2006 Fulkerson Prize

The 2006 Delbert Ray Fulkerson Prize was presented at the 19th International Symposium on Mathematical Programming, held July 30 to August 4, 2006, in Rio de Janeiro, Brazil. Listed below are the names of the authors receiving the Fulkerson Prize, the titles of their prize-winning papers, and the prize citations.

MANINDRA AGRAWAL, NEERAJ KAYAL, and NITIN SAXENA, "PRIMES is in P", *Annals of Mathematics*, volume 160, issue 2, 2004, pages 781–93.

Testing whether an integer is a prime number is one of the most fundamental computational and mathematical problems. The existence of short certificates for both compositeness and primality was known since the 1970s and suggested that primality testing might be in P. Yet, despite numerous efforts and a flurry of algorithms, it was not until 2002 that Agrawal, Kayal, and Saxena devised the first deterministic polynomial-time algorithm for primality testing. Earlier algorithms had either assumed the generalized Riemann hypothesis, or been randomized or had been only subexponential. This is a stunning development. This result is a true masterpiece, combining algebraic and number theoretic results in a seemingly simple way.

Manindra Agrawal is in the Computer Science Department of the Indian Institute of Technology in Kanpur. Neeraj Kayal and Nitin Saxena are students of Agrawal's.

MARK JERRUM, ALISTAIR SINCLAIR, and ERIC VIGODA, "A polynomial-time approximation algorithm for the permanent of a matrix with nonnegative entries", *Journal of the ACM*, Volume 51, Issue 4, 2004, pages 671–97.

The permanent of a matrix has been studied for over two centuries and is of particular importance to statistical physicists as it is central to the dimer and Ising models. For a 0-1 matrix, it represents the number of perfect matchings in the corresponding bipartite graph. Although polynomial-time computable for planar graphs, the computation of the permanent is #P-complete for general graphs as shown by Valiant in 1979. This opened the search for approximation schemes. In this paper, Jerrum, Sinclair, and Vigoda give the first Fully Polynomial Randomized Approximation Scheme for computing the permanent of any 0-1 matrix or any nonnegative matrix. This is a remarkable result. Their algorithm is based on updating a Markov chain in a way that quickly converges to a rapidly mixing non-uniform Markov chain on perfect matchings and near-perfect matchings. Their work builds upon the earlier pioneering work of Jerrum and Sinclair who initiated the use of rapidly mixing Markov chains for combinatorial problems.

Mark Jerrum is in the School of Mathematics at the University of Edinburgh. Alistair Sinclair is in the Computer Science Division of the University of California, Berkeley. Eric Vigoda is in the College of Computing at the Georgia Institute of Technology.

NEIL ROBERTSON and PAUL D. SEYMOUR, "Graph Minors. XX. Wagner's conjecture", *Journal of Combinatorial Theory, Series B*, volume 92, issue 2, 2004, pages 325–57.

Kuratowski's theorem says that a graph is planar if and only if it does not contain K_5 or $K_{3,3}$ as a minor. Several other excluded minor characterizations are known, and Wagner conjectured that any minor-closed graph property can be characterized by a finite list of excluded minors. Restated, this says that for any infinite family of finite graphs, one of its members is a minor of another one. In a remarkable tour de force, Robertson and Seymour proved Wagner's conjecture, and this paper appeared as part 20 of their monumental work on the theory of graph minors. Their proof of the Graph

Minor Theorem required the development of many graph theoretic concepts, such as linkages and tree-width. This is a spectacular achievement in graph theory with far reaching consequences. It shows, for example, that embeddability in any fixed surface can be characterized by a finite list of excluded minors, or that the disjoint paths problem can be solved in polynomial time for a fixed number of terminals.

Neil Robertson is in the Department of Mathematics at the Ohio State University. Paul D. Seymour is in the Department of Mathematics at Princeton University.

About the Prize

The Delbert Ray Fulkerson Prize recognizes outstanding papers in the area of discrete mathematics. Established in 1979, the prize is sponsored jointly by the Mathematical Programming Society (MPS) and the AMS. Up to three awards of US$1,500 each are made every three years at each (triennial) international symposium of the MPS. The prize is made possible by a memorial fund established by friends of the late Delbert Ray Fulkerson to encourage mathematical excellence in the fields of research in which he worked.

The prize is presented for papers published during the six calendar years preceding the year in which the prize is given. The prize is given for single papers, not series of papers or books, and in the event of joint authorship the prize is divided. The topics of papers considered for the prize include graph theory, networks, mathematical programming, applied combinatorics, and related subjects.

The selection committee for the 2006 Fulkerson Prize consisted of Noga Alon, William Cunningham, and Michel Goemans (chair).

—Announcement of the Fulkerson Prize Committee