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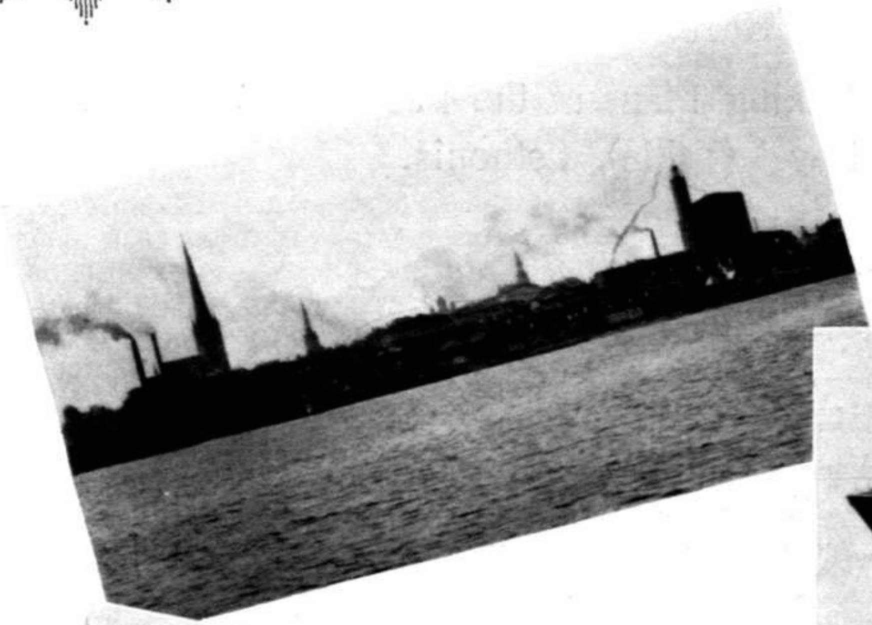
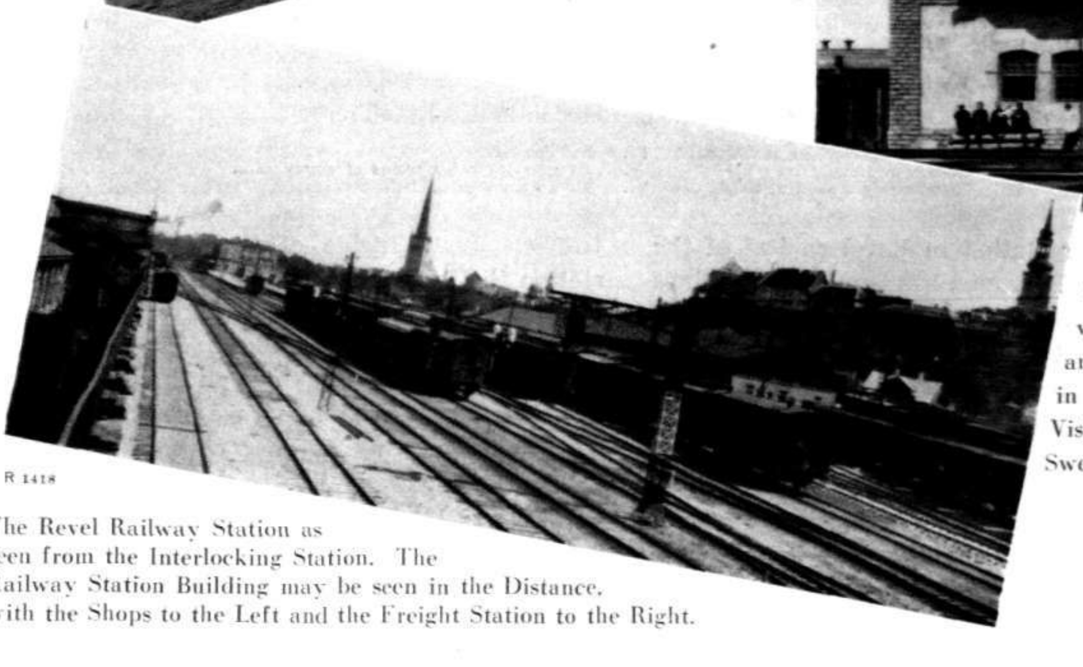


Harbour View of Revel, Capital of Esthonia.

The Interlocking Station, Decorated with the Swedish and Esthonian Flags in Honour of the Visit of the King of Sweden on June 5th, 1929.



R 1418

The Revel Railway Station as seen from the Interlocking Station. The Railway Station Building may be seen in the Distance, with the Shops to the Left and the Freight Station to the Right.

ENGLISH EDITION

L. M. Ericsson

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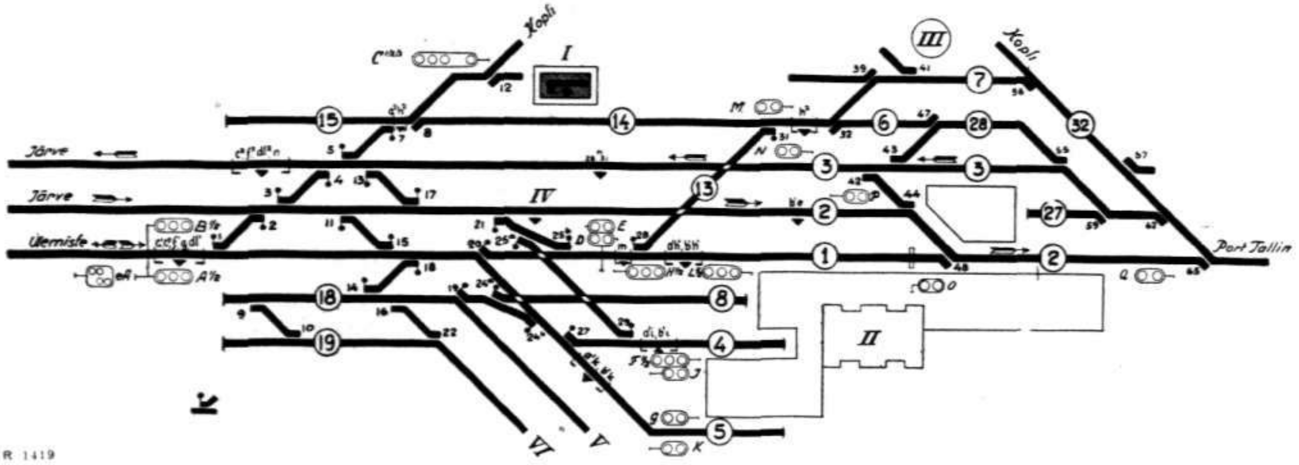
The Electric Interlocking Plant at the Passenger Station in Revel (Tallin), Esthonia.



Track Plan of the Passenger Station in Revel (Tallin).

I. Electric interlocking machine.
II. Station building.
III. To shunting station at Kopli.
IV. Rail contact.
V. To spur track for trains of empty cars.
VI. To freight station.

The passenger station in Revel capital of the republic of Esthonia, has two incoming lines from the southwest, viz. the single track line from Ülemiste (from the directions of Leningrad, Pskow and Riga) and the line from Nömme — recently made over into a double track line and electrified — (from the directions of Hapsal and Baltischport). From a northeasterly direction we have incoming tracks from the docks as well as a number of private spur tracks.

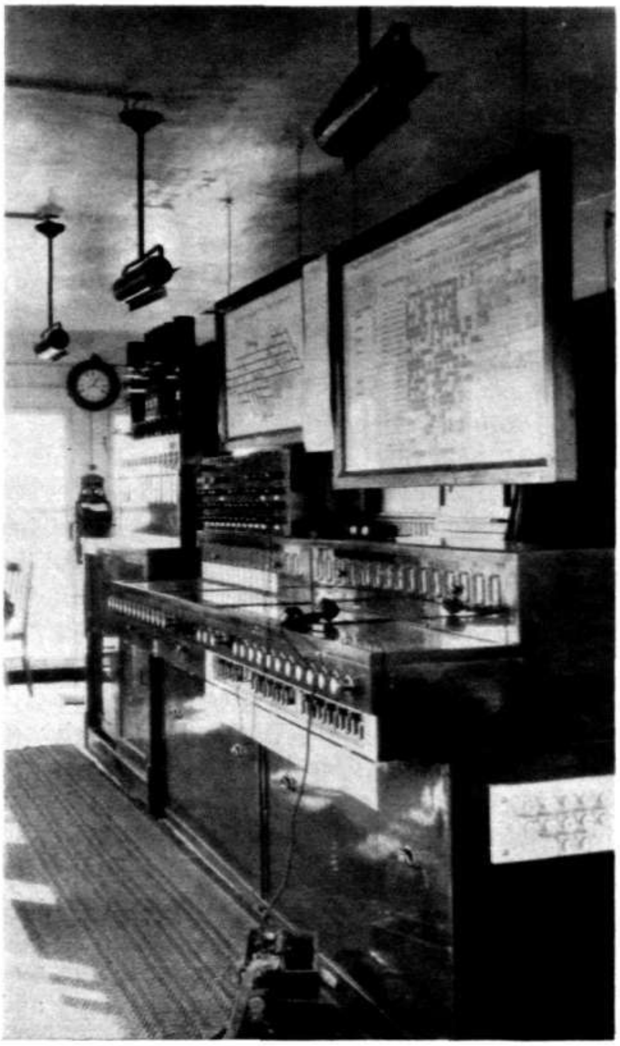
The passenger and freight traffic over the Ülemiste line is light at present, but over the Nömme line there is a heavy and growing suburban traffic. Just outside of the Revel passenger station the freight trains from the Ülemiste line as well as trains from the spur track of the A. M. Luther woodworking factory — which enters the Ülemiste track out on the line 3.3 km. from the Revel station — are switched out to the Kopli shunting station, both of the main tracks of the Nömme line being crossed at track grade. The track system of the recently rebuilt passenger station at Revel — constructed in part as through station and in part as terminal — still has its disadvantages which could not be entirely removed, however, except through a complete and costly reconstruction of the whole installation. Thus, within the area occupied by the passenger station, all of the main tracks are crossed by the many freight and shunting trains passing between the Kopli shunting station, the spur tracks in the northeast part of the track yard, the docks and the freight yard as well as by all the locomotives shunting between the round-house and the tracks of arrival and departure of the passenger trains.

The above-mentioned conditions constituted a serious menace to the safety of the traffic in and about the Revel passenger station, where all of the points, independently of the signals, were manoeuvered by hand, besides which a large personnel was required for the tedious local, manual manoeuvering of the points at the Revel station as well as for the telegraphic train dispatching system used for directing the traffic between Nömme, Revel and Ülemiste as well as between Ülemiste and the Luther spur track on the one hand and Kopli on the other, improvement of the service as well as a reduction of the personnel being called for.

After the completed reconstruction of the previously quite unsuitable track and train clearing arrangements at the Revel passenger station and after the extension of the second main track to Nömme, the above disadvantages forced the Esthonian Gov't Railway Administration to take steps for the adoption of modern means for safeguarding the traffic, i. e. the centralization of the manoeuvering of the points and signals at the Revel passenger station and the introduction of electric section blocking on the stretch Nömme—Revel—Ülemiste. Here, however, it was necessary to figure with the additional and very extensive work of rebuilding the track system, a project which must be accomplished as economically as possible.

The following basic principles were to be applied for the projected interlocking and blocking plant.

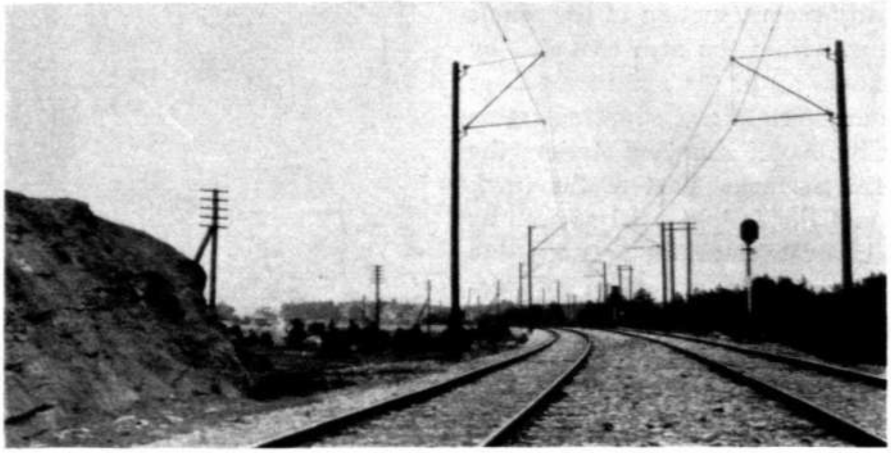
An electric interlocking plant should be built for the Revel passenger station and electric section blocking provided for the stretches Nömme—Revel and Revel—Ülemiste, while all counter points at the Revel passenger station which entered into the regular through tracks and the necessary protective switches and all signals should have central manoeuvering, which latter should also serve for the section blocking so as to provide absolute safety for the traffic.



R 1408 Lock-and-Block Apparatus and Interlocking Machine. Above the latter are mounted repeating lamps for all the day light-signals.

Since an ample supply of electrical energy was available at all stations between Nömme, Revel and Ülemiste — there being even two separate sources of energy at Revel —, all signals were to consist of day light-signals, thereby giving the desired uniformity between the day and night signal light combinations and a simplicity in the plant which would be accompanied by lower costs for both installation and maintenance. As a result, a special source of energy within the interlocking station could be dispensed with.

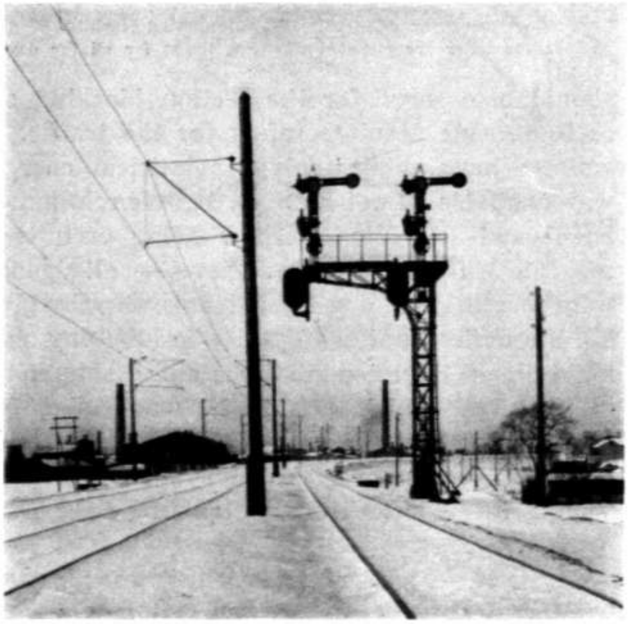
At the Revel passenger station, it was planned to erect a single interlocking machine close to the common center for all the points with central manoeuvering and for the greater part of all



R 1412 Approach to the Järve Station.

the switching operations. The train dispatcher at the interlocking station was to handle the clearing of all tracks outside and within the passenger station, while the station master was only to direct the travellers, dispatch the passenger trains and give the interlocking operator necessary information by telephone as to cleared tracks, switching operations etc. Emergency keys by means of which a signal might be set to 'stop' were to be installed in the office of the station master, while repeating lamps for the home signals were to be placed on the main platform.

Feed current for the switching machine motors, the testing battery, magnets, relays and signals was to be obtained direct or through transformers, rectifiers or the like from the Ellamaa generating station or from the city mains with a 50 cycle 220 volt current and triple and single phase respectively.

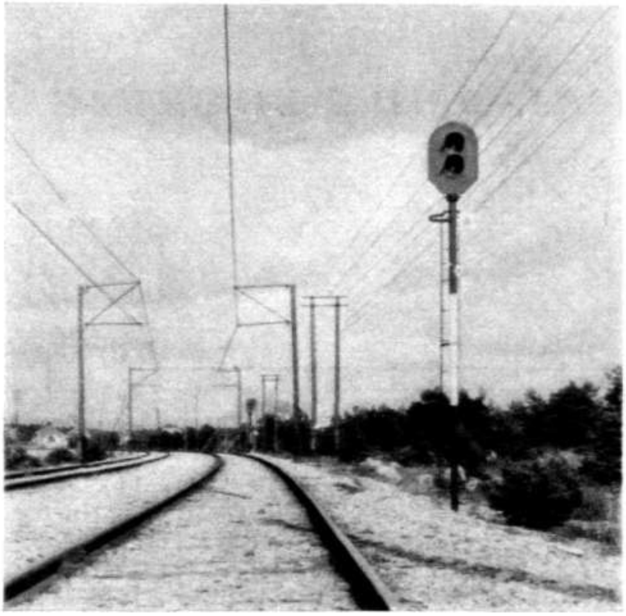


R 1409 Main Home Signals B $^1/_2$ and A $^1/_2$ at Revel.
New day light-signals installed; the old semaphores have not yet been removed.

With due consideration for the unfavourable climatic and local conditions, numerous rainfalls, thaws, gravel ballast and inefficient draining, it was necessary to reduce the operating current to a minimum. In most places only one rail was available as a conductor for the signal current, due to the electric traction.

On the double track line to Nömme it was decided to install electric section blocking with four sections and with five sections for the single track line to Ülemiste; the Luther spur track was to have three blocking sections and form a part of the entire blocking system.

On account of the electric traction over the Nömme line (a traction current of 1200 volts D. C. in the overhead wire) and the aerial power line from Ellamaa (35000 volts triple phase A. C.) which for the most part run parallel to each other and at a distance of up to twenty-five metres from the tracks, special return lines for the blocking system would be necessary in order to avoid eventual disturbances in the lock and block apparatus. At some points, where both railway lines run on the same bank and where the lines for the blocking system numbered as many as eighteen, these lines were to be run in a cable and laid in the ground. The points at the neighbouring stations Nömme and Ülemiste were to be retained with local manual manœuvering until further notice, as there are plans afoot to rebuild these stations within the



R 1410 Main Day Light-Signal. Home Signal A at Järve.

near future. The points at Järve and in the Luther spur track were to be provided with locks which — in the latter case — were to be constructed as point locks and included in the section blocking system.

An apparatus for controlling the traffic over the tracks in question was to be installed in the station building at the Kopli shunting station.

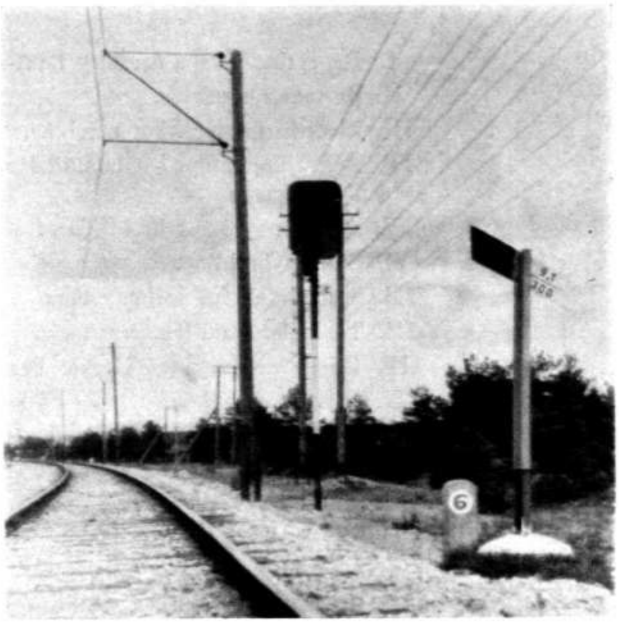
The track gauge of the Esthonian Gov't Railways being the same as in Russia (1.524 m.) and the signal devices according to German design, it was necessary that the new devices be suited thereto. Since the electric switching machines were to be provided with tongue control and inside locking, it was decided that the catch locks were superfluous and could be removed.

Both of the lines from Nömme and Ülemiste have towards Revel, a long, steep incline of up to 1.1 per cent; the track yard at Järve and the branching for the Luther spur track lie on an incline of .8 per cent, thereby rendering more difficult the installation of the various apparatus.

On the submitting of tenders in the spring of 1928, Signalbolaget — a Stockholm subsidiary of the Ericsson Company — was successful in obtaining the contract for the Revel passenger station and adjoining tracks. On account of intervening changes (the manner of executing the second main track to Nömme and the corresponding reconstruction of the track systems at Revel, Järve and Nömme was not finally decided until after tenders had been requested for this work) the whole project underwent a radical change, so that the contract was not signed until the 4th of July 1928. The work of installation was commenced in the middle of September 1928, and in spite of the extremely rigorous winter of 1928—29, which considerably hampered this work, the entire plant was completed and put in operation on April 5, 1929, according to contract, i. e. after nine months.

Since that time and up till now (October 1929) the plant has functioned most satisfactorily, no trouble of any kind having occured.

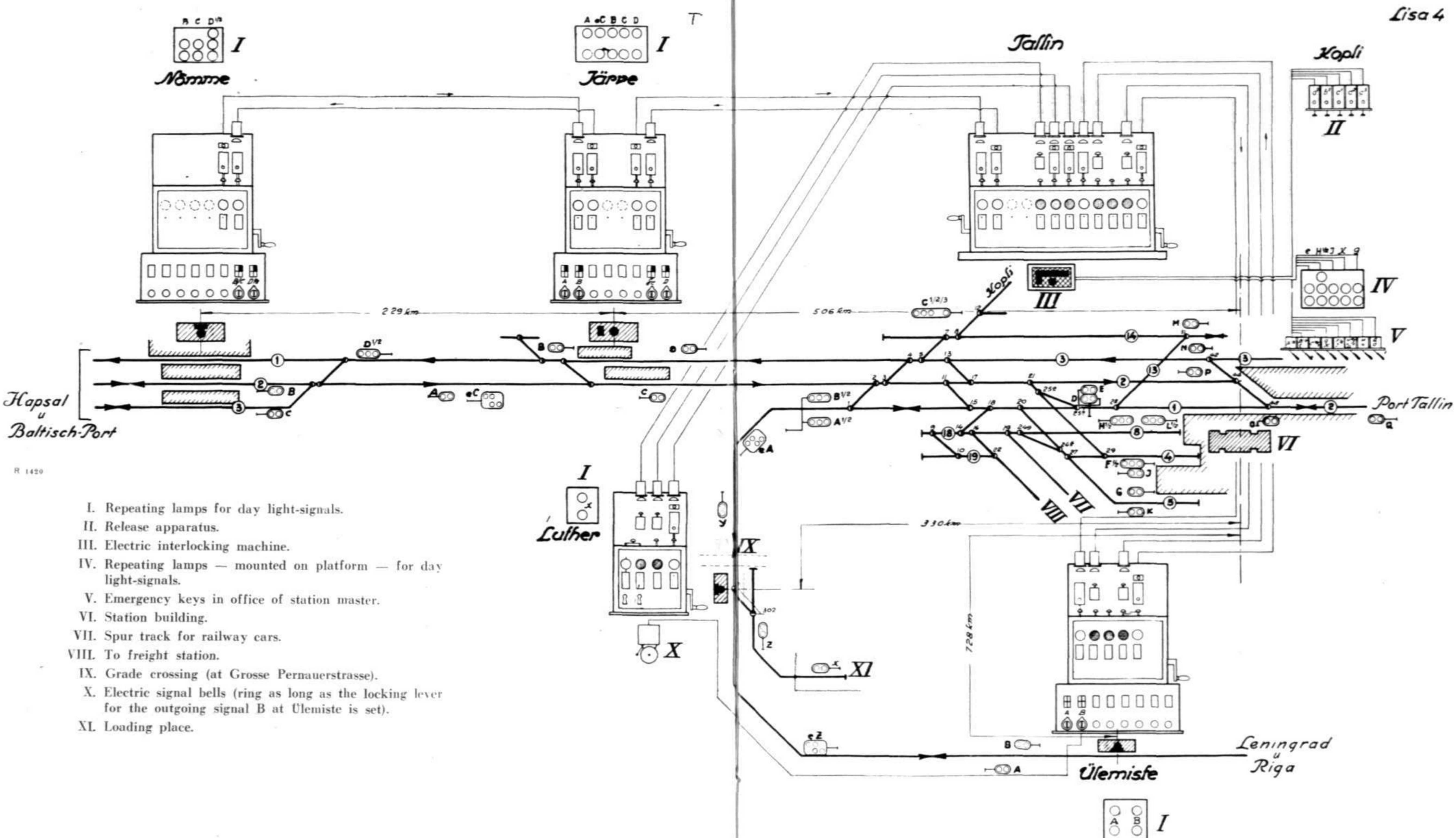
The building for the interlocking station was erected in two stories (the lower of masonry and the upper of timber). The floor of the upper story is on a level with the top of the loading profile for the freight cars, or 5.25 m. over the top of the rail. The interlocking room has wide windows on every side with two bay-windows between which there is a small open balcony, permitting a wide and unobstructed view of the entire track system and all the switching operations, since most of the latter take place in the vicinity of the interlocking station.



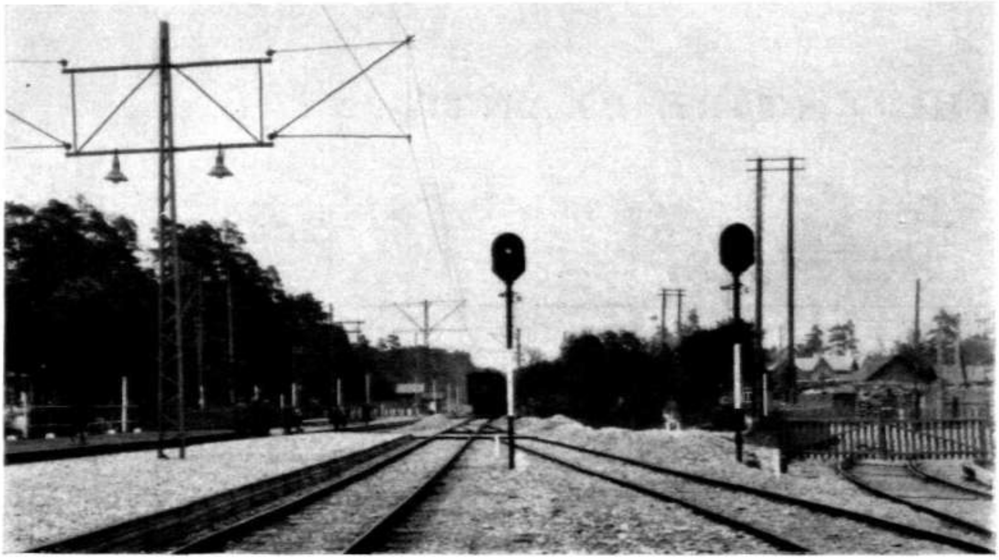
R 1411 Distance Day Light-Signal. Outgoing Signal eC at Järve.

L. M. Ericsson

SIGNAL AND BLOCKING DIAGRAM FOR THE ENTIRE PLANT.



I. Repeating lamps for day light-signals.
II. Release apparatus.
III. Electric interlocking machine.
IV. Repeating lamps — mounted on platform — for day light-signals.
V. Emergency keys in office of station master.
VI. Station building.
VII. Spur track for railway cars.
VIII. To freight station.
IX. Grade crossing (at Grosse Pernauerstrasse).
X. Electric signal bells (ring as long as the locking lever for the outgoing signal B at Ülemiste is set).
XI. Loading place.

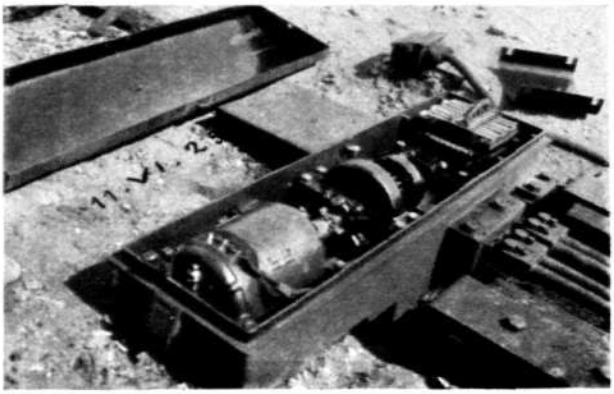


R 1412 Outgoing Signals B and C at Nömme.

The distances from the interlocking station to various points are as follows

	N. E. direction.	*S. W. direction.*
a. to the most distant signal	703 m.	2860 m.
b. to the most distant point with centralized manoeuvering	235 m.	453 m.

The upper story has a floor space of $10 \times 4 = 40$ sq. m. and contains the interlocking machine, lock-and-block apparatus, relay cabinet, instrument board with metal rectifiers for connecting up to the A. C. power net, a cabinet for high-precision mensuration instruments, a writing desk for the train dispatcher, a table with Morse receiver and the necessary boards, furniture and heating apparatus. Space is provided for a 1.95 m. extension of the interlocking machine.



R 1415 Electric Switching Machine.

An electric signal horn, by means of which the train dispatcher gives acoustic switching signals, is mounted on the balcony. Special care has been given the problem of adequately lighting the interlocking room at night, so as to provide good lighting of the apparatus, clock and writing desk without in any way impairing the clarity of the view from the windows. The interlocking operator is able to get into immediate communication with the station master by means of a loudspeaker and with the nearest railway stations, the more important groups of switches and the telephone exchange by means of the telephone.

All of the incoming and outgoing cables and wires terminate in the space provided for this purpose between the two stories.

The lower story consists of two rooms, one of them being used for an electrical repair shop and the other for housing the storage battery and the stock of material and spare parts.

The stairway to the upper story is inside the building, which is provided with a heating plant, water, fire hydrant and two lightning rods. Hand fire extinguishers are placed at suitable places in both stories.

The interlocking station is manned by a train dispatcher and a signal operator.

The lock-and-block apparatus are made with fields for alternating current. The interlocking machine at Revel, besides being provided with electric control by means of locking magnets, has a mechanical cross locking gear. 24 tracks

R 1414
Electric Switching Machines with Tongue Control and Enclosed Points Lock.

can be cleared and 1 advance signal, 21 signals and 21 points (of which two are simple crossing points) can be manoeuvered from the interlocking machine by means of 13 track signal levers and 14 point levers. Shunt supervisory lamps for all of the signals are mounted in an extension to the interlocking machine. A true picture of all the signals may thus be obtained by the train dispatcher.

Since most of the switching operations take place in the immediate vicinity of the interlocking station and may be easily supervised by the train dispatcher, no special arrangements were made in order to prevent the setting of points occupied by rolling stock. The more dangerous switching operations which include the crossing of a main track and which take place at a comparatively great distance from the interlocking station, however, are considered as separate tracks and are provided with the corresponding safety arrangements.

Electrical energy may be obtained according to necessity, either from the Ellamaa generating station or the city mains.

A D. C. supervisory current of about 40 volts' tension for the magnets and relays is obtained from a 220 volt A. C. net by means of a 6 amp. dry rectifier. The direct current thus obtained might of course be carried direct to the interlocking machine for supervisory purposes, and this is also done when necessary, for instance when the storage battery is disconnected. For a regular supply of supervisory current, however, there is a storage battery with a capacity of 34 amp. h. and composed of 30 nife cells. This battery is placed on a shelf with a surface of but .26 sq. m. and therefore requires no special room.

All the signals are day light-signals. The separate lamps have double lenses and simple 127 volt single wire incandescent lamps. Each light and colour has its own separate lamp. Since the visibility of the lights is much diminished by the smoke from slate fuel, especially at the Revel station, comparatively strong lamps were selected for the day light-signals, viz. 40 watts for main and shunting signals and 20 watts for advance signals. The necessary A. C. for the day light-signals is obtained from the 380 volt triple phase net at Revel by means of a star connection, after which it is transformed down to 220 volts. The transformers are placed in the respective station buildings.

The switching machines are provided with tongue control and inside locking. The motors for these machines are for .6 HP and are fed direct with a 220 volt A. C.

220 volt A. C. direct from the service mains is also used for the lamps in the switch lanterns installed by the railway. Those points which are not used for shunting operations but only

R 1416 Cable Distribution Boxes.
Distribution boxes of this type are placed wherever more than three cables are brought together.

for the regular train traffic have not been provided with switch lanterns, as this was considered superfluous.

The insulated rails are fed with a 6 volt alternating current transformed down from the 220 volt service current in the interlocking machine. By introducing resistances in the circuit, their purpose being to limit the intensity of the current on the passage of a train, the tension is reduced from 6 volts to 3.8 volts in the insulated track sections lying nearest the interlocking station, and to 1.2 volts in the more distant ones. At these latter sections the voltage on the relay side is transformed up from 1.2 volts to a tension more suitable for the relays.

In spite of the above-mentioned disadvantages, the arrangements have proved to be entirely reliable, no trouble whatever having occured in the releasing of the tracks.

The energy consumed by the Revel interlocking plant amounts to about 20.7 kWh per 24-hour day. Of this amount, about 19 kWh are used by the signal and repeating lamps and for the lighting of the interlocking station, leaving only 1.7 kWh per day for the control and manoeuvering current, corresponding to a consumption of energy of .074 kWh per motor and 24-hour day.

We specially wish to emphasize that it was possible to carry on this work smoothly, quickly and efficiently thanks to the very kind cooperation of the administration of the Esthonian Gov't Railways, this statement being upheld by the fact that not one single deviation from the original project was found necessary.

R 1418

New Swedish Carrier Current Telephone and Telegraph Systems on Telephone Lines.*

By H. Sterky.

The most effective use of the products and forces of nature in the service of mankind constitutes the basic principle for all engineering work.

The aim of an electrical engineer is to make use of electricity in the most effective manner, either in the field of power supply and distribution — by seeking to obtain a more favourable distribution and equalization of the load in order the better to suit the production or to increase the efficiency and power factor of transformers and generators — or in the adaptation of low tension electricity, more especially within the fields of telephony and telegraphy — by seeking to make the best possible use of the various existing means of communication, by increasing the speed of the service or by creating new means for the increase of the number of possible conversations or messages over existing lines or wave lengths.

Developments in the wireless art during the last decade have in more ways than one influenced wire telephony and telegraphy. The introduction of carrier current telephony over existing telephone and telegraph lines has provided telephone engineers with an excellent and reliable means for the most efficient use of these lines.

The interest for the introduction of carrier current transmission seems at the present time to be large in practically all parts of the world. It is most noticeable, however, in thinly populated countries with extended areas and in which aerial telephone lines are used. Nevertheless, it has been found to advantage for economic reasons to use multiplex carrier current telephony also in very thickly populated countries, such as Java, for instance.

* Paper read before the Electrical Section of The Swedish Technical Society at a meeting held October 4th, 1929.

Actual carrier current telephony is possible — for technical reasons — only over aerial lines, and these should preferably be of copper or aluminium. In countries extending over large areas it is economically advantageous to establish carrier current telephony over existing telephone lines instead of erecting a new line between two localities for this purpose. Another useful field for carrier telephony or telegraphy is where the operation of a large radio station is to be centralized to a nearby city. Modern large radio stations often handle the receiving and sending of several telegrams simultaneously. It is usually necessary to build both sending and receiving stations at quite some distance from the cities which are to profit from their service and at points to which the construction of the necessary number of lines of communication would be an expensive proposition.

One example of the application of multiplex carrier telephony on a large scale over open wire lines is to be found in the telephone net now in course of erection in Mexico by the Mexican subsidiary of the Ericsson telephone company. This net has been planned with a view towards the establishing of quite a number of carrier telephone channels. The building of the terminal equipment for the eighteen stations required for the nine carrier current channels forming the first stage in the Mexican project has been entrusted to Svenska Radioaktiebolaget (The Swedish Radio Company), since some two years back a subsidiary of the Ericsson company, this latter — together with the Marconi company — being the main shareholders in the same.

The Java telephone administration has quite a number of radio stations for the wireless traffic with Holland, Europe in general and other parts of the world. In order to centralize the service, a main radio office from which both transmitters and receivers are controlled has been established in Batavia. The transmitters are located in Malabar — among other places — and the receivers in Rantja Ekek, both of which are located near the city of Bandoeng up in the mountains. Telephone lines are already in existance between Bandoeng and Weltevreden, Batavia's European suburb, but they are already heavily stressed by the regular telephone traffic. One of these lines is now being used for ten separate carrier current telegraph channels in addition to the regular voice frequency telephone channel. For reasons which will be dealt with in the following the frequency range available for carrier current telegraphy was confined between 5000 and 12,000 cycles per sec. and it was necessary for this band to accomodate the abovementioned ten channels, each one devised for a transmitting speed of 200 words per min. In competition with both German and American firms, Radioaktiebolaget was successful in obtaining the contract for the erection of this plant for the Java Telephone Administration.

From theoretical as well as practical and technical points of view the construction of the carrier current installations in Mexico and Java have offered quite a number of problems which may now be considered as solved. This paper will be devoted to a closer study of the special features which characterize Radioaktiebolaget's carrier current system and to the initiation of the reader in the functioning and construction of such plants. It has been claimed that the time is not yet ripe for giving publicity to descriptions of plants from which actual figures resulting from extended practical operation of the same are as yet unavailable. The new system for carrier current telephony especially devised for Ericsson's Mexican subsidiary, however, has been evolved from Radioaktiebolaget's older system for carrier current telephony as used for a number of years in Finland — among other places —, between Helsingfors and Tammerfors and here in Sweden between Sundsvall and Örebro and between Stockholm and Umeå, and which has given reliable service. The new connections and changes which have been made, however, must be considered as constituting improvements of decided value and tests have provided ample proof of their meeting the expectations which were placed upon them from the very first.

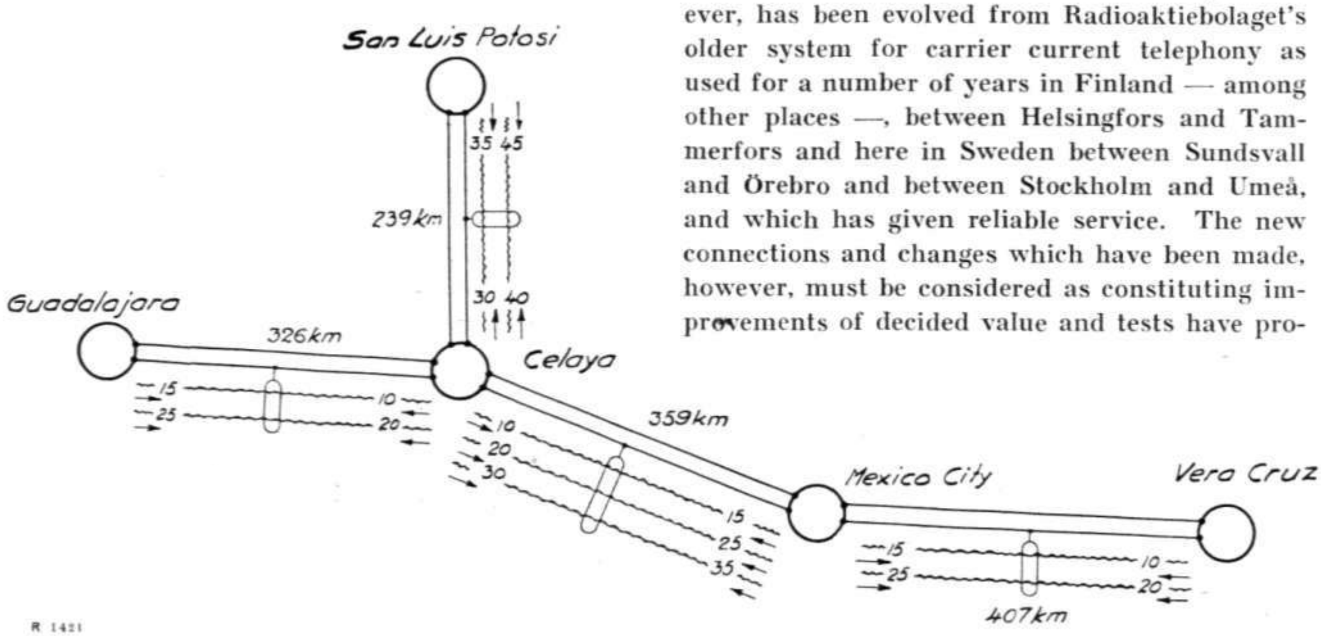


Fig. 1. Location Diagram of Carrier Current Channels in Mexico (Initial plant). Carrier frequencies in kilocycles per sec.

The Java plant, on the other hand, has forced the solution of a number of problems which have arisen specially in connection with the construction of plants for carrier current telegraphy. Just a few of the most important of these problems will be mentioned. Thus, the problem of holding ten carrier frequencies lying between five and twelve thousand cycles per sec. to constant values with an accuracy of better than 1 pro mille has been solved; filters with a band width of not more than 200 cycles per sec. at a frequency of 7000 cycles per sec. have been constructed; the iron core coils designed for these filters have inductances which in actual operation do not vary by more than $\frac{1}{3}$ per cent in spite of a considerable anode D. C. flow through them. At the same time the decrement, for instance, is only about .017 for a frequency of 5250 cycles per sec. Quite a number of other problems of a more practical nature have also found their solutions, to which we will return further on.

THE MEXICO PLANT.

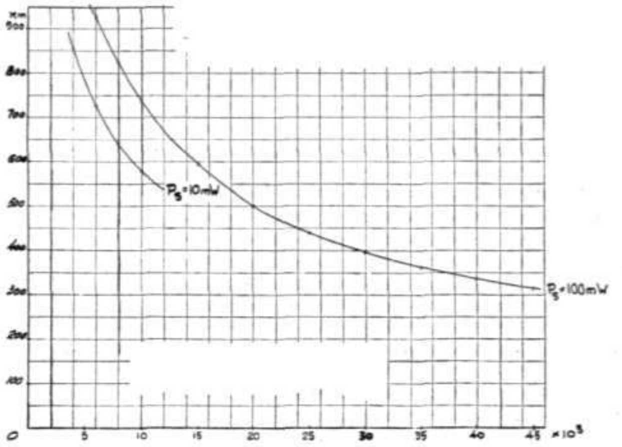
PLANNING OF THE TOLL NET.

When preparing the project for toll communications in Mexico it was first planned to string twisted pairs between the cities Mexico City and Vera Cruz, Mexico City and Celaya, Celaya and Guadalajara, and Celaya and San Luis Potosi. Mexico City, as we are aware, is the capital of the Mexican republic. Vera Cruz is the chief port and Guadalajara and San Luis Potosi two relatively large cities on the Mexican plateau. The city of Celaya is much smaller than the others but has now become a city of very special importance as regards telephone communications, being a junction point for the lines of communication running East and West and North and South.

While carrying on the work of erection of these twisted pairs, however, it was found that is would be much more economical to erect four twisted lines between all these points from the very outset. Moreover, the initial plant comprised nine carrier channels, arranged as shown in fig. 1 and routed over one of the main conductors of the respective quads.

Two frequencies are required for each carrier channel, the one as carrier frequency in one direction and the other as carrier frequency in the other direction. The sending frequencies which have been adopted for the Mexico plant are given on fig. 1 in kilocycles per second.

When calculating the attenuation of a carrier current circuit it will not do to simply apply the constants for resistance and leakance which hold good for ordinary voice frequency transmission. Inductances and capacities per km. loop, on the other hand, are practically the same for high as for low frequencies. The resistance of the conductor per km. loop increases rapidly with the frequency due to the skin effect. The leakance also increases with the frequency on account of dielectric losses at the insulators, and the influence of the leakance on the attenuation for high frequency increases with the frequency.



R 1422

Fig. 2. Range Capacity over a 3 mm. Copper Line with Varying Frequencies. $t = 62^\circ$ C.
Level: + 2.3 to -3 nepers for telephony.
+ 1.15 » -3 nepers for telegraphy.

Two curves showing the ranges for carrier current telephony and telegraphy respectively are plotted in the graph shown in fig. 2. These curves show that the range for the higher frequencies is decidedly shorter than for the lower frequencies, depending on the above stated fact that the attenuation increases with the frequency. The two curves in fig. 2 hold good for a 3 mm. copper wire in a four-twist and with a distance of 40 cm. between the centres of the conductors in the square formed by them. These curves are also applicable to an aluminium cable consisting of a core wire of steel around which are cabled six aluminium wires with a conductivity equal to that of No. 8 AWG copper wire, aluminium cable being used for all of the toll lines in Mexico. The reason for this is quite characteristic for the conditions under which Mexican telephone companies must operate. It has often happened that copper wires have been cut down and carried away i. e. deliberately stolen. In order to avoid such 'operation disturbances', copper lines have been replaced by aluminium cables, the market value of aluminium being much lower than that of copper.

The normal transmitting power of Radioaktiebolaget's installations for carrier current telephony amounts to 100 mW carrier current power on the line. With carrier current telegraphy, on the other hand, the range is sufficient with a carrier current power of only 10 mW, due to the lower frequency used. The ranges given in fig. 2 have been calculated for an attenuation of 5.3 nepers for telephonic and

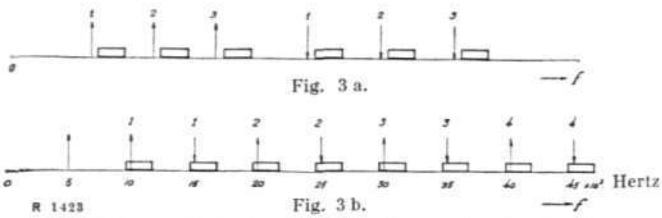


Fig. 3. Different Methods of Spacing Frequencies for Carrier Current Telephony.

4.15 nepers for telegraphic transmission. These values for the attenuation mean that the power at the receiving end is not allowed to fall below a value corresponding to a level of -3 nepers. Experience has proved that this level does not come in dangerous proximity of the disturbance level provided that the lines are so well balanced as to permit voice frequency phantom communication without danger of cross talk.

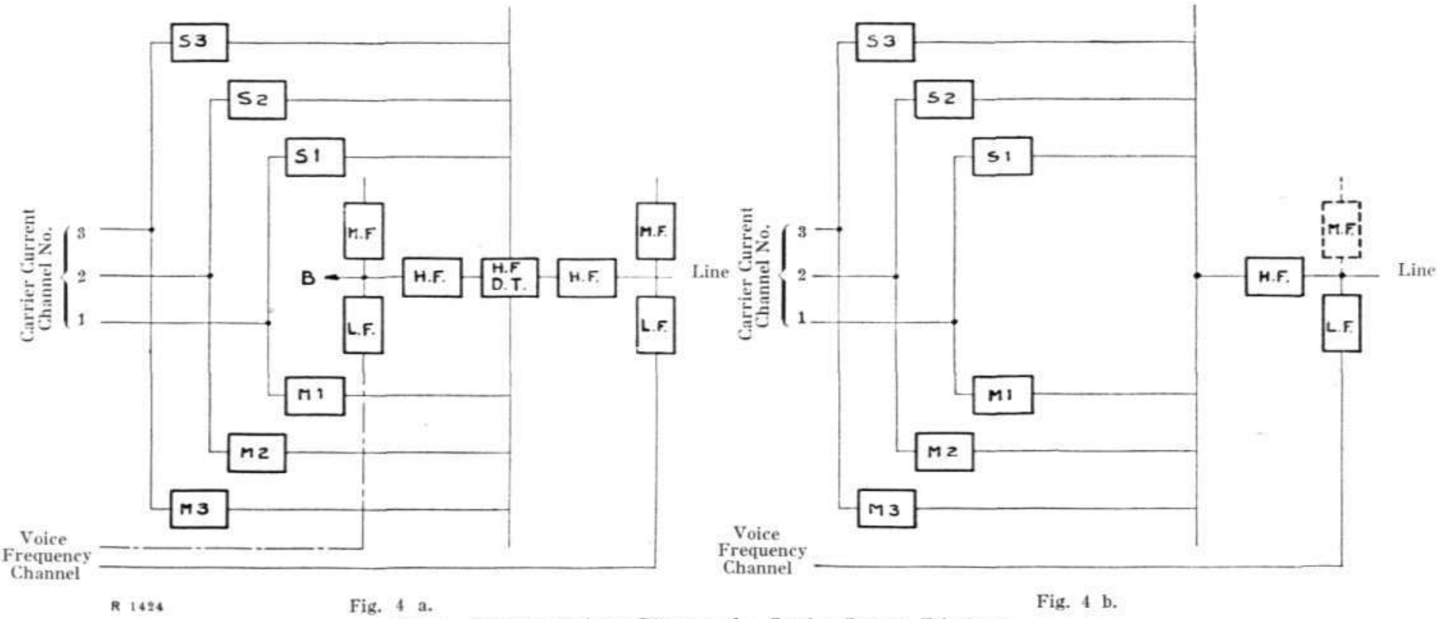


Fig. 4. Different Skeleton Diagrams for Carrier Current Telephony.

CHOICE OF CARRIER FREQUENCIES FOR THE MEXICO PLANT.

As has already been mentioned, two carrier frequencies — one in each direction — are required for one carrier transmission channel, and each terminal must consequently be equipped with one transmitter and one receiver for each channel. The reason why two frequencies are required is that the transmitting power is so much greater than that which arrives at the receiver — the difference in levels can sometimes amount to 5.3 nepers — that if the same frequency were used one could not prevent — in the usual way, by means of a hybrid coil — the transmitter from influencing the receiver. Under such conditions reaction would cause the receiver and transmitter for one transmission channel to oscillate, thereby making all transmission impossible. Consequently, a certain frequency is always chosen for transmission and one for reception. The distance between these frequencies depends upon the frequency range considered necessary for transmission and upon the quality of the filters.

If we assume that a frequency range of from 200 to 2500 cycles per second is necessary in order to obtain good speach transmission, experience has proved that the two carrier frequencies must have a difference of at least 4000 cycles per sec., but ony on condition that not more than one of the side bands, which arise from modulation, is used, according to current practice. Radioaktiebolaget has chosen a difference of 5000 cycles per sec. between the two frequencies used for transmitting and receiving. Figure 3 shows two different arrangements for the spacing of carrier frequencies and side bands in carrier current telephone transmission. According to fig. 3a the three lower carrier frequencies are used for transmission in the same direction within three different channels. The three higher carrier frequencies are used for transmission in the other direction. According to figure 3b two adjacent frequencies are used as carrier frequencies for one channel, the one for transmission in one direction and the other for transmission in the opposite direction. In carrier current telephone systems with a spacing of frequencies according to fig. 3a, it is customary to arrange the various transmitters and receivers and the segregating filters which form a part of the so-called line filter bay — to which we will revert in the following — as shown in fig. 4b. The three transmitting filters must be dimensioned so as to prevent the different transmitters from influencing each other. The receiving filters must not only prevent the wrong frequencies from entering the receivers, but they must also prevent the transmitting frequencies from influencing the receivers of their own or of other channels. It is necessary, therefore, that these filters meet requirements of a most stringent nature. In order to make the filtering process somewhat less difficult the frequencies are grouped as in fig. 3a with a wider spacing between the three higher and the three lower carrier frequencies.

In Radioaktiebolaget's system for carrier current telephony, however, a specially constructed high frequency hybrid coil (H. F. D. T. in fig. 4a) is used. This H. F. hybrid coil possesses — in similarity with other hybrid coils — the quality of being able to differentiate between the transmitting and the receiving sides, when correctly balanced. This function is independent of the frequency, and the hybrid coil acts as though an attenuation were introduced between the transmitting and the receiving sides. With perfect balancing, this attenuation is infinitely great.

In actual practice, and especially with high frequency currents, it is impossible to perfectly balance the line. The hybrid coil gives good service, however, even though the balancing is not perfect; so, for instance, an apparent attenuation of 3 nepers is introduced between the transmitter and the receiver if the balancing deviates 10 per cent from the correct value. A considerable attenuation is thus introduced between the sender and the receiver by means of the hybrid coil. As a result, very expensive band filters are not required in this system, despite the fact that the transmitting and receiving frequencies do not lie further apart than 5000 cycles per sec.

The frequency distribution adopted by Svenska Radioaktiebolaget as shown in fig. 3b is accompanied by still another advantage. The range for a certain channel is naturally determined by the range of the highest carrier frequency in this same channel. If a frequency distribution according to the system illustrated in fig. 3a is chosen, the range of channel 3, for instance, is determined by the highest frequency, or about 35 000 cycles per sec. The range for the second carrier frequency for this same channel 3, then, will not be made use of to its full extent. According to the second method of selecting frequencies (see fig. 3b) the ranges of the carrier frequencies for both directions will be made use of in the most effective manner. One obtains a wider range for channels operating with lower frequencies, this being of great advantage, especially in Mexico where the distance between different cities is sometimes so great that the highest carrier frequencies have an insufficient range.

SYNCHRONISATION.

The selection of frequencies adopted by Svenska Radioaktiebolaget for its installations is characterized also by the fact that the different carrier frequencies are all multiples of a certain master frequency of 5000 cycles per sec. This master frequency is generated at a central point in Mexico City and distributed to other cities with terminal equipment for carrier current telephony. In these cities the master frequency is used for the generating, in so-called multiple generators, of odd or even harmonics of the master frequency. These harmonics are then used as carrier frequencies for the different transmitters in the respective cities.

A simple solution has thus been obtained for the problem of holding the different carrier frequencies to correct values within their different frequency bands. Only the master generator in Mexico City requires to be tuned to the correct frequency, i. e. 5000 cycles per sec., and if this frequency is correct, correct frequencies are obtained for all the carrier currents of all the transmission channels.

The advantages obtained by means of the synchronization arrangement may be summed up as follows.

1. All harmonics of a certain frequency coincide with other carrier frequencies or their harmonics. Consequently, the frequency of the interference tones which arise equals zero and these tones are not heard. Other interference tones — for example between $2 \times 10{,}000$ and 25,000 cycles per sec. —, on the other hand, cause no disturbance because voice frequency filters eliminate such high frequencies as 5000 cycles per sec.
2. Only *one* generator need be tuned and maintained at the correct frequency. This single tuning causes all the carrier frequencies to take their correct positions in the filter bands.
3. Two groups of carrier current calls with the same carrier frequencies may be transmitted over the two lines of a four-twist, the disturbances caused by cross talk being smaller on account of the choice of frequencies (compare point 1).
4. All the channels in the entire net always have the correct carrier frequencies.

CARRIER CURRENT TELEPHONE TERMINAL EQUIPMENT.

Before going in to a more detailed description of the various units of the terminal equipment, it may be of interest to mention one of the essential differences between Radioaktiebolaget's system and that of the Western Electric Company, for instance. The rectangles in fig. 3a and b indicate the areas in which the band filters of the various transmitters and receivers admit frequencies with low attenuation. Fig. 3a, which represents the Western Electric System, shows that here only one — the upper — of the side bands is admitted by the filter. Moreover, in this system, the carrier frequency is suppressed by means of special modulators.

In Radioaktiebolaget's system the carrier frequency is not suppressed. The modulators and demodulators used permit, it is true, the suppression of the carrier frequency only through the changing of the grid voltage, but the reason for its being nevertheless retained is that then there is no necessity for providing the demodulator locally with a new carrier frequency, a procedure which is necessary with the first-mentioned system.

If the carrier frequency is not suppressed one must, it is true, figure with the amplification of the upper side band as well as of the carrier in the different high frequency amplifiers of the installation. This involves a certain danger for overloading the high frequency amplifier of the transmitter, among other things. One may, however, obtain an ample transmitting power, i. e. 100 mW on the line, without any such danger, the reason for this being that the valves used (Marconi's LS 5 series) give considerable spare power.

The main equipment for a carrier current telephone terminal station comprises *line filters* for that line which is to be equipped for carrier current transmission, and a number of transmitting and receiving units corresponding to the number of simultaneous carrier current calls (maximum four) to be transmitted over this line. One transmitting and one receiving unit together constitute the *terminal station equipment for one channel*. In addition to this we have equipment common for several channels and consisting of a *synchronization unit* and *current distribution apparatus*. Each of the above-mentioned four main groups of apparatus fill a 570 mm. wide bay on a standard rack with a height of 3120 mm. (compare fig. 6).

Line filters.

According to the skeleton diagram in fig. 4a, the line filters consist of a double set of low pass, band pass and high pass filters, one high frequency hybrid coil — the function of which has already been described — and a line balancing network, in general consisting of a resistance in series with a condenser. One of the line filter sets is connected to the line side, the other to the balance side. This provides an easy way of balancing the line side, for the same filters are to be found on the balance side as well as on the line side. Furthermore it is possible — without auxiliary apparatus and complicated balancing networks — to obtain from the low pass filter on the balance side a good balance for a two-wire repeater which is eventually used for the voice frequency channel. The following functions are filled by the different line filters. The low pass filter suppresses currents of intermediate and high frequencies but transmits the low frequency oscillations which form the speech in the physical channel over the line in question. The band pass filter is used to separate master frequency of 5000 cycles per sec. from the mixture of frequencies occurring on the line. Lastly, the high pass filter suppresses the low and intermediate frequencies, meanwhile passing high frequencies without appreciable attenuation.

Terminal equipment for one channel.

A skeleton diagram for the terminal equipment for one channel is shown in fig. 5. The audio frequency speech current for a certain high frequency channel coming from the switchboard or subscriber's line enters first a supervisory arrangement for the purpose of supervision and listening in. After this it passes on to a voice frequency hybrid coil and from there in the customary manner to a low frequency amplifier. The switchboard line or subscriber's line does not have to be perfectly balanced, for the function of the hybrid coil is merely to introduce a certain attenuation between the receiver and the transmitter and to direct the major portion of the incoming speech to the low frequency amplifier of transmitter.

After having been amplified in the low frequency amplifier, the speech current is admitted to the modulator which is fed by a carrier frequency from the previously mentioned multiple generator. In the band filter which follows after the modulator the lower side band is filtered out, the carrier frequency and the upper side band being admitted to the high frequency amplifier. After this follows still another band filter, from which the carrier frequency and side band pass to the high frequency hybrid coil, designated in fig. 4a by the letters HFDT.

An incoming carrier frequency with its side band is carried from the high frequency hybrid coil to a receiver band filter, which eliminates other incoming frequencies intended for other channels. After this receiver band filter we pass on to a high frequency amplifier which functions simply as a voltage amplifier and increases the voltage of the incoming frequencies before they are fed in to the demodulator. The demodulator functions as a detector and in its anode circuit we get back the voice frequency which modulated the carrier frequency at the transmitting station. This voice frequency is

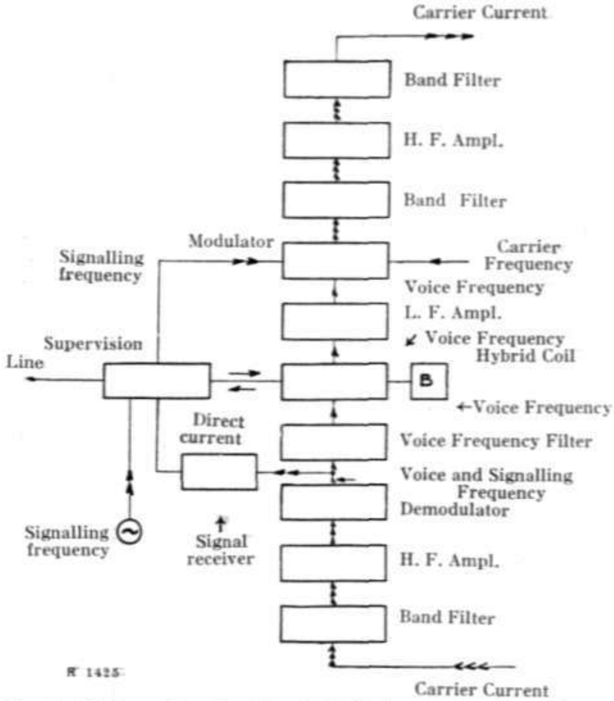
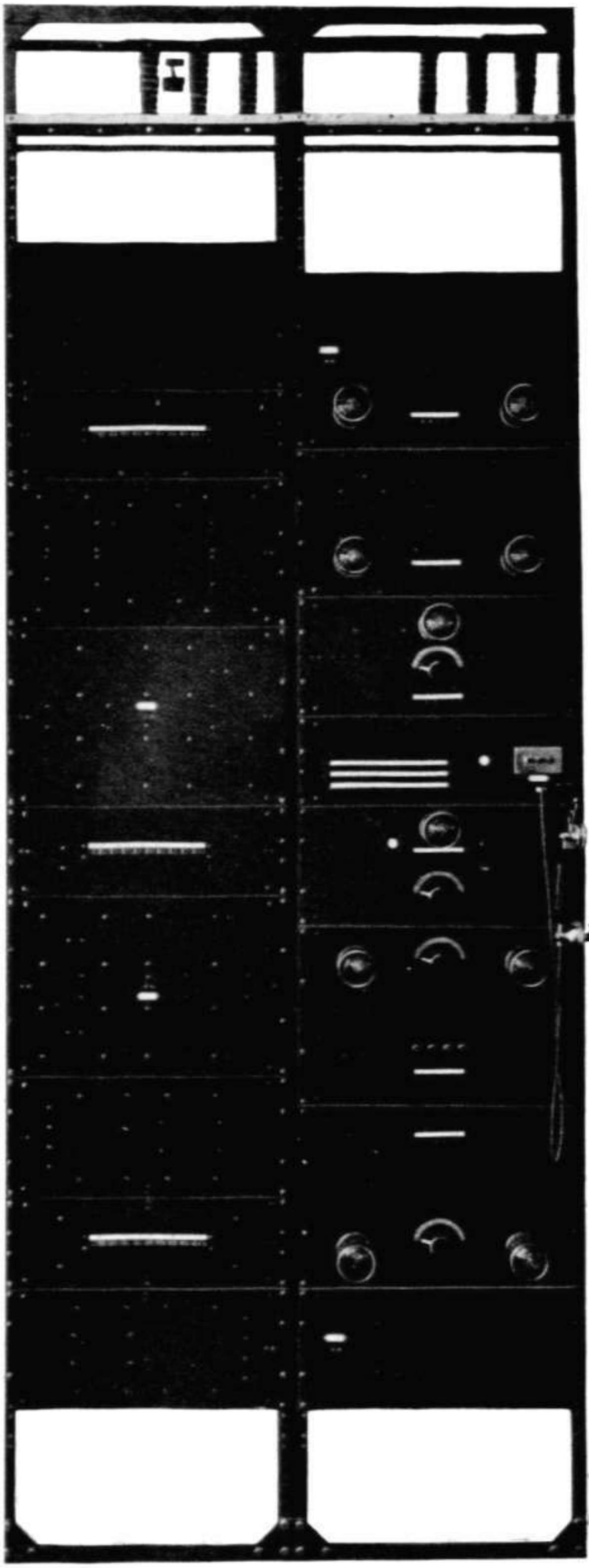


Fig. 5. Diagram Showing Terminal Equipment for One Channel.

fed into the low frequency hybrid coil and subsequently to the subscriber's line.

For signalling in its plants for carrier current telephony, Radioaktiebolaget uses common signalling current with a frequency of 20 to 25 cycles per sec. This signalling frequency is obtained from the regular ringing machine at the telephone exchange. If a signal arrives from the switchboard or from some subscriber on the exchange side, the signalling current actuates a relay in the supervisory arrangement, and this relay admits a signalling frequency from the ringing machine to the modulator. By using a ringing current from the regular ringing machine at the exchange — and not the incoming signalling current — for the feeding of the modulator we gain the advantage that this latter always gives a constant signalling power independently of the length of the subscriber's line. In the modulator the carrier wave is modulated with 20 to 25 cycles per sec. and the signals are transmitted to the distant receiver by means of carrier current. At the receiving station, after demodulation by the demodulator, we again obtain signalling frequency which is fed into a signal receiver. This signal receiver consists of one valve, connected as a rectifier. The incoming signalling frequency is rectified and actuates a polarized relay in the anode circuit of the signal



R 1426
Fig. 6. Line Filter Bay and Terminal Equipment Bay (front view).

receiver valve. In this manner a D. C. circuit is closed and actuates the supervisory arrangement, this latter, in turn, transmitting signalling frequency of sufficient intensity to the subscriber.

The line filters (at left) and terminal station equipment for one channel (at right) are shown in fig. 6. At the extreme top of the line filter bay we notice the two band pass filters, below this the low pass filters on either side of the line and balancing panel, and at the bottom the high pass filters on both sides of the high frequency hybrid coil panel. This photograph of the terminal equipment for one channel shows the different bays arranged in the same sequence as in the skeleton diagram in fig. 5.

SPECIAL CONNECTIONS AND DESIGNS OF RADIOAKTIEBOLAGET'S SYSTEM FOR CARRIER CURRENT TELEPHONY.

Space does not permit of a complete description of the circuit diagrams for the terminal equipment for one channel. Certain apparatus used in the installations by Radioaktiebolaget differ considerably from what is customary in general practice, however. In the following description of the various terminal equipment units we will therefore hold ourselves chiefly to these new designs.

The modulator.

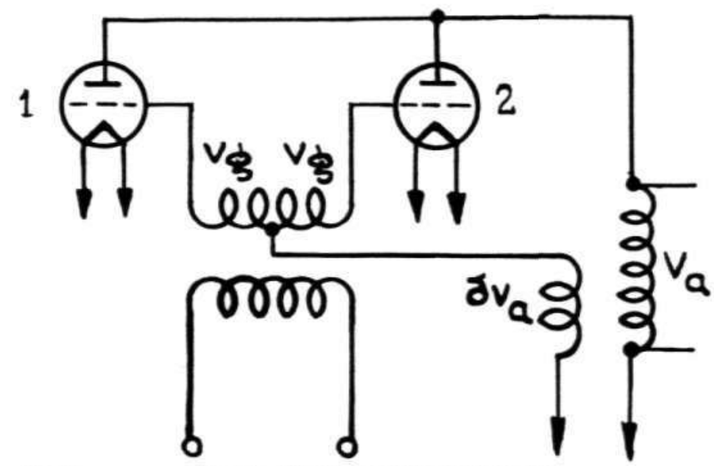
The principle for the modulator is based on a patent held by Dr. Vos, which in the following will be called the compound patent.[1] This patent, the purpose of which is the neutralization of the reaction of the anode voltage on the grid circuit of *one* thermionic vacuum tube, has been adapted to a modulator circuit with *two* tubes, on which arrangement a patent has been applied for.[2]

The circuit diagram for the modulator is shown in fig. 7a. According to this diagram both of the vacuum tubes 1 and 2 are connected with the anodes in parallel and the grids in push pull. The A. C. anode voltage is obtained on a coil in the anode circuit, this coil being included either in a resonance circuit or in a filter.

The relation between the anode current and the resulting A. C. voltage for both valves may

[1] Swed. patent No. 62633.

[2] Application No. 4231/1928.



R 1427 Fig. 7a. Circuit Diagram for Modulator.

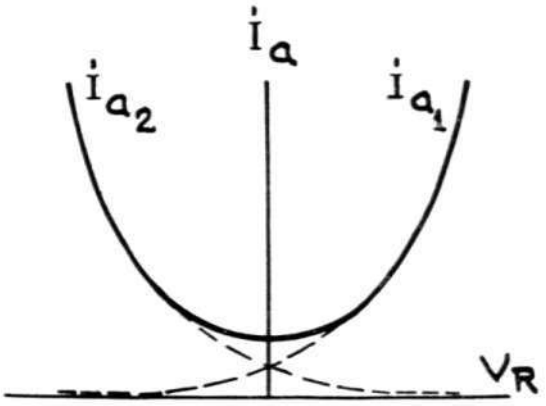


Fig. 7b. Characteristic for Modulator.

be mathematically expressed according to the following equations

$$1\ldots\ldots \begin{cases} i_{a1} = c_0 + c_1 v_{R1} + c_2 v^2{}_{R1} + \ldots \\ i_{a2} = c_0 + c_1 v_{R2} + c_2 v^2{}_{R2} + \ldots \end{cases}$$

These equations are valid on condition that the two valves are identical, i. e. that the coefficients c_0, c_1, c_2 etc. are the same for both valves and that the grid voltage has been chosen so that the resulting D. C. voltage $v_{R0} = v_{g0} + \frac{v_{a0}}{\mu}$, equals zero. The two expressions for i_{a1} and i_{a2} are graphically represented in fig. 7b.

The A. C. voltages v_{R1} and v_{R2} may be expressed as follows:

$$2\ldots\ldots \begin{cases} v_{R1} = \quad v_g + \frac{v_a}{\mu} + \delta\, v_a \\ v_{R2} = -\, v_g + \frac{v_a}{\mu} + \delta\, v_a \end{cases}$$

The first term in these expressions represents the applied A. C. grid voltage v_g. On account of the push pull connection of the grids this tension must receive a negative sign in the latter equation because the voltage on the grid of valve 2 is 180° out of phase with the voltage on the grid of valve 1. The middle term represents the reaction of the A. C. anode voltage, while the last term is a voltage δv_a which is led to both the grids with the same phase. This voltage is but a small fraction δ of the A. C. anode voltage and it is suitably obtained by means of a coil inductively connected to the anode coil. This firstmentioned coil, which we will call the compound coil, is connected so that the voltage δv_a will be 180° out of phase with the A. C. anode voltage v_a. Furthermore, δ is made equal to the inverse value of the amplification factor of the valve.

Thus we have

$$3\ldots\ldots \delta = -\frac{1}{\mu}$$

The two equations 2 may then be simplified to

$$4\ldots\ldots \begin{cases} v_{R1} = v_g \\ v_{R2} = -\, v_g \end{cases}$$

and by inserting equations 4 in the equation system 1 and adding the same we obtain

$$5\ldots\ldots i_a = i_{a1} + i_{a2} = 2c_0 + 2c_2\, v^2{}_g + \ldots$$

This is the basic equation for the special modulator used in Radioaktiebolaget's system. Ignoring terms of the 4th and higher orders, this basic equation is the equation for a parabola. This approximation is permitted for practically all cases occuring in actual practice.

If the modulator is to be used purely for modulating purposes, the grids must be fed with a voltage as follows

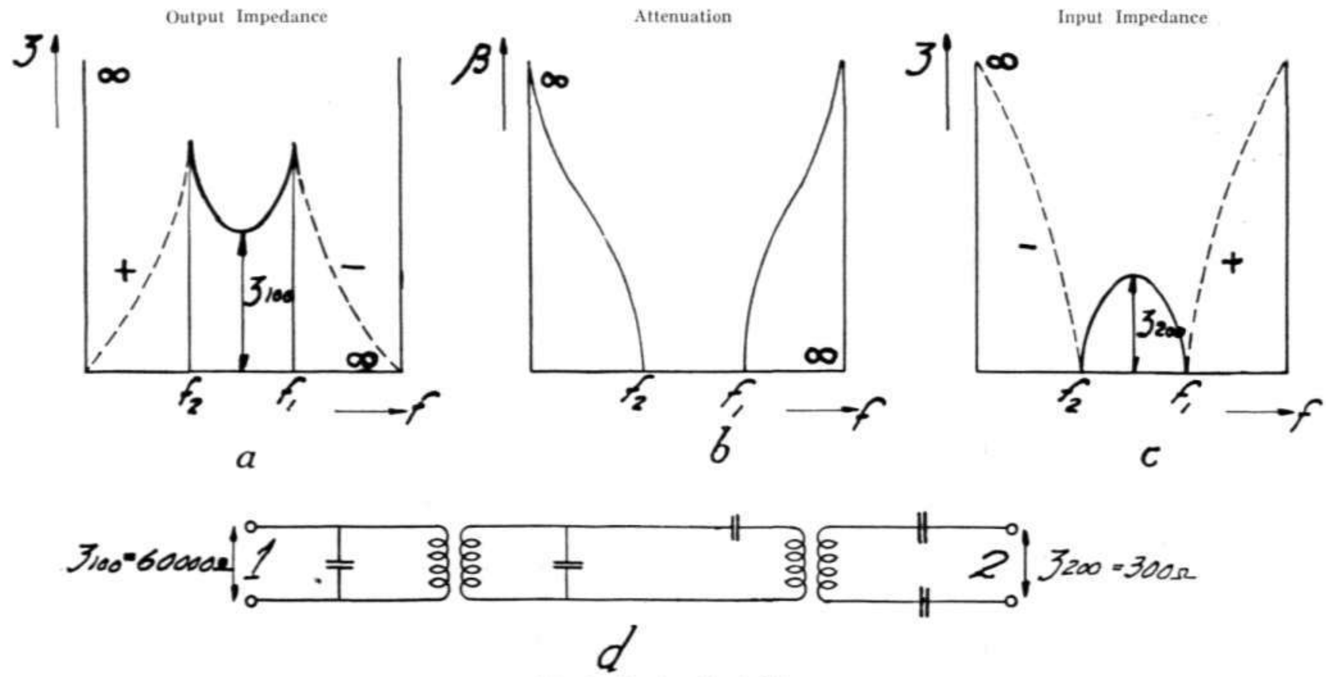
$$6\ldots\ldots v_g = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$$

which is composed of two sinusoidal voltages, the one representing the carrier frequency ω_1 and the other the modulating frequency ω_2.

If we introduce equation 6 in the basic equation 5, we obtain after reduction

$$7\ldots \begin{cases} i_a = 2c_0 + c_2[A_1^2 + A_2^2] - c_2[A_1^2 \cos 2\omega_1 t + \\ \quad + A_2^2 \cos 2\omega_2 t] + 2c_2 A_1 A_2 [\cos (\omega_1 - \omega_2) \\ \quad t - \cos (\omega_1 + \omega_2) t] \end{cases}$$

This equation proves that in the anode circuit of the modulator there arise a direct cur-



R 1428

Fig. 8. Receiver Band Filter.

rent as well as alternating currents with the frequencies $2\omega_1$, $2\omega_2$, $\omega_1 - \omega_2$ and $\omega_1 + \omega_2$. If we introduce a filter in the anode circuit and this filter is dimensioned so that its impedance practically equals zero for the frequencies $2\omega_1$, $2\omega_2$ and $\omega_1 - \omega_2$ but has a positive value for the frequency $\omega_1 + \omega_2$, a voltage with the latter frequency only is obtained in the anode circuit of the modulator. Thus we have obtained the upper side frequency of a modulated carrier frequency.

The above derivation of the modulator theory is valid on condition that it is desirable to suppress the carrier frequency ω_1. If such is not the case, one may — by displacing the working point to the side of the vertex of the parabola according to fig. 7b — also obtain in the anode circuit a voltage with the frequency ω_1, which is just the carrier frequency.

The advantages which the modulator here described possesses as compared with other modulators may be summed up as follows.

1. The characteristic of the modulator is a square law function of the applied A. C. grid voltage, and for this reason there appear during modulation only sums and differences of the input frequencies (the frequency $2\omega_1$ may be considered as the sum of the frequency ω_1 with itself and direct current terms arise in the same manner as the difference between two equal frequencies ω_1).

2. The anode current in the modulator is constant and independant of all reactions from the anode circuit. The A. C. anode voltage which arises in the anode circuit, therefore, is directly proportional to the impedance of the anode circuit.

In other words, the modulator gives a constant anode current independently of the load. The justification of the name 'compound modulator' is apparent if one compares the described modulating arrangement with a compound generator for direct current which supplies constant voltage independently of the current drain.

3. Thanks to its square law curve and the compound principle, the useful power provided by the compound modulator is considerably greater than for an ordinary modulator.

4. Modulation is possible no matter how low the frequency.

5. The mathematical treatment for the dimensioning of the modulator is relatively simple, for according to equation 5 the expression for the anode current is an explicit function of the A. C. grid voltage. This is not the case when figuring with vacuum tubes in general, the resulting A. C. voltage being dependent upon the anode current, for which an implicit expression is therefore usualy obtained.

Band filters.

It may be of interest to go into a more detailed analysis of those points of view on which are based the choice and calculation of band filters for the carrier current system in question. As a type suitable for discussion we will take the receiver band filter.

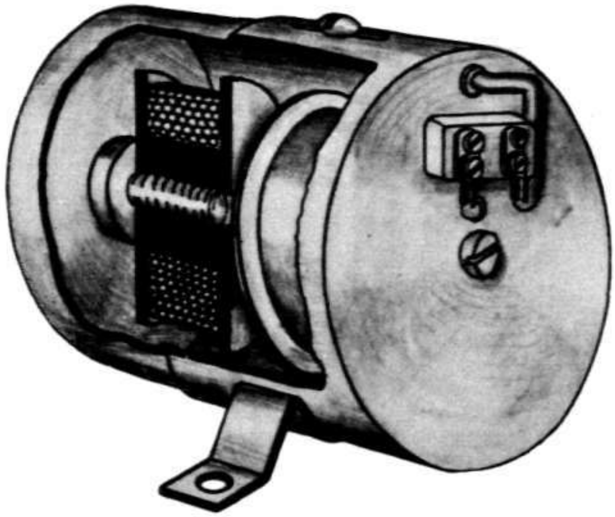
As already mentioned (see fig. 4a), all of the receiver band filters are connected in parallel. In order to prevent the shortcircuiting of one of the frequencies, it is important that the different band filters, outside of their bands, have impedances which are very high as compared with the impedance of the filter through which the frequency shall pass with the least possible attenuation. Consequently, the input impedance of the band filter should have the appearance as shown in fig. 8c.

In this figure, f_1 represents the upper and f_2 the lower cut off frequency. The curves in fig. 8 are plotted according to the symbolic method, first presented by Johnson and Shea.[1] Z_{200} is the impedance of the filter for that frequency at which the filter shall be adjusted to the line impedance. The unbroken curves in fig. 8 a and c indicate that the filter impedance is real, while the dashed lines indicate an imaginary impedance. The plus sign denotes that the impedance is inductive and increases with the frequency, while the minus sign denotes a capacitive impedance.

After having chosen a filter section with the W-impedance as shown in fig. 8c, a number of filter sections are conected in cascade after the first one until sufficient attenuation outside of the band has been obtained. The theoretical attenuation curve for the receiver band filter and with non dissipative condensers and inductance coils is shown in fig. 8b. The new filter sections which are introduced must be of such types as to permit of their being connected in cascade without any reflection losses at the connecting points.

On the output side an M-impedance as shown in fig. 8a is desirable since here it is advantageous if the impedance of the filter outside of the band approaches zero, all the frequencies outside of the band being shortcircuited and only those frequencies lying within the desired band being admitted to the apparatus which follows after the filter. On the output side, this latter is matched to a load Z_{100} which is considerably larger than Z_{200}. In this manner a voltage transformation $= \sqrt{\frac{Z_{100}}{Z_{200}}}$ is obtained in the receiver band filter. Such a transformation in a band filter is possible only when special filter sections, designed by Radioaktiebolaget, are used.

Fig. 8d shows the principle for the connections of the receiver band filter. This filter is composed of several cascade connected sections, chosen according to what has already been stated. By combining capacities and inductances in the different sections the required number of inductance coils and condensers is materially reduced and the filter made cheaper than if the different sections were built separately and connected up afterwards.



R 1429 Fig. 9. Copper Box Coil.

In order to make the attenuation in a filter as low as possible it is important that the condensers as well as the inductance coils have very small losses. The condensers used in Radicaktiebolaget's filters, therefore, are designed as mica condensers, the very best quality of mica being used as dielectric. The inductance coils are made as shown in fig. 9, illustrating a so-called copper box coil with two spools. The two spools of ebonite are wound with litz-wire and affixed one to each end of the copper box. The one half of the box telescopes into the other half thereby making it possible to vary the distance between the coils. The dimensioning of the spools and copper box has required research

[1] K. S. Johnson and T. F. Shea: Mutual Inductance in Wave Filters with an Introduction on Filter Design. Bell System Technical Journal, Vol. IV, No. 1, Jan. 1925.

work of a very special nature, although no detailed account of the same will be given here. The mean diameter and the width of the spools as compared with the main dimensions of the box, the type of wire used and the material of which the box is made all exercise considerable influence on the losses in this inductance coil.

Voltage regulating device.

The incoming frequencies are led from the receiver band filter to a high frequency amplifier, acting as a pure voltage amplifier. It consists of two cascade-connected valves, connected with each other by means of filters with transformation for the purpose of increasing the voltage before it is admitted to the grid of the next valve. Since the amplification in the high frequency amplifier of the receiver must be rather high, the connection is made with the neutralizing of the anode grid capacity, thus preventing self-oscillation. The high frequency amplifier is provided with a device for regulating the degree of amplification and of a special type and will therefore be described in detail in the following.

Potentiometer. T-Network.

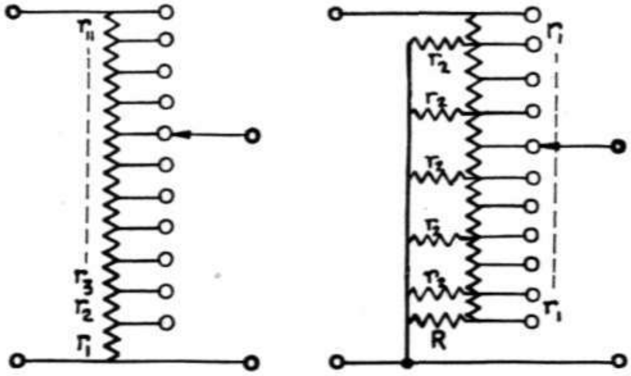


Fig. 10a. Fig. 10b.

R 1430 Fig. 10. Voltage Regulating Device.

In telephony, it is generally considered undesirable for amplifiers to have a linear regulation of the voltage, but the regulating should be exponential with a certain attenuation per step (expressed in nepers or fractions of a neper).

Figure 10a shows a potentiometer in general use for the voltage regulation. The different resistances r_1 to r_{11} are so dimensioned as to give a voltage between the output terminals which varies in an exponential manner when the sliding contact is moved step by step from one contact to another on the potentiometer. If we have voltage transformation in the filter, the output impedance of the filter will be high and consequently also the total resistance of the potentiometer will be high.

For high frequency purposes one must place rather strict requirements on the different resistances which form a part of such a potentiometer. The resistances must be purely ohmic and the capacities between the different contacts on the potentiometer must not exceed some few micro-microfarads. The resistances r_1 to r_{11} usually vary between some few thousand and some few ten thousand ohms. It has been found very difficult to manufacture such resistances

R 1431

Fig. 11. Potentiometer.

in the usual manner with wire wound on a spool. The grid leaks used in wireless are better suited for this purpose, but here we strike another drawback for it is practically impossible to manufacture such resistances at a reasonable price and with predetermined resistance values.

In order to avoid the above disadvantages when designing a voltage regulation device, this device is made in the form of a T-network as shown in fig. 10b. This figure shows five interconnected T-sections each one of which gives a certain attenuation. Each section consists of two resistances r_1 in series and one shunt resistance r_2. At the output side of the fifth section a loading resistance R is connected, corresponding to the impedance for which the T-network is designed. The voltage is regulated by tapping off at contact points at the middle as well as at the end of each section. The advantage with such an arrangement for voltage regulation is that one only needs resistances with three different values, while with a device as shown in fig. 10a eleven different resistance values are required.

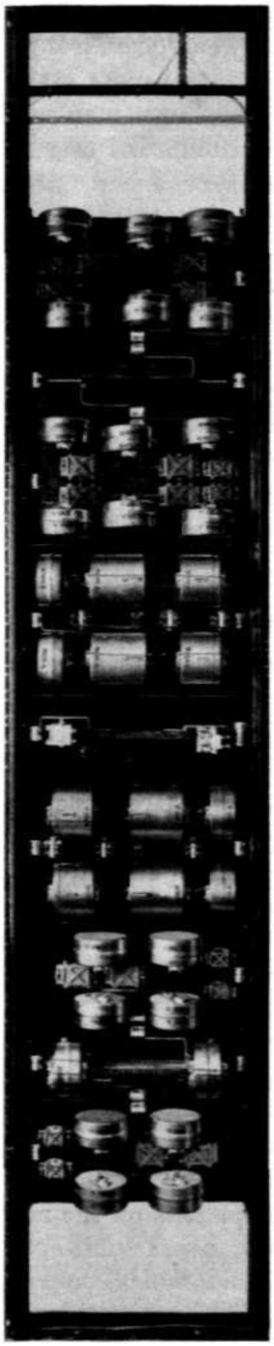
A device of this kind for regulating voltages is shown in fig. 11, the one illustrated being designed as an H-network, i. e. a symmetrical T-network. The different resistances are of the same type as those used for grid leaks in wireless. These resistances are held by spring clips constructed so as to automatically give the correct connection according to fig. 10b. Connecting wires lead from the spring clips to the respective brush contacts, 2×11 in number, over which the contact brushes move.

The advantage with the device for regulating voltages as here described is that the capacity between the different taps is small and that by using standard resistances easily obtainable in the regular market one may still obtain a simple device for the exponential regulation of the voltage.

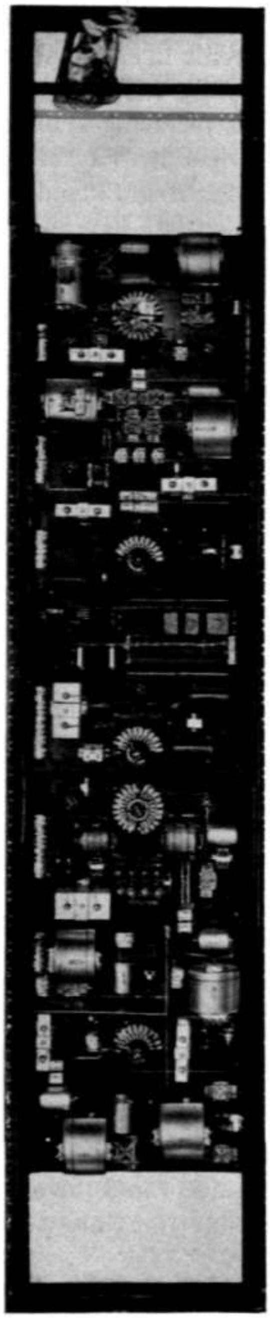
Demodulator.

The incoming voltage is carried direct from the high frequency amplifier to a voltage regulator of the above-described type placed before the grids of the demodulator. The demodulator is wired according to the same principle as the modulator. If the carrier frequency and the upper side band of a modulated wave is fed into the grid circuit of the demodulator, a voice frequency current with the same frequency as the voice frequency which has modulated the carrier frequency in the modulator of the transmitter is formed in the anode circuit of the demodulator. This voice frequency passes from the anode circuit of the demodulator through a voice frequency band filter with the cut off frequencies of 150 and 3000 cycles per sec. and to the low frequency hybrid coil, and from here out over the line to the switchboard or to the subscriber.

If, in the modulator of the transmitter, the carrier frequency is modulated with signalling frequency instead of with voice frequency, an anode current with the same frequency as the modulating signalling voltage is obtained in the demodulator of the receiver. Still another filter, which passes frequencies between 12 and 30 cycles per sec. is inserted in the anode circuit of the demodulator. This filter is designed for voltage transformation, whereby the voltage of the signalling frequency is increased before being fed to the signal receiver.



R 1432 Fig. 12. Line Filter Bay (rear view).



R 1433 Fig. 13. Terminal Equipment Bay (rear view).

The signal receiver.

The signal receiver is designed as a regular vacuum tube rectifier with anode rectification. A polarized relay is included in the anode circuit of the signal receiver and this is influenced by the anode D. C. resulting from the rectification. When the polarized relay energizes, a D. C. circuit is closed which, in turn, transmits a signalling frequency from the ringing machine of the receiving station to the switchboard or subscriber by way of the supervisory device.

Figures 12 and 13 show a rear view of the line filter bay and terminal equipment bay for one channel. The covers which usually protect the equipment from injury and dust are removed so as to give a clear view of the various apparatus. The different types of copper boxes included are clearly discernible, as are the voltage regulating devices of the previously described type which are provided for the adjustment of the voltage in the various equipment units.

SYNCRONISATION.

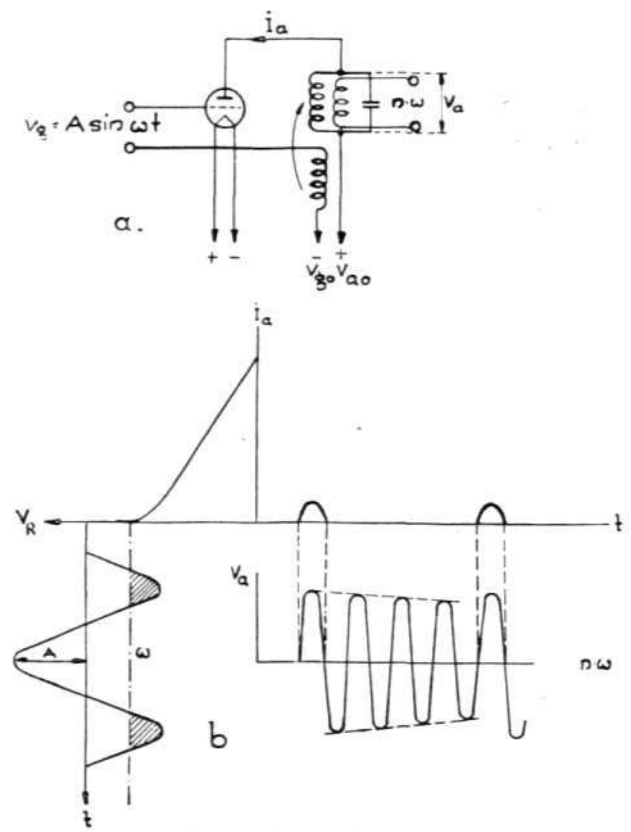
As has already been mentioned, the different channels in the carrier current telephone system obtain their carrier frequencies from a synchronizing device fed from the master generator for 5000 cycles per sec. This master frequency is transmitted to the different terminal stations where it passes through a band pass filter to so-called master frequency amplifiers which amplify the voltage of the master frequency up to the value which is required for feeding the sc-called multiple generators.

These multiple generators are of two different types, viz. *multiple generator I or impulse generator* and *multiple generator II.* The theory for these two types of multiple generators has been worked out by Doctor Vos (of Radioaktiebolaget) and the author, and patents on the new connections are pending.[1]

Multiple generator I or Impulse generator.

The multiple generator I works according to a principle based also in this case on the compound patent. Since lack of space does not permit a closer study of the complicated mathematical treatment of the theory for the impulse generator, this will have to be substituted by a more popular description which will nevertheless give the reader a clear conception of the functioning of the generator.

The wiring diagram for multiple generator I is shown in fig. 14a. The generator consists of



R 1434 Fig. 14. Impulse Generator.

a vacuum tube, in the anode circuit of which is connected a parallel resonant circuit which is tuned to the n^th harmonic of that frequency which is introduced to the grid of the tube. A compound coil, which is connected in the same manner as in the previously described modulator, also supplies the grid with a voltage which is but a fraction of the A. C. anode voltage. We have already shown that, when the compounding is correct, this voltage shall be 180° out of phase with the A. C. anode voltage. Thus, the compounding acts in the same phase as the feed back coil in a common valve generator but the compound voltage is lower than that voltage which is required in order to force the generator to self-oscillate. If a grid voltage is chosen so large that an anode current flows in the anode circuit of the generator only during certain short moments, however, oscillations will arise in the tuned anode circuit (see fig. 14b). The length of time during which these anode current pulsations last may be varied if a suitable grid voltage and a suitable amplitude *A* for the oscillations of the input current are chosen. If the current pulsations in the anode circuit occur

[1] Application No. 4324/1928.

in phase with the voltage of the oscillating circuit, this latter obtains new power all along and continuous oscillations with the frequency $n \cdot w$ are sustained.

Fig. 14b shows the curves for current and voltage for the generation of the 4th harmonic of the input frequency. If the resonance circuit is slightly damped, the oscillations in the circuit will not have time to entirely die out before they are given a new impulse from a current pulsation in the anode circuit. Thus one obtains practically continuous oscillations in the anode circuit with a frequency corresponding to a certain harmonic of the input frequency. A suitable choice of grid voltage, of amplitude for the incoming oscillations and of the impedance of the resonant circuit will give quite a large power for the harmonic in the anode circuit of the impulse generator. It is true that this power diminishes with the ordinal of the harmonic, but thanks of the compounding the power is many times greater than when common connections are used for the generating of harmonics.

In the carrier current system in Mexico, impulse generators are used for generating frequencies of 10,000, 20,000 and 40,000 cycles per sec. through the doubling, quadrupling and octupling of the master frequency of 5000 cycles. The frequency of 40,000 cycles is not generated direct as the 8th harmonic of the master frequency, but two impulse generators are here made use of in cascade. A frequency of 20,000 cycles is generated in the first tube, and this frequency is then fed into the second tube where it is doubled to a frequency of 40,000 cycles per sec.

Multiple generator II.

This generator is used principally when the simultaneous generation of several frequencies which all are multiples of a certain master frequency is required. The different frequencies appear each with the same power, which is not the case with hitherto known devices for the same purpose.

For its carrier current telephone systems, Western Electric at one time also used a multiple generator in which, however, the various overtones were obtained with effects which diminished rapidly with the ordinal of the harmonic. For this reason a great number of amplifiers were required, especially for the higher harmonics, in order to obtain sufficient power. In these generators, as well as in others of similar design — as used in wireless for the governing of transmitters, for instance — harmonics arise on account of the curvature of the characteristic curve, for from a mathematical point of view the characteristic curve for a vacuum tube may be represented by a power series. The coefficients for terms of a higher order gradually become smaller and smaller, however. Since the amplitude of a certain harmonic is directly proportional to the value of the corresponding coefficient, the higher harmonics become very weak.

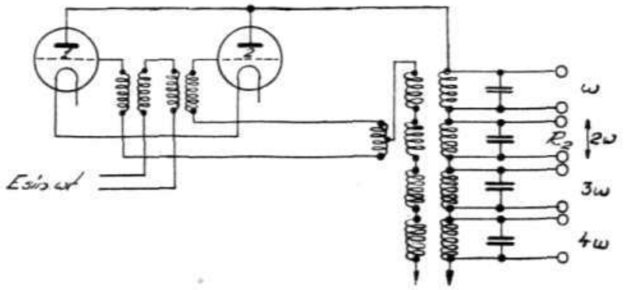
The multiple generator II is connected in about the same manner as the previously described modulator (see fig. 7a). The following schedule indicates the frequencies which arise in the anode circuit of the multiple generator II, if three frequencies are fed to its grid (see page 9).

Input frequency cycles per second	Frequency of anode current, cycles per second		
5,000	0	15,000	
20,000	10,000	25,000	20,000
40,000	40,000	35,000	60,000
	80,000	45,000	

Of these frequencies which arise, it is only those that are odd multiples of the master frequency of 5000 cycles per sec. which are of interest. These four frequencies — 15,000, 25,000, 35,000 and 45,000 cycles — may be obtained from the anode circuit of the modulator through four different filters connected in series and which permit the passage of one frequency each. The frequencies are just those which are used as carrier frequencies for the different transmitters in Mexico City, Guadalajara and San Luis Potosi. As previously mentioned, all the frequencies appear with the same power in the anode circuit, and since this power is sufficient for feeding the different modulators no carrier frequency amplifiers need be inserted between multiple generator II in the synchronization unit and the different modulators.

In Celaya and Vera Cruz even multiples of the master frequency must be used as carrier frequencies, and these might be generated by feeding frequencies of 10,000 and 20,000 cycles, for instance, to a modulator, after which oscillations with frequencies of 10,000, 20,000 and 30,000 cycles per sec. would be obtained in the anode circuit.

There is a much simpler manner of proceeding, however (see fig. 15). If, according to this diagram, we introduce an oscillation with a frequency ω corresponding to a master frequency of 5000 cycles per sec., an oscillation with the frequency 2ω will arise in the anode circuit. With the aid of a coupling coil one may now pick out



R 1435 Fig. 15. Multiple Generator II.

a voltage with this frequency and introduce this voltage to the two grids of the multiple generator in push pull. This arrangement will then function as if two voltages with the frequencies ω and 2ω had been introduced direct to the two grids. In multiple generator II according to fig. 15, therefore, it has been possible, merely by introducing the frequency ω to the grid, to obtain all of the four desired frequencies in the anode circuit, i. e. ω, 2ω, 3ω and 4ω or 10,000, 20,000, 30,000 and 40,000 cycles per sec.

The synchronization unit in Celaya may be considered a typical synchronization unit for the Mexico plant, photographic reproductions of this unit being shown in figures 16 and 17. At the top of the unit we notice a multiple generator II and beneath this the impulse generator for the generation of 10,000 cycles per sec. The impulse generator obtains the master frequency of 5000 cycles per sec. from the master frequency amplifier I, which latter is placed just below the impulse generator. As a rule, the master frequency amplifier I is fed with master frequency from the master generator in Mexico City via the line between the above-mentioned

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Fig. 16. Synchronization Unit at Celaya (front view).

cities. Should this line be taken out of service, however, there is a spare master generator — the panel in fig. 16 with visible ammeter to the left. Below this master generator may be seen two similar panels which serve as master frequency amplifiers II for outgoing master frequency to the both towns Guadalajara and San Luis Potosi.

DISTRIBUTION OF CURRENT.

A description of the apparatus which serves to distribute the current for the filament, plate

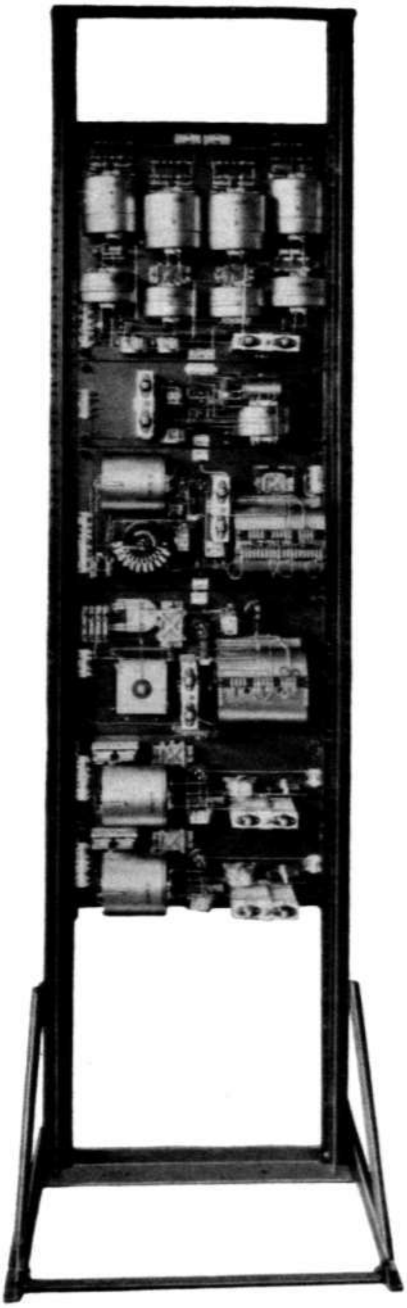

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Fig. 17. Synchronization Unit (rear view).

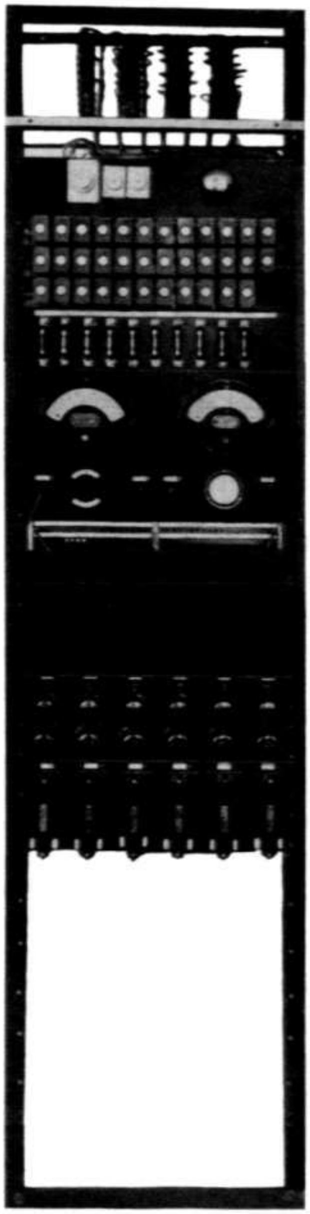

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Fig. 18. Current Distribution Bay (front view).

and grid circuits of the vacuum tubes does not really fall within the scope of this article. The reader is referred to fig. 18 for an idea as to the appearance of the distribution panel. A few words as to the sources of power used for the operation of the plant, however, are not out of place.

The filament current has a tension of 6 volts, the alarm relays etc. working with a tension of 12 volts. These tensions are obtained from two lead storage batteries of three cells each, connected in series. The anode tension is 240 volts, supplied by a lead storage battery with 120 cells.

The filament current for two vacuum tubes passes through the respective windings of a differential relay. These two windings counteract each other so that the relay is not actuated if the two tubes draw equal amounts of current. Should a fault occur in the one or the other of the tubes, the balance of the differential relay is disturbed, the relay attracts its armature and

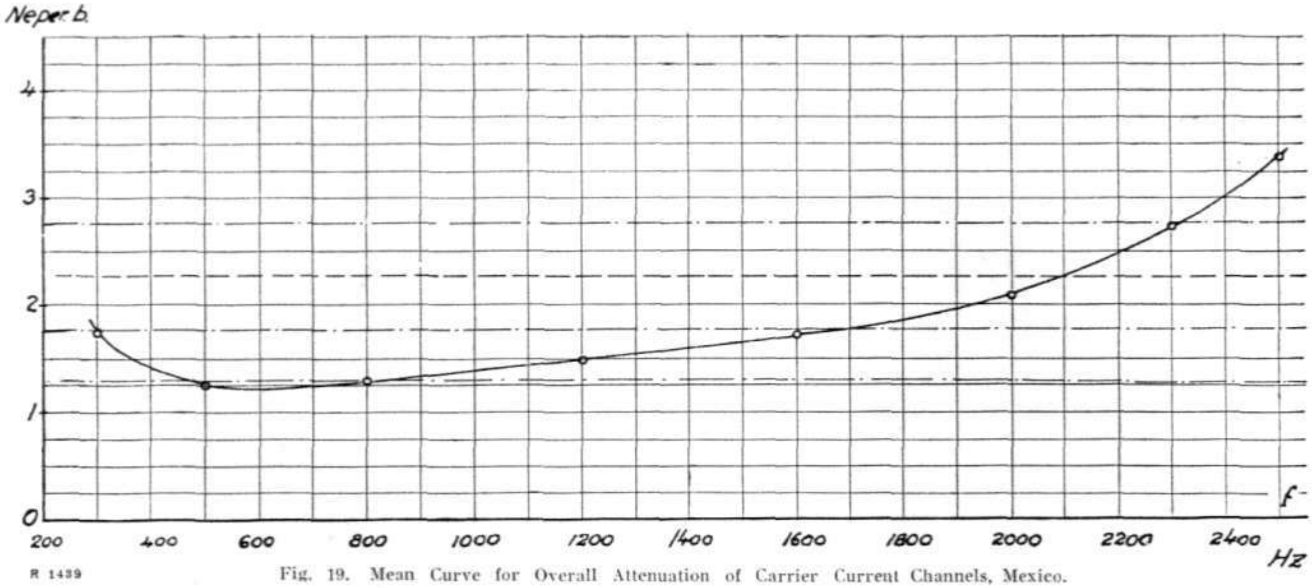

Fig. 19. Mean Curve for Overall Attenuation of Carrier Current Channels, Mexico.

closes an alarm contact, thereby notifying the station watchman that a tube, for instance, has burned out.

All grid tensions are obtained from a dry battery through high ohmic resistances which protect the battery from direct short circuiting. A galvanometer relay, which closes an alarm circuit as soon as a grid current flows through any of the tubes, is inserted in the branch which connects the positive pole of the grid battery with the negative pole of the filament battery.

OVERALL ATTENUATION.

The transmission of speech in a carrier current channel is fully comparable to the transmission over a loaded cable provided with two-wire repeaters.

Fig. 19 shows the mean value of the overall attenuation curves for nine different carrier current channels forming a part of the Mexican system. When obtaining this curve the attenuation for 800 cycles per sec. was set to the value 1.3 nepers, after which the attenuation for the other frequencies was measured in the usual manner. According to the requirements formulated by the 'Comité Consultatif International des Communications Téléphoniques à Grande Distance' (C. C. I.) for international two-wire lines, the attenuation at 300 cycles per sec. must not exceed .5 nepers more than at 800 cycles per sec. At 2000 cycles per sec. the corresponding increase of the attenuation must be less than 1.5 nepers. The horizontal lines in fig. 19 are spaced at a distance of .5 nepers from each other with the starting point at the attenuation for 800 cycles per sec. From the above graph we see that carrier current transmission adequately fills the requirements formulated by C. C. I.

(Cont'd in next issue.)

The Influence of Condensers on the Functioning of Relays with Respect to the Periodic Case.

By I. Frischauf, Vienna.

In the following investigation we will take two typical examples in order to demonstrate the changes which take place in the time current curve of a relay for different values of a condenser which is connected in parallel with the winding of the relay or in parallel with a resistance connected in series with the same winding. The investigation will include the establishing of the frequency conditions as well as the calculating of the maximum and minimum values of the current in the relay winding when the impedance has reached its minimum value and their comparison with the current intensities which characterize the sensitiveness of the relay, i. e. the respective minimum and maximum intensities of current at which the relay energizes and de-energizes. Further, an investigation will be made of tensions which arise at the exact moment of disconnection of the battery. The various possibilities will be illustrated by means of a few examples.

It is a known fact that oscillations arise when a current with inductance, capacity and resistance is connected to or disconnected from a source of energy in the form of direct current.

For the kind of relay which is used for low tension work, Breisig[1], among others, has set up equations for the relay current as a function of time. Also, he has pointed out that, under certain conditions, the intensity of the relay current can be an *e*-function superimposed by a damped oscillation. Chechelovsky[2] makes a more thorough investigation of various combinations of connections and groups together in tables the empirical values — obtained as the result of extensive experimental work — for the influence of condensers on the degree of retardation (lag) in the functioning of relays.

Consequently, we will now — according to the above-mentioned program — investigate two separate cases, viz. with a condenser connected in parallel with the relay winding and with a condenser connected in parallel with a resistance which, in turn, is connected in series with the said relay winding. For the sake of completeness, the time current equation for the first case will be derived, which derivation will indicate how one should proceed in similar cases.

I. *The condenser in parallel with the relay winding.*

The calculation in based on a connection as shown in fig. 1. R_r and L designate the resist-

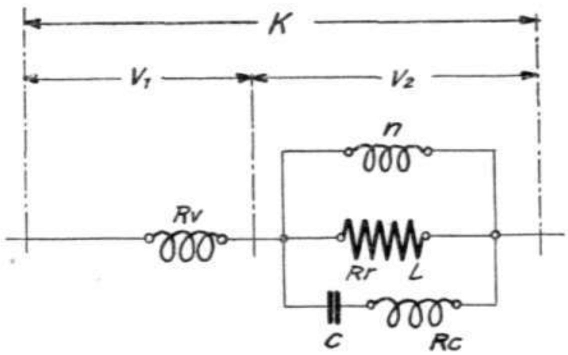

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Fig. 1.

ance of the relay and the mean value of its coefficient of induction respectively, C designates the capacity of the condenser, R_c the resistance of the condenser together with that of a resistance connected in series with the same, R_v is a series resistance and n designates a resistance connected in parallel with the winding of the relay coil. In the present case all the enumera-

[1] Breisig, F., *Theoretische Telegraphie*, Brunswick 1924.

[2] Chechelovsky, *Etudes sur le temps de fonctionnement et de relâchement des relais téléphoniques*, Bell Téléphones, Antwerp.

ted values are assumed to be uninfluenced by time, which also is the case with currents of not too great intensity. The resistances R_v and n are to be free from induction as well as capacity. The unvarying tension of the source of direct current is designated with K, while the variable drops in voltage in the relay winding and in the series resistance are designated with V_2 and V_1 respectively. The currents passing through the resistance n, the condenser, the relay and the series resistance are designated with i_n, i_c, i_r and i_v respectively.

Under these conditions we will begin by establishing the equations for the intensity of the current as a function of time when the relay operates and releases its armature.

Time current curves for operating.

For an arbitrary moment t the following equations apply:

$$K = V_1 + V_2 \qquad i_v = i_r + i_c + i_n \quad \ldots\ldots (1)$$

$$V_1 = i_v R_v = i_r R_v + i_c R_v + i_n R_v \quad \ldots\ldots (2)$$

$$V_2 = i_r R_r + L\frac{di_r}{dt} = i_c R_c + \frac{1}{C}\int i_c\, dt = i_n n \ldots\ldots (3)$$

If we express i_n in terms of i_r according to equation (3) and insert this expression in equation (2) we obtain

$$K = i_r\left(R_r + R_v + \frac{R_r R_v}{n}\right) + L\left(1 + \frac{R_v}{n}\right)\frac{di_r}{dt} + i_c R_v \quad (4)$$

Further

$$i_r R_r + L\frac{di_r}{dt} - i_c R_c - \frac{1}{C}\int i_c\, dt = 0 \quad \ldots\ldots (5)$$

These two simultaneous differential equations (4) and (5) can also be written in another form, if we consider the fact that $\int i_c dt$ is the charge which has passed through the condenser up to the moment t. Therefore if we make $\int i_c dt = i_k$, we find that

$$\left(R_r + R_v + \frac{R_r R_v}{n}\right)i_r + L\left(1 + \frac{R_v}{n}\right)\frac{di_r}{dt} + R_v\frac{di_k}{dt} = K \quad (6)$$

$$R_r i_r + L\frac{di_r}{dt} - R_c\frac{di_k}{dt} - \frac{1}{C}i_k = 0 \quad \ldots\ldots (7)$$

New differential equations will be formed from (6) and (7) by multiplying (7) by $\left(1 + \frac{R_v}{n}\right)$ and subtracting the result from (6), also by multiplying (6) by R_c and (7) by R_v, after which the equations thus obtained are added together. After further reduction we obtain

$$\frac{di_k}{dt} + \frac{1 + \frac{R_v}{n}}{C\left(R_v + R_c + \frac{R_v R_c}{n}\right)}\cdot i_k + \frac{R_v}{R_v + R_c + \frac{R_v R_c}{n}}\cdot i_r = \frac{K}{R_v + R_c + \frac{R_v R_c}{n}} \quad \ldots\ldots (8)$$

$$\frac{di_r}{dt} + \frac{R_r R_v + R_c\left(R_r + R_v + \frac{R_r R_v}{n}\right)}{L\left(R_v + R_c + \frac{R_v R_c}{n}\right)}\cdot i_r - \frac{R_v}{CL\left(R_v + R_c + \frac{R_v R_c}{n}\right)}\cdot i_k = \frac{K \cdot R_c}{L\left(R_v + R_c + \frac{R_v R_c}{n}\right)} \quad (9)$$

For the sake of simplicity we will designate the coefficients for i_k and i_r in (8) with A_1 and B_1 respectively, the expression to the right of the sign of equality in (8) with D_1, further the coefficients for i_k and i_r in (9) with A_2 and B_2 respectively, and the expression $\frac{K R_c}{L\left(R_v + R_c + \frac{R_v R_c}{n}\right)}$ with D_2.

Consequently, we can write

$$\frac{di_k}{dt} + A_1 i_k + B_1 i_r = D_1,$$

$$\frac{di_r}{dt} - A_2 i_k + B_2 i_r = D_2.$$

The second of these equations must now be multiplied by a constant factor Φ and added to the the first one. This gives

$$\frac{di_k}{dt} + \Phi\frac{di_r}{dt} + (A_1 - A_2\Phi)\ i_k + (B_1 + B_2\Phi)\, i_r = D_1 + \Phi D_2 \ldots\ldots (10)$$

If we make $i_k + \Phi i_r = I$, in which I is a new variable, we obtain

$$i_k = I - \Phi i_r \quad \ldots\ldots (11)$$

and

$$\frac{di_k}{dt} + \Phi\frac{di_r}{dt} = \frac{dI}{dt} - \Phi\frac{di_r}{dt} - i_r\frac{d\Phi}{dt} + \Phi\frac{di_r}{dt}. \ldots (12)$$

Since Φ was assumed to be constant, the right half of this equation is reduced to $\frac{dI}{dt}$, after which equation (10) obtains the following appearance

$$\frac{dI}{dt} + (A_1 - A_2\Phi).\ I - i_r\ [\Phi\,(A_1 - \Phi\,A_2) - (B_1 + B_2\Phi)] = D_1 + \Phi D_2 \quad \text{(10 a)}$$

If the factor Φ is so determined that

$$\Phi(A_1 - \Phi A_2) - (B_1 + \Phi B_2) = 0 \quad (12)$$

the following equation remains

$$\frac{dI}{dt} + (A_1 - A_2\Phi)\cdot I = D_1 + \Phi D_2 \quad (13)$$

which is readily integrated.

From (12) we obtain the value for Φ

$$\Phi_{12} = -\frac{B_2 - A_1}{2A_2} \pm \sqrt{\frac{(B_2 - A_1)^2}{4A_2^2} - \frac{B_1}{A_2}} \quad (14)$$

the integral of equation (13) being

$$I_{12} = \frac{D_1 + \Phi_{12}\,D_2}{A_1 - \Phi_{12}\,A_2} - c_{12}\,e^{-(A_1 - A_2\Phi_{12})\cdot t} \quad (15)$$

Values for i_r, i_k and consequently also for i_c, i_n and i_o are obtained from equations (11), (14) and (15)

$$i_k + \Phi_1\,i_r = \frac{D_1 + \Phi_1 D_2}{A_1 - \Phi_1\,A_2} - c_1\,e^{-(A_1 - \Phi_1 A_2)\cdot t}$$

$$i_k + \Phi_2\,i_r = \frac{D_1 + \Phi_2 D_2}{A_1 - \Phi_2\,A_2} - c_2\,e^{-(A_1 - \Phi_2 A_2)\cdot t}$$

$$i_r = \frac{D_1\,A_2 + D_2\,A_1}{B_1\,A_2 + B_2\,A_1} - \frac{c_1}{\Phi_1 - \Phi_2}\,e^{-(A_1 - \Phi_1 A_2)\cdot t} + \frac{c_2}{\Phi_1 - \Phi_2}\,e^{-(A_1 - \Phi_2 A_2)\cdot t}$$

$$i_k = \frac{D_1\,B_2 - D_2\,B_1}{B_1\,A_2 + B_2\,A_1} + \frac{c_1\Phi_2}{\Phi_1 - \Phi_2}\,e^{-(A_1 - \Phi_1 A_2)\cdot t} - \frac{c_2\,\Phi_1}{\Phi_1 - \Phi_2}\,e^{-(A_1 - \Phi_2 A_2)\cdot t}$$

According to our assumptions, however, i_k is the charge in the condenser at the moment t. When $t = 0$ then $i_r = i_k = 0$. Consequently, the two integration constants have the following values

$$c_1 = \frac{\Phi_1\,(D_1\,A_2 + D_2\,A_1) + (D_1\,B_2 - D_2 B_1)}{B_1\,A_2 + B_2\,A_1}$$

and

$$c_2 = \frac{\Phi_2\,(D_1\,A_2 + D_2\,A_1) + (D_1\,B_2 - D_2\,B_1)}{B_1\,A_2 + B_2\,A_1}.$$

If we substitute the original expressions for A_1, A_2, B_1, B_2, D_1 and D_2, we obtain

$$\frac{D_1\,A_2 + D_2\,A_1}{B_1\,A_2 + B_2\,A_1} = \frac{K}{R_r + R_o + \frac{R_r R_o}{n}},$$

$$c_1 = \frac{K}{R_r + R_o + \frac{R_r R_o}{n}}\,(\Phi_1 + R_r C),$$

$$c_2 = \frac{K}{R_r + R_o + \frac{R_r R_o}{n}}\,(\Phi_2 + R_r C),$$

and consequently

$$i_r = \frac{K}{R_r + R_o + \frac{R_r R_o}{n}}\left\{1 - \left[\frac{\Phi_1 + R_r C}{\Phi_1 - \Phi_2}\,e^{-(A_1 - A_2\Phi_1)\cdot t} - \frac{\Phi_2 + R_r C}{\Phi_1 - \Phi_2}\,e^{-(A_1 - A_2\Phi_2)\cdot t}\right]\right\} \quad (16)$$

Further, we find that

$$i_k = \frac{K}{R_r + R_o + \frac{R_r R_o}{n}}\left\{R_r C + \left[\frac{\Phi_2(\Phi_1 + R_r C)}{\Phi_1 - \Phi_2}\,e^{-(A_1 - \Phi_1 A_2)\cdot t} - \frac{\Phi_1(\Phi_2 + R_r C)}{\Phi_1 - \Phi_2}\,e^{-(A_1 - \Phi_2 A_2)\cdot t}\right]\right\}$$

therefore

$$i_c = \frac{K}{R_r + R_o + \frac{R_r R_o}{n}}\left\{\frac{\Phi_1(\Phi_2 + R_r C)(A_1 - \Phi_2 A_2)}{\Phi_1 - \Phi_2}\,e^{-(A_1 - \Phi_2 A_2)t} - \frac{\Phi_2(\Phi_1 + R_r C)\,(A_1 - \Phi_1 A_2)}{\Phi_1 - \Phi_2}\,e^{-(A_1 - \Phi_1 A_2)\cdot t}\right\}$$

Lastly, from relation $i_n = i_r \frac{R_r}{n} + \frac{L\,di_r}{n\,dt}$

we obtain the following value for i_n

$$i_n = \frac{K\frac{R_r}{n}}{R_r + R_o + \frac{R_r R_o}{n}} \times$$

$$\times \left\{1 - \left[\frac{\left[1 - \frac{L}{R_r}(A_1 - \Phi_1 A_2)\right](\Phi_1 + R_r C)}{\Phi_1 - \Phi_2}\,e^{-(A_1 - \Phi_1 A_2)\cdot t} - \frac{\left[1 - \frac{L}{R_r}(A_1 - \Phi_2 A_2)\right](\Phi_2 + R_r C)}{\Phi_1 - \Phi_2}\,e^{-(A_1 - \Phi_2 A_2)\cdot t}\right]\right\}$$

Of these equations, it is principally the one for i_r which interest us. The above method of presentation is not very clear for the following investigations. By giving equation (16) another form and by disregarding the intermediate calculations, i_r can in part be expressed in hyperbolic functions.

$$i_r = \frac{K}{R_r + R_e + \frac{R_r R_e}{n}} \left\{ 1 - e^{-\frac{A_1 + B_2}{2} \cdot t} \times \right.$$

$$\left. \left[\frac{A_1 - B_2 + 2A_2 R_r C}{w} \cdot \sinh \frac{w}{2} \cdot t + \cosh \frac{w}{2} \cdot t \right] \right\} \quad (16a)$$

in which w means

$$w = \sqrt{(B_2 - A_1)^2 - 4 A_2 B_1}.$$

At the time $t = 0$, $\sinh \frac{w}{2} t = 0$, $\cosh \frac{w}{2} t = 1$ and

$$e^{-\frac{A_1 + B_2}{2} \cdot t} = 1.$$

Consequently, the expression enclosed in brackets and, therefore, also i_r equals zero. For $t = \infty$ the expression

$$e^{-\frac{A_1 + B_2}{2} \cdot t} \left[\frac{A_1 - B_2 + 2A_2 R_r C}{w} \sinh \frac{w}{2} t + \cosh \frac{w}{2} \cdot t \right]$$

takes the form $\frac{\infty}{\infty}$. If the e functions are inserted instead of sinh and cosh, the determining of the limit value for this undetermined form is reduced to the determining of the limit values for the expressions

$$\frac{1}{e^{\frac{A_1 + B_2 - w}{2} \cdot t}} \quad \text{and} \quad \frac{1}{e^{\frac{A_1 + B_2 - w}{2} \cdot t}}.$$

If the expression under the radical sign is positive, the latter of the above two expressions will equal 0, since, when $t = \infty$, the value of the denominator is infinitely great. On the other hand, $\frac{1}{e^{\frac{A_1 + B_2 - w}{2} \cdot t}}$ equals 0 only when the exponent in the denominator is positive. This means that

$$A_1 + B_2 > w$$

or

$$A_1 B_2 + A_2 B_1 > 0$$

which actually is the case, since A_1, B_1, A_2 and B_2 are positive quantities.

When $t = \infty$, the intensity of the current in the relay winding is therefore

$$i_{r\infty} = \frac{K}{R_r + R_e + \frac{R_r R_e}{n}}.$$

This is true — as we have already stated — for a positive expression under the radical sign. It may also happen, however, that the expression w becomes equal to 0 or imaginary through a suitable choice of L, C, R_c, R_e and n. This would also change the time current equation. A change in this latter, however, would mean a change in the increase of the current intensity and, consequently, of the operating lag. We will therefore investigate the influence of the radical expression on the time current equation. Three separate cases will be encountered, viz.

1. The expression under the radical sign is positive.
2. The expression under the radical sign equals 0.
3. The expression under the radical sign is negative.

For positive values of the expression under the radical sign, equation (16 a) holds good. If the value of this expression equals zero, $\cosh \frac{w}{2} t = 1$ for $w = 0$, since $\sinh \frac{\frac{w}{2} t}{w}$ takes an undertermined form. By differentiating the numerator and denominator with reference to w we obtain $\frac{t}{2} \cdot \cosh \frac{w}{2} \cdot t$, from which we obtain $\frac{t}{2}$ as the limit value of the fraction. The equation for i_r is then

$$i_r = \frac{K}{R_r + R_e + \frac{R_r R_e}{n}} \left\{ 1 - e^{-\frac{A_1 + B_2}{2} \cdot t} \times \right.$$

$$\left. \times \left(\frac{A_1 - B_2 + 2A_2 R_r C}{2} \cdot t + 1 \right) \right\} \quad (16\text{ b})$$

For reasons which will be touched on in the following, this case is called the 'boundary case'.

In the third case, lastly, when the expression under the radical sign is negative, w becomes an imaginary quantity. If we assume $w = j \cdot w_1$, in which w_1 is the real quantity, instead of the hyperbolic functions we obtain cyclic functions and the equation for i_r will be as follows

$$i_r = \frac{K}{R_r + R_e + \frac{R_r R_e}{n}} \left[1 - e^{-\frac{A_1 + B_2}{2} \cdot t} \times \right.$$

$$\left. \times \left(\frac{A_1 - B_2 + 2A_2 R_r C}{w_1} \sin \frac{w_1}{2} \cdot t + \cos \frac{w_1}{2} \cdot t \right) \right] \quad (16\text{ c})$$

From this equation we find that for a negative expression under the radical sign there appear periodic variations of the current intensity in the relay winding varitions which gradually diminish in proportion to the magnitude of the factor $e^{-\frac{A_1 + B_2}{2} \cdot t}$. On account of these periodic variations of intensity, we will — in the following — call such a case, where $w = j_1 \cdot w_1$, a 'periodic case'. Similarly, with a positive expression under the radical sign and when, consequently, no cyclic functions occur in the time current equation, we will use the appellation 'aperiodic case'. The time current curve represented by the equation (16 b) is a case which lies just on the boundary between the periodic and the aperiodic case. The progress of the current intensity would in this case be in the last stage of aperiodicity and it is for this reason that this case is called the 'boundary case', as mentioned in the foregoing. The time current curves which represent these three cases are called the 'periodic curve', the 'aperiodic curve' and the 'boundary curve', respectively.

Which it is of these three cases that occurs for given values of L, C, R_r, R_c, R_e and n, is evident from the value of the relation $\frac{L}{C}$, obtained through the reforming of the expression under the radical sign. In the aperiodic, periodic and boundary cases we find that

$$4\, A_2\, B_1 \gtreqless (B_2 - A_1)^2$$

respectively, from which we obtain

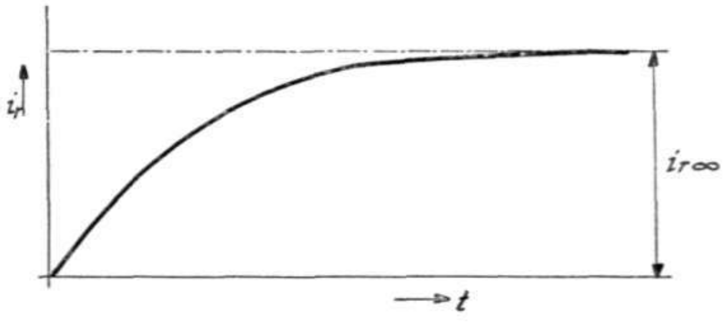
$$\frac{L}{C} \gtreqless \left[\frac{R_e n}{R_e+n} + \sqrt{\left(R_r + \frac{R_e n}{R_e+n}\right)\left(R_c + \frac{R_e n}{R_e+n}\right)}\right]^2 \quad (17)$$

This equation constitutes the necessary and sufficient condition in order that the progress of the curve shall be respectively aperiodic, periodic or in the last stage of aperiodicity.

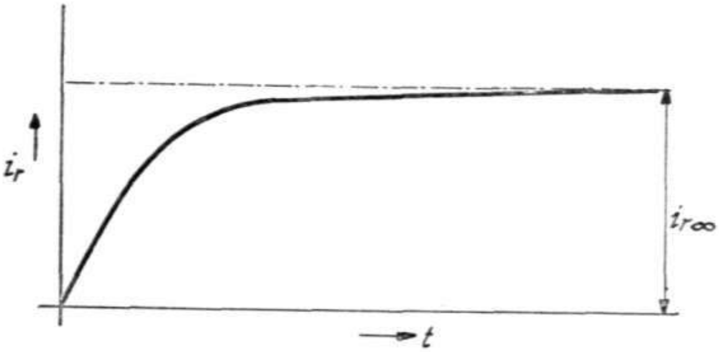
Just as in the aperiodic case, we find that in the periodic and boundary cases there is a limit value which, for $t = \infty$, can be calculated according to

$$i_{r\infty} = \frac{K}{R_r + R_e + \frac{R_r R_e'}{n}}$$

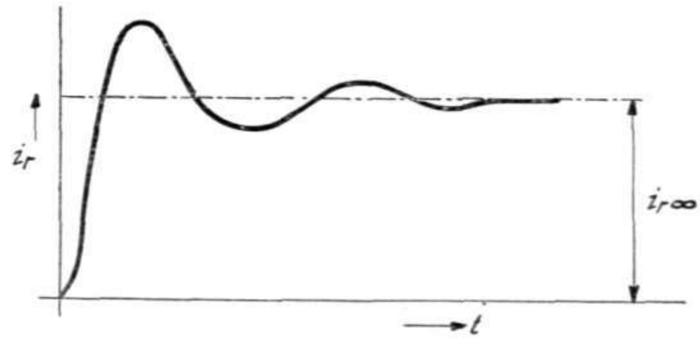
a fact which is easily proved.



R 1255 Fig. 2.



R 1256 Fig. 3.



R 1257 Fig. 4.

According to the calculation, the time current curves which correspond to the three cases would progress as shown in figs 2, 3 and 4.

From these curves we are able to draw the conclusion that, in the periodic case, a relay, after once having operated, could again release its armature if only the first minimum value is sufficiently small. The following investigations will throw further light upon this subject.

The periodic case.

In order to be able to determine whether — in the periodic case — a relay which has operated its armature can again release the same, it is necessary to have a knowledge of the minimum values for the time current curves. In case the first minimum value is smaller than the intensity i_f, at which the relay just barely holds its armature, the armature will be released if the intensity of the current falls below i_f and it is not again attracted until the current has reached a certain, somewhat higher value. An exactly similar condition will arise also if the second, third, etc. minimum value falls below i_f.

The minimum and maximum values of the curve are obtained in the usual way by making the first derivative $\frac{di_r}{dt} = 0$, which gives us

$$\frac{K}{R_r + R_v + \frac{R_r R_v}{n}} \left\{ \frac{A_1 + B_2}{2} e^{-\frac{A_1 + B_2}{2} \cdot t} \times \right.$$

$$\left(\frac{A_1 - B_2 + 2A_2 R_r C}{w} \sin \frac{w_1}{2} t + \cos \frac{w_1}{2} t \right) -$$

$$- e^{-\frac{A_1 + B_2}{2} \cdot t} \left(\frac{A_1 - B_2 + 2A_2 R_r C}{2} \cos \frac{w_1}{2} t - \right.$$

$$\left. \left. - \frac{w_1}{2} \sin \frac{w_1}{2} t \right) \right\} = 0$$

This immediately gives us a value for t, since

$$e^{-\frac{A_1 + B_2}{2} \cdot t} = 0.$$

i. e. $t = \infty$. In this case, however, the existing current has unvarying intensity — as previously mentioned — and the curve will run parallel with the t axis. This value does not interest us in our further investigations, but only the values for t obtained from the remaining equation, which has the following form.

$$\frac{A_1 + B_2}{2} \left(\frac{A_1 - B_2 + 2A_2 R_r C}{w_1} \sin \frac{w_1}{2} \cdot t + \cos \frac{w_1}{2} \cdot t \right) -$$

$$- \left(\frac{A_1 - B_2 + 2A_2 R_r C}{2} \cos \frac{w_1}{2} \cdot t - \frac{w_1}{2} \sin \frac{w_1}{2} \cdot t \right) = 0$$

or

$$\sin \frac{w_1}{2} \cdot t \cdot [2A_2 R_r C (A_1 + B_2) + 4A_2 B_1 + 2A_1 B_2 -$$

$$- 2B_2^2] = w_1 \cdot (2A_2 R_r C - 2B_2) \cos \frac{w_1}{2} \cdot t$$

from which we obtain

$$\tan \frac{w_1}{2} \cdot t = \tan \left(\frac{w_1}{2} \cdot t - m \cdot \pi \right) =$$

$$= \frac{w_1 (A_2 R_r C - B_2)}{2A_2 B_1 + A_1 B_2 + A_2 R_r C (A_1 + B_2) - B_2^2}$$

and

$$t = \frac{2}{w_1} \left(\operatorname{arc\,tan} \frac{w_1 (A_2 R_r C - B_2)}{2A_2 B_1 + A_1 B_2 + A_2 R_r C (A_1 + B_2) - B_2^2} + m \cdot \pi \right).$$

Here, m is a positive integer.

If we should introduce the values for A_1, A_2, B_1, and B_2, and consequently also for w_1, we would obtain t as a function of R_r, R_c, R_v n, L and C. The expressions for A_1, A_2 etc. will be still further altered by putting $\frac{R_v n}{R_v + n} = \alpha$, and $\frac{\alpha R_r}{\alpha + R_r} = \varepsilon$. Mathematically, this means that α, for instance, is the harmonic mean value between R_v and n. From an electric point of view, it means that α is the total resistance for two resistances R_v and n connected in parallel. Consequently, the above quantities may be defined as follows,

$$A_1 = \frac{1}{C(\alpha + R_c)},\ B_1 = \frac{\alpha}{\alpha + R_c},\ A_2 = \frac{\alpha}{CL(\alpha + R_c)},$$

$$B_2 = \frac{R_c \alpha + R_r (R_c + \alpha)}{L(R_c + \alpha)}$$

First of all, we will assume that $R_c = 0$. The above equations may then be written as follows,

$$A_1 = \frac{1}{C \cdot \alpha},\ B_1 = 1,\ A_2 = \frac{1}{C \cdot L},\ B_2 = \frac{R_r}{L}$$

and also

$$t = \frac{2\, m\, \pi}{w_1}$$

since the numerator of the arc tangent and therefore also the arc tangent itself equals zero. If this value for t is inserted in equation (16 c) we obtain

$$i_r = \frac{K \cdot \varepsilon}{R_r R_v} \left(1 - e^{-\frac{A_1 + B_2}{w_1} \cdot m \cdot \pi} \left[\frac{A_1 - B_2 + 2A_2 R_r C}{w_1} \times \right. \right.$$

$$\left. \left. \times \sin m \cdot \pi + \cos m \cdot \pi \right] \right).$$

For $m = 0, 1, 2 \ldots \sin m \pi = 0$.

For $m = 0, 2, 4, 6 \ldots \cos m \pi = +1$.

For $m = 1, 3, 5, 7 \ldots \cos m \pi = -1$.

For even values of m, therefore, the factor for $\frac{K \cdot \varepsilon}{R_r R_v}$ is smaller than 1, and for odd values of m it is greater than 1. This means that i_r has a minimum value when m is an even number, and a maximum value, on the other hand when m is an odd number. Furthermore, since $2\pi\left(\frac{A_1 + B_2}{w_1}\right)$ is the logarithmic decrement of attenuation — designated by ϑ —, the minimum values may be expressed by the equation

$$i_{r\,\min} = \frac{K \cdot \varepsilon}{R_r R_v}\left(1 - e^{-\vartheta\,(0,\ 1,\ 2\ \ldots)}\right)$$

and the maximum values by the equation

$$i_{r\,\max} = \frac{K \cdot \varepsilon}{R_r R_v}\left(1 - e^{-\vartheta\,(\frac{1}{2},\ \frac{3}{2},\ \frac{5}{2}\ \ldots)}\right)$$

These values for $i_{r\,\min}$ vary for different C values. It is possible to conceive $i_{r\,\min}$ as a function of C, and a curve for $i_{r\,\min} - C$, corresponding to the equation $i_{r\,\min} = \frac{K \cdot \varepsilon}{R_r R_v}(1 - e^{-\vartheta})$, can be plotted. As to this curve, we will investigate whether or not there is a certain value of C, for which $i_{r\,\min}$ reaches a minimum. This calculation begins with the equation

$$\frac{di_{r\,\min}}{dC} = \frac{K \cdot \varepsilon}{R_r R_v} \cdot e^{-\vartheta}\,\frac{d\vartheta}{dC} = 0$$

This equation is satisfied either if $e^{-\vartheta} = 0$, which means that ϑ must equal ∞, or if $\frac{d\vartheta}{dC} = 0$. In the latter case we find that

$$2\pi\,\frac{R_r\alpha[4\alpha^2CL - (CR_r\alpha L)^2] - (L + CR_r\alpha)[2\alpha^2L - R_r\alpha(CR_r\alpha - L)]}{[4\alpha^2CL - (CR_r\alpha - L)^2]^{\frac{3}{2}}} = 0$$

and consequently $C = \frac{L}{R_r\alpha}$.

If, for this C, we wish to find a minimum for $i_{r\,\min}$, $\frac{d^2 i_{r\,\min}}{dC^2}$ must be positive, which is the case if

$$\frac{K \cdot \varepsilon}{R_r R_v}\left[e^{-\vartheta}\,\frac{d^2\vartheta}{dC^2} - e^{-\vartheta}\left(\frac{d\vartheta}{dC}\right)^2\right] > 0$$

Since $\frac{d\vartheta}{dC} = 0$ and $\frac{d^2\vartheta}{dC^2}$ becomes positive when $C = \frac{L}{R_r\alpha}$, the above difference holds good and a minimum for $i_{r\,\min}$ is obtained when

$$C = \frac{L}{R_r\alpha}$$

If we introduce this expression in the equation

$$\vartheta = 2\pi\,\frac{A_1 + B_2}{w_1}$$

we obtain $\vartheta = 2\pi\sqrt{\frac{R_r}{\alpha}}$. This is also a minimum value since, according to the above, also $\frac{d\vartheta}{dC} = 0$ and $\frac{d^2\vartheta}{dC^2} > 0$.

If — with an arrangement according to fig. 1 — $R_c = 0$, for each combination of finite values for R_r, R_v, n and L, in the periodic case there is one certain capacity for which the logarithmic decrement of attenuation is a minimum and at which, consequently, the greatest variations in the intensity of the current make their appearance.

These maximum and minimum values for the intensity of the current are given in the following equations

$$i_{r\,\max} = \frac{K \cdot \varepsilon}{R_r R_v}\left(1 + e^{-\pi\sqrt{\frac{R_r}{\alpha}}\,(1,\ 3,\ 5\ \ldots)}\right)$$

and

$$i_{r\,\min} = \frac{K \cdot \varepsilon}{R_r R_v}\left(1 - e^{-\pi\sqrt{\frac{R_r}{\alpha}}\,(0,\ 2,\ 4\ \ldots)}\right)$$

Theoretically, i. e. if we consider the relay as being devoid of mass, the armature of the relay would be released when

$$i_{r\,\min} < i_f,$$

where i_f, as previously mentioned, denotes the intensity of current at which the relay begins to release its armature. This is again attracted when $i_r = i_s$, i. e. when i_r becomes equal to that intensity of current at which the operation of the armature begins.

The influence of the mass of the armature as well as of other masses will not be made the subject of any investigation here. It is quite apparent, however, that with a high frequency, should $i_{r\,\min}$ fall below i_f even for a very short time, the armature will not be released. This frequency at which the armature is no longer released although $i_{r\,\min}$ falls below i_f, will be called the critical frequency. It is best obtained through tests.

If we make $C = \frac{L}{R_r\alpha}$, the frequency $\nu = \frac{w_1}{4\pi}$ of the current passing through the relay winding is

$$\nu = \frac{\sqrt{R_r\,\alpha}}{2\pi L}.$$

As a result, we find that for large values for α and with a constant R_r, the frequency ν is high at the same time as there is a diminution in the logarithmic decrement of the attenuation.

One is prone to wonder whether, by the judicious choice of R_r, R_v, n, L and C, a relay might not be constructed which, after having operated its armature, releases the same for a short time only to attract it again and hold it in this position. We will illustrate this in the following.

The final value $i_{r\infty}$ of the intensity of the current in the relay winding must — in order to guarantee a certain functioning — depass by a certain percentage the value i_f for the intensity of current at the releasing of the armature, and the latter, in turn, must depass the minimum value $i_{r\,\min}{}'$ during the the first cycle. The minimum value during the second cycle is then a certain percentage lower than $i_{r\infty}$. Thus we find that

$$i_f = \frac{1}{q} \cdot i_{r\infty} = \frac{1}{q}\frac{K \cdot \varepsilon}{R_r\,R_v}$$

$$i_f = p \cdot i_{r\,\min}{}' \text{ and } i_{r\,\min}{}'' = \frac{i_{r\infty}}{s}$$

from which

$$i_{r\,\min}{}' = i_{r\infty}\left(1 - e^{-2\pi\sqrt{\frac{R_r}{\alpha}}}\right) = \frac{i_{r\infty}}{p \cdot q}$$

$$i_{r\,\min}{}'' = i_{r\infty}\left(1 - e^{-4\pi\sqrt{\frac{R_r}{\alpha}}}\right) = \frac{i_{r\infty}}{s}.$$

From this we obtain

$$2\pi\sqrt{\frac{R_r}{\alpha}} = ln\,\frac{p \cdot q}{p \cdot q - 1} \text{ and } 4\pi\sqrt{\frac{R_r}{\alpha}} = ln\,\frac{s}{s-1}.$$

Thus we have a relation between p, q and s, namely $s = \frac{p^2q^2}{2\,pq - 1}$. The logarithmic decrement of the attenuation, $\vartheta = 2\pi\sqrt{\frac{R_r}{\alpha}}$ is then, for the newly derived value of s,

$$2\pi\sqrt{\frac{R_r}{\alpha}} = ln\sqrt{\frac{s}{s-1}}$$

from which we obtain the following relation between R_r and α:

$$R_r = \alpha\left(\frac{1}{2\pi}\,ln\sqrt{\frac{s}{s-1}}\right)^2.$$

After introducing this expression for R_r in the equation for the frequency, we obtain

$$\nu = \frac{\alpha}{4\pi^2 L}\,ln\sqrt{\frac{s}{s-1}}.$$

The inductance L and the resistance R_r for a given relay winding both increase approximately with the square of the number of windings and are therefore about proportional, but only *about*, since the section factor, i. e. the relation between the space occupied by the copper and the free space required for the winding, is not constant. Thus, if we write $L = .004\ R_r$, this really does not hold good except for an Ericsson relay with a resistance of 500 ω, the coil of which is full wound.

With the previously obtained value for R_r, $L = .004\,\alpha\left(\frac{1}{2\pi}\ ln\sqrt{\frac{s}{s-1}}\right)^2$ and the frequency

$$\nu = \frac{1}{.004\,ln\sqrt{\frac{s}{s-1}}}.$$

If we figure with a case already rather unfavourable and assume one half cycle to be not more than twenty-five thousandths of a second, within which time the relay shall release and again operate its armature, we find that

$$\frac{1}{.004\ ln\sqrt{\frac{s}{s-1}}} = 20$$

and therefore

$$s = \frac{e^{25}}{e^{25} - 1} \cong 1.$$

For $s \cong 1$, however, $p \cong q \cong 1$, i. e. the following equation would hold good

$$i_f \cong i_{r\,\min}{}' \cong i_{r\,\min}{}'' \cong i_{r\infty}$$

or, in other words, there could be a difference of but a small fraction of one per mille between the operating, releasing and final current intensities. This proves the utter futility of prevailing on a relay in such a combination with a condenser, series resistance and parallel resistance to release its armature for $R_c = 0$, when it has previously operated the same. Even though it would be possible to increase the inductance of the relay ten-fold, so as to make $L = .04\ R_r$, the value of pq would still not be greater than 1.09.

This result is not much better than the previous one and is of no practical importance for the present problem.

The investigations made thus far have been limited to that case where the relay, after having operated its armature, releases the same but once. There is a possibility, however, of the armature being released a number of times. Without any calculation it is easy to see, however, that the logarithmic decrement of the attenuation then must be less than if the relay should release its armature but once. This, however, requires a smaller value for R_r and a greater for C, respectively.

From a mathematical point of view the problem appears in the following light.

We assume the releasing value of the current intensity to be still barely reached during the μ^{th} cycle.

We will assume that it is not until the μ^{th} cycle that the intensity of the current does not fall below the releasing value, but still barely reaches the same. Then we obtain

$$i_f = \frac{i_{r\infty}}{q} = i_{r\infty}\,(1 - e^{-\mu \cdot \vartheta})$$

from which

$$\vartheta = \frac{1}{\mu}\, ln\, \frac{q}{q-1},$$

$$R_r = \alpha\left(\frac{1}{2\mu\pi}\, ln\, \frac{q}{q-1}\right)^2,\; L = \frac{R_r \mu}{\nu\; ln\, \frac{q}{q-1}} \text{ and}$$

$$C = \frac{\mu}{\alpha\, \nu\, ln \frac{q}{q-1}}.$$

For $q = 2$ and $\nu = 20$, we get

$$\vartheta \cong \frac{0.7}{\mu},\; R_r \cong 124\,\frac{\alpha}{\mu^2}\cdot 10^{-4}, L \cong 72\cdot R_r \mu \cdot 10^{-3},$$

$$C \cong 72\,\frac{\mu}{\alpha}\cdot 10^{-3}.$$

For $\alpha = 72 \cdot 10^3\ \Omega$

$$\vartheta \cong \frac{0.7}{\mu},\; R_r \cong \frac{900}{\mu^2},\; L \cong \frac{65}{\mu},\; C \cong \mu \cdot 10^{-6}.$$

With increasing values for μ, therefore, C must increase while L and R_r decrease. Since R_r is inversely proportional to μ^2, and L, on the other hand, inversely proportional to μ, the relation $\frac{L}{R_r}$ is directly proportional to μ, i. e. the inductance would also have to increase for constant R_r and increasing μ, making the practical construction of such a relay still more impossible.

Up till now we have only investigated those cases in which R_c could be assumed to equal zero, so that $i_{r\,min}$ was exclusively a function of C. For $R_c > 0$ the equation for $i_{r\,min}$ would obtain the following form

$$i_{r\,min} = i_{r\infty}\,(1 - e^{-Y} \cdot X)$$

in which Y and X have the following values

$$Y = \frac{A_1 + B_2}{w_1}\,(\text{arc tan}\,\varphi + m\cdot\pi)$$

$$\varphi = \frac{w_1\,(A_2 R_r C - B)}{2A_2B_1 + A_1B_2 + A_2R_rC(A_1 + B_2) - B_2^2}$$

$$X = \frac{A_1 - B_2 + 2A_2R_rC}{w_1}\,\sin \text{arc tan}\,(\varphi + m\pi) +$$
$$+ \cos \text{arc tan}\,(\varphi + m\cdot\pi).$$

As long as $R_c = 0$, φ also equalled 0 and therefore also arc tan $\varphi = 0$ and $X = 1$. For $m = 0$, $i_{r\,min}$ equalled 0, i. e. the t-coördinate in the $i_r - t$ graph was a tangent to the curve in the point of origin. On the other hand, if $R_c > 0$ and $m = 0$, then $i_{r\,min}$ equals infinity. Consequently, the first minimum value no longer coincides with the point of origin and the t-coördinate is therefore no longer a tangent to the curve in that point.

The angle formed between the t-coördinate and the tangent with $t = 0$ is given by $\frac{di_r}{dt}$ and will be designated by ψ.

Since

$$\frac{di_r}{dt} = i_{r\infty}\, e^{-\frac{A_1+B_2}{2}\cdot t}\left\{\frac{(A_1+B_2)(A_1-B_2+2A_2R_rC)+w_1^2}{2\,w_1}\times\right.$$
$$\left.\times \sin\frac{w_1}{2}\cdot t + (B_2 - A_2R_rC)\cos\frac{w_1}{2}\cdot t\right\}$$

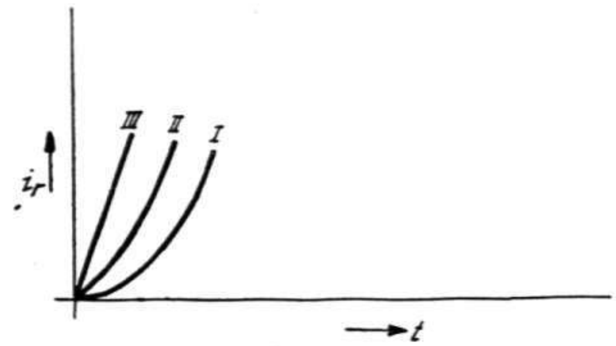
we find that

$$\tan\psi = i_{r\infty}\,(B_2 - A_2R_rC) = \frac{K\cdot R_c\alpha}{L\cdot R_v(\alpha + R_c)}.$$

Thus, the curve rises more sharply against the t-coördinate with an increasing R_c (see fig. 5) and finally, with $R_c = \infty$ in the point of origin, reaches a maximum incline determined by

$$\tan\psi = \frac{K\cdot\alpha}{L\cdot R_v}.$$

In this connection we should emphasize that for relays with which no resistance or condenser is connected in parallel and for which, consequently, α equals R_o, the rise of the curve in



R 1258 Fig. 5.

origin is determined solely by the admitted voltage and by the resistance L, but is entirely unaffected by the series resistance. This last resistance exercises an influence only when a current passes through the relay. From the formula we also find that the increase in the current for a time $t = 0$ is so much the quicker the smaller we make the inductance and the greater we make the voltage K between the terminals. Furthermore, it is important that the progress of the time current curve in the point of origin is unaffected by the size of the parallel condenser. It follows that no conclusions as to the time of attraction are to be drawn from the rise of the time current curve for a time $t = 0$.

If the time current curve for a finite R_c — according to the results obtained from the above calculation — rises to positive values from the point of origin, a maximum must first be reached since the curve is continuous from 0 to ∞. The second derivative $\frac{d^2 i_r}{dt^2}$ denotes that the curve has a maximum for all odd m values and a minimum for all even m values including zero. Since the first value actually reached is a maximum, it follows that the first minimum, which arises for $m = 0$, lies on the negative side of the t-coördinate. However, this can be the case only if φ is negative, which actually is the case if the corresponding values for the constants A_1, B_1, A_2 and B_2 are inserted. Furthermore, we find that this first minimum must correspond to a negative $i_{r\ \min}$, giving

$$H = e^{-\frac{A_1 + B_2}{w_1} \operatorname{arc} \tan \varphi} \cdot X > 1.$$

By inserting this values for H in the equations which express the conditions for the minimum and maximum values respectively, it is also possible to express these under the form

$$i_{r\ \max} = i_{r\,\infty} \left(1 + H \cdot e^{-\frac{A_1 + B_2}{w_1} \cdot \pi\,(1, 3, 5 \ldots)}\right)$$

$$i_{r\ \min} = i_{r\,\infty} \left(1 - H \cdot e^{-\frac{A_1 + B_2}{w_1} \cdot \pi\,(0, 2, 4 \ldots)}\right)$$

or

$$i_{r\ \max} = i_{r\,\infty} \left(1 + H \cdot e^{-\vartheta \frac{2\mu + 1}{2}}\right)$$

$$i_{r\ \min} = i_{r\,\infty} \left(1 - H \cdot e^{-\vartheta \mu}\right)$$

where μ takes the values 0, 1, 2 etc. and ϑ, as heretofore, denotes the logarithmic decrement of the attenuation.

Although these equations are of a similar type to those for $R_c = 0$, the previously used method of calculation cannot be applied here, since here H is also a function of the variable C, considered as independent.

The differentation $\frac{di_{r\ \min}}{dC} = 0$ leads to an expression arc tan $f\,(C) = F\,(C)$, from which it is possible to calculate the value of C only by approximation.

We will therefore apply the following line of thought (see fig. 6).

If we try to make the successive values i', i'' etc. differ from each other as little as possible, we also find that the difference between the successive values

$$i_{r\,\infty}\,(1 - H), \quad i_{r\,\infty}\,(1 - e^{-\vartheta} \cdot H), \ \ldots \text{ etc.}$$

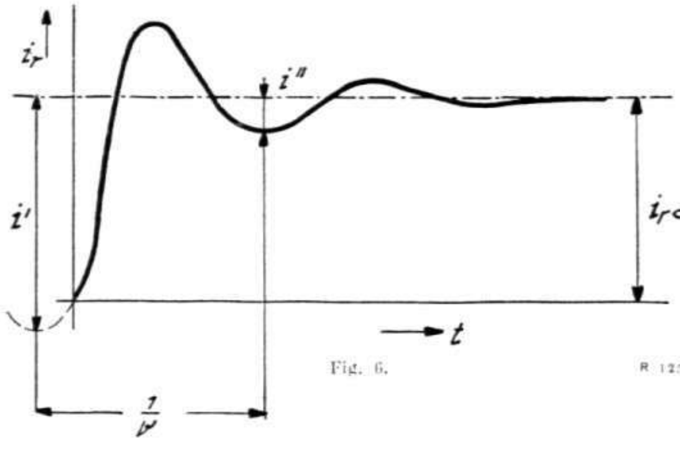


Fig. 6. R 1259

becomes as small as possible. This takes place the closer the value of the expression

$$\frac{i'}{i''} = \frac{i_{r\infty} H}{i_{r\infty} e^{-\vartheta} \cdot H} = e^{\vartheta}$$

approaches 1 and the closer ϑ approaches zero. With given R_c, R_r, L, R_o and n and a variable C, however, ϑ can only reach a certain minimum which we will find when $\frac{d\vartheta}{dC}$ equals zero and $\frac{d^2\vartheta}{dC^2}$ is greater than zero. From $\frac{d\vartheta}{dC}$ we obtain the following two values for C,

$$C = \infty \text{ and } C = \frac{L}{r^2}$$

(where $r^2 = R_c\,\alpha + R_r\,\alpha + R_r\,R_c$), both of which satisfy the requirement $\frac{d^2\vartheta}{dC^2} > 0$. The first value is of no consequence for our further investigations, as it merely denotes that with an infinitely great capacity the attenuation equals zero.

With the second value we find that $\vartheta = 2\pi\frac{r}{\alpha}$ and

$$H = \frac{\alpha}{r}\sqrt{\frac{R_r + R_c}{\alpha + R_c}}\; e^{-\frac{r}{\alpha}\left(\pi - \text{arc tan}\frac{R_c}{r}\right)}.$$

The frequency ν coinciding with this value for the capacity is

$$\nu = \frac{\alpha \cdot r}{2\pi L\,(\alpha + R_c)}.$$

For $R_c = 0$, we obtain from these equations as special cases the cases previously calculated by us. With an increasing R_c, ϑ also increases. If R_c finally reaches a value depassing every given finite value, ϑ as well increases beyond all bounds.

We will now investigate if, by means of suitable assumptions, it will not be possible to produce a case which will permit a relay, after once having operated its armature, to release the same and after attracting it again to hold it until the operating circuit is broken.

The present line of thought resembles the former one. According to our assumption we again find that

$$i_f = p \cdot i_{r\,\text{min}},\quad i_{r\infty} = q \cdot i_f \text{ and } i_{r\,\text{min}}' = \frac{i_{r\infty}}{s}.$$

from which we find that

$$e^{-\vartheta} \cdot H = \frac{p \cdot q - 1}{p \cdot q} \text{ and } e^{-2\vartheta} \cdot H = \frac{s - 1}{s}.$$

After eliminating H from these two equations we obtain

$$\vartheta = ln\frac{s\,(p \cdot q - 1)}{p \cdot q\,(s - 1)}.$$

In contrast to the previously obtained results, according to which — after having chosen p and q — s and ϑ were unequivocally determined, here the choice between the two quantities s and ϑ is still open. Not having fixed the value for R_c, we have one more degree of liberty than previously. This is apparent also from the following discussions. When we made $R_c = 0$, the first minimum value for the time current curve, i. e. in origin for the $i_r - t$ graph, was determined. Thus if also another value for $i_{r\,\text{min}}$ is fixed, ϑ is thereby unequivocally determined. On the other hand, if we determine that $p \cdot q, \cdot i_{r\,\text{min}}$ is equal to $i_{r\infty}$ for $R_c > 0$ according to previous assumptions, thereby determining a minimum value for the curve, we will find that — since, as already mentioned, there now appears no $i_{r\,\text{min}}$ - value in origin - the choice is free for some other value for $i_{r\,\text{min}}$.

$r = \frac{\alpha}{2\pi} \cdot \vartheta$ is now calculated from $\vartheta = 2\pi\frac{r}{\alpha}$.

This expression is inserted in the equations for C and ν, from which we then obtain

$$C = \frac{4\pi^2 L}{\alpha^2\vartheta^2} = 40\frac{L}{\alpha^2\vartheta^2}$$

and

$$\nu = \frac{\vartheta\,(\alpha + R_r)}{L\,(\vartheta^2 + 4\pi^2)} = \frac{\vartheta\,(\alpha + R_r)}{L\,(\vartheta^2 + 40)}.$$

With due consideration for the cost as well as for the good functioning of the relay, we will make the following assumptions with respect to C, ν and L,

$$C = 4 \cdot 10^{-6}\,\text{F},\ \nu = 20,\ L = .004\ R_r$$

The following five conditional equations are then obtained for the six unknown quantities ϑ, R_r, R_c, α, $p \cdot q$ and s,

$$20 = \frac{\vartheta\,(\alpha + R_r)}{.004\ R_r\,(\vartheta^2 + 40)},$$

$$4 \cdot 10^{-6} = \frac{.16\ R_r}{\alpha^2\,\vartheta^2},$$

$$R_c = \frac{.025\,\alpha^2\,\vartheta^2 - R_r\,\alpha}{R_r + \alpha},$$

$$\vartheta = ln\frac{s\,(pq - 1)}{p \cdot q\,(s - 1)},$$

$$H = \frac{p \cdot q - 1}{p \cdot q} e^{\vartheta} = \frac{\alpha}{r} \sqrt{\frac{R_r + R_c}{\alpha + R_c}}\, e^{\frac{r}{\alpha} \arctan \frac{R_c}{r}}.$$

We can give one of these unknown quantities an arbitrary value, after which the other five may be calculated. What interests us most is whether an increased value of R_c will simplify the problem of obtaining a relay which will again release its armature efter a first operating of the same. We have a feeling that the conditions will merely be increasedly unfavorable, since for $R_c > 0$ we obtain an increase in the logarithmic decrement of the attenuation. This is verified by calculation. We obtain as follows

$$\text{for } R_c = 0 \quad : p \cdot q = \frac{e^{8.9}}{e^{8.9} - 1}$$

$$\text{and for } R_c = 100\,\Omega: p \cdot q = \frac{e^{12.2}}{e^{12.2} - 1}$$

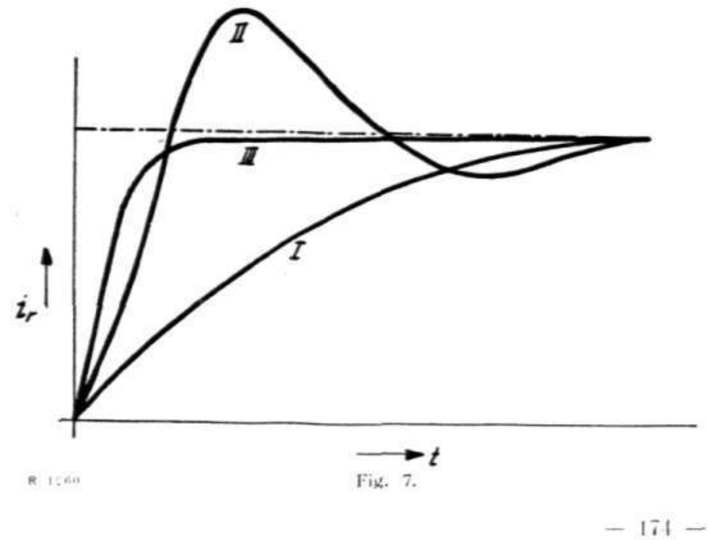
Thus for increasing R_c, the value of $p \cdot q$ approaches closer and closer to 1.

All that now remains is to show how the time current curve varies for different values of C.

In order to determine this, we will take equation (17), which can also be written under the following form,

$$C \gtreqless \frac{L}{\left(\alpha + \sqrt{(R_r + \alpha)(R_c + \alpha)}\right)^2} \quad \ldots\ldots\ldots (17\text{a})$$

in which the signs of unequality and of equality, read from top to bottom, hold good for the aperiodic, periodic and boundary cases respectively. From the derivation of equation (17), however, we also obtain other values, namely



R 1560

Fig. 7.

$$C \lesseqgtr \frac{L}{\left(\alpha - \sqrt{(R_r + \alpha)(R_c + \alpha)}\right)^2} \quad \ldots\ldots\ldots (17\text{b})$$

Thus, the time current curve follows the periodic case if we have

$$\frac{L}{\left(\alpha + \sqrt{(R_r + \alpha)\,(R_c + \alpha)}\right)^2} < C < \frac{L}{\left(\alpha - \sqrt{(R_r + \alpha)\,(R_c + \alpha)}\right)^2}.$$

From equations (17 a) an (17 b) it follows that there are two boundary cases, one upper and one lower. Within these boundary curves, the current in the relay progresses periodically; beyond them it progresses aperiodically (see fig. 7).

In the boundary cases the frequency is zero. It is not difficult to comprehend, therefore, that for a certain value for C, ν will reach a maximum. The differentiation of w_1 with respect to C gives the following value for C, after the expression has been made equal to zero,

$$C_{\nu\,\max} = \frac{L}{\alpha^2 + (\alpha + R_r)\,(\alpha + R_c)}.$$

After a comparison with the capacites for the upper and lower boundary curves C_o and C_μ, we obtain the following relation between the three values,

$$C_{\nu\,\max} = 2\, \frac{C_o \cdot C_\mu}{C_o + C_\mu},$$

which means that the value of C, for which ν is a maximum, is the double harmonic mean value between C_o and C_μ. Furthermore, we will readily find that the value of C, for which ϑ is a minimum, is the geometrical mean value between C_o and C_μ,

$$C_{\vartheta\,\min} = \sqrt{C_o \cdot C_\mu}.$$

The highest obtainable frequency for $C_{\nu\max}$ is then

$$\nu_{\max} = \frac{\alpha}{2\pi L} \sqrt{\frac{R_r + \alpha}{R_c + \alpha}}.$$

Time current curves for release.

When a certain time has elapsed after the moment when the relay operates, the intensity of the current in the relay winding has reached a value i_{rt} and the condenser has a charge i_{kt}. If we assume that the voltage between the terminals becomes zero, the charge of the condenser will be equalized over the relay and the resistance, and also the current which arises in the relay as a result of the disappearance of the magnetic field will flow out through the resistance n and the condenser. According to Kirchhoff's both laws, therefore

$$i_r = i_n + i_c\,,$$

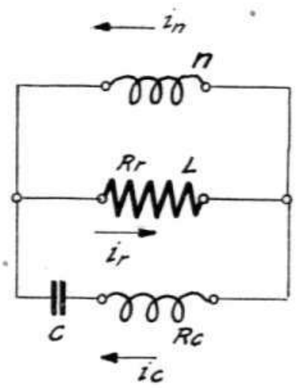
$$i_r R_r + L\frac{di_r}{dt} + i_n \cdot n = 0,$$

$$i_c R_c = \frac{1}{C}\int i_c\,dt - i_n \cdot n = 0.$$

In the last equation, the current is to be again replaced by the charge and also i_n is to be expressed in i_r and i_c. Two simultaneous differential equations of the following form are then obtained,

$$L\frac{di_r}{dt} - n\frac{di_k}{dt} + (n + R_r)\,i_r = 0,$$

$$(R_c + n)\frac{di_k}{dt} - n\,i_r + \frac{1}{C}\cdot i_k = 0.$$



R 1261 Fig. 8.

If, instead of $\frac{di_k}{dt}$ in the first equation, we insert the value obtained from the second, we obtain a relation between $\frac{di_r}{dt}$, i_r and i_k. In addition to this new equation, however, the second of the above two equations must also still be valid. The two differential equations thus obtained are of a similar type to (8) and (9), the only difference being that the constants have other values. We obtain

$$\frac{di_k}{dt} + i_k\frac{1}{C\,(R_c + n)} - i_r\frac{n}{R_c + n} = 0$$

and

$$\frac{di_r}{dt} + i_k\frac{n}{LC(R_c + n)} + i_r\frac{n\left(R_r + R_c + \frac{R_r R_c}{n}\right)}{L\,(R_c + n)} = 0.$$

The solution with respect to i_k and i_r is carried out in the same way as before so as to obtain the following (disregarding the intermediate calculation),

$$i_r = \frac{c_1}{\Phi_1 - \Phi_2}e^{-(A_1 + \Phi_1 A_2)\cdot t} - \frac{c_2}{\Phi_1 - \Phi_2}e^{-(A_1 + \Phi_2 A_2)\cdot t}$$

and

$$i_k = \frac{c_2\,\Phi_1}{\Phi_1 - \Phi_2}e^{-(A_1 + \Phi_2 A_2)\cdot t} - \frac{c_1 \Phi_2}{\Phi_1 - \Phi_2}e^{-(A_1 + \Phi_1 A_2)\cdot t}$$

in which

$$A_1 = \frac{1}{C(R_c + n)},\ B_1 = \frac{n}{R_c + n},\ A_2 = \frac{n}{LC(R_c + n)},$$

$$B_2 = \frac{n\left(R_r + R_c + \frac{R_r R_c}{n}\right)}{L\,(R_c + n)}$$

and

$$\Phi_{12} = \frac{B_2 - A_1}{2A_2} \pm \sqrt{\left(\frac{B_2 - A_1}{2A_2}\right)^2 - \frac{B_1}{A_2}}.$$

The constants c_1 and c_2, again, are determined by the original conditions. If τ seconds have elapsed from the beginning of the make to the beginning of the break, the current intensity in the relay winding will have reached a value $i_{r\tau}$ and the condenser charge a value $i_{k\tau}$. Then, for $t = 0$

$$c_1 = i_{k\tau} + \Phi_1\,i_{r\tau}\,,$$

$$c_2 = i_{k\tau} + \Phi_2\,i_{r\tau}\,.$$

If we insert these constants, we obtain

$$i_r = \frac{i_{k\tau} + \Phi_1\,i_{r\tau}}{\Phi_1 - \Phi_2}e^{-(A_1 + \Phi_1 A_2)\cdot t} - \frac{i_{k\tau} + \Phi_2\,i_{r\tau}}{\Phi_1 - \Phi_2}e^{-(A_1 + \Phi_2 A_2)\cdot t}$$

or, written in another form

$$i_r = e^{-\frac{A_1 + B_2}{2}\cdot t}\left\{\frac{(A_1 - B_2)\,i_{r\tau} - 2\,A_2\,i_{k\tau}}{w}\sinh\frac{w}{2}\cdot t + i_{r\tau}\cosh\frac{w}{2}\cdot t\right\} \quad \ldots\ldots\ (18)$$

Should τ be so large, as to permit — with a sufficiently good approximation — of its being written

$$i_{r\tau} = i_{r\infty} = \frac{K}{R_r + R_c + \frac{R_r R_c}{n}}$$

then one may also write

$$i_{k\tau} = R_r\,C i_{r\infty}$$

from which we obtain

$$i_k = i_{r\infty}\, e^{-\frac{A_1+B_2}{2}\cdot t} \left\{ \frac{A_1 - B_2 - 2A_2 R_r C}{w} \sinh\frac{w}{2}\cdot t + \cosh\frac{w}{2}\cdot t \right\} \ldots\ldots (18\,a)$$

The radical w is then

$$w = \sqrt{(B_2 - A_1)^2 - 4A_2 B_1}\;.$$

As with attraction, three different cases may here be distinguished according to whether

$$4\,A_2\,B_1 \lesseqgtr (B_2 - A_1)^2.$$

By inserting the corresponding values we obtain for C equations similar to the previous ones,

$$C \leqq \frac{L}{\left(n + \sqrt{(n + R_r)\,(n + R_c)}\right)^2}$$

and

$$C \geqq \frac{L}{\left(n - \sqrt{(n + R_r)\,(n + R_c)}\right)^2}.$$

Instead of the quantity α we here have only n, corresponding to $R_c = \infty$ and consequently a break in the battery feed. Since n is greater

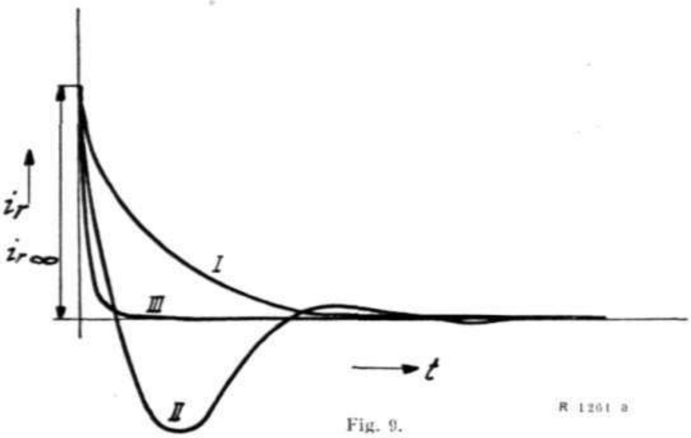


Fig. 9.

than α, the C values obtained for the boundary cases lie further appart than the C values for attraction. The periodic zone for release, therefore, is much greater than for operation (see fig. 9).

Such a result was to be predicted, since the constants A_1, A_2, B_1 and B_2 are identical with the previous constants A_1, A_2, B_1 and B_2 for $R_c = \infty$.

Just as for operation, the current time equation (18 a) obtains different forms depending on whether the course of the current follows the aperiodic, periodic or boundary condition.

In the periodic case — when the circular functions replace sinh and cosh in the equation (18 a) — the current changes direction in certain points. The times when this reversing of the direction of the current takes place are obtained by making equation (18 a) $= 0$ for the periodic case

$$t' = \frac{2}{w_1}\left(\arctan\frac{w_1}{B_2 - A_1 + 2\,A_2\,R_r\,C} + m_1\pi\right).$$

This reversal in the direction of the current enables us to obtain a sure release of the armature for relays in which the value of the intensity of the releasing current i_f is small and in which the load on the armature from the contact springs and the adjusting spring is more or less insignificant.

From the hysterisis curve we obtain the number of ampere turns required to make the remanent magnetic field disappear. If N is the number of turns in the relay, in order to obtain this, we choose $i_{r\,\min}$ so that

$$AW = N \cdot i_{r\,\min}$$

where $i_{r\,\min}$ is the first minimum releasing value for the current time curve. This minimum value appears to be negative, which may also be confirmed by calculation. After the first minimum value has been passed, the curve rises to a maximum value, only to descend again to a minimum value etc. Thus the current oscillates about a zero value. We have already located the points where the current passes zero; we will now attempt to fix the maximum and minimum values. The times at which these values appear are obtained as before by writting $\frac{di_r}{dt} = 0$

$$t = \frac{2}{w_1}\left(\arctan\frac{(B_2 + A_2R_rC)\cdot w_1}{B_2^{\,2} + (A_1 + B_2)A_2R_rC - 2A_2B_1 - A_1B_2} + m\cdot\pi\right).$$

We find again — when these t values are inserted in $\frac{d^2 i_r}{dt^2}$ — that i_r is a minimum for all even m values including zero, and a maximum value, on the other hand, for all odd m values. The equations in question obtain the following form

$$i_{r\,\min} = -\,i_{r\infty}\, e^{-\vartheta\mu}\cdot H,$$

$$i_{r\,\max} = +\,i_{r\infty}\, e^{-\vartheta\frac{2\mu+1}{2}}\cdot H,$$

where μ passes through the values 0, 1, 2 etc. and

$$H = e^{-\frac{A_1 + B_2}{w_1} \operatorname{arc\,tan} \varphi'} \sqrt{\frac{n[CR_r(nR_r + r'^2) - L(R_r - n)]}{L(n^2 + r'^2)}}$$

and

$$\varphi' = \frac{(n R_r + r'^2)\sqrt{4n^2CL - (Cr'^2 - L)^2}}{Cr'^2(n R_r + r'^2) - L(2n^2 - R_r n + r'^2)}.$$

In the last two equations $r' = n R_r + n R_c + R_r R_c$, analogously with the previously used abbreviation r. The logarithmic decrement ϑ of the attenuation is obtained from the equation

$$\vartheta = 2\pi \frac{Cr'^2 + L}{\sqrt{4n^2CL - (Cr'^2 - L)^2}}.$$

With the same trend of thought as before we obtain here also a value for C for which ϑ is a minimum. Here $C = \frac{L}{r'^2}$ and

$$\vartheta_{\min} = 2\pi \frac{r'}{n}.$$

It is easily proved that $\frac{r'}{n} < \frac{r}{\alpha}$ and that consequently, the attenuation during the release is less than during operation, for the same values of R_e, R_r, R_c and n.

For $R_c = 0$, the form for ϑ is similar to the one previously obtained, namely $\vartheta = 2\pi\sqrt{\frac{R_r}{n}}$, while, on the other hand, H does not equal 1, as was the case during operation. Neither could this be the case, for then the first minimum value would coincide with $t = 0$ and be equal to $-i_{r\infty}$; however, this would mean that in the moment of breaking the circuit, the current would suddenly change direction from $+i_{r\infty}$ to $-i_{r\infty}$. The coefficient of direction $\tan\psi'$ of the curve in the moment when the circuit is broken is obtained if we make $t=0$ in $\frac{di_r}{dt}$. We then find that

$$\tan\psi' = -\frac{K\cdot\alpha\,[2R_r n + R_c(R_r + n)]}{(\alpha + R_r)\,R_e L\,(n + R_c)}$$

from which it is apparent that the curve drops from $i_{r\infty}$ to lesser values. For $R_c = 0$, we find that $\tan\psi' = -\frac{2K\cdot\alpha\cdot R_r}{LR_e(\alpha + R_r)}$ at which inclination the curve begins to drop towards zero.

Further, it would seem as if $\vartheta_{\min}$ for $n = \infty$ would equal zero. This is not the case, however, for we find that for $n = \infty$, $\frac{d^2\vartheta}{dC^2} = 0$ and that consequently there exists no value for C, for which ϑ is a minimum or maximum. The equation for ϑ, with $n = \infty$, is

$$\vartheta_{n=\infty} = \frac{C(R_r + R_c)}{\sqrt{4CL - C^2(R_r + R_c)}}\cdot 2\pi$$

and it is easily seen that $\vartheta_{n=\infty}$ is directly dependent upon the magnitude of the radical expression. If we consider the radical expression, we find that for $C \gtreqless \frac{L}{\left(\frac{R_r + R_c}{2}\right)^2}$ the current time curve will belong to the aperiodic, periodic or boundary case according to whether the uppermost, intermediate or lower sign in the equation is valid. In this instance there is but one aperiodic zone and one single boundary case. For the smallest C values the curve already is periodic, and consequently the first aperiodic zone and the first boundary case disappear if no ohmic resistance is connected in parallel with the relay.

For $C = 0$, ϑ equals zero, thereafter increasing quickly in order to asymptotically approach the value $C = \frac{L}{\left(\frac{R_r + R_c}{2}\right)^2}$. For $C = 0$, however, ϑ has no maximum but in the $\vartheta - C$ graph for $C = 0$ the curve indicates a point of inflexion, which also is apparent from the calculation, for then $\frac{d\vartheta}{dC} = 0$ and $\frac{d^2\vartheta}{dC^2} = 0$.

If we return to the conditional equation $\vartheta = 2\pi\frac{r'}{n}$ we find that ϑ increases for an increasing R_c. Consequently, we will base the following investigations on the assumption that $R_c = 0$, from which we obtain the smallest possible value for ϑ.

We will now investigate which value may be reached by $i_{r\min}$ in the most advantageous case. If we make $i_{r\min} = -\frac{1}{\nu}\,i_{r\infty}$, then — since for $R_c = 0$ and $C = \frac{L}{R_r n}$ in H, the expression under the radical sign equals 1 and $\varphi' = \frac{\sqrt{n R_r}}{R_r - n}$ — we find that

$$\nu = e^{\vartheta\left(1+\frac{1}{2\pi}\text{ arc tan }\frac{\sqrt{n R_r}}{R_r - n}\right)}.$$

The frequency ν, for which the general formula is

$$\nu = \frac{\sqrt{4n^2 CL - [C_r'^2 - L]^2}}{4\pi CL\,(n + R_c)}$$

will, if we again make $L \cong .004\ R_r$, be

$$\nu = \frac{1}{2\pi \cdot .004}\sqrt{\frac{n}{R_r}} \cong 40\sqrt{\frac{n}{R_r}}$$

from which

$$n = \left(\frac{\nu}{40}\right)^2 \cdot R_r.$$

If we introduce this value in ϑ and arctan φ', we obtain

$$\vartheta = \frac{80\,\pi}{\nu}$$

and

$$V = e^{\frac{80\,\pi}{\nu}\left(1+\frac{1}{2\pi}\text{ arc tan }\frac{40\,\nu}{1600-\nu^2}\right)}.$$

The larger we make the frequency for $\vartheta_{\min}$, the smaller will V become and the closer does $i_{r\,\min}$ approach the value $-\,i_{r\,\infty}$. For $\nu = \infty$, n would also equal ∞. We have found, however, that for $n = \infty$ there exists no $\vartheta_{\min}$, value, for which reason this case must be omitted.

If we make $\nu = 20$, then $V = 10^6$ and $i_{r\min} = -\,i_{r\infty} \cdot 10^{-6}$; with $\nu = 80$ we obtain $V = 175$ and $i_{r\,\min} = -\,.00572\ i_{r\infty}$.

In calculating values for V, when ν is greater than 40, one must consider that φ' is negative. We figure with the positive φ', arctan φ' and instead of arctan $(-\varphi')$ we write π — arctan φ'.

Further, from the last assumption we obtain $n = 4\,R_r$, $C = \frac{1}{R_r} \cdot 10^{-3}$ F and $\vartheta = \pi$, and for $R_r = 500\,\Omega$, we obtain $C = 2\,\mu$F and $n = 2000\,\Omega$. Thus, if a frequency of 80 is permitted, the value of the current in the first minimum value does not amount to more than .6 percent of the negative original value $i_{r\,\infty}$.

Further it is not difficult to see that a large part of the condenser current flows out through the resistance. If we should close this means of exit and make $n = \infty$, the above-mentioned percentage would be increased.

In order to investigate this by calculation, we will take the foregoing equations and introduce the values $R_c = 0$, $n = \infty$, $CR_r = a$ and $L = .004\ R_r$. We then obtain

$$\nu = \frac{\sqrt{.016\ a - a^2}}{.05 \cdot a},\ \frac{A_1 + B_2}{w_1} = \frac{a}{\sqrt{.016\ a - a^2}},$$

$$\varphi' = \frac{\sqrt{.016\ a - a^2}}{a - .004},$$

and the radical expression in H

$$\sqrt{\frac{2\,a + .004}{.004}}.$$

If we express a in ν and introduce this value in the other expressions, we can write, as previously,

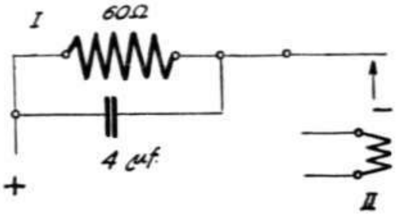
$$V = \frac{e^{\frac{40\,\pi}{\nu}\left(1+\frac{1}{2\pi}\text{ arc tan }\frac{\sqrt{370+1025\,\nu^2}}{4.8-.004\,\nu^2}\right)}}{\sqrt{\frac{14.4 + .004\,\nu^2}{1.6 + .004\,\nu^2}}}.$$

For $\nu = 20$ we obtain $V = 1025$, and for $\nu = 80$ we obtain $V = 6.25$. In the latter case $i_{r\min} = -\,.16\ i_{r\infty}$, thus 16 % as against .6 % for a finite n. For $\nu = 80$ we obtain $a = 9.42 \cdot 10^{-4}$, and for $R_r = 500\ \Omega$ we get $C = 1.88\ \mu$ F.

Thus we find that, when $R_r = 500\ \Omega$ and $C \cong 2\,\mu$ F, the first minimum value of the relay current is increased about 27-fold if we make $n = \infty$ instead of $n = 2000\ \Omega$.

From the above discussion we can make the following deduction, viz. if we want to obtain a powerful re-operation of a relay and also be sure that the relay will with absolute certainty release its armature, a condenser — but no ohmic resistance — is connected in parallel with the relay coil.

In order to obtain an idea of the progress of the current in relays with a low ohmic resistance, we will figure a case which offers a decided similarity to the connections of the selector magnets for an OL 20 P. A. X. and which is illustrated by the diagram in fig. 10. Exact results are not to be expected, since the equations which we have used up till now have not taken into consideration certain conditions, such as the alteration of the inductance during the release of the armature, losses etc., and are based on simplified assumptions ($L = .004\ R_r$, for instance) which are not absolutely correct here.



R 1262 Fig. 10.

We will make $R_r = 60\,\Omega$, $C = 2\,\mu$ F and $n = \infty$. If we assume R_c to be so small that we need not give it any consideration, we can write $R_c = 0$. Then $a = 1.2 \cdot 10^{-4}$, arctan $\varphi' = .889 \cdot \pi$, $\nu = 730$ and finally

$$\nu = 1.28$$

i. e. for $i_{r\,\infty} = 400$ ma, the first minimum value $i_{r\,\min} = -.783 \cdot 400 = -313$ ma. Consequently, the logarithmic decrement ϑ is relatively small. If the relay, for instance, was the rotating magnet of a selector, which is actuated by relay II, another circumstance must be taken into consideration.

On account of the small attenuation of the circuit it is necessary to make the time during which the circuit is broken sufficiently long, as otherwise the operating current will vary between too great values. It may happen, that the following connecting up of K takes place just when the tension at the ends of the coil winding counteracts the admitted battery tension K. If, in addition, relay II is a very quick working relay in which the duration of the makes, consequently, is very short, it is no longer possible to obtain an accurate functioning of the selector. In order to actuate relay I with greater speed, therefore, it would be necessary to increase the attenuation of the circuit which may be accomplished by increasing R_r or by introducing a resistance R_c or n.

From our latest calculations we also find that a relay, after having released its armature, is not likely to again attract the same unless $i_{r\,\infty}$ is many times greater than i_s, the value of the current intensity for operation, thereby making the frequency sufficiently low to permit the operating of the relay.

Furthermore, we will now investigate the value of the tension at the ends of the relay winding at the moment when the relay is disconnected from K. This tension is obtained by means of the equation

$$V' = i_r R_r + L \frac{di_r}{dt}.$$

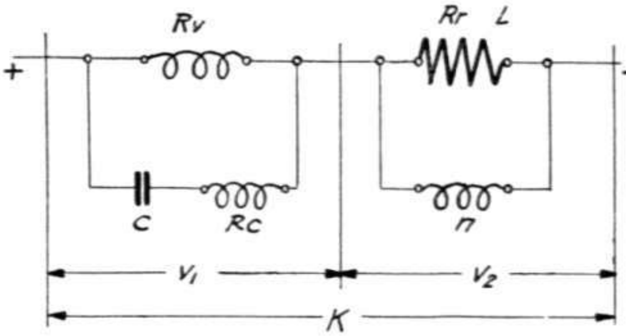
At the time $t = 0$, $i_r = i_{r\,\infty}$ and $\frac{di_r}{dt} = \tan \psi'$, for which reason we obtain

$$V' = -\frac{n(R_r + R_c)}{R_c + n} \cdot i_{r\,\infty}.$$

The tension at the time $t = 0$ is therefore dependent only upon the size of the ohmic resistance. $V' = i_{r\,\infty} \cdot R_r$ for $R_c = 0$, consequently independent of n. Actually it must be so, since at the first moment the condenser constitutes a short circuit. We also find, however, that for $R_c = \infty$ and $n = \infty$, V' will surpass all limits and there will be danger of flashing over in the coil. Actually, however, it is probably but a break spark which occurs in the majority of cases. Sparking depends not only on the tension at the relay but also on the form, material and surface of the contacts, on the speed of the break, on the density of the current at the contacts and on the percentage of ions in the air as well as on the air-pressure. A detailed investigation of these conditions, however, does not fall within the scope of this article.

II. *The condenser in parallel with the series resistance.*

We will carry out our calculations with the arrangements as in fig. 11. The designations are the same as under I, viz. L is the induct-



R 1263 Fig. 11.

ance of the relay at H, R_r the ohmic resistance of the relay, C the capacity of the relay at F, R_c the resistance of the condenser increased with an ohmic series resistance. R_v and n are resistances free from both induction and capacity. K is the constant tension of the battery, and V_1 and V_2 the variable tensions of the condenser and the relay respectively. The following conditions hold good according to the diagram,

$$K = V_1 + V_2$$

$$i_v + i_c = i_r + i_n$$

$$V_1 = i_c R_c + \frac{1}{C}\int i_c\,dt = i_v R_v$$

$$V_2 = i_r R_r + L\frac{di_r}{dt} = i_n \cdot n$$

from which, if i_k again signifies the charge passing through the condenser up to the time t, the following simultaneous differential equations are obtained,

$$L\frac{di_r}{dt} + R_c\frac{di_k}{dt} + R_r i_r + \frac{1}{C}\cdot i_k = K,$$

$$L\left(1 + \frac{R_v}{n}\right)\frac{di_r}{dt} - R_v\frac{di_k}{dt} + \left(R_r + R_v + \frac{R_r R_v}{n}\right) i_r = K.$$

Both of these equations are solved in the same manner as in case I, so that — disregarding the intermediate figuring — the following expression for i_r is obtained,

$$i_r = \frac{D_2 A_1 - D_1 A_2}{A_1 B_2 + A_2 B_1} - \frac{c_1}{\Phi_1 - \Phi_2} e^{-(A_1 + \Phi_1 A_2)\cdot t} + \\ + \frac{c_2}{\Phi_1 - \Phi_2}\cdot e^{-(A_1 + \Phi_2 A_2)\cdot t} \quad \ldots\ldots (19)$$

In the same way we obtain the expression for i_k

$$i_k = \frac{D_1 B_2 + D_2 B_1}{A_1 B_2 + A_2 B_1} - \frac{c_2 \Phi_1}{\Phi_1 - \Phi_2} e^{-(A_1 + \Phi_2 A_2)\cdot t} + \\ + \frac{c_1 \Phi_2}{\Phi_1 - \Phi_2} e^{-(A_1 + \Phi_1 A_2)\cdot t} \quad \ldots\ldots (20)$$

In these two equations we have

$$A_1 = \frac{1 + \frac{R_v}{n}}{C\left(R_v + R_c + \frac{R_v R_c}{n}\right)},\quad A_2 = \frac{R_v}{CL\left(R_v + R_c + \frac{R_v R_c}{n}\right)},$$

$$B_1 = \frac{R_v}{R_v + R_c + \frac{R_v R_c}{n}},$$

$$B_2 = \frac{R_c R_v + R_r\left(R_v + R_c + \frac{R_v R_c}{n}\right)}{L\left(R_v + R_c + \frac{R_v R_c}{n}\right)},$$

$$D_1 = \frac{K\cdot R_v}{n\left(R_v + R_c + \frac{R_v R_c}{n}\right)},\quad D_2 = \frac{K(R_v + R_c)}{L\left(R_v + R_c + \frac{R_v R_c}{n}\right)}$$

$$\Phi_{12} = \frac{B_2 - A_1}{2A_2} \pm \sqrt{\left(\frac{B_2 - A_1}{2A_2}\right)^2 - \frac{B_1}{A_2}}.$$

The integration constants c_1 and c_2 are determined by the original conditions. For the time $t = 0$, the relay current $i_r = 0$ and also the condenser charge $i_k = 0$. By introducing these values in equations (19) and (20), the expressions for c_1 and c_2 are obtained. The equation for i_r is then as follows,

$$i_r = \frac{K}{R_r + R_v + \frac{R_r R_v}{n}}\left\{1 - \left[\frac{\Phi_1 + CR_v\left(1 + \frac{R_r}{n}\right)}{\Phi_1 - \Phi_2} e^{-(A_1 + \Phi_1 A_2)\cdot t} - \right.\right. \\ \left.\left. - \frac{\Phi_2 + CR_v\left(1 + \frac{R_r}{n}\right)}{\Phi_1 - \Phi_2} e^{-(A_1 + \Phi_2 A_2)\cdot t}\right]\right\} \quad \ldots (19a)$$

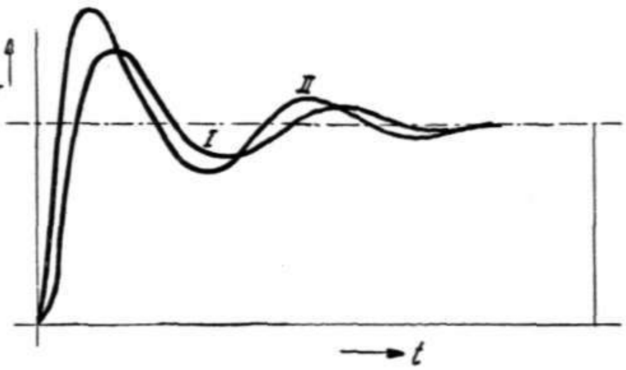
If, on the other hand, the hyperbolic functions are introduced, the equation may be written as follows,

$$i_r = \frac{K}{R_r + R_v + \frac{R_r R_v}{n}}\left\{1 - e^{-\frac{A_1 + B_2}{2}\cdot t} \times \right. \\ \left. \left[\frac{A_1 - B_2 - 2A_2 R_v C\left(1 + \frac{R_r}{n}\right)}{w}\cdot \sinh\frac{w}{2}\cdot t + \cosh\frac{w}{2}\cdot t\right]\right\} \quad (19b)$$

The constants A_1, B_1, A_2, and B_2 are identical with the corresponding constants obtained under I in the equation for the curve of operation. Furthermore, since here also

$$w = \sqrt{(B_2 - A_1)^2 - 4\,A_2\,B_1},$$

the equations (17 a) and (17 b) are valid here too, these equations determining whether the course of the current in the relay is periodic, aperiodic or on the boundary of aperiodicity. Also, the logarithmic decrement as well as the frequency remain unchanged when C and R_c are connected in parallel with the series resistance instead of in parallel with the relay winding. We will find, however, that the magnitude of i_r, between the values zero and ∞, is influenced by this change (see fig. 12). This is apparent already from the factor for sin $\frac{w_1}{2}\cdot t$, this being the only quantity which is different



R 1264 Fig. 12.

in the two equations for i_r. If the difference between equations (16 c) and (19 b) is developed, the latter for the periodic case, and if we designate this difference with Δi_r, we obtain the following

$$\Delta i_r = K\cdot e^{-\frac{A_1+B_2}{2}\cdot t}\times$$
$$\times\frac{2A_2C\left(R_r+R_v+\frac{R_r\,R_v}{n}\right)}{w_1\left(R_r+R_v+\frac{R_r\,R_v}{n}\right)}\cdot\sin\frac{w_1}{2}\cdot t$$

or simplified,

$$\Delta i_r=\frac{2K\cdot A_2C}{w_1}\cdot e^{-\frac{A_1+B_2}{2}\cdot t}\sin\frac{w_1}{2}\cdot t\,.$$

From this we find that the difference between the two currents consists of a damped sine oscillation. From that moment when the circuit is closed, for an increasing t the difference will be increased up to a maximum value. The calculation aims to obtain maximum and minimum values of Δi_r. From $\frac{d^2\Delta i_r}{dt^2}$ we find that the first maximum, when

$$t=\frac{2}{w_1}\left(\text{arc tan}\,\frac{2\pi}{\vartheta}+m\cdot\pi\right),$$

occurs when $m=0$. The following is then true,

$$\Delta i_{r\max}=K\cdot e^{-\frac{\vartheta}{2\pi}\,\text{arc tan}\,\frac{2\pi}{\vartheta}}\times$$
$$\times\frac{2\alpha C}{\sqrt{4\alpha^2LC-(Cr^2-L)^2}}\cdot\frac{2\pi}{\sqrt{4\pi^2+\vartheta^2}}$$

in which, again, $r^2=R_r\alpha+R_c\alpha+R_r\,R_c$. $\Delta i_{r\max}$ becomes greater for a decreasing ϑ. Consequently, the greatest difference is obtained if we make $\vartheta=2\pi\frac{r}{\alpha}$ and $C=\frac{L}{r^2}$. Then

$$\Delta i_{r\max}=K\cdot e^{-\frac{\vartheta}{2\pi}\,\text{arc tan}\,\frac{2\pi}{\vartheta}}\cdot\frac{2\pi}{r\sqrt{4\pi^2+\vartheta^2}}$$

and, if we make $R_c=0$,

$$\Delta i_{r\max}=K\cdot e^{-\sqrt{\frac{R_r}{\alpha}}\,\text{arc tan}\sqrt{\frac{\alpha}{R_r}}}\,\frac{1}{\sqrt{R_r(\alpha+R_r)}}.$$

This occurence means that the operating lag is much shorter when the condenser is connected in parallel with R_v instead of with R_r, since the current reaches its value for operating earlier. Already from the point of origin of the i_r-t graph, i_r makes a more precipitate rise in case II than in case I. This appears from $\frac{di_r}{dt}$ for $t=0$, which — in case II — gives

$$\tan\psi=\frac{K(R_c+R_v)\cdot\alpha}{LR_v(\alpha+R_c)}\,.$$

Since, in this equation, only the numerator is greater than in case I, tan ψ must also be greater. The two angles of inclination differ from each other all the more, the smaller R_c is. This is explained by the fact that, depending on the magnitude of R_c, in case I R_r will be shunted during the first moment, while in case II it will be R_v, so that — if specially R_c equals zero — the relay, at the moment it is connected in circuit, is short circuited or obtains the full voltage K respectively.

If, instead of resistance R_v, we imagine a relay II with the same ohmic resistance as our first relay in case I, according to what has just been stated we must expect a shorter operating lag for relay I than for relay II, in spite of the fact that the previously derived equations cannot be applied. This is a convenient way of obtaining a large difference between the operating lags for equally adjusted relays.

For this connecting arrangement we will also investigate whether the relay, once it has operated, can again release its armature, i. e. whether i_r can drop below the releasing value i_f. The calculation is carried out in the same manner as previously and shows — if we immediately make $C=\frac{L}{r^2}$, for which ϑ reaches its smallest value — that minimum and maximum are given by the following equations,

$$i_{r\,\min} = i_{r\,\infty}\left(1 - e^{-\vartheta\frac{2\mu+1}{2}} \cdot H\right)$$

and

$$i_{r\,\max} = i_{r\,\infty}\left(1 + e^{-\vartheta\mu} \cdot H\right)$$

respectively, in which

$$H = e^{-\frac{r}{\alpha}\arctan\frac{\alpha}{r}\,\frac{R_c + R_v}{R_v - \alpha}} \cdot \frac{\alpha}{r}\sqrt{\frac{r^2 + R_v^2\left(1 + \frac{R_r}{n}\right)^2}{r^2 + \alpha^2}}$$

and where μ succesively takes the values 0, 1, 2 etc. If we assume $i_f = p \cdot i_{r\,\min}'$, $q \cdot i_f = i_{r\,\infty}$ and the second minimum value $i_{r\,\min}'' = \frac{i_{r\,\infty}}{s}$, and also, in order to obtain the lowest possible minimum, $R_c = 0$, we obtain

$$\vartheta = 2\pi\sqrt{\frac{R_r}{\alpha}},\ \nu = \frac{\sqrt{\alpha R_r}}{2\pi L} \text{ and } H = e^{-\sqrt{\frac{R_r}{\alpha}}\arctan\frac{R_v}{R_v - \alpha}} \cdot \sqrt{\frac{\alpha}{R_r}} \times$$

$$\times\sqrt{\frac{\alpha}{R_r}}\sqrt{\frac{R_r\alpha + R_v^2\left(1 + \frac{R_r}{n}\right)^2}{R_r\alpha + \alpha^2}}.$$

If we now select $L \cong .004\ R_r$ and $\nu = 40$, then

$$\sqrt{\frac{R_r}{\alpha}} \cong 1 \text{ and } \vartheta = 2\pi.$$

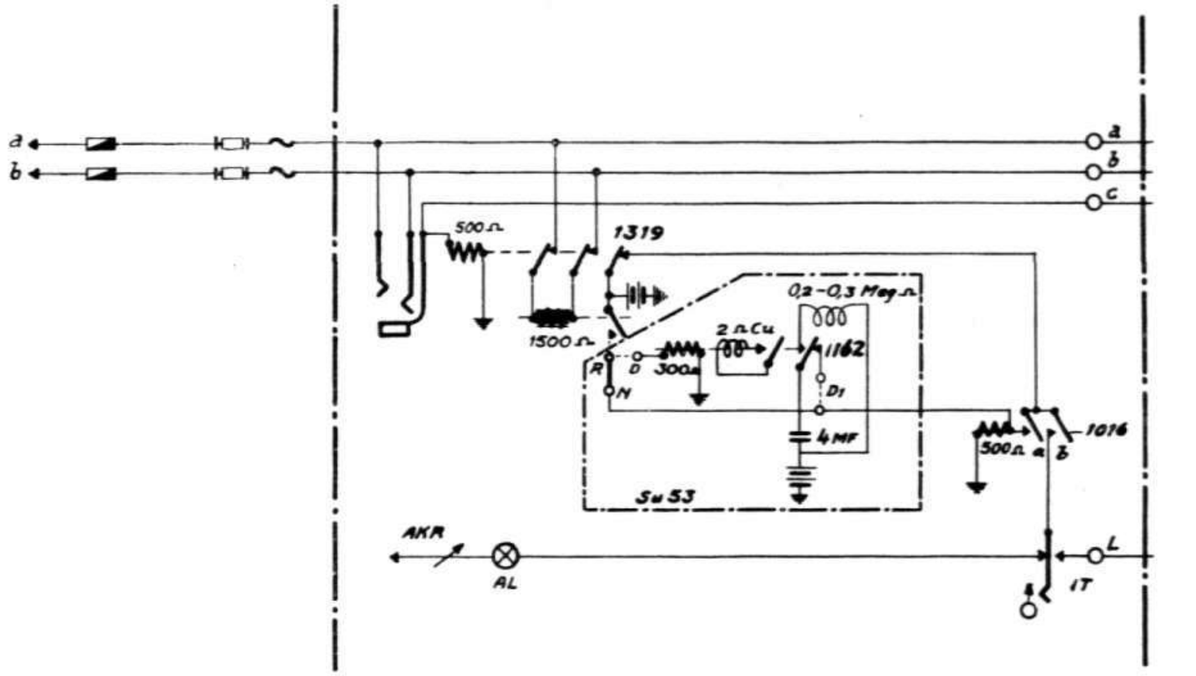
If now $R_v = 20\ R_r$, we obtain the values $H = 12.2$ and $p \cdot q = \frac{e^{25}}{e^{25} - 12.2} = 2.11$. For a 400 ohm relay which attracts at 12 ma. and whose $i_{r\,\infty}$ is 16 ma., we obtain the following values, $L = 1.6\ H$; $C = 10 \cdot 10^{-6}$ F; $n = 420\ \Omega$; $R_v = 8000\ \Omega$; $K = 216$ V; $i_{r\,\min} = 7.6$ ma.

The voltage is rather high and cannot be tolerated for low tension installations; $i_{r\,\min}$ will differ but slightly from i_f and since the duration of one half cycle is not more than 12.5 millisec. it will fall but slightly below i_f. Also, the condenser is quite large. In order to reduce the voltage it would be necessary to reduce R_r, which, on the other hand, would increase C. A reduction of the attenuation, again, would bring about an increase of the frequency and, consequently, a further shortening of the half cycle. If we assume R_v to be smaller than 20 R_r — say $R_v = 10\ R_r$ — then $p \cdot q$ will not be more than 1.22. For this reason it is not necessary here either, under normal conditions, to consider the possibility of the relay again releasing its armature after having operated or that — even for a very short time — it will make an unreliable contact.

We will now investigate the appearance of the operating time current curve for $R_v = \infty$. The discharged condenser with its resistance R_c then lies ahead of the relay with the shunt resistance n. When the circuit is closed, the condenser is charged via R_r and n, thereby permitting the relay to operate its armature for a short time on condition, however, that the intensity of the current is sufficiently high. If the condenser should again be discharged, this procedure might be repeated any number of times. A practical example of this is shown in fig. 13, in which are shown the arrangements made by Ericsson's Vienna subsidiary at the Wiener-Neustadt, Leoben and Gastein toll exchanges for indicating calls by means of long signals. Several exchanges which call each other by means of different A. C. signals are connected in parallel on the same toll line. The toll exchange is called only when a long signal is given. The connections indicated by means of dotted lines are made, after which the connection between the terminals R and N is broken. For each calling signal over the line a 1500-ohm choking relay energizes and connects up a 300-ohm relay. Between the 300-ohm winding and the core of the coil there is a damping winding (2 ohms) which is connected in circuit at the energizing of the relay and which prevents the releasing of the armature during the conversation. This relay brings about (by means of an alternating contact) the discharge of a 4 μ F condenser over a resistance of .3 megohms or — in rest position — charging over the 500-ohm relay respectively. When this relay makes contact a, it remains in this position until the 500-ohm relay on the c-conductor energizes on the introduction of the plug. The condenser, which meanwhile has been completely discharged, is again charged via the 500-ohm relay. Its de-energizing is thereby retarded, it is true, but this is of no consequence, since the relay has only to connect up the calling lamp.

Mathematically, the progress of the discharging and charging of the condenser is as follows.

Considering that in this case $R_v = \infty$ and $n = \infty$; also, that R_c is very small in relation to the other resistances, so that one may — with suf-



R 1253

Fig. 13.

ficient accuracy — make $R_c = 0$, the constants A_1, B_1 etc. obtain the following values,

$$A_1=0,\ A_2=\frac{1}{CL};\ B_1=1;\ B_2=\frac{R_r}{L};\ D_1=0,\ D_2=\frac{K}{L}.$$

If we insert these values in equations (19) and (20) and determine the constants for the original condition when the time $t = 0$ and also i_r and i_k equal zero, we obtain $c_1 = c_2 = K \cdot C$, from which, after simplifying the equation, we obtain

$$i_r = \frac{2KCA_2}{w}\cdot e^{-\frac{A_1+B_2}{2}\cdot t}\cdot \sinh\frac{w}{2}\cdot t.$$

Here, again $w=\sqrt{(B_2-A_1)^2-4A_2B_1}$. Furthermore, since $\frac{D_1B_2+D_2B_1}{A_1B_2+A_2B_1}=K\cdot C$, this expression may be removed from the three terms in the equation for i_k, from which we find that the charge of the condenser at the time $t=\infty$ reaches the value $K\cdot C$.

When a calling signal is given over the toll line, C is short circuited over the ohmic resistance ϱ (.3 megohms), during the entire duration of the calling signal, and the condenser is discharged. If the duration of the calling signal is τ seconds, the charging of the condenser — when this latter is connected up to the relay — is no longer zero, as in the original condition, but $K\cdot C\cdot e^{-\frac{1}{\varrho C}\cdot\tau}$. In the equation for the relay current, therefore, instead of $K\cdot C$ we insert $K\cdot C\left(1-e^{-\frac{1}{\varrho\cdot C}\cdot\tau}\right)$ so that the equation obtains the following form,

$$i_r=\frac{2A_2}{w}\cdot K\cdot C\left(1-e^{-\frac{1}{\varrho C}\cdot\tau}\right)\cdot e^{-\frac{A_1+B_2}{2}\cdot t}\cdot\sinh\frac{w}{2}\cdot t.$$

After inserting the values of the constants we obtain

$$i_r=2K\cdot\sqrt{C}\,\frac{1-e^{-\frac{1}{\varrho\cdot C}\cdot\tau}}{\sqrt{R_r^2C-4L}}\cdot e^{-\frac{R_r}{2L}\cdot t}\times$$

$$\times\sinh\sqrt{\frac{R_r^2C-4L}{4L^2C}}\cdot t.$$

Since in our special case $4\,L$ is greater than CR_r^2, we here have the periodic case. For this reason we will here replace $\sinh\frac{w}{2}\cdot t$ with $\sin\frac{w_1}{2}\cdot t$. If we again seek the maximum and minimum values for the functions thus obtained, we will find that the first-mentioned occur when in the equation

$$t=\frac{2}{w_1}\left(\text{arc tan}\frac{2\pi}{\vartheta}+m\cdot\pi\right)$$

m becomes equal to 0, 2, 4 etc. and that i_r becomes a minimum when m equals 1, 3, 5 etc. The first maximum value

$$i_{r\,\max} = \frac{2A_2}{w_1}\cdot K\cdot C\left(1-e^{-\frac{1}{\varrho\cdot C}\cdot\tau}\right)\cdot e^{-\frac{\vartheta}{2\pi}\operatorname{arc\,tan}\frac{2\pi}{\vartheta}}\,\frac{2\pi}{\sqrt{4\pi^2+\vartheta^2}}$$

must then — in order to prevent the operation of the relay — not exeed the value of the intensity of current i_s for operation.

If we insert the values which were applied in the case for practical demonstration purposes, viz. $R_r = 500\ \Omega$; $L = 2$ H; $C = 4\cdot 10^{-6}$ F, $K = 24$ V; $\varrho = .3\cdot 10^{-6}\ \Omega$; then

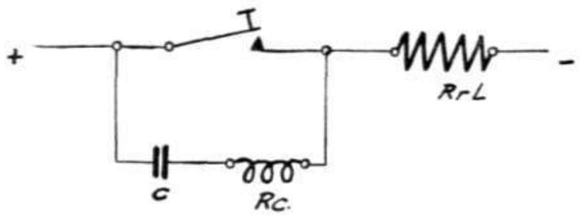
$\vartheta = 2\pi\cdot .378$, $\operatorname{arc\,tan}\frac{2\pi}{\vartheta} = 1.22$ and therefore

$$i_{r\,\max} = 21\left(1-e^{-\frac{1}{1.2}\cdot\tau}\right)$$

in which $i_{r\,\max}$ is expressed in ma. Since i_s amounts to 15 ma., 15 must be smaller than $21\cdot\left(1-e^{-\frac{1}{1.2}\cdot\tau}\right)$, or $\tau < 1.04$ sec. If the calling signal lasts longer than 1.04 sec., the relay will consequently energize and light a calling indicator lamp.

From the general equation for i_r we also arrive at the conclusion that i_r is dependent on the time constant ϱC. The larger ϱ becomes, the smaller is i_r for an equal time of discharge τ. Thus, by varying ϱ, we have the possibility of immediately varying τ. For a twice as large ϱ we obtain the same $i_{r\,\max}$, if τ also is twice as large. A change in C means that also the other terms in the equation for $i_{r\,\max}$ undergo a change, for which reason the influence of C is not visible at a glance. However, the calculations show — as could be expected — that a greater $i_{r\,\max}$ corresponds to a greater C under the assumption that the other quantities remain unchanged.

Lastly we will make an investigation of another connection, as show in fig. 14. This connection may be considered as a special case under II and is often used in applied low tension electricity, when a reversal of the current in the relay winding is desired in order to make sure of a certain release of the armature or



R 1265 Fig. 14.

to prevent sparking at the break point. R_ϱ is here replaced by a contact and n is assumed to be inifinitely great.

The constants are then

$$A_1 = \frac{1}{CR_c}:\ A_2 = 0;\ B_1 = 0;\ B_2 = \frac{R_r}{L}\,D_1 = 0$$

and $D_2 = \frac{K}{L}$, from which it follows that

$$\frac{D_2A_1 - D_1A_2}{B_2A_1 + B_1A_2} = \frac{K}{R_r} \text{ and } \frac{D_1B_2 + D_2B_1}{B_2A_1 + B_1A_2} = 0.$$

Since at the time $t = 0$, also i_r equals zero, while the condenser has a certain charge i_k, the integration constants are

$$c_1 = \frac{K}{R_r}\Phi_1 - i_k' \text{ and } c_2 = \frac{K}{R_r}\Phi_2 - i_k'.$$

After inserting these values in equation (19), the intensity of the current in the relay winding is obtained as follows

$$i_r = \frac{K}{R_r}\left(1-e^{-\frac{R_r}{L}\cdot t}\right).$$

The current time curve of the relay will therefore progress as if the relay was connected direct to the terminal voltage K.

The condenser charge diminishes from the moment the contact is closed. If the above values are inserted in equation (20), the momentary charge is obtained as follows,

$$i_k = i_k'\,e^{-\frac{1}{CR_c}\cdot t}.$$

If we now break the contact, the condenser is charged via R_r and R_c. The constants will then have the following values,

$$A_1 = 0;\ A_2 = \frac{1}{C\cdot L};\ B_1 = 1;\ B_2 = \frac{R_r + R_c}{L};$$
$$D_1 = 0;\ D_2 = \frac{K}{L}.$$

From this is further obtained, however,

$$\frac{D_2A_1 - D_1A_2}{A_1B_2 + A_2B_1} = 0 \text{ and } \frac{D_1B_2 + D_2B_1}{A_1B_2 + A_2B_1} = K\cdot C.$$

In order to find the integration constants we will return to the original conditions. If $t = 0$, i_r has a certain value i_r' which — on condition that the time constant $\frac{L}{R_r}$ for the relay is small and the time t, during which the relay was in circuit, is sufficiently long — may be assumed equal to $\frac{K}{R_r}$. If we take this assumption as the basis for the calculation, we obtain the following equation for the relay current,

$$i_r = \frac{K}{R_r} e^{-\frac{R_r + R_c}{2L} \cdot t} \left[\frac{R_r - R_c}{L \cdot w} \cdot \sinh\frac{w}{2} \cdot t + \cosh\frac{w}{2} \cdot t \right] \quad (21)$$

in which $w = \frac{\sqrt{(R_r + R_c)^2 - 4\frac{L}{C}}}{L}$.

The tension V' at the moment of closing the circuit is given by the equation

$$V'' = i_r R_r + L\frac{di_r}{dt}.$$

If we insert the values for i_r and $\frac{di_r}{dt}$, we obtain

$$V'' = - i_{r\infty}(R_c - R_r).$$

It follows from this equation that, when R_c is made equal to R_r, the tension at the ends of the relay winding is zero at the moment the connection is broken. As previously shown, this condition cannot be obtained when the condenser is connected in parallel with the relay. The lowest tension which could then be obtained for $n = \infty$, was K.

From equation (21) it follows that the current passes through zero when

$$t'' = \frac{2}{w_1} \text{arc tan} \frac{\sqrt{4CL - C^2(R_r + R_c)^2}}{C(R_c - R_r)}.$$

If we compare the values for t', which were obtained during the releasing movement — when the condenser was in parallel with the relay — for the zero values in the time current curve, with those now obtained we find that t' is always greater than t''. If we assume that part of the curve which lies between $t = 0$ and t' and t'' respectively to be a straight line, we can say that — when the condenser is connected in parallel with the make contact — the time of release is greater than when the condenser is in parallel with the relay.

Conclusion.

We have found that with connections according to cases I and II, the intensity of current in the periodic current time curve for standard Ericsson relays can never assume values which, for any length of time, fall below the value of the current intensity for release so as to cause the release of the relay armature. The actually occuring maximum and minimum values deviate still less from the final value than what has appeared from the calculation, since no consideration for the losses has been taken in this latter. Neither has any consideration been given the fact that the induction previous to the beginning of the armature movement is smaller than when the armature is attracted. This change in the inductance takes place during the operating and releasing lags and is responsible — according to oscillographic photographs — for the more or less pronounced hump in the rising as well as in the falling branch. For this reason the higher values for L have always been used in the calculations. In general, the calculations give a clear conception and represent the most unfavourable possibilities in respect to release after the operating movement and attraction after the releasing movement. Also the results obtained for the special case with the indicating of calling signals by means of a long signal coincide very well with the actual conditions.

The calculations for the selector magnet are not in exact accord with the connection in the Ericsson P. A. X. switchboards, this connection not being completely broken in the last selector, but remaining connected to K over a 1500 ohm relay so that the current does not oscillate about zero but about a value $\frac{K^{\text{volts}}}{1560}$ ma., whereby the conditions during quick operation become much more favourable with respect to the losses.

Furthermore, the results obtained are important in that, at the break moment, the tension at the ends of the relay winding is uninfluenced by the inductance of the relay and influenced by the ohmic resistance only. The tension can be zero only in the one case, when the condenser is in parallel with the make contact and the value for R_c is chosen suitably large.

Lastly, I wish to extend my warmest thanks to Mr. Tschepper, of the Austrian Ministry for Comerce and Communications in Vienna, for his friendly assistance in the oscillographic investigations, and to the Ericsson works in Vienna for the material placed at my disposal as well as for the diagram in fig. 13.

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Kurt Lindberg, Boktryckeriaktiebolag, Stockholm 1930.