and e^2, e'^2, k, m, q_o, q_o' are derived constants for which equations will be given later in this Chapter. Comparisons were made between values of $\bar{\gamma}$, γ_e, γ_p and $\bar{\gamma}'$, γ_e', γ_p', respectively, where $\bar{\gamma}'$, γ_e', γ_p' were computed using values of GM_A ranging from

$$GM_A = 3.5 \times 10^8 m^3 s^{-2} \quad \text{to} \quad GM_A = 3.6 \times 10^8 m^3 s^{-2}.$$

The results are shown in Table 3.4. It is noted that a GM_A value of

$$GM_A = 3.54 \times 10^8 m^3 s^{-2}$$

produces a difference of approximately 0.87 milligal (mgal) in theoretical gravity.

An alternative approach for determining which value of GM_A is implied by the atmospheric corrections to observed gravity, is to solve for GM_A, utilizing the above equations for γ_e and γ_p in the form:

$$GM = ab\gamma_e /(1 - m - \frac{m}{6} \frac{e' q_o'}{q_o}) \qquad (3\text{-}7)$$

$$GM = a^2 \gamma_p / (1 + \frac{m}{3} \frac{e' q_o'}{q_o}) . \qquad (3\text{-}8)$$

Replacing γ_e and γ_p in Equations (3-7) and (3-8), rspectively, by the atmospheric correction of 0.87 mgal, the derived values for GM_A are:

$$GM_A = 3.546 \times 10^8 m^3 s^{-2}$$

$$GM_A = 3.527 \times 10^8 m^3 s^{-2}.$$

Thus, it is seen that a GM_A value of approximately

$$GM_A = 3.54 \times 10^8 m^3 s^{-2}$$

is implied by the atmospheric correction to observed gravity. The preceding value, when rounded to two significant digits, is

$$GM_A = 3.5 \times 10^8 m^3 s^{-2} ,$$

which agrees with (is) the value of GM_A adopted for use with WGS 84, Equation (3-2). These results illustrate the consistency that exists between the various WGS 84 parameters, correction terms, etc.

3.3.2.3 GM With Earth's Atmosphere Excluded (GM')

The earth's gravitational constant, with the mass of the earth's atmosphere excluded (GM'), was obtained by subtracting GM_A, Equation (3-2), from GM, Equation (3-1); i.e.:

$$GM' = (3986005 \times 10^8 m^3 s^{-2}) - (3.5 \times 10^8 m^3 s^{-2})$$

$$GM' = (3986001.5 \pm 0.6) \times 10^8 m^3 s^{-2} \qquad (3\text{-}9)$$

The fact that the WGS 84 value for GM', Equation (3-9), is given to one more digit than the WGS 84 value for GM, Equation (3-1), does not imply that GM' is known more accurately than GM. The additional digit used with GM' only reflects a desire to maintain consistency between the various WGS 84 parameters and correction terms. In fact, GM' is known less well, due to the uncertainty introduced via GM_A. The lack of a more realistic accuracy value for GM_A prevents acknowledgment of this in the above one-sigma accuracy estimate for GM'.

3.3.3 Normalized Second Degree Zonal Gravitational Coefficient ($\bar{C}_{2,0}$)

Another defining parameter of the WGS 84 Ellipsoid is the normalized second degree zonal gravitational coefficient, $\bar{C}_{2,0}$, which has the following value and assigned accuracy (one sigma):

$$\bar{C}_{2,0} = (-484.16685 \pm 0.00130) \times 10^{-6}. \qquad (3\text{-}10)$$

This $\bar{C}_{2,0}$ value was obtained from the adopted GRS 80 value for J_2 [3.1], $(J_2=J_{2,0})$,

$$J_2 = 108263 \times 10^{-8} \tag{3-11}$$

by using the mathematical relationship

$$\bar{C}_{2,0} = -J_2/(5)^{1/2} \tag{3-12}$$

and truncating the result to eight significant digits.

The GRS 80 value for J_2 is representative of the second degree zonal gravitational coefficient of several earth gravitational model (EGM) solutions. These EGMs and their respective J_2 values are listed in Table 3.5. Since earth gravitational models are usually expressed in $\bar{C}_{n,m}$, $\bar{S}_{n,m}$ form, the second degree zonal gravitational coefficient is expressed as $\bar{C}_{2,0}$ (as well as J_2) in Table 3.5.

In keeping with the GRS 80 value for J_2, the $\bar{C}_{2,0}$ value for the WGS 84 Ellipsoid also does not include the permanent tidal deformation. This effect, usually represented by δJ_2, is due to the attraction of the earth by the sun and moon. It has the magnitude [3.5]:

$$\delta J_2 = 9.3 \times 10^{-9} \tag{3-13}$$

or, equivalently

$$\delta\bar{C}_{2,0} = -4.16 \times 10^{-9}. \tag{3-14}$$

This quantity would be added to $\bar{C}_{2,0}$, Equation (3-10), if it were desired to have $\bar{C}_{2,0}$ include the permanent tidal deformation.

The question of whether to include the permanent tidal deformation is discussed in some detail in the Appendix to [3.4]. In that discussion, it is stated that:

> "Neither J_2 nor the geoid as determined in geodesy should contain the permanent tidal deformation unless explicitly stated otherwise."

The reason is that Stokes' formula presupposes the gravitational potential to be everywhere harmonic outside the earth. However, this does not hold true for the tidal potential, which is not everywhere harmonic, because it has a singularity at infinity.

3.3.4 Angular Velocity of the Earth

The value of ω used as one of the defining parameters of the WGS 84 (and GRS 80) Ellipsoid and its accuracy estimate (one sigma) are:

$$\omega = 7292115 \pm 0.1500 \times 10^{-11} \text{ radians/second.} \qquad (3\text{-}15)$$

This value, for a standard earth rotating with a constant angular velocity, is an IAG adopted value for the true angular velocity of the earth which fluctuates with time. However, for most geodetic applications which require angular velocity, these fluctuations do not have to be considered.

Although suitable for use with a standard earth and the WGS 84 Ellipsoid, it is the IAU version of this value (ω')

$$\omega' = 7292115.1467 \times 10^{-11} \text{ radians/second,} \qquad (3\text{-}16)$$

that is consistent with the new definition of time [3.17]. This value was derived in [3.18] using the formula

$$\omega' = \frac{2\pi}{86400} \frac{s + 86400 - \mu/1500}{s} \qquad (3\text{-}17)$$

where

$$s = 31556925.9747$$

$$\mu = \rho \cos \varepsilon - 12.473'' \sin^2 \varepsilon \qquad (3\text{-}18)$$

$$\rho = 5025.64''$$

$$\varepsilon = 23°27'08.26'' = 84428.26''.$$

Inserting these values for ρ and ε in Equation (3-17), the resultant equation is, to 12 significant digits,

$$\omega' = \frac{2\pi}{86164.0989041} = 7292115.14667 \times 10^{-11} \text{ radians/second}$$

or, when rounded:

$$\omega' = 7292115.1467 \times 10^{-11} \text{ radians/second.} \qquad (3\text{-}19)$$

Historically, by virtue of the values used in the calculation, ω' is also consistent with the System of Astronomical Constants adopted by the IAU in 1964 [3.18].

For precise satellite applications, the IAU value of the earth's angular velocity (ω'), rather than ω, should be used in the formula

$$\omega^* = \omega' + m \qquad (3\text{-}20)$$

to obtain the angular velocity of the earth in a precessing reference frame (ω^*). In the above equation [3.9] [3.17]:

m = new precession rate in right ascension

$$m = (7.086 \times 10^{-12} + 4.3 \times 10^{-15} T_U) \text{ radians/second} \quad (3\text{-}21)$$

T_U = Julian Centuries from Epoch J2000.0

$T_U = d_U/36525$

d_U = Number of days of Universal Time (UT) from Julian Date (JD) 2451545.0 UT1, taking on values of ± 0.5, ± 1.5, ± 2.5,...

d_U = JD - 2451545.

Therefore, the angular velocity of the earth in a precessing reference frame, applicable for precise satellite applications, may be written:

$$\omega^* = (7292115.1467 \times 10^{-11} + 7.086 \times 10^{-12} + 4.3 \times 10^{-15} T_U) \text{ radians/second}$$

or

$$\omega^* = (7292115.8553 \times 10^{-11} + 4.3 \times 10^{-15} T_U) \text{ radians/second} \quad (3\text{-}22)$$

As stated earlier, the earth's angular velocity fluctuates with time. This fluctuation is rather significant, as is apparent from Table 3.6 and Figure 3.1, which show the high, low, and yearly average values of the earth's angular velocity for years 1967 through 1985. During this time span, the lowest and highest angular velocities (averaged over a five-day period) were:

$$\omega \text{ (lowest)} = 7292114.832 \times 10^{-11} \text{ radians/second}$$

$$\omega \text{ (highest)} = 7292115.099 \times 10^{-11} \text{ radians/second.}$$

This data was taken from the Annual Reports of the BIH [3.19].

The angular velocity values adopted for WGS 72 and WGS 84 are also shown in Figure 3.1. Note that the ω value adopted for use with the WGS 84 Ellipsoid (Equation 3-15) agrees more closely with recent values of ω than does the value adopted for use with the WGS 72 Ellipsoid.

3.4 Derived Geometric and Physical Constants

3.4.1 General

Many parameters associated with the WGS 84 Ellipsoid, other than the four defining parameters (Table 3.1), are needed for geodetic and gravimetric applications. Using the four defining parameters, it is possible to derive these associated constants. The more commonly used geometric and physical constants associated with the WGS 84 Ellipsoid, and the formulas used in their derivation, are presented here for user convenience and information. Unless otherwise indicated, the formulas used in the calculation of the constants are from [3.1] and [3.18].

3.4.2 Fundamental Derived Constant

The fundamental derived constant is the square of the first eccentricity, e^2, normally defined by the equation

$$e^2 = \frac{a^2 - b^2}{a^2} \qquad (3\text{-}23)$$

where

a = semimajor axis of the ellipsoid
b = semiminor axis of the ellipsoid.

The basic equation which relates e^2 of the WGS 84 Ellipsoid to the four defining parameters (a, GM, $\bar{C}_{2,0}$, and ω) is

$$e^2 = -3\ (5^{1/2})\ \bar{C}_{2,0} + \frac{4}{15}\frac{\omega^2 a^3}{GM}\frac{e^3}{2q_0} \tag{3-24}$$

where

$$2q_0 = [1 + \frac{3}{e'^2}] \arctan (e') - \frac{3}{e'}, \tag{3-25}$$

e' = second eccentricity

$$e' = e\ (1 - e^2)^{-1/2}\ . \tag{3-26}$$

Equation (3-24) is solved iteratively for e^2.

3.4.3 Derived Geometric Constants

Having the four defining parameters and knowing e^2, it is possible to determine the other geometric constants for the WGS 84 Ellipsoid.

3.4.3.1 Semiminor Axis

The semiminor axis b is defined by the Equation

$$b = a\ (1 - e^2)^{1/2} \tag{3-27}$$

or by

$$b = a\ (1 - f) \tag{3-28}$$

where f is the flattening (ellipticity) of the ellipsoid.

3.4.3.2 Flattening

The formula for the flattening in terms of e^2 is

$$f = 1 - (1 - e^2)^{1/2}. \qquad (3\text{-}29)$$

A more familiar formula for f is the one expressed in terms of the semiaxes a and b:

$$f = \frac{a - b}{a} . \qquad (3\text{-}30)$$

3.4.3.3 Linear Eccentricity

Linear eccentricity, E, can be determined using either the formula

$$E = (a^2 - b^2)^{1/2} \qquad (3\text{-}31)$$

or

$$E = ae . \qquad (3\text{-}32)$$

3.4.3.4 Polar Radius of Curvature

The equation for the polar radius of curvature, c, is

$$c = \frac{a^2}{b} . \qquad (3\text{-}33)$$

Expressed in terms of a and e^2, this equation becomes

$$c = a\ (1 - e^2)^{-1/2}. \qquad (3\text{-}34)$$

(Note that c is also used in Section 3.4.5. and Table 3.9 to denote the velocity of light in a vacuum.)

3.4.3.5 Meridian Arc Distances

The length of the meridian arc from the equator to the pole (meridian quadrant), Q, can be determined using the equation

$$Q = c \int_0^{\pi/2} \frac{d\phi}{(1 + e'^2 \cos^2\phi)^{3/2}} \qquad (3\text{-}35)$$

where ϕ is the geodetic latitude. This integral can be evaluated by the series expansion

$$Q = c \frac{\pi}{2} [1 - \frac{3}{4} e'^2 + \frac{45}{64} e'^4 - \frac{175}{256} e'^6 + \frac{11025}{16384} e'^8 - \frac{43659}{65536} e'^{10} + \ldots]. \qquad (3\text{-}36)$$

Knowing the meridian quadrant, Q, the pole to pole meridian arc distance is 2Q. The total meridian distance around the earth is 4Q. It is important to remember that meridian arc distances will vary from ellipsoid to ellipsoid, as a function of c and e', Equation (3-36).

3.4.3.6 Circumference of the Equator

The circumference of the equator, C, of the WGS 84 Ellipsoid (or any other ellipsoid of revolution) is

$$C = 2\pi a . \qquad (3\text{-}37)$$

(Note that C is also used in Section 3.4.6 and in Tables 3.10, 3.11, and 3.12 to denote the moment of inertia of the earth with respect to the Z-axis.)

3.4.3.7 Mean Radius

There are several methods for determining a value for the mean radius of the ellipsoid. First, there is the arithmetic mean

(R_1) of the three semiaxes (a, a, b):

$$R_1 = \frac{a + a + b}{3} = a \left(1 - \frac{f}{3}\right) . \qquad (3\text{-}38)$$

When expressed in terms of a and e^2, Equation (3-38) becomes

$$R_1 = \frac{1}{3} a \left[2 + (1 - e^2)^{1/2}\right] . \qquad (3\text{-}39)$$

Second, the radius of a sphere (R_2) having the same surface area as the WGS 84 Ellipsoid is:

$$R_2 = c \left[\int_0^{\pi/2} \frac{\cos\phi}{(1 + e'^2 \cos^2\phi)^2} \, d\phi \right]^{1/2} \qquad (3\text{-}40)$$

$$R_2 = c \left(1 - \frac{2}{3} e'^2 + \frac{26}{45} e'^4 - \frac{100}{189} e'^6 + \frac{7034}{14175} e'^8 - \frac{220652}{467775} e'^{10} + \ldots \right) . \qquad (3\text{-}41)$$

An alternative formula for R_2 is [3.20]:

$$R_2 = \frac{1}{2} a \left(2 + \frac{1 - e^2}{e} \ln \frac{1 + e}{1 - e}\right)^{1/2} . \qquad (3\text{-}42)$$

A third method for determining the mean radius of the WGS 84 Ellipsoid is to find the radius of a sphere having the same volume as the ellipsoid. The equation for this radius, R_3, is [3.20]:

$$R_3 = (a^2 b)^{1/3} = a \, (1 - e^2)^{1/6} . \qquad (3\text{-}43)$$

3.4.3.8 Surface Area and Volume of the WGS 84 Ellipsoid

Occasionally, it is necessary to know either the surface area of the reference ellipsoid or its volume. The surface area S of the reference ellipsoid can be calculated directly from the semimajor axis (a) and eccentricity (e) using the closed form Equation [3.20]:

$$S = \pi a^2 \left(2 + \frac{1-e^2}{e} \ln \frac{1+e}{1-e}\right). \tag{3-44}$$

The surface area can also be calculated from R_2 using the expression

$$S = 4 \pi R_2^2 . \tag{3-45}$$

The mathematical expression for the volume (V) of the reference ellipsoid, in terms of a and e, is

$$V = \frac{4}{3} \pi a^3 (1-e^2)^{1/2} . \tag{3-46}$$

An alternative method for determining V is the equation

$$V = \frac{4}{3} \pi R_3^3 . \tag{3-47}$$

3.4.3.9 Other Derived Geometric Constants

There are a few other derived geometric constants which sometimes appear in a listing of ellipsoid constants, or which are used in other equations. These constants, defined by their equations, are:

$$m' = \frac{a^2 - b^2}{a^2 + b^2} = \frac{e^2}{2 - e^2} \tag{3-48}$$

$$n' = \frac{a - b}{a + b} = \frac{f}{2 - f} \tag{3-49}$$

$$q_0' = 3 \left(1 + \frac{1}{e'^2}\right)\left(1 - \frac{1}{e'} \arctan e'\right) - 1 . \tag{3-50}$$

3.4.3.10 Numerical Results

Using the preceding formulation, numerical values were computed for the above discussed geometric constants associated with the WGS 84 Ellipsoid. These values are listed in Table 3.7. For ease of reference, the four defining parameters of the WGS 84 Ellipsoid are also included in the table.

The defining parameters are considered to be exact. On the other hand, the derived geometric constants are as stated-derived. Users are reminded that the derived geometric constants cannot be arbitrarily truncated if consistency between the magnitudes of the various parameters is to be maintained. These constants should always be calculated to, and used with, the number of digits required to maintain the consistency needed for each specific application.

3.4.4 Derived Physical Constants

Having the four defining parameters and knowing the first eccentricity (e), it is possible to determine various physical constants for the WGS 84 Ellipsoid.

3.4.4.1 Theoretical (Normal) Potential of the WGS 84 Ellipsoid

As was stated earlier, the WGS 84 Ellipsoid is defined to be an equipotential ellipsoid, a surface of constant theoretical gravity potential, $U = U_0$. This constant, U_0, the theoretical gravity potential of an ellipsoid, is defined by the expression

$$U_0 = \frac{GM}{E} \arctan (e') + \frac{1}{3} \omega^2 a^2 \qquad (3\text{-}51)$$

$$U_0 = \frac{GM}{b} [1 + \sum_{n=1}^{\infty} (-1)^n \frac{e'^{2n}}{2n+1} + \frac{1}{3} m] \qquad (3\text{-}52)$$

where

GM = earth's gravitational constant

E = linear eccentricity

e' = second eccentricity

ω = earth's angular velocity

a = semimajor axis

b = semiminor axis

$m = \omega^2 a^2 b/GM.$ (3-53)

3.4.4.2 Zonal Gravitational (Spherical Harmonic) Coefficients

The zonal gravitational coefficients, J_2, J_4, J_6,..., are constants which appear in the spherical harmonic expansion for the theoretical (normal) gravitational potential (V'):

$$V' = \frac{GM}{r} [1 - \sum_{n=1}^{\infty} J_{2n} (\frac{a}{r})^{2n} P_{2n}(\sin \phi')] \qquad (3\text{-}54)$$

where

r = radius vector

ϕ'= geocentric latitude.

The theoretical gravitational potential (V') represents the theoretical gravity potential (U) minus the potential of centrifugal force (of the earth's rotation).

The coefficient J_2 for WGS 84 is calculated from $\bar{C}_{2,0}$ using Equation (3-12)

$$J_2 = -(5)^{1/2}\,\overline{C}_{2,0} \quad . \tag{3-55}$$

The general equation for the other coefficients expressed in terms of J_2 is

$$J_{2n} = (-1)^{n+1}\frac{3e^{2n}}{(2n+1)(2n+3)}\left(1 - n + 5n\frac{J_2}{e^2}\right) \tag{3-56}$$

Using this equation, the zonal harmonic coefficients through n = 5 are:

$$n = 2,\ J_4 = -\frac{3e^4}{35}\left(-1 + 10\frac{J_2}{e^2}\right) \tag{3-57}$$

$$n = 3,\ J_6 = \frac{e^6}{21}\left(-2 + 15\frac{J_2}{e^2}\right) \tag{3-58}$$

$$n = 4,\ J_8 = -\frac{e^8}{33}\left(-3 + 20\frac{J_2}{e^2}\right) \tag{3-59}$$

$$n = 5,\ J_{10} = \frac{3e^{10}}{143}\left(-4 + 25\frac{J_2}{e^2}\right) . \tag{3-60}$$

If the normalized $\overline{C}_{n,0}$ coefficients are desired, they can either be calculated from the J_n coefficients using the mathematical relationship

$$\overline{C}_{n,0} = -J_n/(2n+1)^{1/2} \tag{3-61}$$

or they can be calculated directly from $\overline{C}_{2,0}$ using an expression analogous to Equation (3-56):

$$\overline{C}_{2n,0} = (-1)^n\frac{3e^{2n}}{(2n+1)(2n+3)(4n+1)^{1/2}}\left(1 - n - 5^{3/2}n\frac{\overline{C}_{2,0}}{e^2}\right) \tag{3-62}$$

Equation (3-62) was derived from Equations (3-55), (3-56), and (3-61).

3.4.4.3 Theoretical Gravity at the Equator and the Poles

Theoretical gravity at the equator, γ_e, and theoretical gravity at the poles, γ_p, can be calculated using the expressions

$$\gamma_e = \frac{GM}{ab}\left(1 - m - \frac{m}{6}\frac{e'q_o'}{q_o}\right) \qquad (3\text{-}63)$$

$$\gamma_p = \frac{GM}{a^2}\left(1 + \frac{m}{3}\frac{e'q_o'}{q_o}\right) \qquad (3\text{-}64)$$

where

$$q_o' = 3\left(1 + \frac{1}{e'^2}\right)\left(1 - \frac{1}{e'} \arctan e'\right) - 1 \qquad (3\text{-}65)$$

$$q_o = \frac{1}{2}\left[\left(1 + \frac{3}{e'^2}\right) \arctan e' - \frac{3}{e'}\right]. \qquad (3\text{-}66)$$

3.4.4.4 Gravity Flattening

The expression for the constant f*, called gravity flattening, is

$$f^* = \frac{\gamma_p - \gamma_e}{\gamma_e}\,. \qquad (3\text{-}67)$$

3.4.4.5 Mean Value of Theoretical Gravity

The general expression for the average or mean value of theoretical gravity ($\overline{\gamma}$) on (at) the surface of the ellipsoid is:

$$\overline{\gamma} = \int_0^{\pi/2} \frac{\gamma \cos\phi \, d\phi}{(1 - e^2 \sin^2\phi)^2} \Big/ \int_0^{\pi/2} \frac{\cos\phi \, d\phi}{(1 - e^2 \sin^2\phi)^2} \,. \qquad (3\text{-}68)$$

Transforming Equation (3-68) using series expansions [3.2], it becomes

$$\overline{\gamma} = \gamma_e \Big(1 + \frac{1}{6} e^2 + \frac{1}{3} k + \frac{59}{360} e^4 + \frac{5}{18} e^2 k \qquad (3\text{-}69)$$

$$+ \frac{2371}{15120} e^6 + \frac{259}{1080} e^4 k + \frac{270229}{1814400} e^8 + \frac{9623}{45360} e^6 k\Big)$$

where

$$k = \frac{b\gamma_p - a\gamma_e}{a\gamma_e} \,. \qquad (3\text{-}70)$$

3.4.4.6 Mass of the Earth

The mass of the earth (M), or mass of the WGS 84 Ellipsoid, can be determined from the earth's gravitational constant (GM), provided a value for the universal constant of gravitation (G) is known. The appropriate equation is

$$M = \frac{GM}{G} \,. \qquad (3\text{-}71)$$

The value of G adopted for use with WGS 84 is [3.7]:

$$G = 6.673 \times 10^{-11} \ m^3 \ s^{-2} \ kg^{-1} \,.$$

More information on G is available in Section 3.3.2.2.1 and Table 3.3.

3.4.4.7 Numerical Results

Using the preceding formulation, numerical values were computed for the above discussed physical constants associated with the WGS 84 Ellipsoid. These values are listed in Table 3.8. For ease of reference, the four defining parameters of the WGS 84 Ellipsoid are also included in the table.

3.4.5 Relevant Miscellaneous Constants/Conversion Factors

In addition to the four defining parameters of the WGS 84 Ellipsoid (Table 3.1), necessary for describing (representing) the ellipsoid geometrically and gravimetrically, and the derived sets of commonly used geometric and physical constants associated with the WGS 84 Ellipsoid (Tables 3.7 and 3.8), two other important constants are an integral part of the definition of WGS 84. These constants are the velocity of light (c) and the dynamical ellipticity (H).

3.4.5.1 Velocity of Light

The currently accepted value for the velocity of light in a vacuum (c) is [3.21]:

$$c = (299792458 \pm 1.2) \text{ m s}^{-1} .$$

This value is officially recognized by both the IAG [3.7] and IAU [3.9], and has been adopted for use with WGS 84.

3.4.5.2 Dynamical Ellipticity

The dynamical ellipticity, H, is necessary for determining the principal moments of inertia of the earth, A, B, and C. In the literature, H is variously referred to as dynamical ellipticity, mechanical ellipticity, or precessional constant. It is a factor in the theoretical value of the rate of precession of the equinoxes, which is well known from observation. In a recent IAG report on fundamental geodetic constants [3.5], the following value for the reciprocal of H was given in the discussion of moments of inertia:

$$1/H = 305.4413 \pm 0.0005 \ .$$

For consistency, this value has been adopted for use with WGS 84.

3.4.5.3 Numerical Values

Values of the velocity of light in a vacuum and the dynamical ellipticity adopted for use with WGS 84 are listed in Table 3.9 along with other WGS 84 associated constants used in special applications; e.g., the earth's principal moments of inertia (Section 3.4.6.2 and Table 3.12, dynamic solution). Factors for effecting a conversion between meters, feet, and/or nautical and statute miles are also given in the table.

3.4.6 Moments of Inertia

The moments of inertia of the earth with respect to the X, Y, Z axes are defined by the Equations [3.22]

$$A = \iiint_{earth} (Y'^2 + Z'^2)dM \qquad (3\text{-}72)$$

$$B = \iiint_{earth} (Z'^2 + X'^2)dM \qquad (3\text{-}73)$$

$$C = \iiint_{earth} (X'^2 + Y'^2)dM \qquad (3\text{-}74)$$

where

A = moment of inertia with respect to the X axis
B = moment of inertia with respect to the Y axis
C = moment of inertia with respect to the Z axis

X', Y', Z' = rectangular coordinates of the variable mass element dM.

The moments of inertia are related to the second degree gravitational coefficients $J_{2,0}$ and $J_{2,2}$ by the formulas [3.23]

$$J_{2,0} = \frac{1}{Ma^2}\left(C - \frac{A+B}{2}\right) \tag{3-75}$$

$$J_{2,2} = \frac{1}{4Ma^2}\left(A - B\right) \tag{3-76}$$

where

a = semimajor axis of the ellipsoid
M = mass of the earth.

It is possible to determine A, B, and C either geometrically, using only the defining parameters (a, GM, $\bar{C}_{2,0}$, ω) of an ellipsoid, or dynamically, using earth gravitational model coefficients. Both approaches are used here to determine the moments of inertia. Although normalized gravitational coefficients are usually used for most applications, it is easier to express the equations for the moments of inertia in terms of conventional (unnormalized) coefficients, either $J_{n,m}$ or $C_{n,m}$. Therefore, gravitational coefficients in conventional form are used in the following development. (However, equations expressed in terms of normalized coefficients are introduced at the end.)

3.4.6.1 Geometric Solution

In the geometric solution, the moments of inertia are calculated from the defining parameters of an ellipsoid, which for WGS 84 are a, GM, $\bar{C}_{2,0}$, and ω. Due to the symmetry of the rotational ellipsoid,

$$A = B, \tag{3-77}$$

so that Equations (3-75) and (3-76) reduce to

$$J_{2,0} = \frac{1}{Ma^2} (C - A) \tag{3-78}$$

$$J_{2,2} = 0 \tag{3-79}$$

where

C = moment of inertia with respect to the axis of rotation (Z - axis)

A = moment of inertia with respect to any axis in the equatorial plane.

Thus, in the geometric solution, there are only two moments of inertia to be solved for, A and C.

The geometric solution for C is [3.23]:

$$C = \frac{2}{3} Ma^2 \left[1 - \frac{2}{5} \left(\frac{5m}{2f} - 1\right)^{1/2} \right] \tag{3-80}$$

or

$$\frac{C}{Ma^2} = \frac{2}{3} \left[1 - \frac{2}{5} \left(\frac{5m}{2f} - 1\right)^{1/2}\right] \tag{3-81}$$

where, as before:

$$m = \frac{\omega^2 a^2 b}{GM} \tag{3-82}$$

f = ellipsoidal flattening

ω = earth's angular velocity

b = semiminor axis

GM = earth's gravitational constant.

Knowing M, a, C, and $J_{2,0}$, Equation (3-78) can be used to obtain A, i.e.:

$$A = C - Ma^2 J_{2,0} \tag{3-83}$$

or

$$\frac{A}{Ma^2} = \frac{C}{Ma^2} - J_{2,0} \ . \tag{3-84}$$

Moments of inertia are often given in terms of their differences

$$C - A, \ C - B, \text{ or } B - A$$

rather than in terms of their individual values (A, B, and C). In the geometric solution, due to Equation (3-77), there is only one difference to be concerned with:

$$C - A \ .$$

This difference is readily obtained from Equation (3-78), which yields

$$C - A = Ma^2 J_{2,0} \tag{3-85}$$

or

$$\frac{C-A}{Ma^2} = J_{2,0} \ . \tag{3-86}$$

Although the dynamical ellipticity (H) is more accurately determined using other techniques, as was discussed in Section 3.4.5.2, it can also be solved for geometrically from A and C, using the equation [3.22]:

$$H = \frac{C - A}{C} \ . \tag{3-87}$$

3.4.6.2 Dynamic Solution

Discussion of the dynamic solution for the moments of inertia also starts with Equations (3-75) and (3-76):

$$J_{2,0} = \frac{1}{Ma^2}\left(C - \frac{A+B}{2}\right) \tag{3-88}$$

$$J_{2,2} = \frac{1}{4Ma^2}(A - B) \; . \tag{3-89}$$

These equations can be rewritten as:

$$C_{2,0} = \frac{1}{Ma^2}\left(\frac{A + B}{2} - C\right) \tag{3-90}$$

$$C_{2,2} = \frac{1}{4Ma^2}(B - A) \tag{3-91}$$

where

$$C_{n,m} = - J_{n,m} \; . \tag{3-92}$$

It is assumed that the $C_{2,0}$ and $C_{2,2}$ (or $J_{2,0}$ and $J_{2,2}$) coefficients are from an earth gravitational model that is referenced to the same reference ellipsoid as that used for the geometric solution.

Solving Equations (3-90) and (3-91) simultaneously for the moment of inertia differences, (C-A, C-B, B-A):

$$\frac{C - A}{Ma^2} = - \left(C_{2,0} - 2C_{2,2}\right) \tag{3-93}$$

$$\frac{C - B}{Ma^2} = - \left(C_{2,0} + 2C_{2,2}\right) \tag{3-94}$$

$$\frac{B - A}{Ma^2} = 4C_{2,2} \tag{3-95}$$

or

$$C - A = - Ma^2(C_{2,0} - 2C_{2,2}) \qquad (3\text{-}96)$$

$$C - B = - Ma^2 (C_{2,0} + 2C_{2,2}) \qquad (3\text{-}97)$$

$$B - A = 4Ma^2 C_{2,2} . \qquad (3\text{-}98)$$

If a value is known for the moment of inertia C, then A and B can be obtained from Equations (3-90) and (3-91). Although it is possible to determine C using the geometric solution, Equation (3-80), subsequent values obtained for A and B will differ significantly from the various values published for these constants, e.g., [3.5]. An alternative approach is to determine C from the dynamical ellipticity (H), Section 3.4.5.2. The mathematical relationship between C and H is:

$$\frac{C}{Ma^2} = - \frac{C_{2,0}}{H} \qquad (3\text{-}99)$$

or

$$C = - Ma^2 \frac{C_{2,0}}{H} . \qquad (3\text{-}100)$$

Inserting Equation (3-100) into Equations (3-96) and (3-97), the expressions obtained for A and B are:

$$A = C + Ma^2 (C_{2,0} - 2C_{2,2})$$

$$A = Ma^2 [C_{2,0}(1 - \frac{1}{H}) - 2C_{2,2}] \qquad (3\text{-}101)$$

$$B = C + Ma^2 \left(C_{2,0} + 2C_{2,2}\right)$$

$$B = Ma^2 \left[C_{2,0}\left(1 - \frac{1}{H}\right) + 2C_{2,2}\right] \qquad (3\text{-}102)$$

or

$$\frac{A}{Ma^2} = C_{2,0} \left(1 - \frac{1}{H}\right) - 2C_{2,2} \qquad (3\text{-}103)$$

$$\frac{B}{Ma^2} = C_{2,0} \left(1 - \frac{1}{H}\right) + 2C_{2,2}. \qquad (3\text{-}104)$$

Since earth gravitational model coefficients are usually given in normalized form, it is convenient to also have the equations for the moments of inertia expressed in terms of normalized coefficients ($\bar{C}_{2,0}$ and $\bar{C}_{2,2}$). The mathematical relationships between the conventional and normalized coefficients are:

$$\bar{C}_{2,0} = \frac{1}{5^{1/2}} C_{2,0} = -\frac{1}{5^{1/2}} J_{2,0} \qquad (3\text{-}105)$$

$$\bar{C}_{2,2} = 2\left(\frac{3}{5}\right)^{1/2} C_{2,2} = -2\left(\frac{3}{5}\right)^{1/2} J_{2,2}\ . \qquad (3\text{-}106)$$

For convenience, the equations for calculating the moments of inertia are given in Tables 3.10 and 3.11, respectively, for both conventional and normalized gravitational coefficients.

3.4.6.3 Numerical Results

Numerical values are given in Table 3.12 for all the moment of inertia parameters. The geometrically determined parameters, considered to be less accurate than those determined from a dynamic solution, are included for comparison purposes only. For information and convenience, several of the constants needed for calculating moment of inertia parameters are also included in the table.

3.4.7 Geocentric Radius and Radii of Curvature

It is often helpful to have equations readily available for the geocentric radius (to the surface of the ellipsoid), the radius of curvature in the meridian, and the radius of curvature in the prime vertical (Figure 3.2). The equations for these parameters are [3.24]:

$$r = \frac{a(1 - e^2)^{1/2}}{(1 - e^2\cos^2\phi')^{1/2}} \tag{3-107}$$

$$R_M = \frac{a(1 - e^2)}{(1 - e^2\sin^2\phi)^{3/2}} \tag{3-108}$$

$$R_N = \frac{a}{(1 - e^2\sin^2\phi)^{1/2}} \tag{3-109}$$

where

r = geocentric radius to the surface of the ellipsoid
R_M = radius of curvature in the meridian
R_N = radius of curvature in the prime vertical
a = semimajor axis
e = first eccentricity
ϕ' = geocentric latitude
ϕ = geodetic latitude.

and the mathematical relationship between the geocentric and geodetic latitudes is [3.20]:

$$\phi' = \arctan\left[(1 - e^2)\tan\phi\right] . \tag{3-110}$$

The above equations have been used to compute values of r, R_M, and R_N at 1° intervals of geodetic latitude from 0° to 90° for the WGS 84 Ellipsoid. These quantities are given in [3.25].

3.4.8 Ellipsoidal Arc Distances

For user convenience, formulas and numerical values are given for arc distances along meridians and parallels of the WGS 84 Ellipsoid. These arcs are depicted in Figure 3.2.

3.4.8.1 Meridian Arc Distance

Meridian arc distance, S_ϕ, for a small increment of latitude $\Delta\phi$ (less than 45 kilometers in length), can be calculated using the equation [3.20]:

$$S_\phi = R_M \Delta\phi \qquad (3\text{-}111)$$

where, as before:

R_M = radius of curvature in the meridian

$$R_M = \frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}} \qquad (3\text{-}112)$$

a = semimajor axis of the ellipsoid

e = first eccentricity of the ellipsoid

ϕ = geodetic latitude.

3.4.8.2 Arc Distance Along a Parallel of Latitude

Arc distance along a parallel of latitude, S_λ, for an increment of longitude $\Delta\lambda$ at latitude ϕ, can be calculated using the equation [3.20]:

$$S_\lambda = R_N \cos\phi \; \Delta\lambda \qquad (3\text{-}113)$$

where

R_N = radius of curvature in the prime vertical

$$R_N = \frac{a}{(1-e^2\sin^2\phi)^{1/2}} \cdot \qquad (3\text{-}114)$$

3.4.8.3 Numerical Values

Meridian arc distances, S_ϕ, corresponding to one arc second in latitude ($\Delta\phi$ = 1") were calculated at latitude intervals of 5° from 0° to 90° for the WGS 84 Ellipsoid using Equation (3-111). Similarly, arc distances along a parallel of latitude, S_λ , corresponding to one arc second in longitude ($\Delta\lambda$ = 1") were calculated at latitude intervals of 5° from 0° to 90° using Equation (3-113). These S_ϕ and S_λ values, provided in [3.25], are the number of meters in one arc second of geodetic latitude and geodetic longitude, respectively, on the WGS 84 Ellipsoid. The increase in S_ϕ with geodetic latitude reflects the effect of the flattening of the ellipsoid. The decrease in S_λ with geodetic latitude reflects the effect of the convergence of the meridians towards the poles.

3.5 Summary/Comments

The defining parameters of the WGS 84 Ellipsoid are the same as those of the internationally sanctioned GRS 80 Ellipsoid with one minor exception. To maintain consistency with the coefficient form used with the WGS 84 EGM, the defining parameter J_2 of the GRS 80 Ellipsoid, given to six significant digits in [3.1], was converted to $\bar{C}_{2,0}$, truncated to eight significant digits, and used with WGS 84. As such, this converted value ($\bar{C}_{2,0}$) is a defining parameter of the WGS 84 Ellipsoid and the second degree zonal coefficient of the WGS 84 EGM (Chapter 5).

The four defining parameters (a, $\bar{C}_{2,0}$, ω, GM) of the WGS 84 Ellipsoid were used to calculate the more commonly used geometric and physical constants associated with the WGS 84 Ellipsoid. As a result of the use of $\bar{C}_{2,0}$ in the form described, the derived WGS 84 Ellipsoid parameters are slightly different from their GRS 80 Ellipsoid counterparts. Although these minute parameter differences and the conversion of the GRS 80 J_2-value to $\bar{C}_{2,0}$ are insignificant from a practical standpoint, it has been more appropriate to refer to the ellipsoid used with WGS 84 as the WGS 84 Ellipsoid.

In contrast, since NAD 83 does not have an associated EGM, the J_2 to $\bar{C}_{2,0}$ conversion does not arise and the ellipsoid used with NAD 83 by NGS is in name and in both defined and derived parameters the GRS 80 Ellipsoid. Although it is important to know that these small undesirable inconsistencies exist between the WGS 84 and GRS 80 Ellipsoids, from a practical application standpoint the parameter differences are insignificant. This is especially true with respect to the defining parameters. Therefore, as long as the preceding is recognized, it can be stated that WGS 84 and NAD 83 are based on the same ellipsoid.

With continuing research, new values will become available for the ellipsoid defining parameters discussed above. Although there is often a temptation to replace an existing parameter with a new ("improved") value when the latter appears, this should not be done with WGS 84. It is technically inappropriate to use such an "improved" value in the context of WGS 84 since the defining and derived parameters of the WGS 84 Ellipsoid form an internally consistent set of parameters. Since replacement of any of the defining parameters by an "improved" value has an effect on the derived parameters, disturbing this consistency, organizations involved in a DoD application that may require a WGS 84-related parameter of better accuracy than that presented in this Chapter should not substitute an "improved" parameter value but make the requirement known to the address provided in the PREFACE.

It is anticipated that at some point in the future, DMA will need to address the time varying nature of $\bar{C}_{2,0}$ and ω [3.26] [3.27], the inclusion of additional significant digits in both, and the possible need for an improved GM (WGS 84 scale) for high altitude satellite applications. The international geodetic community can assist in this endeavor by keeping these possible improvements in mind in any future update (replacement) of GRS 80.

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3.26 Yoder, C.F.; Williams, J.G.; Dickey, J.O.; Schutz, R.E.; Eanes, R.J.; and B. D. Tapley; "Secular Variation of Earth's Gravitational Harmonic J_2 Coefficient From LAGEOS and Non-Tidal Acceleration of Earth Rotation"; Nature; Vol. 303; 1983.

3.27 Dickey, J.O. and T. M. Eubanks; Atmospheric Excitation of the Earth's Rotation - Progress and Prospects; JPL Geodesy and Geophysics Preprint No. 149; California Institute of Technology, Jet Propulsion Laboratory (JPL); Pasadena, California; October 1986.

Table 3.1

Defining Parameters of the WGS 84 Ellipsoid
and Their Accuracy Estimates (1σ)

Parameters	Notation	Value/(Accuracy, 1σ)
Semimajor Axis	a	6378137 m (±2 m)
Normalized Second Degree Zonal Gravitational Coefficient	$\bar{C}_{2,0}$	$-484.16685 \times 10^{-6}$ ($\pm 1.30 \times 10^{-9}$)
Angular Velocity of the Earth	ω	7292115×10^{-11} rad s^{-1} ($\pm 0.1500 \times 10^{-11}$ rad s^{-1})
Earth's Gravitational Constant (Mass of Earth's Atmosphere Included)	GM	$3986005 \times 10^{8}\ m^3 s^{-2}$ ($\pm 0.6 \times 10^{8}\ m^3 s^{-2}$)

Table 3.2

Reference Ellipsoid Constants

Reference Ellipsoids	a (Meters)	f
Airy	6377563.396	1/299.3249646
Modified Airy	6377340.189	1/299.3249646
Australian National	6378160	1/298.25
Bessel 1841	6377397.155*	1/299.1528128
Clarke 1866	6378206.4	1/294.9786982
Clarke 1880	6378249.145	1/293.465
Everest	6377276.345	1/300.8017
Modified Everest	6377304.063	1/300.8017
Fischer 1960 (Mercury)	6378166	1/298.3
Modified Fischer 1960 (South Asia)	6378155	1/298.3
Fischer 1968	6378150	1/298.3
Geodetic Reference System 1967	6378160	1/298.247167427
Geodetic Reference System 1980	6378137	1/298.257222101
Helmert 1906	6378200	1/298.3
Hough	6378270	1/297
International	6378388	1/297
Krassovsky	6378245	1/298.3
South American 1969	6378160	1/298.25
WGS 60	6378165	1/298.3
WGS 66	6378145	1/298.25
WGS 72	6378135	1/298.26
WGS 84	6378137	1/298.257223563

* In Namibia, use a = 6377483.865 meters for the Bessel 1841 Ellipsoid.

Table 3.3

Estimated Values for the Universal Gravitational Constant (G) and Mass of the Earth's Atmosphere (M_A)

G	M_A	References
6.6720 ± 0.0041	5.136 ± 0.007	[3.3]
6.672 ± 0.0041	5.24 ± 0.02	[3.8]
6.672		[3.9]
6.6726 ± 0.0005		[3.5]
6.673 ± 0.001		[3.7]
6.6745 ± 0.0008		[3.4]

[Each tabular entry for G, above, must be multiplied by $10^{-11} m^3 s^{-2} kg^{-1}$. Each tabular entry for M_A, above, must be multiplied by 10^{18}kg.]

Table 3.4

Effect of GM_A on Theoretical Gravity

GM_A	Effect of GM_A on Theoretical Gravity		
	$\delta\gamma_e$	$\delta\gamma_p$	$\delta\bar{\gamma}$
3.50	0.86176	0.86036	0.86127
3.51	0.86422	0.86282	0.86373
3.52	0.86668	0.86528	0.86619
3.53	0.86915	0.86773	0.86865
3.54	0.87161	0.87019	0.87111
3.55	0.87407	0.87265	0.87357
3.56	0.87653	0.87511	0.87604
3.57	0.87899	0.87757	0.87850
3.58	0.88146	0.88002	0.88096
3.59	0.88392	0.88248	0.88342
3.60	0.88638	0.88494	0.88588

[Each GM_A tabular value, above, must be multipled by $10^8 m^3 s^{-2}$.
γ_e = theoretical gravity at the equator; γ_p = theoretical gravity at the poles; $\bar{\gamma}$ = average value of theoretical gravity; all values based on the WGS 84 Ellipsoidal Gravity Formula; units = milligals for $\delta\gamma_e$, $\delta\gamma_p$, $\delta\bar{\gamma}$ tabular values (above).]

Table 3.5

Comparison of Second Degree Zonal Gravitational Coefficients

$\bar{C}_{2,0}$	J_2	Source/Reference	
-484.1605×10^{-6}	1082.6158×10^{-6}	WGS 72	[3.3]
$-484.16140 \times 10^{-6}$	1082.6178×10^{-6}	GRIM3-L1	[3.10]
$-484.16289 \times 10^{-6}$	1082.6211×10^{-6}	GRIM3B	[3.11]
$-484.16474 \times 10^{-6}$	1082.6252×10^{-6}	GEM-T1	[3.12]
$-484.16499 \times 10^{-6}$	1082.6258×10^{-6}	GEM-L2	[3.13]
-484.1653×10^{-6}	1082.6265×10^{-6}	OSU 81	[3.14]
-484.1655×10^{-6}	1082.6269×10^{-6}	GEM 9	[3.4]
$-484.16551 \times 10^{-6}$	1082.6269×10^{-6}	GEM 10C	[3.15]
$-484.16602 \times 10^{-6}$	1082.6281×10^{-6}	PGS-1331	[3.16]
$-484.16685 \times 10^{-6}$	1082.6300×10^{-6}	WGS 84	---
-484.1691×10^{-6}	1082.6350×10^{-6}	GRIM 2	[3.4]
-484.1700×10^{-6}	1082.6370×10^{-6}	SAO SE III	[3.4]

Table 3.6

Yearly Angular Velocity Values of the Earth's Rotation

Year	Angular Velocity (Units = 10^{-11} rad s^{-1})		
	Highest*	Lowest*	Average**
1967	7292115.018	7292114.903	7292114.946
1968	114.999	114.894	114.937
1969	114.989	114.875	114.921
1970	114.998	114.870	114.918
1971	114.953	114.836	114.901
1972	114.946	114.845	114.882
1973	114.952	114.836	114.889
1974	114.982	114.852	114.917
1975	114.991	114.875	114.920
1976	114.961	114.855	114.901
1977	114.993	114.867	114.912
1978	114.995	114.832	114.903
1979	114.997	114.876	114.926
1980	115.018	114.909	114.952
1981	115.052	114.893	114.964
1982	115.031	114.918	114.964
1983	115.034	114.877	114.954
1984	115.096	114.975	115.019
1985	115.099	114.977	115.024

* Averaged over a five-day period.

** Averaged over a year.

Table 3.7

WGS 84 Ellipsoid

- Defining Parameters and Derived Geometric Constants -

Parameter	Symbol	Numerical Value
Defining Parameters		
Semimajor axis	a	6378137 m
Earth's gravitational constant	GM	$3986005 \times 10^8 m^3 s^{-2}$
Normalized second degree zonal gravitational coefficient	$\bar{C}_{2,0}$	$-484.16685 \times 10^{-6}$
Earth's angular velocity	ω	7292115×10^{-11} rad s^{-1}
Derived Geometric Constants		
Semiminor axis	b	6356752.3142 m
Linear eccentricity	E	521854.0084 m
Polar radius of curvature	c	6399593.6258 m
First eccentricity (e) squared	e^2	0.00669437999013
	e	0.0818191908426
	$1-e^2$	0.993305620010
	$(1-e^2)^{1/2}$	0.996647189335
Second eccentricity (e') squared	e'^2	0.00673949674227
	e'	0.0820944379496
Flattening (ellipticity)	f	0.00335281066474
Reciprocal of the flattening	f^{-1}	298.257223563
Axis ratio	b/a	0.996647189335
Meridian quadrant	Q	10001965.7293 m
Pole-to-pole meridian distance	$2Q$	20003931.4586 m
Total meridian distance	$4Q$	40007862.9173 m
Circumference of the equator	C	40075016.6856 m
Mean radius of semiaxes	R_1	6371008.7714 m
Radius of sphere of same surface*	R_2	6371007.1809 m
Radius of sphere of same volume*	R_3	6371000.7900 m
Surface area of ellipsoid	S	$5.10065621724 \times 10^{14}$ m^2
Volume of ellipsoid	V	$1.08320731980 \times 10^{21}$ m^3
$m' = (a^2-b^2)/(a^2+b^2)$	m'	0.00335843130272
$n' = (a-b)/(a+b)$	n'	0.00167922038638
$q_0 = (1/2)[(1+3/e'^2)\arctan(e')-3/e']$	q_0	0.0000733462578707
$q_0'=3(1+1/e'^2)[1-(1/e')\arctan e']-1$	q_0'	0.00268804130046

* As the WGS 84 Ellipsoid

Table 3.8

WGS 84 Ellipsoid

-Defining Parameters and Derived Physical Constants-

Constant	Symbol	Numerical Value
Defining Parameters		
Semimajor axis	a	6378137 m
Earth's gravitational constant	GM	$3986005 \times 10^{8} m^{3} s^{-2}$
Normalized second degree zonal gravitational coefficient	$\bar{C}_{2,0}$	$-484.16685 \times 10^{-6}$
Earth's angular velocity	ω	$7292115 \times 10^{-11} rad\ s^{-1}$
Derived Physical Constants		
Theoretical gravity potential of ellipsoid	U_0	$62636860.8497\ m^{2} s^{-2}$
$m = \omega^2 a^2 b/GM$	m	0.00344978600313
Theoretical gravity at the equator	γ_e	$9.7803267714\ m\ s^{-2}$
Theoretical gravity at the poles	γ_p	$9.8321863685\ m\ s^{-2}$
Gravity flattening	f^*	0.00530244012894
$k = (b\gamma_p - a\gamma_e)/a\gamma_e$	k	0.00193185138639
Mean value of theoretical gravity	$\bar{\gamma}$	$9.7976446561\ m\ s^{-2}$
Mass of the earth (including the atmosphere)	M	5.9733328×10^{24} kg

Geometrically Derived Gravitational Coefficients

Degree (n)	$\bar{C}_{n,0}$	J_n
2	- - -	0.00108262998905
4	0.000000790304054	-0.00000237091216
6	-0.000000001687251	0.00000000608347
8	0.000000000003461	-0.00000000001427
10	-0.000000000000003	0.00000000000001

Table 3.9
Relevant Miscellaneous Constants
and Conversion Factors

Constant	Symbol	Numerical Value
Velocity of light (in a vacuum)	c	$299792458 \text{ m s}^{-1}$
Dynamical ellipticity	H	1/305.4413
Earth's angular velocity [for satellite applications; see Equation (3-22).]	ω^*	$(7292115.8553 \times 10^{-11} + 4.3 \times 10^{-15}\, T_U)$ rad s^{-1}
Universal constant of gravitation	G	$6.673 \times 10^{-11} m^3 s^{-2} kg^{-1}$
GM of the earth's atmosphere	GM_A	$3.5 \times 10^8 m^3 s^{-2}$
Earth's gravitational constant (excluding the mass of the earth's atmosphere)	GM'	$3986001.5 \times 10^8 m^3 s^{-2}$
Earth's principal moments of inertia (dynamic solution)	A B C	8.0091029×10^{37} kg m^2 8.0092559×10^{37} kg m^2 8.0354872×10^{37} kg m^2

Conversion Factors

1 Meter	= 3.28083333333 US Survey Feet
1 Meter	= 3.28083989501 International Feet
1 International Foot	= 0.3048 Meter (Exact)
1 US Survey Foot	= 1200/3937 Meter (Exact)
1 US Survey Foot	= 0.30480060960 Meter
1 Nautical Mile	= 1852 Meters (Exact) = 6076.10333333 US Survey Feet = 6076.11548556 International Feet
1 Statute Mile	= 1609.344 Meters (Exact) = 5280 International Feet (Exact)

T_U = Julian Centuries from Epoch J2000.0

Table 3.10

Principal Moments of Inertia Equations
- Geometric Solution -

Moment of Inertia Parameters	Equations	
	Using Conventional Coefficients	Using Normalized Coefficients
$(C-A)/Ma^2 =$	$J_{2,0}$	$-5^{1/2}\,\tau_{2,0}$
$C-A =$	$Ma^2\,J_{2,0}$	$-5^{1/2}\,Ma^2\,\tau_{2,0}$
$C/Ma^2 =$	$\frac{2}{3}[1 - \frac{2}{5}\,(\frac{5m}{2f} - 1)^{1/2}]$	$\frac{2}{3}[1 - \frac{2}{5}\,(\frac{5m}{2f} - 1)^{1/2}]$
$A/Ma^2 =$	$(C/Ma^2) - J_{2,0}$	$(C/Ma^2) + 5^{1/2}\,\tau_{2,0}$
$C =$	$\frac{2}{3}Ma^2[1 - \frac{2}{5}(\frac{5m}{2f} - 1)^{1/2}]$	$\frac{2}{3}Ma^2[1 - \frac{2}{5}(\frac{5m}{2f} - 1)^{1/2}]$
$A =$	$C - Ma^2\,J_{2,0}$	$C + 5^{1/2}\,Ma^2\,\tau_{2,0}$
$H =$	$(C - A)/C$	$(C - A)/C$

Table 3.11

Principal Moments of Inertia Equations
- Dynamic Solution -

Moment of Inertia Parameters	Equations	
	Using Conventional Coefficients	Using Normalized Coefficients
$(C-A)/Ma^2 =$	$-(C_{2,0} - 2\ C_{2,2})$	$-5^{1/2}(\bar{C}_{2,0} - \bar{C}_{2,2}/3^{1/2})$
$(C-B)/Ma^2 =$	$-(C_{2,0} + 2\ C_{2,2})$	$-5^{1/2}(\bar{C}_{2,0} + \bar{C}_{2,2}/3^{1/2})$
$(B-A)/Ma^2 =$	$4\ C_{2,2}$	$2(5/3)^{1/2}\ \bar{C}_{2,2}$
$C-A =$	$-Ma^2(C_{2,0} - 2\ C_{2,2})$	$-5^{1/2}Ma^2(\bar{C}_{2,0} - \bar{C}_{2,2}/3^{1/2})$
$C-B =$	$-Ma^2(C_{2,0} + 2\ C_{2,2})$	$-5^{1/2}Ma^2(\bar{C}_{2,0} + \bar{C}_{2,2}/3^{1/2})$
$B-A =$	$4\ Ma^2\ C_{2,2}$	$2(5/3)^{1/2}\ Ma^2\ \bar{C}_{2,2}$
$C/Ma^2 =$	$-C_{2,0}/H$	$-5^{1/2}\ \bar{C}_{2,0}/H$
$A/Ma^2 =$	$(1 - 1/H)\ C_{2,0} - 2\ C_{2,2}$	$5^{1/2}[(1-1/H)\bar{C}_{2,0} - \bar{C}_{2,2}/3^{1/2}]$
$B/Ma^2 =$	$(1 - 1/H)C_{2,0} + 2\ C_{2,2}$	$5^{1/2}[(1-1/H)\bar{C}_{2,0} + \bar{C}_{2,2}/3^{1/2}]$
$C =$	$-Ma^2\ C_{2,0}/H$	$-5^{1/2}\ Ma^2\ \bar{C}_{2,0}/H$
$A =$	$Ma^2[(1 - 1/H)C_{2,0} - 2\ C_{2,2}]$	$5^{1/2}Ma^2[(1-1/H)\bar{C}_{2,0} - \bar{C}_{2,2}/3^{1/2}]$
$B =$	$Ma^2[(1 - 1/H)C_{2,0} + 2\ C_{2,2}]$	$5^{1/2}Ma^2[(1-1/H)\bar{C}_{2,0} + \bar{C}_{2,2}/3^{1/2}]$

A, B, C = Principal Moments of Inertia

$C_{2,0}, C_{2,2}; \bar{C}_{2,0}, \bar{C}_{2,2}$ = Second Degree Gravitational Coefficients

H = Dynamical Ellipticity
a = Semimajor Axis
M = Earth's Mass

Table 3.12
WGS 84 - Related
Moment of Inertia Values

Parameters	Geometric Solution	Dynamic Solution
Input Data		
$\bar{C}_{2,0}$	$-484.16685 \times 10^{-6}$	$-484.16685 \times 10^{-6}$
$\bar{C}_{2,2}$	---	2.4395796×10^{-6}
H	---	1/305.4413
a	6378137 m	6378137 m
G	$6.673 \times 10^{-11} m^3 s^{-2} kg^{-1}$	$6.673 \times 10^{-11} m^3 s^{-2} kg^{-1}$
m	0.00344978600313	---
f	0.00335281066474	---
M	$5.9733328 \times 10^{24} kg$	$5.9733328 \times 10^{24} kg$
Ma^2	$2.4299895 \times 10^{38} kg\ m^2$	$2.4299895 \times 10^{38} kg\ m^2$
Calculated Parameters		
$(C-A)/Ma^2 =$	0.0010826300	0.0010857795
$(C-B)/Ma^2 =$	---	0.0010794805
$(B-A)/Ma^2 =$	---	0.0000062989674
C-A =	$2.6307795 \times 10^{35} kg\ m^2$	$2.6384327 \times 10^{35} kg\ m^2$
C-B =	---	$2.6231263 \times 10^{35} kg\ m^2$
B-A =	---	$1.5306425 \times 10^{33} kg\ m^2$
$C/Ma^2 =$	0.33228868	0.33067991
$A/Ma^2 =$	0.33120605	0.32959413
$B/Ma^2 =$	---	0.32960043
C =	$8.0745801 \times 10^{37} kg\ m^2$	$8.0354872 \times 10^{37} kg\ m^2$
A =	$8.0482723 \times 10^{37} kg\ m^2$	$8.0091029 \times 10^{37} kg\ m^2$
B =	---	$8.0092559 \times 10^{37} kg\ m^2$
H =	1/306.92728	---

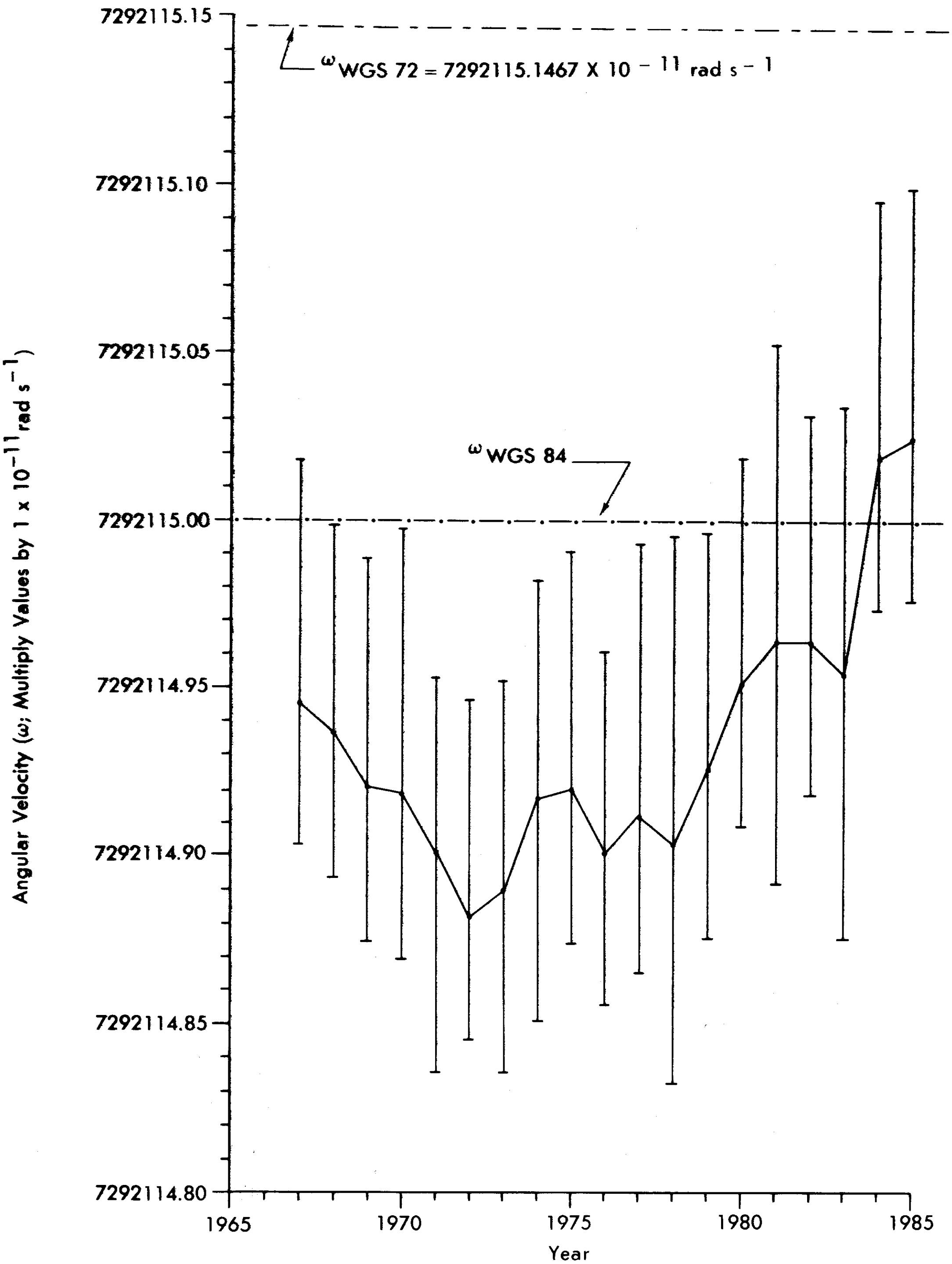

Figure 3.1. High, Low, and Yearly Average Angular Velocity (ω) Values

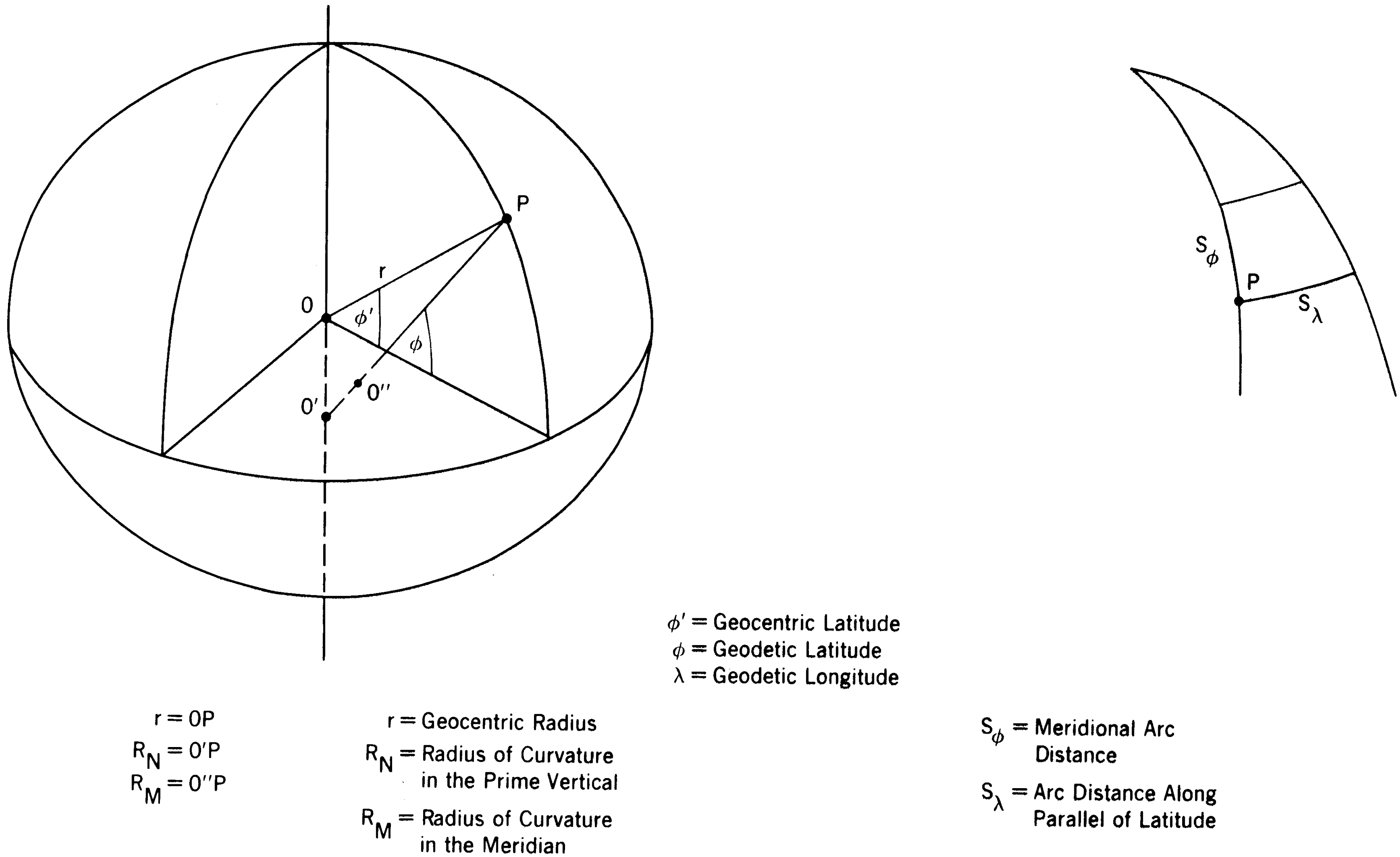

Figure 3.2. Geocentric Radius, Radii of Curvature, and Ellipsoidal Arc Distances

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