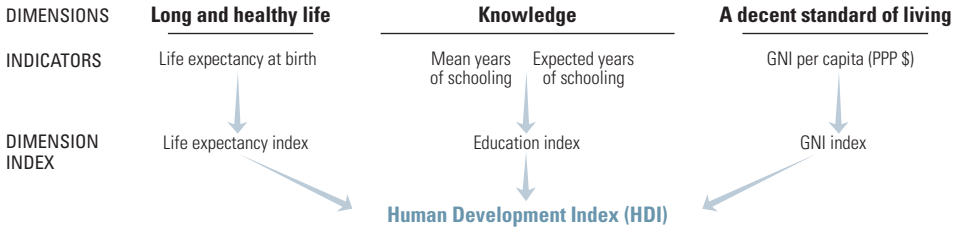
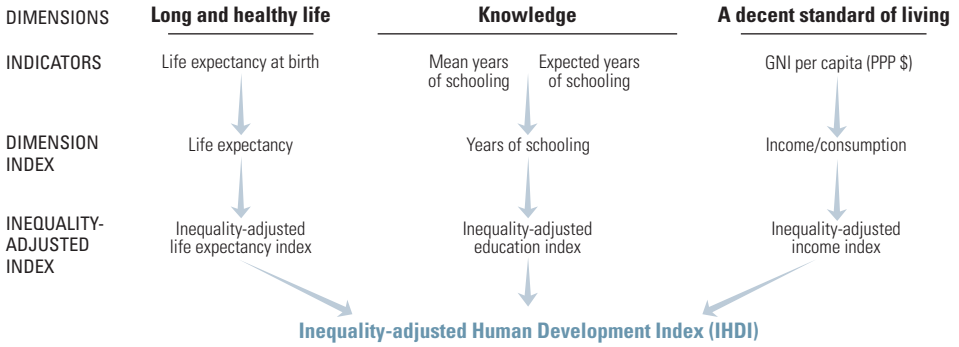
Technical notes

Calculating the human development indices—graphical presentation

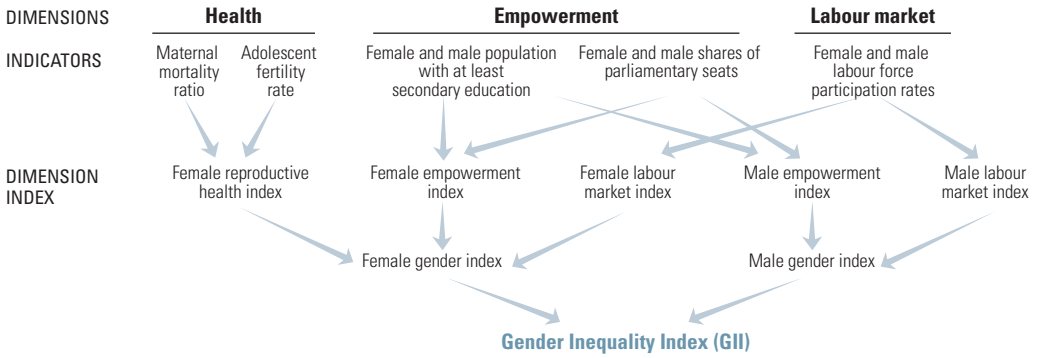
Human Development Index (HDI)



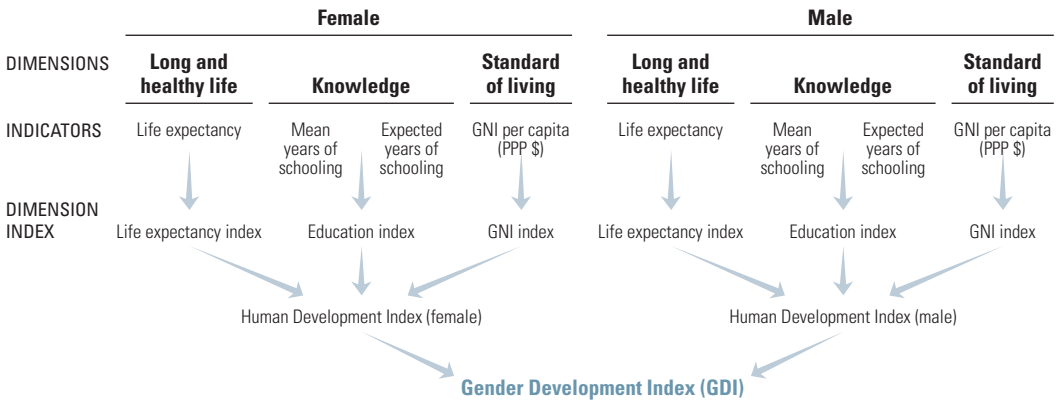
Inequality-adjusted Human Development Index (IHDI)



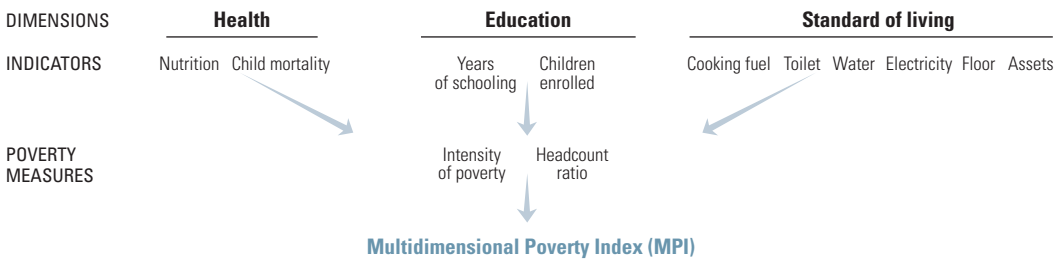
Gender Inequality Index (GII)



Gender Development Index (GDI)



Multidimensional Poverty Index (MPI)



Technical note 1. Human Development Index

The Human Development Index (HDI) is a summary measure of achievements in key dimensions of human development: a long and healthy life, access to knowledge and a decent standard of living. The HDI is the geometric mean of normalized indices for each of the three dimensions. This technical note describes the steps to calculating the HDI, data sources and the methodology used to estimate missing values.

Steps to calculate the Human Development Index

There are two steps to calculating the HDI.

Step 1. Creating the dimension indices

Minimum and maximum values (goalposts) are set in order to transform the indicators expressed in different units into indices between 0 and 1. These goalposts act as the 'natural zeroes' and 'aspirational goals', respectively, from which component indicators are standardized.[1] They are set at the following values:

Dimension	Indicator	Minimum	Maximum
Health	Life expectancy (years)	20	85
Education	Expected years of schooling	0	18
	Mean years of schooling	0	15
Standard of living	Gross national income per capita (PPP 2011 $)	100	75,000

The justification for placing the natural zero for life expectancy at 20 years is based on historical evidence that no country in the 20th century had a life expectancy of less than 20 years (Oeppen and Vaupel 2002; Maddison 2010; Riley 2005).

Societies can subsist without formal education, justifying the education minimum of 0 years. The maximum for mean years of schooling, 15, is the projected maximum of this indicator for 2025. The maximum for expected years of schooling, 18, is equivalent to achieving a master's degree in most countries.

The low minimum value for gross national income (GNI) per capita, $100, is justified by the considerable amount of unmeasured subsistence and nonmarket production in economies close to the minimum, which is not captured in the official data. The maximum is set at $75,000 per capita. Kahneman and Deaton (2010) have shown that there is a virtually no gain in human development and well-being from annual income beyond $75,000. Assuming annual growth rate of 5 percent, only three countries are projected to exceed the $75,000 ceiling in the next five years.

Having defined the minimum and maximum values, the dimension indices are calculated as:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}. \quad (1)$$

For the education dimension, equation 1 is first applied to each of the two indicators, and then the arithmetic mean of the two resulting indices is taken.

Because each dimension index is a proxy for capabilities in the corresponding dimension, the transformation function from income to capabilities is likely to be concave (Anand and Sen 2000)—that is, each additional dollar of income has a smaller effect on expanding capabilities. Thus for income, the natural logarithm of the actual, minimum and maximum values is used.

Step 2. Aggregating the dimensional indices to produce the Human Development Index

The HDI is the geometric mean of the three dimensional indices:

$$HDI = (I_{Health} \cdot I_{Education} \cdot I_{Income})^{⅓} \quad (2)$$

Example: Costa Rica

Indicator	Value
Life expectancy at birth (years)	79.93
Mean years of schooling	8.37
Expected years of schooling	13.50
Gross national income per capita (PPP 2011 $)	13,011.7

Note: Values are rounded.

$$\text{Health index} = \frac{79.93 - 20}{85 - 20} = 0.922$$

$$\text{Mean years of schooling index} = \frac{8.37 - 0}{15 - 0} = 0.558$$

$$\text{Expected years of schooling index} = \frac{13.50}{18} = 0.750$$

$$\text{Education index} = \frac{0.558 + 0.750}{2} = 0.654$$

$$\text{Income index} = \frac{\ln(13{,}011.7) - \ln(100)}{\ln(75{,}000) - \ln(100)} = 0.735$$

$$\text{Human Development Index} = (0.922 \cdot 0.654 \cdot 0.735)^{⅓} = 0.763$$

Data sources

- Life expectancy at birth: UNDESA (2013).
- Mean years of schooling: Barro and Lee (2013), UNESCO Institute for Statistics (2013) and Human Development

Report Office updates based on UNESCO Institute for Statistics (2013).
- Expected years of schooling: UNESCO (2013).
- GNI per capita: World Bank (2014), IMF (2014), UNSD (2014) and UNDESA (2013).

Methodology used to express income

The World Bank's 2014 World Development Indicators database contains estimates of GNI per capita in 2011 purchasing power parity (PPP) terms for many countries. For countries missing this indicator (entirely or partly), the Human Development Report Office calculates it by converting GNI from current to constant terms using two steps. First, the value of nominal GNI per capita is converted into PPP terms for the base year (2011). Second, a time series of GNI per capita in 2011 PPP terms is constructed by applying the real growth rates to the GNI per capita in PPP terms for the base year. The real growth rate is implied by the ratio of the nominal growth of current GNI per capita in local currency terms to the GDP deflator.

To obtain the income value for 2013, International Monetary Fund (IMF)–projected GDP growth rates (based on growth in constant terms) are applied to the most recent GNI values in constant PPP terms. The IMF-projected growth rates are calculated based on local currency terms and constant prices rather than in PPP terms. This avoids mixing the effects of the PPP conversion with those of real growth of the economy.

Official PPP conversion rates are produced by the International Comparison Program, whose surveys periodically collect thousands of prices of matched goods and services in many countries. The last round of this exercise refers to 2011 and covered 180 countries.

Estimating missing values

For a small number of countries missing one of the four indicators, the Human Development Report Office has estimated the missing values using cross-country regression models. The details of the models used are available at http://hdr.undp.org.

In this Report expected years of schooling were estimated for Côte d'Ivoire, Haiti, Liberia, Federated States of Micronesia, Papua New Guinea, Sierra Leone, South Africa, Sudan and Turkmenistan, and mean years of schooling were estimated for Antigua and Barbuda, Cape Verde, Dominica, Equatorial Guinea, Eritrea, Grenada, Kiribati, Madagascar, Palau, Saint Kitts and Nevis, Saint Lucia, Saint Vincent and the Grenadines and Solomon Islands.

Country groupings

The 2014 HDI introduces a system of fixed cutoff points for the four categories of human development achievements. The cutoff points (COP) are obtained as the HDI values calculated using the quartiles of the distributions of component indicators. The resulting HDI values are averaged over the 10-year interval (2004–2013):

$$COP_q = HDI\,(LE_q, MYS_q, EYS_q, GNIpc_q),\ q = 1,2,3$$

For example, LE_1, LE_2, LE_3 denote three quartiles of the distribution of life expectancy across countries.

The resulting cutoff points for the country grouping are:

Very high human development (COP_3)	0.800
High human development (COP_2)	0.700
Medium human development (COP_1)	0.550

Technical note 2. Inequality-adjusted Human Development Index

The Inequality-adjusted Human Development Index (IHDI) adjusts the Human Development Index (HDI) for inequality in the distribution of each dimension across the population. It is based on a distribution-sensitive class of composite indices proposed by Foster, Lopez-Calva and Szekely (2005), which draws on the Atkinson (1970) family of inequality measures. It is computed as a geometric mean of inequality-adjusted dimension indices.

The IHDI accounts for inequalities in HDI dimensions by 'discounting' each dimension's average value according to its level of inequality. The IHDI equals the HDI when there is no inequality across people but falls below the HDI as inequality rises. In this sense, the IHDI is the level of human development when inequality is accounted for.

Data sources

Since the HDI relies on country-level aggregates such as national accounts for income, the IHDI must draw on alternative sources of data to obtain insights into the distribution. The distributions are observed over different units—life expectancy is

distributed across a hypothetical cohort, while years of schooling and income are distributed across individuals.

Inequality in the distribution of HDI dimensions is estimated for:

- Life expectancy, using data from abridged life tables provided by UNDESA (2013). This distribution is presented over age intervals (0–1, 1–5, 5–10, ... , 85+), with the mortality rates and average age at death specified for each interval.
- Mean years of schooling, using household survey data harmonized in international databases, including the Luxembourg Income Study, Eurostat's European Union Survey of Income and Living Conditions, the World Bank's International Income Distribution Database, the United Nations Children's Fund's Multiple Indicators Cluster Survey, ICF Macro's Demographic and Health Survey, and the United Nations University's World Income Inequality Database.
- Disposable household income or consumption per capita using the above listed databases and household surveys—and for a few countries, income imputed based on an asset index matching methodology using household survey asset indices (Harttgen and Vollmer 2011).

A full account of data sources used for estimating inequality in 2013 is available at http://hdr.undp.org/en/statistics/ihdi/.

Steps to calculate the Inequality-adjusted Human Development Index

There are three steps to calculating the IHDI.

Step 1. Measuring inequality in the dimensions of the Human Development Index

The IHDI draws on the Atkinson (1970) family of inequality measures and sets the aversion parameter ε equal to 1.[2] In this case the inequality measure is $A = 1 - g/\mu$, where g is the geometric mean and μ is the arithmetic mean of the distribution. This can be written as:

$$A_x = 1 - \frac{\sqrt[n]{X_1 \ldots X_n}}{\bar{X}} \qquad (1)$$

where $\{X_1, \ldots, X_n\}$ denotes the underlying distribution in the dimensions of interest. A_x is obtained for each variable (life expectancy, mean years of schooling and disposable income or consumption per capita).

The geometric mean in equation 1 does not allow zero values. For mean years of schooling one year is added to all valid observations to compute the inequality. Income per capita outliers—extremely high incomes as well as negative and zero incomes—were dealt with by truncating the top 0.5 percentile of the distribution to reduce the influence of extremely high incomes and by replacing the negative and zero incomes with the minimum value of the bottom 0.5 percentile of the distribution of positive incomes. Sensitivity analysis of the IHDI is given in Kovacevic (2010).

Step 2. Adjusting the dimension indices for inequality

The inequality-adjusted dimension indices are obtained from the HDI dimension indices, I_x, by multiplying them by $(1 - A_x)$, where A_x, defined by equation 1, is the corresponding Atkinson measure:

$$I_x^* = (1 - A_x) \cdot I_x .$$

The inequality-adjusted income index, I_{Income}^*, is based on the index of logged income values, I_{Income^*} and inequality in income distribution computed using income in levels. This enables the IHDI to account for the full effect of income inequality.

Step 3. Combining the dimension indices to calculate the Inequality-adjusted Human Development Index

The IHDI is the geometric mean of the three dimension indices adjusted for inequality:

$$IHDI^* = (I_{Health}^* \cdot I_{Education}^* \cdot I_{Income}^*)^{1/3} =$$

$$[(1 - A_{Health}) \cdot 1 - A_{Education}) \cdot (1 - A_{Income})]^{1/3} \cdot HDI.$$

The loss in the Human Development Index due to inequality is:

$$Loss\ \% = 1 - [(1 - A_{Health}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})]^{1/3}.$$

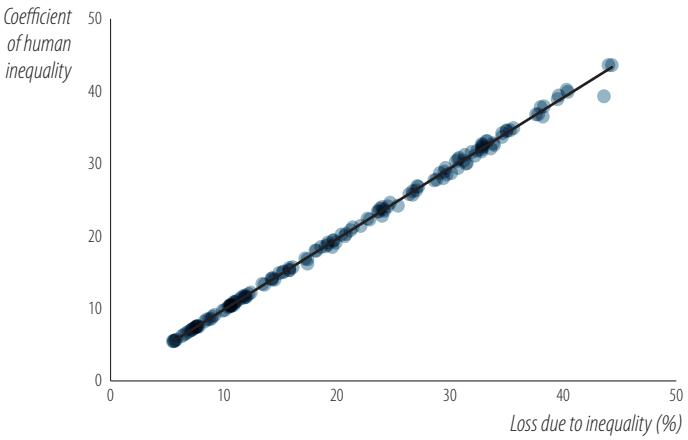
Coefficient of human inequality

An unweighted average of inequalities in health, education and income is denoted as the coefficient of human inequality. It averages these inequalities using the arithmetic mean:

$$Coefficient\ of\ human\ inequality = \frac{A_{Health} + A_{Education} + A_{Income}}{3}.$$

When all inequalities in dimensions are of a similar magnitude the coefficient of human inequality and the loss in HDI differ negligible. When inequalities differ in magnitude, the loss in HDI tends to be higher than the coefficient of human inequality.

Coefficient of human inequality vs. loss due to inequality



Notes on methodology and caveats

The IHDI is based on the Atkinson index, which satisfies subgroup consistency. This ensures that improvements or deteriorations in the distribution of human development within a certain group of society (while human development remains constant in the other groups) will be reflected in changes in the overall measure of human development.

The main disadvantage is that the IHDI is not association sensitive, so it does not capture overlapping inequalities. To make the measure association sensitive, all the data for each individual must be available from a single survey source, which is not currently possible for a large number of countries.

Example: Bosnia and Herzegovina

Indicator	Indicator	Dimension index	Inequality measure[a] (*A*)	Inequality-adjusted index
Life expectancy (years)	76.4	0.827	0.067	(1–0.067) · 0.827 = 0.772
Mean years of schooling	8.3	0.555	0.052	
Expected years of schooling	13.6	0.756		
Education index		0.655	0.052	(1–0.052) · 0.655 = 0.620
Logarithm of gross national income	9.15	0.687		(1–0.192) · 0.687 = 0.555
Gross national income (PPP 2011 $)	9,431		0.192	

Human Development Index	Inequality-adjusted Human Development Index
$(0.827 \cdot 0.655 \cdot 0.687)^{1/3} = 0.731$	$(0.772 \cdot 0.620 \cdot 0.548)^{1/3} = 0.653$
Loss due to inequality (%)	**Coefficient of human inequality (%)**
$100 \cdot \left(1 - \frac{0.653}{0.731}\right) = 10.6$	$\frac{100 \cdot (0.067 + 0.052 + 0.192)}{3} = 10.4$

Note: Values are rounded.
a. Inequalities are estimated from micro data.

Technical note 3. Calculating the Gender Inequality Index

The Gender Inequality Index (GII) reflects gender-based disadvantage in three dimensions—reproductive health, empowerment and the labour market—for as many countries as data of reasonable quality allow. It shows the loss in potential human development due to inequality between female and male achievements in these dimensions. It ranges between 0, where women and men fare equally, and 1, where one gender fares as poorly as possible in all measured dimensions.

The GII is computed using the association-sensitive inequality measure suggested by Seth (2009). It is based on the general mean of general means of different orders—the first aggregation is by a geometric mean across dimensions; these means, calculated separately for women and men, are then aggregated using a harmonic mean across genders.

Data sources

- Maternal mortality ratio (*MMR*): WHO and others (2013).
- Adolescent birth rate (*ABR*): UNDESA (2013).
- Share of parliamentary seats held by each sex (*PR*): IPU (2013).
- Attainment at secondary and higher education (*SE*) levels: Barro and Lee (2013) and UNESCO Institute for Statistics (2013).
- Labour market participation rate (*LFPR*): ILO (2013).

Steps to calculate the Gender Inequality Index

There are five steps to calculating the GII.

Step 1. Treating zeros and extreme values

Because a geometric mean cannot be computed from zero values, a minimum value of 0.1 percent is set for all component indicators. Further, as higher maternal mortality suggests poorer maternal health, for the maternal mortality ratio the maximum value is truncated at 1,000 deaths per 100,000 births and the minimum value at 10. The rationale is that countries where maternal mortality ratios exceed 1,000 do not differ in their inability to create conditions and support for maternal health

and that countries with 10 or fewer deaths per 100,000 births are performing at essentially the same level and that differences are random.

Sensitivity analysis of the GII is given in Gaye and others (2010).

Step 2. Aggregating across dimensions within each gender group, using geometric means

Aggregating across dimensions for each gender group by the geometric mean makes the GII association sensitive (see Seth 2009).

For women and girls, the aggregation formula is:

$$G_F = \sqrt[3]{\left(\frac{10}{MMR} \cdot \frac{1}{ABR}\right)^{½} \cdot (PR_F \cdot SE_F)^{½} \cdot LFPR_F}, \qquad (1)$$

and for men and boys the formula is

$$G_M = \sqrt[3]{1 \cdot (PR_M \cdot SE_M)^{½} \cdot LFPR_M}.$$

The rescaling by 0.1 of the maternal mortality ratio in equation 1 is needed to account for the truncation of the maternal mortality ratio minimum at 10.

Step 3. Aggregating across gender groups, using a harmonic mean

The female and male indices are aggregated by the harmonic mean to create the equally distributed gender index

$$HARM(G_F, G_M) = \left[\frac{(G_F)^{-1} + (G_M)^{-1}}{2}\right]^{-1}.$$

Using the harmonic mean of geometric means within groups captures the inequality between women and men and adjusts for association between dimensions.

Step 4. Calculating the geometric mean of the arithmetic means for each indicator

The reference standard for computing inequality is obtained by aggregating female and male indices using equal weights (thus treating the genders equally) and then aggregating the indices across dimensions:

$$G_{\overline{F,M}} = \sqrt[3]{\overline{Health} \cdot \overline{Empowerment} \cdot \overline{LFPR}}$$

where $\overline{Health} = \left(\sqrt{\frac{10}{MMR} \cdot \frac{1}{ABR}} + 1\right)/2,$

$$\overline{Empowerment} = \left(\sqrt{PR_F \cdot SE_F} + \sqrt{PR_M \cdot SE_M}\right)/2, \text{ and}$$

$$\overline{LFPR} = \frac{LFPR_F + LFPR_M}{2}.$$

$\overline{Health}$ should not be interpreted as an average of corresponding female and male indices but as half the distance from the norms established for the reproductive health indicators—fewer maternal deaths and fewer adolescent pregnancies.

Step 5. Calculating the Gender Inequality Index

Comparing the equally distributed gender index to the reference standard yields the GII,

$$1 - \frac{HARM(G_F, G_M)}{G_{\overline{F,M}}}.$$

Example: Yemen

	Health		Empowerment		Labour market
	Maternal mortality ratio (deaths per 100,000 live births)	Adolescent birth rate (births per 1,000 women ages 15–19	Parliamentary representation (percent)	Attainment at secondary and higher education (percent)	Labour market participation rate (percent)
Female	200	47.0	0.007	0.076	0.252
Male	na	na	0.993	0.244	0.718
$\frac{F+M}{2}$	$\frac{\sqrt{\left(\frac{10}{200}\right)\cdot\left(\frac{1}{47}\right)}+1}{2} = 0.516$		$\frac{\sqrt{0.007 \cdot 0.076} + \sqrt{0.993 \cdot 0.244}}{2} = 0.258$		$\frac{0.252 + 0.718}{2} = 0.485$

Note: na is not applicable.

Using the above formulas, it is straightforward to obtain:

$$G_F \quad 0.058 = \sqrt[3]{\sqrt{\frac{10}{200} \cdot \frac{1}{47}} \cdot \sqrt{0.007 \cdot 0.076} \cdot 0.252}$$

$$G_M \quad 0.707 = \sqrt[3]{1 \cdot \sqrt{0.993 \cdot 0.244} \cdot 0.718}$$

$$HARM(G_F, G_M) \quad 0.107 = \left[\frac{1}{2}\left(\frac{1}{0.058} + \frac{1}{0.707}\right)\right]^{-1}$$

$$G_{\overline{F,M}} \quad 0.401 = \sqrt[3]{0.516 \cdot 0.258 \cdot 0.485}$$

$$\text{GII} \quad 1 - (0.107/0.401) = 0.733.$$

Technical note 4. Gender Development Index

The Gender Development Index (GDI) measures gender inequalities in achievement in three basic dimensions of human development: health, measured by female and male life expectancy at birth; education, measured by female and male expected years of schooling for children and female and male mean years of schooling for adults ages 25 and older; and command over economic resources, measured by female and male estimated earned income.

Data sources

- Life expectancy at birth: UNDESA (2013).
- Mean years of schooling for adults ages 25 and older: data from UNESCO Institute for Statistics (2013) and methodology for female and combined mean years of schooling from Barro and Lee (2012). (Male mean years of schooling is derived from the combined mean years of schooling for both sexes and for women and from the male population ages 25 and older; estimates for some countries are from the United Nations Educational, Scientific and Cultural Organization Institute Statistics.)
- Expected years of schooling: UNESCO Institute for Statistics (2013).
- Estimated earned income: Human Development Report Office estimates based on female and male shares of economically active population, ratio of female to male wage in all sectors and gross national income in 2011 purchasing power parity (PPP) terms for female and male populations from World Bank (2014) and ILO (2013).

Steps to calculate the Gender Development Index

There are four steps to calculating the GDI.

Step 1. Estimating female and male earned incomes

To calculate estimated incomes, the share of the wage bill is calculated for each gender. The female share of the wage bill (S_f) is calculated as follows:

$$S_f = \frac{W_f/W_m \cdot EA_f}{W_f/W_m \cdot EA_f + EA_m}$$

where W_f/W_m is the ratio of female to male wage, EA_f is the female share of the economically active population and EA_m is the male share of the economically active population.

The male share of the wage bill is calculated as:

$$S_m = 1 - S_f$$

Estimated female earned income per capita is obtained from GNI per capita,[3] first by multiplying it by the female share of the wage bill, S_f, and then rescaling it by the female share of the population, $P_f = N_f/N$:

$$GNIpc_f = GNIpc \cdot S_f/P_f.$$

Estimated male earned income per capita is obtained in the same way:

$$GNIpc_m = GNIpc \cdot S_m/P_m.$$

To construct the female and male HDIs, first the indicators, which are in different units are transformed into indices and then dimension indices for each sex are aggregated by taking the geometric mean.

Step 2. Normalizing the indicators

The indicators are transformed into a scale of 0 to 1 using the same goalposts as for the HDI, except life expectancy at birth, which is adjusted for the average of five years biological advantage that women have over men (though in some countries the gap could be greater than 10 years).

Goalposts for the Gender Development Index in this Report

Indicator	Minimum	Maximum
Expected years of schooling	0	18
Mean years of schooling	0	15
Estimated earned income (2011 PPP $, natural log)	100	75,000
Life expectancy at birth (years)		
Female	22.5	87.5
Male	17.5	82.5

Note: For the rationale on the choice of minimum and maximum values, see *Technical note 1.*

Having defined the minimum and maximum values, the subindices are calculated as follows:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}.$$

For education, the dimension index is first obtained for each of the two subcomponents, and then the unweighted arithmetic mean of the two resulting indices is taken.

Step 3. Calculating the female and male Human Development Index values

The male and female HDI values are the geometric means of the three dimensional indices for each gender:

$$HDI_f = (I_{Health_f} \cdot I_{Education_f} \cdot I_{Income_f})^{1/3}$$

$$HDI_m = (I_{Health_m} \cdot I_{Education_m} \cdot I_{Income_m})^{1/3}$$

Step 4: Calculating the Gender Development Index

The GDI is simply the ratio of female HDI to male HDI:

$$GDI = \frac{HDI_f}{HDI_m}$$

Example: Philippines

Indicator	Female value	Male value
Life expectancy at birth (years)	72.24	65.35
Mean years of schooling for adults	8.81	8.51
Expected years of schooling	11.50	11.10
Wage (local currency)	278.6	279.2
Gross national income per capita (2011 PPP $)	6,381.4	
Share of economically active population (percent)	0.391	0.609
Share of population (percent)	0.499	0.501

Female wage bill

Female to male wage ratio = 278.6 / 279.2 = 0.9979

Female wage bill (S_f) = (0.9979 · 0.391) / [(0.979 · 0.391) + 0.609] = 0.3905

Estimated female earned income per capita:
$GNIpc_f$ = 6,381.4 · 0.3905 / 0.4991 = 4,987

Male wage bill

Male wage bill (S_m) = 1 – 0.3905 = 0.6105

Estimated male earned income per capita:
$GNIpc_f$ = 6,381.4 · 0.6105 = 7,771

Female health index = (72.24 – 22.5) / (87.5 – 22.5) = 0.765

Male health index = (65.35 – 17.5) / (82.5 – 17.5) = 0.736

Female education index = [(8.81 / 15) + (11.50 / 18)] / 2 = 0.613

Male education index = [(8.51 / 15) + (11.10 / 18)] / 2 = 0.592

Estimated female earned income index:
[ln(4,987) – ln(100)] / [(ln(75,000) – ln(100)] = 0.591

Estimated male earned income index:
[ln(7,771) – ln(100)] / [(ln(75,000) – ln(100)] = 0.658

Female HDI = $(0.765 \cdot 0.613 \cdot 0.591)^{1/3}$ = 0.652

Male HDI = $(0.736 \cdot 0.592 \cdot 0.658)^{1/3}$ = 0.659

GDI = 0.652 / 0.659 = 0.989

Technical note 5. Multidimensional Poverty Index

The Multidimensional Poverty Index (MPI) identifies multiple deprivations at the household level in education, health and standard of living. It uses micro data from household surveys, and—unlike the Inequality-adjusted Human Development Index—all the indicators needed to construct the measure must come from the same survey. More details about the general methodology can be found in Alkire and Santos (2010). More details about changes in the methodology and the treatment of missing responses and nonapplicable households are given in Klasen and Dotter (2013) and Calderon and Kovacevic (2014).

Methodology

Each person is assigned a deprivation score according to his or her household's deprivations in each of the 10 component indicators. The maximum deprivation score is 100 percent with each dimension equally weighted; thus the maximum deprivation score in each dimension is 33.3 percent. The education and health dimensions have two indicators each, so each indicator is worth 33.3 / 2, or 16.7 percent. The standard of living dimension has six indicators, so each indicator is worth 33.3 / 6, or 5.6 percent.

The indicator thresholds for households to be considered deprived are as follows:

Education:

- School attainment: no household member has completed at least six years of schooling.
- School attendance: a school-age child (up to grade 8) is not attending school.[4]

Health:

- Nutrition: a household member (for whom there is nutrition information) is malnourished, as measured by the body mass index for adults (women ages 15–49 in most of the surveys) and by the height-for-age *z* score calculated using World Health Organization standards for children under age 5.
- Child mortality: a child has died in the household within the five years prior to the survey.[5]

Standard of living:

- Electricity: not having access to electricity.
- Drinking water: not having access to clean drinking water or if the source of clean drinking water is located more than 30 minutes away by walking.
- Sanitation: not having access to improved sanitation or if improved, it is shared.[6]
- Cooking fuel: using 'dirty' cooking fuel (dung, wood or charcoal).
- Having a home with a dirt, sand or dung floor.
- Assets: not having at least one asset related to access to information (radio, TV, telephone[7]) and not having at least one asset related to mobility (bike, motorbike, car, truck, animal cart, motorboat) or at least one asset related to livelihood (refrigerator, arable land,[8] livestock[9]).

To identify the multidimensionally poor, the deprivation scores for each indicator are summed to obtain the household deprivation score, *c*. A cutoff of 33.3 percent, which is equivalent to ⅓ of the weighted indicators, is used to distinguish between the poor and nonpoor. If the deprivation score is 33.3 percent or greater, that household (and everyone in it) is multidimensionally poor. Households with a deprivation score greater than or equal to 20 percent but less than 33.3 percent are considered to be near multidimensional poverty. Households with a deprivation score of 50 percent or higher are severely multidimensionally poor.

The headcount ratio, *H,* is the proportion of the multidimensionally poor in the population:

$$H = \frac{q}{n}$$

where q is the number of people who are multidimensionally poor and n is the total population.

The intensity of poverty, *A,* reflects the proportion of the weighted component indicators in which, on average, poor people are deprived. For poor households only (deprivation score *c* greater than or equal to 33.3 percent), the deprivation scores are summed and divided by the total number of poor people:

$$A = \frac{\sum_i^q c_i}{q},$$

where c is the deprivation score that the ith poor individual experiences.

The deprivation score c of a poor person can be expressed as the sum of deprivations in each dimension j (j = 1, 2, 3), $c = c_1 + c_2 + c_3$.

The MPI value is the product of two measures: the multidimensional poverty headcount ratio and the intensity of poverty.

$$MPI = H \cdot A$$

The contribution of dimension j to multidimensional poverty can be expressed as

$$Contrib_j = \frac{\sum_1^q c_j}{n} / MPI$$

Example using hypothetical data

Indicator	Household 1	2	3	4	Weights
Household size	4	7	5	4	
Education					
No one has completed six years of schooling	0	1	0	1	⅓ ÷ 2 or 16.7%
At least one school-age child not enrolled in school	0	1	0	0	⅓ ÷ 2 or 16.7%
Health					
At least one member is malnourished	0	0	1	0	⅓ ÷ 2 or 16.7%
One or more children have died	1	1	0	1	⅓ ÷ 2 or 16.7%
Living conditions					
No electricity	0	1	1	1	⅓ ÷ 6 or 5.6%
No access to clean drinking water	0	0	1	0	⅓ ÷ 6 or 5.6%
No access to adequate sanitation	0	1	1	0	⅓ ÷ 6 or 5.6%
House has dirt floor	0	0	0	0	⅓ ÷ 6 or 5.6%
Household uses "dirty" cooking fuel (dung, firewood or charcoal)	1	1	1	1	⅓ ÷ 6 or 5.6%
Household has no access to information and has no assets related to mobility or assets related to livelihood.	0	1	0	1	⅓ ÷ 6 or 5.6%
Results					
Household deprivation score, *c* (sum of each deprivation multiplied by its weight)	22.2%	72.2%	38.9%	50.0%	
Is the household poor (*c* > 33.3 percent)?	No	Yes	Yes	Yes	

Note: 1 indicates deprivation in the indicator; 0 indicates nondeprivation.

Weighted deprivations in household 1:

$$(1 \cdot 16.67) + (1 \cdot 5.56) = 22.2 \text{ percent.}$$

Headcount ratio *(H)* =

$$\left(\frac{7+5+4}{4+7+5+4}\right) = 0.800$$

(80% of people live in poor households).

Intensity of poverty *(A)* =

$$\frac{(72.2 \cdot 7) + (38.9 \cdot 5) + (50.0 \cdot 4)}{(7 + 5 + 4)} = 56.3 \text{ percent}$$

(the average poor person is deprived in 56.3 percent of the weighted indicators).

MPI = $H \cdot A = 0.8 \cdot 0.563 = 0.450$.

Contribution of deprivation in:

Education:

$$Contrib_1 = \frac{16.67 \cdot 7 \cdot 2 + 16.67 \cdot 4}{4 + 7 + 5 + 4} \; / \; 45.0 = 33.3\%$$

Health:

$$Contrib_2 = \frac{16.67 \cdot 7 \cdot 5 + 16.67 \cdot 4}{4 + 7 + 5 + 4} \; / \; 45.0 = 29.6\%$$

Living conditions:

$$Contrib_3 = \frac{5.56 \cdot 7 \cdot 4 + 5.56 \cdot 4 \cdot 3}{4 + 7 + 5 + 4} \; / \; 45.0 = 37.1\%$$

Calculating the contribution of each dimension to multidimensional poverty provides information that can be useful for revealing a country's configuration of deprivations and can help with policy targeting.

Notes

1. The indicators were standardized (normalized) as:
2. The inequality aversion parameter affects the degree to which lower achievements are emphasized and higher achievements are de-emphasized.
3. The World Bank's World Development Indicators database contains data for gross national income (GNI) and GNI per capita (in 2011 PPP $) up to 2012 for most of countries. To calculate the HDI, the Human Development Report Office projects GNI per capita to 2013 using growth rates from the International Monetary Fund and the United Nations Statistics Division.
4. Up to one year late enrollment to primary school is allowed for to prevent counting a mismatch between the birthday and the beginning of the school year as a deprivation.
5. Some surveys do not collect information about time when the death of child happened; in such cases any child death reported by a mother age 35 or younger is counted.
6. Drinking water and improved sanitation are as defined in the Millennium Development Goals.
7. Including both land-line and mobile telephones.
8. Any size of land usable for agriculture.
9. A horse, a head of cattle, two goats, two sheep or 10 chickens.

References

Akire, S., and M. Santos. 2010. "Acute Multidimensional Poverty: A New Index for Developing Countries." Human Development Research Paper 2010/11. UNDP-HDRO. New York. http://hdr.undp.org/en/content/acute-multidimensional-poverty.

Anand, S., and A. Sen. 2000. "The Income Component of the Human Development Index." *Journal of Human Development and Capabilities* (1)1: 83–106.

Atkinson, A. 1970. "On the Measurement of Economic Inequality." *Journal of Economic Theory* 2(3): 244–263.

Barro, R.J., and J.W. Lee. 2013. *A New Data Set of Educational Attainment in the World, 1950–2010.* National Bureau of Economic Research Working Paper 15902. Cambridge, MA: National Bureau of Economic Research. www.nber.org/papers/w15902. Accessed 15 November 2013.

Calderon, M.C., and M. Kovacevic. 2014. "The 2014 Multidimensional Poverty Index: New Specification." Human Development Research Paper. UNDP-HDRO, New York. http://hdr.undp.org.

Foster, J., L. Lopez-Calva, and M. Szekely. 2005. "Measuring the Distribution of Human Development: Methodology and an Application in Mexico." *Journal of Human Development and Capabilities* 6(1): 5–25.

Gaye, A., J. Klugman, M. Kovacevic, S. Twigg, and E. Zambrano. 2010. "Measuring Key Disparities in Human Development: The Gender Inequality Index. Human Development Research Paper. UNDP-HDRO, New York. http://hdr.undp.org/sites/default/files/hdrp_2010_46.pdf.

Harttgen, K., and S. Vollmer. 2011. "Inequality Decomposition without Income or Expenditure Data Using an Asset Index to Simulate Household Income." Human Development Research Paper. UNDP-HDRO, New York. http://hdr.undp.org/en/content/inequality-decomposition-without-income-or-expenditure-data.

ILO (International Labour Organization). 2013. *Key Indicators of the Labour Market.* 7th edition. Geneva. www.ilo.org/empelm/what/WCMS_114240/lang--en/index.htm. Accessed 15 December 2013.

IMF (International Monetary Fund). 2014. World Economic Outlook database. April 2014. www.imf.org/external/pubs/ft/weo/2014/01/weodata/index.aspx. Accessed 7 May 2014.

IPU (Inter-Parliamentary Union). 2013. Women in national parliaments. www.ipu.org/wmn-e/classif.htm. Accessed 15 October 2013.

Kahneman, D., and A. Deaton. 2014. "High Income Improves Evaluation of Life But Not Emotional well-being. Psychological and Cognitive Sciences." *Proceedings of National Academy of Sciences* 107(38) 16489–16493.

Klasen, S., and C. Dotter. 2013. "The Multidimensional Poverty Index: Achievements, Conceptual, and Empirical Issues." Human Development Research Paper. UNDP-HDRO, New York. http://hdr.undp.org.

Kovacevic, M. 2010. "Measurement of Inequality in Human Development—A Review." Human Development Research Paper. UNDP-HDRO, New York. http://hdr.undp.org/en/content/measurement-inequality-human-development-%E2%80%93-review.

Maddison, A. 2010. *Historical Statistics of the World Economy, 1–2030 AD.* Paris: Organisation for Economic Co-operation and Development.

Oeppen, J. and J.W. Vaupel. 2002. "Broken Limits to Life Expectancy." *Science* 296: 1029–1031.

Riley, J.C. 2005. *Poverty and Life Expectancy.* Cambridge, UK: Cambridge University Press.

Seth, S. 2009. "Inequality, Interactions, and Human Development. *Journal of Human Development and Capabilities* 10(3): 375–396.

UNDESA (United Nations Department of Economic and Social Affairs). 2013. *World Population Prospects: The 2012 Revision.* New York. http://esa.un.org/unpd/wpp. Accessed 15 October 2013.

UNESCO Institute for Statistics. 2013. Data Centre. http://stats.uis.unesco.org. Accessed 15 May 2013.

United Nations Statistics Division. 2014. National Accounts Main Aggregate Database. http://unstats.un.org/unsd/snaama. Accessed 7 May 2014.

WHO (World Health Organization), UNICEF (United Nations Children's Fund), UNFPA (United Nations Population Fund) and the World Bank. 2013. Trends in estimates of maternal mortality ratio. www.childinfo.org/maternal_mortality_ratio.php. Accessed 15 November 2013.

World Bank. 2014. World Development Indicators database. Washington, D.C. http://data.worldbank.org. Accessed 7 May 2014.