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A method of computing the attenuation in a band pass filter arbitrarily composed of resistances, inductances, capacities and transformers.

By Professor *H. Pleijel.*

A filter circuit with no losses will let stationary alternating currents pass without attenuation within certain ranges or bands of frequencies. In practice, however, losses must always be allowed for in the coils, transformers, and capacities of the filter circuit, and a certain amount of attenuation will always occur. This attenuation is as a rule increased when the frequency approaches the cut-off frequencies, and sometimes the whole width of the band can therefore not be used. It is consequently of interest to determine the attenuation in the centre and at the edges of a band of frequencies in a filter circuit. It is of course always possible to use the generally applicable formulæ obtained from the filter element by applying the theory of what has somewhat inadequately been designated a "quadripole". This, however, is a troublesome way, and the formulæ deduced are not easily surveyable in a general discussion. H. F. Mayer*) has given a formula for the approximate computation of the attenuation in a filter. But this formula is not generic, and the author has therefore attempted to deduce generally applicable formulæ for the attenuation.

To attain this object, a filter consisting of an arbitrary, complex circuit connected between two line contacts and earth is chosen, built up of resistances, inductances, capacities, and transformers. The loss angles of all the appliances are assumed to be small.

In a complex circuit joining two line contacts and earth, the following relations exist between voltages and currents at the ends (line contacts):

$$v' = I \cdot i' - A \cdot i''$$
$$v'' = A \cdot i' - J \cdot i''$$

*) E. N. T. Vol. 2, 1925, p. 335.

I and J are the insulation impedances measured from the two ends of the circuit. The corresponding short-circuit impedances are designated R and K. The complex circuit can, we know, be regarded as a circuit with the characteristics Z' and Z'' from the two end points, and with a total attenuation complex $= \Theta$.

The relations of these quantities are given by the following equations:—

$$I = Z' \coth \Theta$$
$$R = Z' \tanh \Theta$$
$$J = Z'' \coth \Theta$$
$$K = Z'' \tanh \Theta$$
$$A = \frac{\sqrt{Z' Z''}}{\sinh \Theta}$$

By adding and substracting the two first equations we get:—

$$I + R = \frac{Z' \cosh 2\Theta}{\sinh \Theta \cosh \Theta}$$
$$I - R = \frac{Z'}{\sinh \Theta \cosh \Theta}$$

Consequently after division:

$$\frac{I+R}{I-R} = \cosh 2\Theta = \frac{J+K}{J-K}$$

If the circuit is symmetrical as regards its centre point ($I = J$; $R = K$), half the circuit may with advantage be used in the computation, which thus will give us $\cosh \Theta$ instead of $\cosh 2\Theta$.

From the equation obtained we get the attenuation and the phase angle of the device. We introduce the symbol

$$\Theta = b + ja,$$

where b thus is the attenuation and a the phase constant.

Any one of the circuits can now be expressed by the formula.

$$z' = r + j\omega l + \frac{1}{j\omega c}$$

where ω is the frequency, r the resistance, l the inductance, and c the capacity forming part of the circuit.

If a transformer forms part of the complex circuit, its primary and secondary impedances may be referred to the conductors to which they are attached, or else be regarded as independent conductors with resistance and inductance. The mutual inductance will appear in the equations as an independent conductor of the type $r + j\omega m$.

The various conductors forming part of the complex circuit are numbered and their impedances designated $z'_1, z'_2, z'_3 \ldots$ If the resistances be made nil, z' will change into purely imaginary quantities, here designated $z_1, z_2, z_3 \ldots\ldots\ldots$

Both I and R are now homogeneous polynomes of the same degree in $z'_1, z'_2 \ldots\ldots$ The same will be the case with $\frac{I+R}{I-R}$, and its numerator and denominator will be of the same degree after transformation also.

We temporarily introduce:—

$$\frac{I+R}{I-R} = \varphi(z'_1, z'_2, z'_3 \ldots\ldots)$$

The function φ is thus the quotient in $z'_1, z'_2 \ldots$ of two homogeneous polynomes, which are of the same degree.

We have now:

$$z'_1 = z_1 + r_1$$
$$z'_2 = z_2 + r_2$$

where r_1, r_2, etc. (except possibly for certain z-values) are assumed to be small in relation to $z_1, z_2 \ldots\ldots$ respectively.

We expand the function φ in a Taylor's series and get:

$$\varphi(z'_1\, z'_2 \ldots) = \varphi(z_1\, z_2 \ldots) + \frac{\delta\varphi}{\delta z_1} r_1 + \frac{\delta\varphi}{\delta z_2} r_2 + \ldots;$$

$z_1, z_2 \ldots\ldots$ are functions of ω. On the assumption that $r_1, r_2 \ldots\ldots$ are small, and that not all the derivates $\frac{\delta\varphi}{\delta z_1}, \frac{\delta\varphi}{\delta z_2} \ldots\ldots$ are nil, we may terminate the expansion as above. $\varphi(z_1, z_2, \ldots)$ being the quotient of two polynomes which are of the same degree and homogeneous, $\varphi(z_1, z_2, \ldots)$ must be real. But $\frac{\delta\varphi}{\delta z_1}, \frac{\delta\varphi}{\delta z_2}$ are also the quotients of two homogeneous polynomes, but the degree of the numerator is one unit lower than the degree of the denominator; these derivates must therefore be purely imaginary. If we therefore introduce

$$\cosh 2\Theta = \frac{I+R}{I-R} = A + jB,$$

where A and B are real, we thus get:

$$A = \varphi(z_1, z_2, z_3 \ldots\ldots)$$

$$jB = \frac{\delta\varphi}{\delta z_1} \cdot r_1 + \frac{\delta\varphi}{\delta z_2} \cdot r_2 + \ldots\ldots$$

or

$$jB = \frac{\delta A}{\delta z_1} r_1 + \frac{\delta A}{\delta z_2} r_2 + \ldots\ldots$$

A is thus obtained by introducing $z_1, z_2 \ldots\ldots\ldots$ respectively instead of z'_1, z'_2 in the expression $\frac{I+R}{I-R}$ or, in other words, by computing $\frac{I+R}{I-R}$ for the circuit on the assumption of the resistances everywhere being nil. The function A consequently also determines B, which as we have seen is obtained by deriving A with respect to its variables $z_1, z_2 \ldots$

If the losses are small, B will be small in relation to A. If solved, the equation:—

$$\cosh 2(b + ja) = A + jB$$

will give:

$$\begin{cases} \cosh 2b \cdot \cos 2a = A \\ \sinh 2b \cdot \sin 2a = B \end{cases}$$

If we here substitute

$$\sinh 2b = x$$
$$k^2 = A^2 + B^2$$

we get, by eliminating 2a, the formulæ:

$$\left\{\begin{aligned} & x = \sinh 2b = \frac{1}{\sqrt{2}}\left[-(1-k^2) + \sqrt{(1-k^2)^2 + 4B^2}\right]^{1/2} = \frac{\sqrt{2} \cdot B}{\left[1 - k^2 + \sqrt{(1-k^2)^2 + 4B^2}\right]^{1/2}} \\ & \sin 2a = \frac{\sqrt{2} \cdot B}{\left[-(1-k^2) + \sqrt{(1-k^2)^2 + 4B^2}\right]^{1/2}} = \frac{\left[1 - k^2 + \sqrt{(1-k^2)^2 + 4B^2}\right]^{1/2}}{\sqrt{2}} \\ & \cos 2a = \frac{\sqrt{2} \cdot A}{\left[1 + k^2 + \sqrt{(1-k^2)^2 + 4B^2}\right]^{1/2}} \end{aligned}\right.$$

The first formula shows that x or $2b$ will be small if $k^2 < 1$ and large if $k^2 > 1$.

Hence the bands of frequencies which let a stationary alternating current pass with small attenuation will correspond to the values of ω for which $A^2 + B^2 < 1$.

As we have assumed the losses to be small, B^2 must be small in relation to A^2, and hence the bands of frequencies will practically correspond to those frequencies for which

$$-1 < A < 1$$

The function A will consequently determine the bands of frequencies. By introducing $k^2 = A^2$, our formulæ may be abbreviated:—

Within the bands: $(A^2 < 1)$.

$$\left\{\begin{aligned} x = \sinh 2b &= \frac{B}{\sqrt{1-A^2}} \\ \sin\ 2a &= \sqrt{1-A^2} \\ \cos\ 2a &= A \end{aligned}\right\}$$

Outside the bands: $(A^2 > 1)$.

$$\left\{\begin{aligned} \sinh\ 2b &= \sqrt{A^2-1} \\ \sin\ 2a &= 0 \\ \cos\ 2a &= \pm 1 \end{aligned}\right\}$$

For $2\ b$ we may then write:—

$$2b = \log\left[A + \sqrt{A^2-1}\right]$$

$\cos 2\ a$ must be of the same sign as A, as $\cosh 2\ b$ always is positive.

At the edge of the band: $(k^2 = A^2 = 1)$

$$\left\{\begin{aligned} \sinh\ 2b &= \sqrt{B} \\ \sin\ 2a &= \sqrt{B} \\ \cos\ 2a &= \frac{A}{\sqrt{1+B}} = \frac{\sqrt{1-B^2}}{\sqrt{1+B}} = \sqrt{1-B} \end{aligned}\right\}$$

Above we have now found an expression for B, viz:

$$B = \left|\frac{\delta A}{\delta z_1} r_1 + \frac{\delta A}{\delta z_2} r_2 \ldots\right|.$$

For the attenuation *within* a band, the final formula will thus be:

$$2b = \sinh 2b = \frac{\left|\frac{\delta A}{\delta z_1} r_1 + \frac{\delta A}{\delta z_2} r_2 + \frac{\delta A}{\delta z_3} r_3 + \ldots\right|}{\sqrt{1-A^2}} \quad \ldots (1)$$

and at the edge of the band

$$\sinh\ 2b = \sqrt{\left|\frac{\delta A}{\delta z_1} r_1 + \frac{\delta A}{\delta z_2} r_2 + \frac{\delta A}{\delta z_3} r_3 + \right|} \quad \ldots\ldots (2)$$

In practice A will frequently take the form:—

$$A = 1 + U,$$

where U is the quotient of two homogeneous polynomes of the same degree:—

If U is substituted for A, we get the formulæ:—

$$2b = \frac{\left|\frac{\delta U}{\delta z_1} r_1 + \frac{\delta U}{\delta z_2} r_2 + \ldots\right|}{\left|\sqrt{2U}\right| \cdot \sqrt{1 + \frac{U}{2}}} \quad \ldots\ldots\ldots (3)$$

and at the edge:—

$$\sinh\ 2b = \sqrt{\left|\frac{\delta U}{\delta z_1} r_1 + \frac{\delta U}{\delta z_2} r_2 + \ldots\right|} \quad \ldots\ldots (4)$$

U usually occurs divided into certain factors, and it may be of interest to have an expression for the attenuation, taking this division into account.

The cut-off frequencies are obtained by making $U = 0$ and $U = -2$.

For the sake of simplicity we assume only two factors, thus

$$U = U_1\ U_2$$

If we substitute this expression for U, and introduse the abbreviation $r\nabla = r_1 \frac{\delta}{\delta z_1} + r_2 \frac{\delta}{\delta z_2} + r_3 \frac{\delta}{\delta z_3} + \ldots$, formula (3) will be changed into:—

$$2b = \frac{1}{\sqrt{2}} \frac{\sqrt{U_1 U_2}}{\sqrt{1 + \frac{U_1 U_2}{2}}} \cdot \left[\frac{1}{U_1} \cdot r\nabla \cdot U_1 + \frac{1}{U_2} \cdot r\nabla \cdot U_2\right] (5)$$

and formula (4) into:— ?

$$\sinh 2b = \left|\sqrt{U_1 U_2} \cdot \sqrt{\frac{1}{U_1} \cdot r\nabla \cdot U_1 + \frac{1}{U_2} \cdot r\nabla \cdot U_2}\right| (6)$$

Before passing to the application of the formulæ deduced to specific instances, we will examine what happens to the attenuation when a certain ω-value renders both $U_1 = 0$ and $U_2 = 0$. In that case two bands will meet in the point in question, and we get $\frac{\delta U}{\delta z_1} = \frac{\delta U}{\delta z_2} = \ldots\ldots = 0$. In formula (3) both denominator and numerator will then be nil, while formula (5) can be written:—

$$\left.\begin{aligned} 2b &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1 + \frac{U_1 U_2}{2}}} \cdot \\ &\left[\sqrt{\frac{U_2}{U_1}} \cdot r\nabla \cdot U_1 + \sqrt{\frac{U_1}{U_2}} \cdot r\nabla \cdot U_2\right] \end{aligned}\right\} \ldots. (7)$$

Although both U_1 and U_2 are nil, $\frac{U_2}{U_1}$ will as a rule be finite.

(Above, we have assumed $A = 1$ as the limit, but the same argument is valid if the two bands meet in a point where $A = -1$).

Had U been composed of three factors, i. e.

$$U = U_1\, U_2\, U_3$$

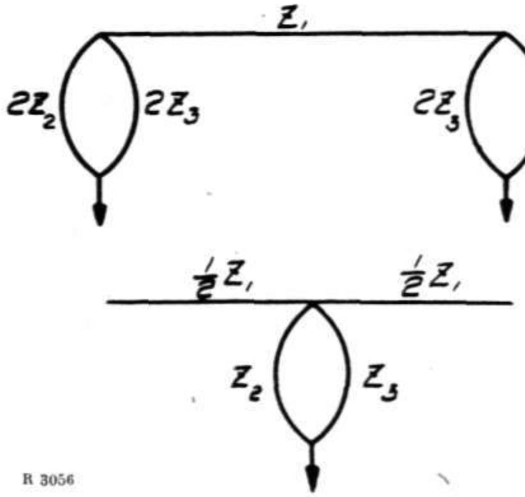
the formula for the attenuation would have been:—

$$2\,b = \frac{\sqrt{U_1 U_2 U_3}}{\sqrt{2}\cdot\sqrt{1+\frac{U_1 U_2 U_3}{2}}}\cdot \left[\frac{1}{U_1}\cdot r\nabla\cdot U_1 + \frac{1}{U_2}\cdot r\nabla\cdot U_2 + \ldots.\right]$$

Application.

We will apply the formulæ derived above to a specific example.

A great number of filters are made up of elements arranged as in the diagram:



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For the sake of brevity we will introduce the symbols:—

$$z_1 = \sigma$$

$$\frac{z_1 z_2}{z_1 + z_2} = \varrho$$

σ is here the impedance in the longitudinal direction of the conductor and ϱ the impedance between the line and earth.

These two filters are symmetric with respect to their centre points, and it is therefore expedient to divide them into two equal halves, and use one of them for the calculations. We take the filter at the top of the figure.

The half circuit then gives us:

$$I = 2\varrho$$

$$R = \frac{\sigma\varrho}{\frac{\sigma}{2} + 2\varrho}$$

and therefore:

$$\cosh\,\Theta = \frac{I+R}{I-R} = 1 + \frac{\sigma}{2\varrho}$$

(The filter circuit at the bottom of the figure will give the same equation.)

Substituting the values of σ and ϱ, we get the following expression for A:—

$$A = 1 + \frac{z_1}{2}\left[\frac{1}{z_2} + \frac{1}{z_3}\right]$$

and

$$U = U_1 U_2 = \frac{z_1}{2}\left[\frac{1}{z_2} + \frac{1}{z_3}\right];\ U_1 = \frac{z_1}{2};\ U_2 = \frac{1}{z_2} + \frac{1}{z_3}$$

The cut-off frequencies are obtained by making $U = 0$ and $U = -2$.

The conditions of the cut-off frequencies will thus be:—

$$z_1 = 0;\ \frac{1}{z_2} + \frac{1}{z_3} = 0;\ \frac{1}{z_2} + \frac{1}{z_3} + \frac{4}{z_1} = 0$$

The number of bands obtained will depend on the composition of z_1, z_2, z_3.

We have:

$$r\nabla\cdot U_1 = \frac{r_1}{2}$$

$$r\nabla\cdot U_2 = -\frac{r_2}{z_2^2} - \frac{r_3}{z_3^2}$$

Formula (5) now gives the attenuation:

$$b = \frac{1}{2}\,\frac{\left| r_1\sqrt{\frac{1}{z_1}\left(\frac{1}{z_2}+\frac{1}{z_3}\right)} - \left(\frac{r_2}{z_2^2}+\frac{r_3}{z_3^2}\right)\sqrt{\frac{z_1}{\frac{1}{z_2}+\frac{1}{z_3}}}\right|}{\sqrt{1+\frac{z_1}{4}\left(\frac{1}{z_2}+\frac{1}{z_3}\right)}} \qquad (8)$$

As a specific example we choose:

$$z_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$z_2 = \frac{1}{j\omega K}$$

$$z_3 = j\omega M$$

consequently:

$$\frac{1}{z_2}+\frac{1}{z_3}=\frac{1-\omega^2 MK}{j\omega M}$$

$$\frac{1}{z_2}+\frac{1}{z_3}+\frac{4}{z_1}=\frac{1-\omega^2 MK}{j\omega M}+\frac{4j\omega C}{1-\omega^2 LC}$$

If we substitute:

$$\omega_1=\frac{1}{\sqrt{LC}};\ \omega_2=\frac{1}{\sqrt{MK}};\ \omega_3=\frac{1}{\sqrt{MC}}$$

we may write:

$$A=1-\frac{1}{2}\frac{\left(1-\frac{\omega^2}{\omega_1{}^2}\right)\left(1-\frac{\omega^2}{\omega_2{}^2}\right)}{\frac{\omega^2}{\omega_3{}^2}}$$

This expression shows that when ω is intermediate between ω_1 and ω_2, A will be positive and larger than 1. The bands will therefore lie on both sides of the range bounded by ω_1 and ω_2. For $\omega=0$ and $\omega=\infty$, A will be negative and infinitely large. In the equation $A=-1$ there will thus be two real roots ω^2, and consequently we obtain two bands.

If we introduce the values of z_1, z_2, z_3 in formula (8), we get the attenuation within the bands according to the formula:

$$b=\frac{1}{2}\frac{r_1\sqrt{\frac{C}{M}}\cdot\sqrt{\frac{1-\omega^2 MK}{1-\omega^2 LC}}+\left(r_2\omega^2K^2+\frac{r_3}{\omega^2M^2}\right)\sqrt{\frac{M}{C}}\cdot\sqrt{\frac{1-\omega^2 LC}{1-\omega^2 MK}}}{\sqrt{1-\frac{1}{4}\frac{(1-\omega^2 LC)\ (1-\omega^2 MK)}{\omega^2 MC}}}$$

If $MK=LC$, ω_1 and ω_2 will coincide and the two bands will be contiguous. The formula for b will then be simplified to:

$$\mathrm{b}=\frac{1}{2}\frac{r_1\sqrt{\frac{C}{M}}+\left(r_2\omega^2K^2+\frac{r_3}{\omega^2M^2}\right)\sqrt{\frac{M}{C}}}{\sqrt{1-\frac{1}{4}\frac{(1-\omega^2 LC)^2}{\omega^2 MC}}}$$

At the edges of the bands the attenuation formula will be:

$$\sinh b=\left|\sqrt{\frac{1-\omega^2 MK}{\omega M}\frac{r_1}{2}+\frac{1-\omega^2 LC}{\omega C}\left(\frac{r_2}{2}\omega^2K^2+\frac{r_3}{2}\frac{1}{\omega^2M^2}\right)}\right|$$

We see here that, at the points corresponding to the cut-off frequencies ω_1 and ω_2, the second and first term respectively under the root mark will disappear. In case the bands meet, the formulæ are not applicable to the point of meeting, as B would be nil at that point. The expansion of $z'_1\ z'_2\ z'_3$ must then include terms of a higher degree. This and related questions will be dealt with in a subsequent paper.

Above, the loss in the condensers has been regarded as represented by resistances. Frequently, however, it may be more expedient to substitute the leakage of the condenser. A condenser may then be introduced as a conductor, the z' value of which is $g+j\omega c$, where g is the leakage of the condenser. The corresponding z will then be $j\omega c$. Although the function A thus obtained will no longer be a homogeneous function of the z-values, j will appear in it in exactly the same way as before, and hence the formulæ given above for A and B will be valid.

To illustrate this method, we will apply it to the simple instance of a Kennelly π-device, the line impedance of which consists of an inductance L connected in series with the condenser C, and where the impedance to earth corresponds to a condenser of the capacity K. We assume L to have a resistance r_1, C a leakage g_2, and K a leakage g_3.

We have here:

$$z_1=j\omega L$$
$$z_2=j\omega C$$
$$z_3=j\omega K$$
$$\sigma=j\omega L+\frac{1}{j\omega C}=z_1+\frac{1}{z_2}$$
$$\frac{1}{\varrho}=z_3$$

The expression for A will consequently be:

$$A=1+\frac{1}{2}\left(z_1+\frac{1}{z_2}\right)z_3 \text{ and}$$

$$U=\frac{1}{2}\left(z_1+\frac{1}{z_2}\right)z_3=\frac{1}{2}\frac{K}{C}[1-\omega^2 LC]$$

The cut-off frequencies are obtained when $U=0$ and $U=-2$. They will be:

$$\omega_1=\frac{1}{\sqrt{LC}}$$

$$\omega_2=\frac{1}{\sqrt{LC}}\cdot\sqrt{1+\frac{4C}{K}}$$

U changes from 0 to -2 as ω changes from ω_1 to ω_2. The transmission band will thus lie between ω_1 and ω_2.

Formula (3) now gives the attenuation. We write this formula as follows:

$$b=\frac{\left|\frac{1}{U}\frac{\delta U}{\delta z_1}r_1+\frac{1}{U}\frac{\delta U}{\delta z_2}g_2+\frac{1}{U}\frac{\delta U}{\delta z_3}g_3\right|}{\sqrt{1+\frac{2}{U}}}$$

We know that

$$\frac{\delta U}{\delta z_1}=\frac{1}{2}z_3$$

$$\frac{\delta U}{\delta z_2}=-\frac{1}{2}\frac{z_3}{z_2^2}$$

$$\frac{\delta U}{\delta z_3}=\frac{1}{2}\left(z_1+\frac{1}{z_2}\right)$$

Substituting these values, a slight transformation gives us:

$$\mathrm{b}=\left|\frac{r_1}{z_1}\cdot\frac{1}{1+\frac{1}{z_1z_2}}-\frac{g_2}{z_2}\cdot\frac{1}{z_1z_2+1}+\frac{g_3}{z_3}\right|\cdot\frac{1}{\sqrt{1+\frac{4z_2}{z_3(z_1z_2+1)}}}$$

or

$$b=\left\{\frac{r_1}{\omega L}\cdot\frac{\omega^2}{\omega^2-\omega_1^2}+\frac{g_2}{\omega C}\cdot\frac{\omega_1^2}{\omega^2-\omega_1^2}+\frac{g_3}{\omega K}\right\}\cdot\frac{1}{\sqrt{1-\frac{4C}{K}\frac{\omega_1^2}{\omega^2-\omega_1^2}}}$$

In his paper, Mayer has assumed the loss factors of all the inductances in the system to be equal and that this is also the case with the capacities forming part of the lines, which assumptions have enabled him to deduce the following simple formula for the attenuation:

$$b=\frac{1}{2}\omega\frac{\delta a}{\delta\omega}[f+h],$$

where a is the phase constant in Θ, f the loss factor of one of the inductances, and h the same factor for one of the condensers. Dr. M. Vos, in computing filters under similar conditions, has used the following instead of the Mayer formula:

$$b=\frac{1}{2}\cdot\frac{\omega\frac{\delta A}{\delta\omega}}{\sqrt{1-A^2}}[f+h]$$

This last formula has the advantage over Mayer's that only loss factors and the function A, which determines the cut-off frequencies, enter on the right hand side.

We will now see how it is possible to deduce from the general formula this special formula, valid for any network linking two line contacts and earth. For a conductor with inductance and capacity in series, we introduce the symbol

$$z_n=z_n'+\frac{1}{z_n''}$$

where

$$z_n'=j\omega L_n$$
$$z_n''=j\omega C_n.$$

(z' has previously been used to signify something else, but this will hardly cause confusion). The function A is now homogeneous in all its z_1, z_2 ......, and its degree is zero, as denominator and numerator are of the same degree. A well known theorem then allows us to write:

$$z_1\frac{\delta A}{\delta z_1}+z_2\frac{\delta A}{\delta z_2}\ldots\ldots=0.$$

If we substitute the above subdivision for z_1 and z_2, we get the equation:

$$\left.\begin{array}{l} z_1'\frac{\delta A}{\delta z_1}+z_2'\frac{\delta A}{\delta z_2}+\ldots\ldots=-\\ -\frac{1}{z_1''}\frac{\delta A}{\delta z_1}-\frac{1}{z_2''}\frac{\delta A}{\delta z_2}-\ldots\ldots\end{array}\right\}\ldots\ldots \quad (10)$$

We pass to the formula for b. We have found:

$$2b=\frac{\left|\frac{\delta A}{\delta z_1}r_1-\frac{\delta A}{\delta z_1}\frac{1}{z_1''^2}g_1+\frac{\delta A}{\delta z_2}r_2-\frac{\delta A}{\delta z_2}\frac{1}{z_2''^2}\cdot g_2+\ldots\right|}{\sqrt{1-A^2}}$$

This formula can be written:

$$2b=\frac{\left|z_1'\frac{\delta A}{\delta z_1}\frac{r_1}{z_1'}+z_2'\frac{\delta A}{\delta z_2}\frac{r_2}{z_2'}+\ldots\ldots-\frac{1}{z_1''}\frac{\delta A}{\delta z_1}\frac{g_1}{z_1''}-\ldots\right|}{\sqrt{1-A^2}}$$

or, if we substitute the loss factors f_n and h_n of the inductances and capacities:

$$2b=\frac{\left|z_1'\frac{\delta A}{\delta z_1}f_1+z_2'\frac{\delta A}{\delta z_2}f_2-\frac{1}{z_1''}\frac{\delta A}{\delta z_1}h_1-\ldots\ldots\right|}{\sqrt{1-A^2}}$$

If all f and all h are alike, equation (10) gives us:

$$2b=\frac{\left|z_1'\frac{\delta A}{\delta z_2}+z_2'\frac{\delta A}{\delta z_2}+\ldots\ldots\right|(f+h)}{\sqrt{1-A^2}}$$

But we have

$$\omega\frac{\delta A}{\delta\omega}=\frac{\delta A}{\delta z_1}\left[z_1'-\frac{1}{z_1''}\right]+\frac{\delta A}{\delta z_2}\left[z_2'-\frac{1}{z_2''}\right]+\ldots\ldots$$

or, according to (10),

$$\omega\frac{\delta A}{\delta\omega}=2\left[z_1'\frac{\delta A}{\delta z_1}+z_2'\frac{\delta A}{\delta z_2}+\ldots\right]$$

The attenuation may therefore be written:

$$2b = \frac{1}{2} \cdot \frac{\omega \left| \frac{\delta A}{\delta \omega} \right|}{\sqrt{1 - A^2}} (f+h)$$

This formula is thus valid for any network, provided all inductances as well as all capacities have the same loss factor.

We will now examine the attenuation in a band at the cut-off points.

We have found that at the cut-off points, which are determined by the condition $k^2 = 1$ (or practically $A^2 = 1$), the attenuation will be $\sqrt{B}$, where B is obtained from the equation:

$$B = \left| \frac{\delta A}{\delta z_1} r_1 + \frac{\delta A}{\delta z_2} r_2 + \ldots . \right|$$

To begin with, we assume that the derivates $\frac{\delta A}{\delta z_1}, \frac{\delta A}{\delta z_2}$ are not all zero at the cut-off point. A has thus no maximum or minimum value at this point, but is rising or falling, and B has a finite value. We call the cut-off frequency ω_o, and will examine the attenuation in points corresponding to the frequency $\omega_o + d\omega$, where $d\omega$ may be either positive or negative. At the cut-off point we may write

$$k^2 = 1 + \frac{\delta k^2}{\delta \omega} \delta \omega,$$

as $k^2 = 1$ when $\omega = \omega_o$.

But $k^2 = A^2 + B^2$, and B^2 is now small in relation to A^2, and $\frac{\delta A}{\delta \omega}$ is not zero at the cut-off point. With some approximation, we may therefore write:

$$\frac{\delta k^2}{\delta \omega} = \frac{\delta A^2}{\delta \omega} = 2A \frac{\delta A}{\delta \omega} = \pm 2 \frac{\delta A}{\delta \omega}$$

We may thus write:

$$1 - k^2 = \pm 2 \frac{\delta A}{\delta \omega} \delta \omega$$

The general attenuation formula may be written (see p. 2).

$$2b = \frac{\sqrt{2}\, B}{[\sqrt{(1-k^2)^2 + 4B^2} + 1 - k^2]^{1/2}} =$$

$$= \frac{\sqrt{B}}{\left[\sqrt{1 + \left(\frac{1-k^2}{2B}\right)^2} + \frac{1-k^2}{2B}\right]^{1/2}} =$$

$$= \frac{B}{\sqrt{1-k^2}} \cdot \frac{\sqrt{2}}{\left[1 + \sqrt{1 + \left(\frac{2B}{1-k^2}\right)^2}\right]^{1/2}} \quad (\text{if } k^2 < 1)$$

$$= \sqrt{k^2 - 1} \cdot \frac{1}{\sqrt{2}} \left[1 + \sqrt{1 + \left(\frac{2B}{k^2-1}\right)^2}\right]^{1/2} \quad (\text{if } k^2 > 1)$$

We now have

$$\frac{1-k^2}{2B} = \pm \frac{\omega_o \frac{\delta A}{\delta \omega} \frac{\delta \omega}{\omega_o}}{\left[\frac{\delta A}{\delta z_1} r_1 + \frac{\delta A}{\delta z_2} r_2 + \ldots .\right]}$$

If, for the sake of simplicity in studying the attenuation in the neighbourhood of the cut-off point, we assume that all coils have the same loss factor and that the same is the case for all capacities, we may, according to the above, write:

$$B = \frac{1}{2} \omega_o \left| \frac{\delta A}{\delta \omega} \right| (f+h)$$

Using this expression, we get:

$$\frac{1-k^2}{2B} = \pm \frac{\frac{\delta \omega}{\omega_o}}{\frac{f+h}{2}}$$

The attenuation can thus be written:

$$2b = \frac{\sqrt{B}}{\left[\sqrt{1 + \left(\frac{2}{f+h} \frac{\delta \omega}{\omega_0}\right)^2} \pm \frac{2}{f+h} \frac{\delta \omega}{\omega_0}\right]^{1/2}}$$

or

$$2b = \frac{B}{\sqrt{1-k^2}} \cdot \frac{\sqrt{2}}{\left[1 + \sqrt{1 + \left(\frac{f+h}{2} \frac{\omega_0}{\delta \omega}\right)^2}\right]^{1/2}} \quad (k^2 < 1)$$

$$2b = \sqrt{k^2 - 1} \cdot \frac{1}{\sqrt{2}} \left[1 + \sqrt{1 + \left(\frac{f+h}{2} \frac{\omega_0}{\delta \omega}\right)^2}\right]^{1/2} \quad (k^2 > 1)$$

The first formula is used when we are so near the cut-off point that $\frac{2}{f+h} \cdot \frac{\delta \omega}{\omega_o} < 1$, the second when $\delta \omega$ is large enough to make this expression larger than 1. The above formulæ can now approximately be written:

$$2b = \sqrt{B} \left[1 \pm \frac{1}{f+h} \frac{\delta \omega}{\omega_0}\right]$$

and

$$2b = \frac{B}{\sqrt{1-k^2}} \cdot \left[1 - \frac{1}{8} \left(\frac{f+h}{2} \frac{\omega_0}{\delta \omega}\right)^2\right] \quad (k^2 < 1)$$

or

$$2b = \sqrt{k^2 - 1} \cdot \left[1 + \frac{1}{8}\left(\frac{f+h}{2}\frac{\omega_0}{\delta\omega}\right)^2\right] (k^2 > 1)$$

The formulæ derived show that the slope of the attenuation curve is determined by the expression:

$$\frac{2}{f+h}\frac{\delta\omega}{\omega_0},$$

where f and h are the loss factors of an inductance and a capacity respectively.

The first of the three formulæ shows the attenuation in the immediate neighbourhood of the cut-off point, and the two latter the transition of the attenuation to the values within and outside the band. The cut-off range may suitably be defined as that range in which $\frac{2}{f+h} \cdot \frac{\delta\omega}{\omega_0}$ is less than 1, or a width half of which is on each side of the cut-off point and equal to

$$\omega_0 (f+h)$$

Outside this range the original formulæ may be used.

Constant Current Instead of Constant Voltage at the Track Transformer in an A.C. Track Circuit.

By *Ture Hård.*

The usual arrangement of an A. C.-fed track circuit is shown in fig. 1. The supply voltage is stepped down in the track transformer to a constant secondary voltage, which is imposed on the track circuit over a series resistance of suitable size. This resistance may be inductive or purely ohmic, and its object is in either case to cause a heavy voltage drop between the track transformer and the rails when a carriage enters the track. The shunt created by the wheel axles causes more current to be fed to the rails, thus producing an increased voltage drop in the series resistance. The secondary voltage of the track transformer being all the time constant, the potential between the rails will at the same time be reduced. The track relay is released when the voltage has dropped so much that it is no longer sufficient to maintain the minimum energizing

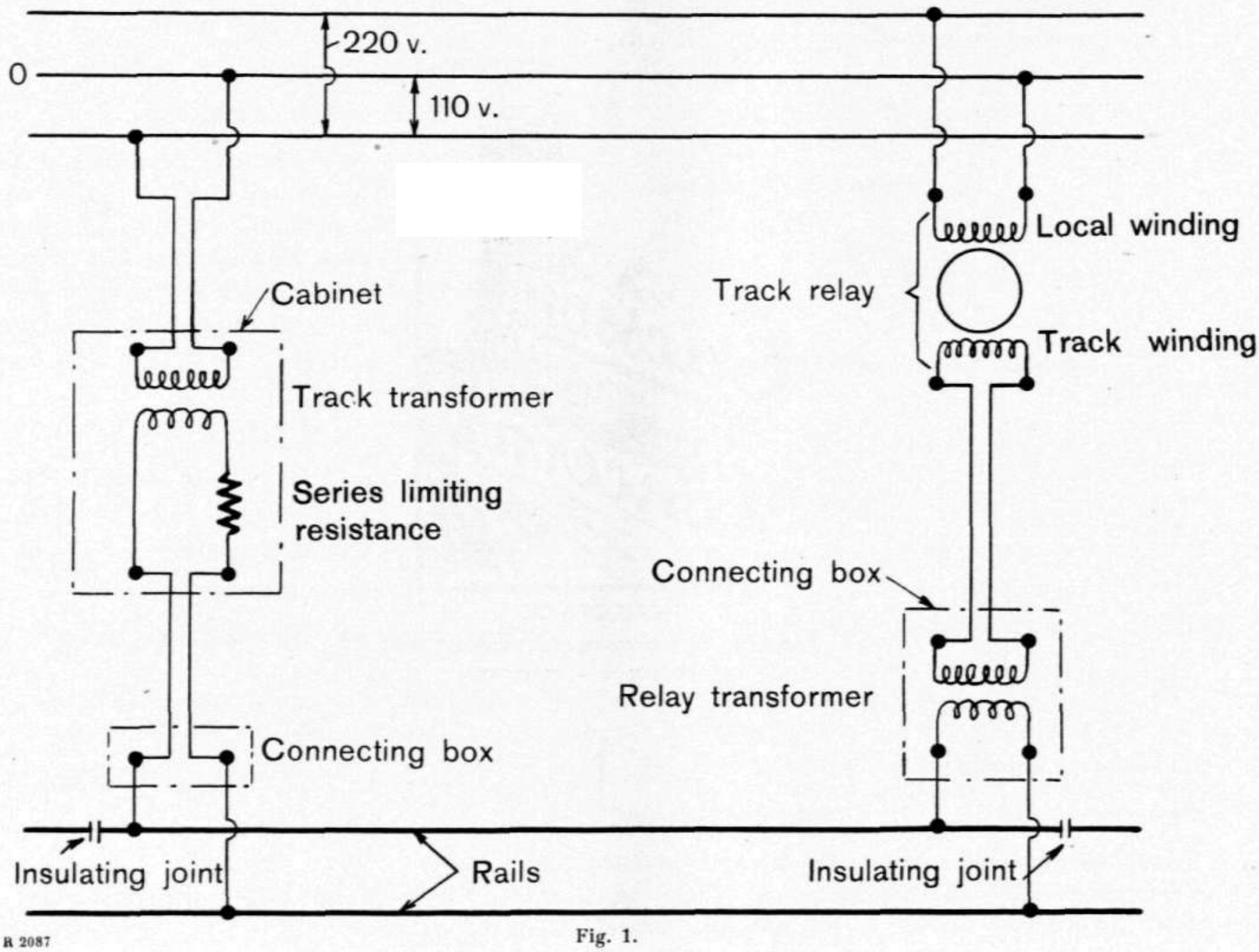


Fig. 1.

current required for the relay. The current in the relay is then reduced to or below a value, characteristic of the relay, which is called the "drop away" value.

Again considering fig. 1, we find that a wheel axle placed between the rails is connected in parallel with the relay transformer primary and the ballast. The current from the track transformer is distributed between the wheel axle, the ballast, and the relay transformer, in inverse proportion to their resistances. The current in each branch increases as the total current is increased. To attain the necessary traffic security, the relay must be released even if the wheel axle offers a certain resistance to the current. The increase of the total current is therefore in itself a drawback, as a more complete shunting through the wheel axle, i. e. a lower resistance, is required to lead away the increase of current also from the relay. If the current from the track transformer could be retained unchanged, release might be obtained with higher resistance, i. e. with greater margin of safety than is possible with any arrangement based on increasing current and series resistance.

We now introduce the following symbols:

u = the normal current output of the track transformer when the track is clear.

u_o = the current taken by the relay and ballast together when the current through the relay is reduced to the "drop away" value.

p_o = the voltage at the track, corresponding to u_o.

d_1 = the wheel axle resistance between the rails, necessary for obtaining p_o.

o = the increase of current in the track transformer caused by shunting with d_1.

We then get:

$$P_o = (u + o - \omega_o)\, d_1;$$

Assuming that the increase of current o may be eliminated and u remain unchanged after the shunting, and denoting the shunt then required d_2, we get

$$p_o = (u - u_o)\, d_1;$$

and hence $\dfrac{d_2}{d_1} = \dfrac{u + o - u_o}{u - u_o} = 1 + \dfrac{o}{u - u_o}$

This expression indicates that the resistance d_2 will always be higher than d_1.

We will now assume that in a track circuit according to fig. 1 a two-phase frequency-selective vane relay, Westinghouse type L with 6 front- and 2 back-contacts, of the following data is to be used:

Local winding: 0.66 amp. at 110 volts.

$$\cos \varphi_L = 0.78,\ \varphi_L = 39°.$$

Track winding: Working current = 0.63 amp. at 7.5 volts.
"Drop away" current = 0.21 amp. at 2.5 volts.

$$\cos \varphi_S = 0.50,\ \varphi_S = 60°.$$

Consequently the relay will not be released until the current has dropped to $^1/_3$ of the value required for full attraction.

The relay is to be connected to the track over a relay transformer of the ratio 1:3, mounted close to the track. The resistance of the leads between the relay transformer and the relay is assumed to be 12 ohms.

By means of the vector diagram shown in fig. 2, the magnitudes and phases of the voltage e and current i required at the primary of the relay transformer in order to obtain 0.63 amp. in the relay track winding are ascertained.

The diagram gives us:

$$i = 2.0\,\underline{|0°}$$

$$e = 5.2\,\underline{|30°}$$

The current i is made the reference-axis.

The following are the data of the track circuit:

Length of the track circuit, km. $l = 1$.

Rail impedance, ohm per km, $z = 0.45\,\underline{|45°}$

Ballast resistance, ohm per km, $r = 2$.

To calculate the track circuit, the auxiliary constants a, b and c, defined in an article with the heading "Some hints on track circuit calculation" printed in the L. M. Ericsson Review No. 4—6, 1928, are now obtained as follows:

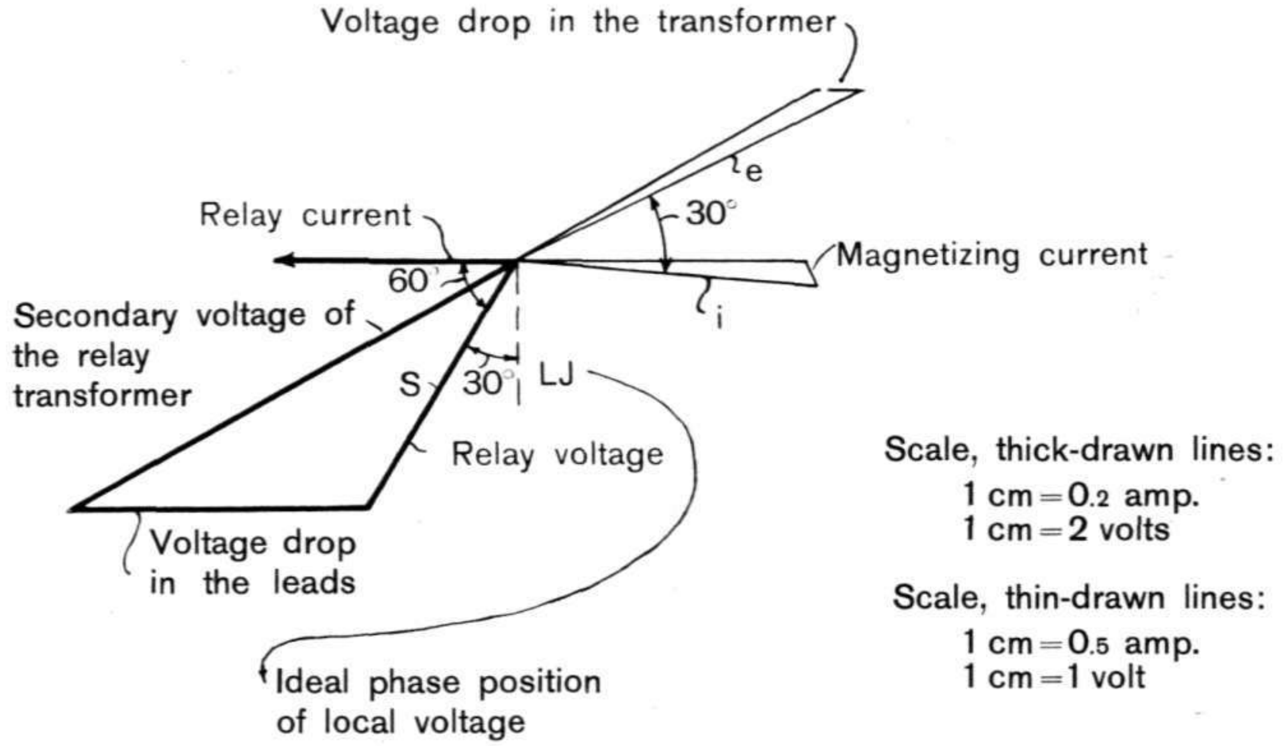
$$a = \sqrt{rz} = 1.23\,\underline{|22.5°}$$

We have: $l\sqrt{\dfrac{z}{r}} = 0.62;\ Z = 45°.$

By the aid of the curves shown in the article, we get:

$$b = 0.6\,\underline{|18°}$$

$$c = 1.15\,\underline{|7.5°}$$



R 2088 Fig. 2.

Hence:

$$e\,\frac{b}{a} = 5.2\,\underline{|30^\circ} \times \frac{0.6\,\underline{|18^\circ}}{1.23\,\underline{|22.5^\circ}} = 2.6\,\underline{|25.5^\circ}$$

$$i\,a\,b = 2\,\underline{|0^\circ} \times 1.23\,\underline{|22.5^\circ} \times 0.6\,\underline{|18^\circ} = 1.5\,\underline{|40.5^\circ}$$

If u and p denote current and voltage at the transformer end of the track circuit, formulæ (1) and (2) in the article mentioned give us:

$$u = \left(i + e\,\frac{b}{a}\right) c = (2.0\,\underline{|0^\circ} + 2.6\,\underline{|25.5^\circ}) \times 1.15\,\underline{|7.5^\circ} =$$
$$= 2.3\,\underline{|7.5^\circ} + 3.0\,\underline{|33^\circ}.$$

$$p = (e + i\,a\,b)\,c = (5.2\,\underline{|30^\circ} + 1.5\,\underline{|40.5^\circ}) \times$$
$$\times\, 1.15\,\underline{|7.5^\circ} = 6.0\,\underline{|37.5^\circ} + 1.75\,\underline{|45^\circ}$$

To determine the magnitudes and phases of u and p, we plot the vector diagram shown in fig. 3, which gives us:

$$u = 5.2\,\underline{|22^\circ}$$
$$p = 7.7\,\underline{|40^\circ}$$

The phase angles refer all the time to the current i as the reference axis:

In fig. 4 the two vectors $OA = u$ and $OB = p$ are drawn with the reference axis i in the direction of the ordinate.

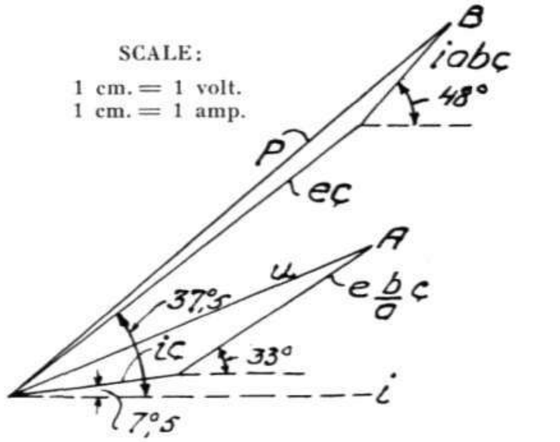
Assuming an ohmic series resistance, and a voltage on the secondary winding of the track transformer of 16 volts, the voltage drop BC in the series resistance can be determined by plotting BC parallel to OA, and making $OC = 16$ volts.

From the vector diagram we get

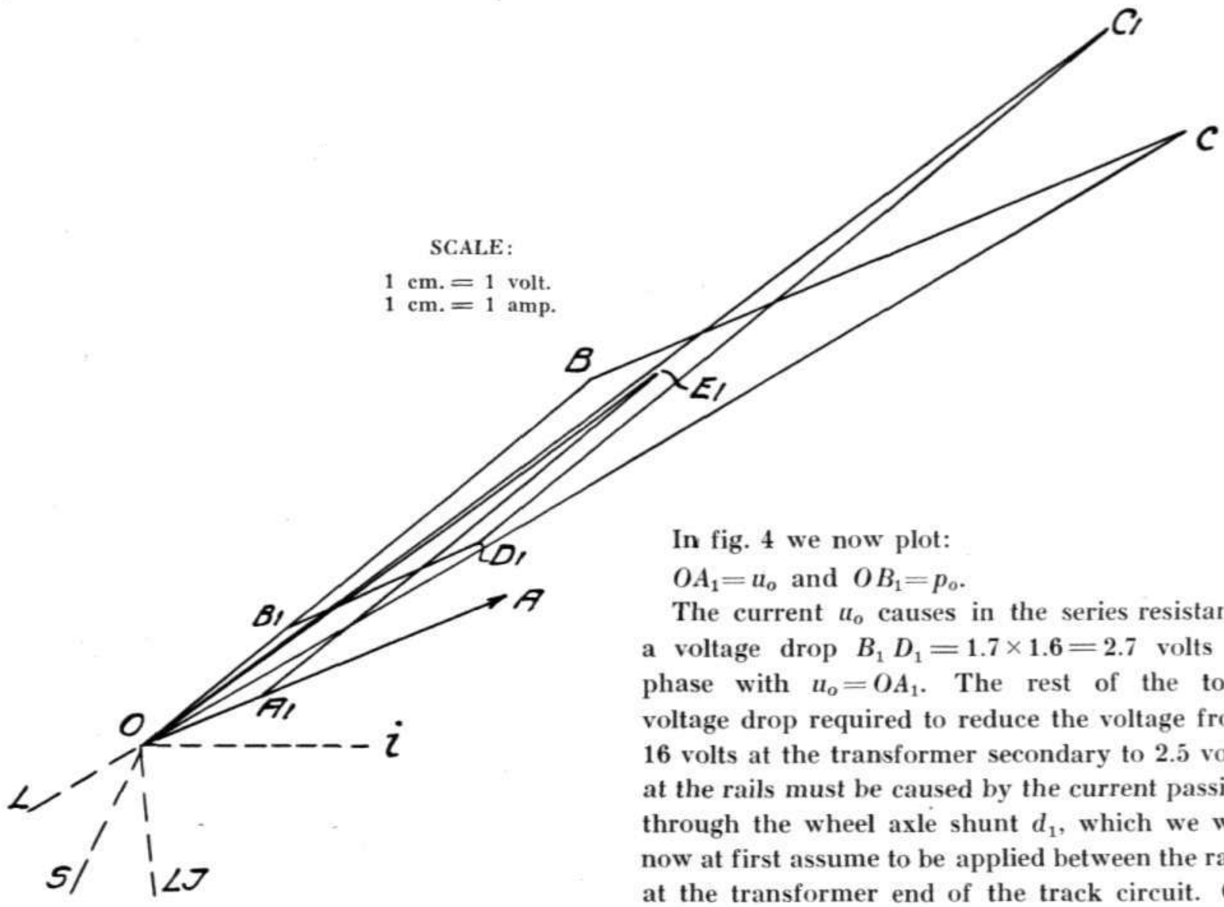
$$BC = 8.5 \text{ volts.}$$

Consequently a series resistance of $= \frac{8.5}{5.2} = 1.6$ ohms may be used for this track circuit.

We will now calculate the value of the wheel axle resistance d_1 required for the release of the



R 2089 Fig 3.



R 2090 Fig. 4.

relay. To de-energize the relay, the voltage at the relay must drop to $^1/_3$ of the value required for full attraction. Assuming d_1 to be applied opposite the track-transformer, the voltage impressed on the rails must be reduced to $^1/_3$ of p, which obviously will reduce the current in the track circuit in the same proportion, i. e. to $^1/_3$ of u.

If e_o and i_o denote the voltage and current in the rails at the relay end, and p_o and u_o the corresponding current and voltage in the rails at the transformer end, the release of the relay will occur with:

$$e_o \leqq \frac{1}{3} \times 5.2; \quad i_o \leqq \frac{1}{3} \times 2.0$$

$$p_o \leqq \frac{1}{3} \times 7.7; \quad u_o \leqq \frac{1}{3} \times 5.2$$

Consequently we may assume:

$e_o = 1.7$ volts	30°
$i_o = 0.65$ amp.	0°
$p_o = 2.5$ volts	40°
$u_o = 1.7$ amps.	22°

In fig. 4 we now plot:

$OA_1 = u_o$ and $OB_1 = p_o$.

The current u_o causes in the series resistance a voltage drop $B_1 D_1 = 1.7 \times 1.6 = 2.7$ volts in phase with $u_o = OA_1$. The rest of the total voltage drop required to reduce the voltage from 16 volts at the transformer secondary to 2.5 volts at the rails must be caused by the current passing through the wheel axle shunt d_1, which we will now at first assume to be applied between the rails at the transformer end of the track circuit. On account of the nature of this shunt, the shunt current may be assumed to be in phase with the impressed voltage $p_o = OB_1$. The vector of the voltage drop $D_1 C_1$ caused by the shunt current may therefore be plotted in parallel with OB, its length being determined by OC_1 having to be 16 volts.

The diagram gives us $D_1 C_1 = 10.7$ volts.

As the series resistance was 1.6 ohms, the shunt current will be $\frac{10.7}{1.6} = 6.7$ amps.

A wheel axle resistance $d_1 = \frac{2.5}{6.7} = 0.37$ ohm will thus be required for release.

Completing now also the current diagram in fig. 4 by plotting the vector of the shunt current $A_1 E_1 = 6.7$ amps. in phase with the voltage OB, we get the total current $= OE_1 = 8.3$ amps.

Thus the increase of current

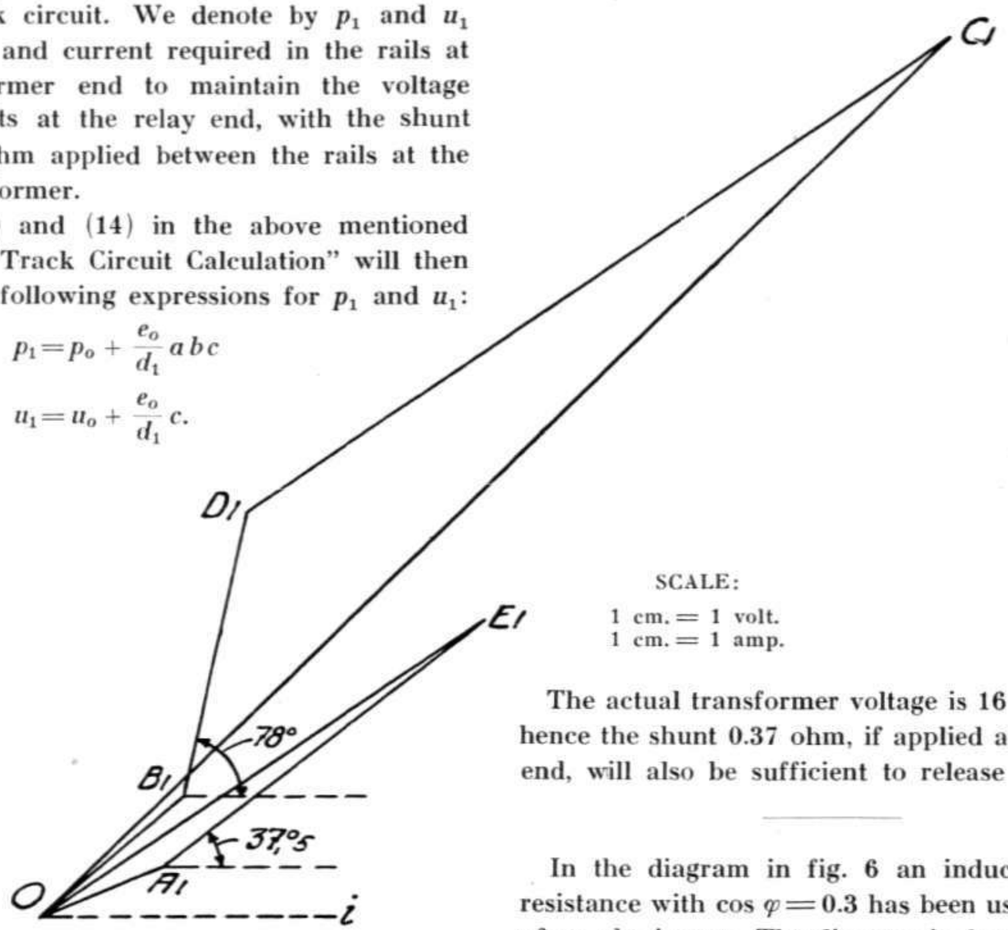
$$o = 8.3 - 5.2 = 3.1 \text{ amps.}$$

The object of the diagram drawn in fig. 5 is to show the effect of the same shunt $d_1 = 0.37$ ohm, applied between the rails at the relay end of the track circuit. We denote by p_1 and u_1 the voltage and current required in the rails at the transformer end to maintain the voltage $e_o = 1.7$ volts at the relay end, with the shunt $d_1 = 0.37$ ohm applied between the rails at the relay transformer.

Equ. (13) and (14) in the above mentioned article "On Track Circuit Calculation" will then give us the following expressions for p_1 and u_1:

$$p_1 = p_o + \frac{e_o}{d_1} a\, b\, c$$

$$u_1 = u_o + \frac{e_o}{d_1} c.$$



R 2091 Fig. 5.

Thus, with the current i as reference axis:

$$p_1 = p_o + \frac{1.7}{0.37}\underline{|30^\circ} \times 1.23\,\underline{|22.5^\circ} \times 0.6\,\underline{|18^\circ} \times$$
$$\times\, 1.15\,\underline{|7.5^\circ} = p_o + 3.8\,\underline{|78^\circ}$$

$$u_1 = u_o + \frac{1.7}{0.37}\underline{|30^\circ} \times 1.15\,\underline{|7.5^\circ} = u_o + 5.3\,\underline{|37.5^\circ}$$

As before, the vectors $OA_1 = u_o$ and $OB_1 = p_o$ are plotted.

To OA_1 we add the vector $A_1 E_1 = 5.3$, its phase being 37.5° in advance of the reference axis i.

From the diagram we get:

$u_1 = OE_1 = 7.0$ amps.

To OB_1 we add the vector $B_1 D_1 = 3.8$, its phase being 78° in advance of the reference axis i.

The voltage drop in the series resistance is now plotted as the vector $D_1 C_1 = 7 \times 1.6 = 11.2$ volts, in phase with the current vector OE_1.

The vector OC_1 will then be the voltage required at the track transformer. From the diagram we get $OC_1 = 16.7$ volts.

SCALE:
1 cm. = 1 volt.
1 cm. = 1 amp.

The actual transformer voltage is 16 volts, and hence the shunt 0.37 ohm, if applied at the relay end, will also be sufficient to release the relay.

In the diagram in fig. 6 an inductive series resistance with $\cos\varphi = 0.3$ has been used instead of an ohmic one. The diagram is drawn in analogy with fig. 4, except that the voltage drop in the series resistance is always drawn at an angle of 72° in advance of the current causing the voltage drop.

The diagram first gives us $BC = 10.2$ volts, from which we get the series resistance $= \frac{10.2}{5.2} =$ $= 1.9$ ohms.

As the final result we get $D_1 C_1 = 11.8$ volts, and the shunt current $\frac{11.8}{1.9} = 6.2$ amps.

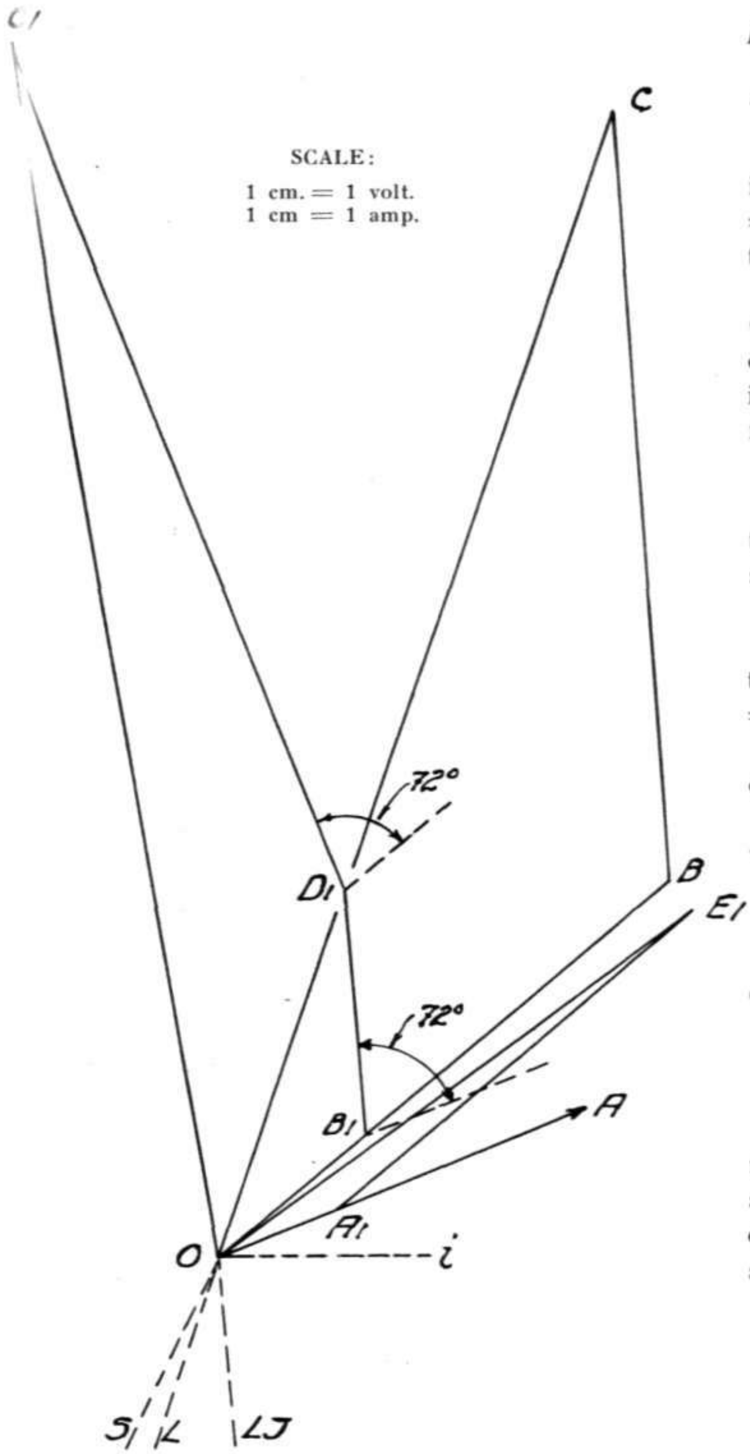
Heuce the wheel axle resistance required for releasing the relay:

$$d_1 \leqq \frac{2.5}{6.2} = 0.40 \text{ ohms.}$$

The total current $OE_1 = 7.7$ amps.

Consequently the increase of the current $o = 7.7 - 5.2 = 2.5$ amps.

Fig. 7 is showing the diagram when the shunt $d_1 = 0.40$ ohm is applied at the relay end.



R 2092

Fig. 6.

We have, as in fig. 5:

$$p_1 = p_o + \frac{e_o}{d_1}\,abc$$

$$u_1 = u_o + \frac{e_o}{d_1}\,c$$

Hence:

$$p_1 = p_o + \frac{1.7 \times 1.23 \times 0.6 \times 1.15}{0.40}\,\underline{\vert 78^\circ} = p_o + 3.6\,\underline{\vert 78^\circ}$$

$$u_1 = u_o + \frac{1.7 \times 1.15}{0.40}\,\underline{\vert 37.5^\circ} = u_o + 4.9\,\underline{\vert 37.5^\circ}$$

The diagram is obtained in the same way as in fig. 5, except that the voltage $D_1\,C_1 = 6.5 \times 1.9 = 12.4$ volts is now drawn 72° in advance of the current $OE_1 = 6.5$ amps.

In this case also we find the sought voltage $OC_1 > 16$ volts and consequently a shunt of 0.4 ohm will be sufficient to release the relay even if applied at the relay end instead of the transformer end of the track circuit.

We will now assume that the total current from the track transformer can be kept constant in spite of the shunting of the track, i. e. at 5.2 amps.

Fig. 8 again shows the vector $OA_1 = u_o$.

The vector $A_1\,E_1$ of the shunt current is plotted in phase with the voltage p_o, making $OE_1 = 5.2$.

From the vector diagram we then get the shunt current $A_1\,E_1 = 3.6$ amps.

The shunt resistance d_2 required for release will thus be

$$d_2 \leqq \frac{2.5}{3.5} = 0.71 \text{ ohm}.$$

Assuming instead the shunt applied at the relay end, we have as before

$$u_1 = u_o + \frac{e_o}{d_2}\,c.$$

The current $\frac{e_o}{d_2}\,c$ is in the diagram, fig. 8, represented by the vector $A_1\,E_2$ being 37.5° in advance of the reference vector i. The length of AE_2 is determined by $u_1 = OE_2$ being 5.2 amps.

From the diagram we get:

$$AE_2 = \frac{e_o}{d_2}\,c = 3.5.$$

Consequently $d_2 = \dfrac{1.7 \times 1.15}{3.5} = 0.56$ ohm.

With constant current, the wheel axle resistance required for release will thus, even in the most unfavourable circumstances, be considerably higher than the one required when using a track transformer with constant voltage.

Some methods of obtaining constant current by a combination of inductive and condensive reactances in series are discussed in Steinmetz: "Theory and Calculation of Alternating Current Phenomena."

In fig. 9 a) and b), two devices taken from Steinmetz book are shown, by means of which a constant A. C. voltage t impressed between 1 and 2, can be transformed into a constant current y_1 between 3 and 4, irrespective of the impedance of the current consumer, provided that the condenser and the inductive resistance of the device are selected so that $\omega L = \frac{1}{\omega C}$, i. e. that current resonance obtains between the inductive and condensive reactances L and C.

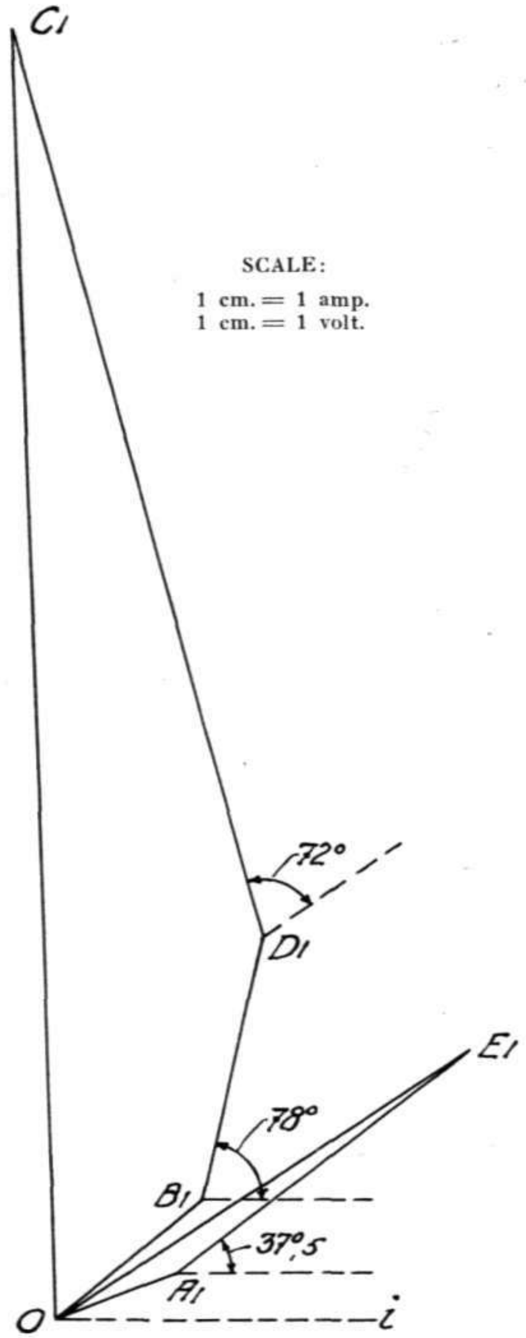
The value of the constant current y_1 depends on the voltage t and the magnitude of the reactances, according to the formula

$$y_1 = \frac{t}{\omega L} = t \omega C.$$

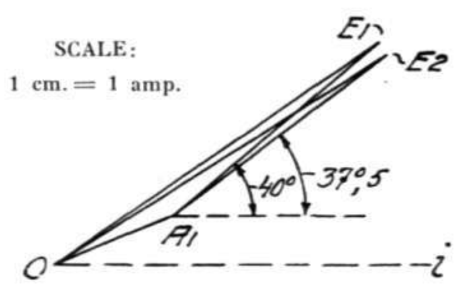
With an impressed voltage $t = 110$ volts, 50 cycles, a capacity $C = \frac{5.2}{110 \times 2 \times 3.14 \times 50} = 0.000150$ farad would thus be necessary for obtaining the constant current $y_1 = 5.2$ amps. Consequently the condenser will be of a magnitude which makes the arrangement according to fig. 9 rather unsuitable for direct practical use in connexion with track circuits.

By *inserting a current transformer* between 3 and 4, as shown in fig. 10 a) and b), the size of the condenser can be reduced in proportion to the ratio of the current transformer. A considerable portion of the constant current in the primary of the current transformer, however, is now consumed for magnetizing the transformer. This magnetizing current varies, on account of the variations in the primary voltage of the current transformer, and the current y_1, transformed to the track circuit, can therefore no longer be constant.

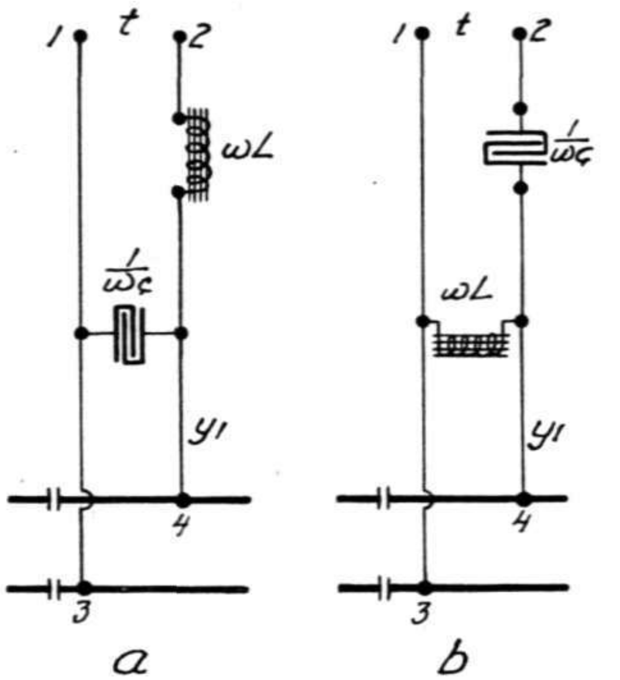
A method more fitted for practical purposes is obtained by the modification of the fig. 10 b) arrangement shown in fig. 11, utilizing the primary winding of the track transformer as inductive reactance. The current which according to fig. 10 b) passes through the reactance ωL is according to fig. 11 utilized for the magnetization of the iron core of the transformer. If the selfinduction of the transformer is chosen to make current resonance with the capacity reac-



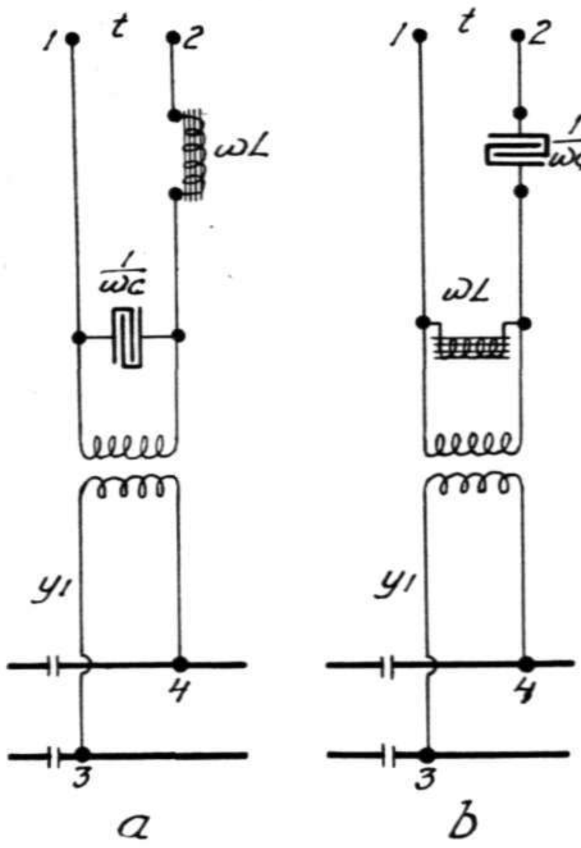
R 2093 Fig. 7.



R 2094 Fig. 8.

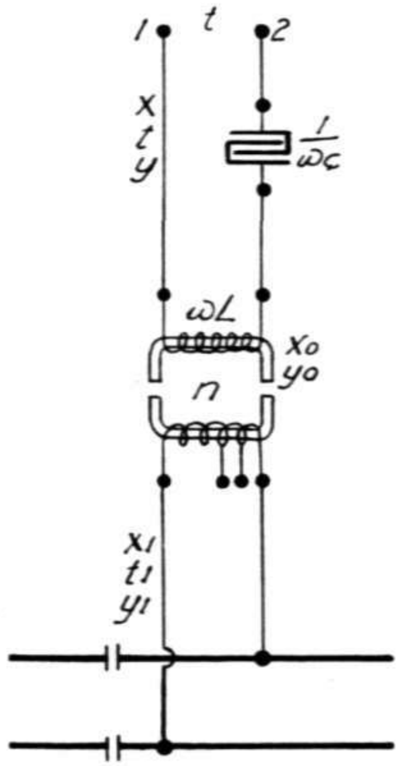


R 2095 Fig. 9.



R 2096 Fig. 10.

tance connected in series, it should be possible to obtain a constant current from the secondary winding of the current transformer. The amount of this current is determined by the voltage impressed between 1 and 2, as well as the ratio of the transformer, and can without changing any other part of the arrangements be regulated by using different taps on the secondary winding. The current is decreased by connecting more windings and increased by utilizing a smaller number of windings in the secondary of the transformer.



R 2097 Fig. 11.

To explain the functioning of the arrangement in fig. 11, the following symbols, shown in fig. 11, are introduced.

x = the total impedance of the primary circuit 1—2 with the secondary winding short-circuited. Thus x is the vector sum of the condenser reactance, the line resistance, and the short-circuit impedance of the transformer.

x_0 = the reactance of the track transformer with the secondary winding open.

x_1 = the total impedance of the secondary circuit.

n = ratio between the number of the primary turns to the number of secondary turns.

t = the voltage impressed between 1 and 2.

t_1 = the secondary current.

y = the primary current.

y_o = the magnetizing current.

y_1 = the secondary voltage of track transformer.

The following relations apply:

$$y_1 = \frac{t_1}{x_1}$$

$$y_o = \frac{nt_1}{x_o}$$

$$y = \frac{y_1}{n} + y_o = nt_1\left(\frac{1}{n^2 x_1} + \frac{1}{x_o}\right)$$

$$t = nt_1 + xy = nt_1\left(1 + \frac{x}{n^2 x_1} + \frac{x}{x_o}\right)$$

$$t_1 = \frac{t}{n\left(1 + \frac{x}{n^2 x_1} + \frac{x}{x_o}\right)}$$

$$y_1 = \frac{t_1}{x_1} = \frac{nt}{x + n^2 x_1\left(1 + \frac{x}{x_o}\right)}$$

If the ohmic and inductive components of the impedance x are assumed to be small compared to the condensive reactance, we may put $x = \frac{1}{j\omega C}$, where C = the capacity, ω = the frequency of the alternating current, and $j = \sqrt{-1}$.

For the track transformer, we may also put $x_o = j\omega L$, where L = the self-inductance of the primary winding.

Hence:

$$\frac{x}{x_o} = \frac{1}{j\omega C \times j\omega L} = \frac{1}{\omega^2 CL}$$

$$y_1 = \frac{nt}{x + n^2 x_1\left(1 - \frac{1}{\omega^2 CL}\right)}$$

If $\omega^2 CL = 1$, i. e. with series resonance between the condensive and inductive reactances, we get $y_1 = \frac{nt}{x}$. Therefore y_1 is constant for a given voltage t, condenser reactance x and ratio n. x being a condensive reactance, the phase angle of y_1 is rotated 90° in relation to the impressed voltage.

In figs. 12 a), b) and c) the functioning of an arrangement for constant current transformation according to fig. 11 is illustrated by means of vector diagrams. The diagrams a), b) and c) are plotted for the same values of n, t, and $x = x_o$, but for different, arbitrary values of x_1. The diagram will be of the shape indicated in fig. 12 a), b) and c) respectively, according to whether x_1 is an inductive, condensive, or a purely ohmic impedance. The current y_1 obtained in the secondary circuit is in all three cases of the same magnitude, and its phase is 90° behind the voltage t.

In plotting the diagrams, the given voltage t has first been drawn, and afterwards the vector xy, representing the voltage drop in the condenser, has been plotted at an arbitrary angle to t. The vector nt_1, representing the voltage drop in the primary winding of the transformer, is then obtained by combining the vectors xy and t.

x_o being given, the magnetizing current $y_o = \frac{nt_1}{x_o}$ can be plotted at an angle of 90° to nt_1. The current vector y is further plotted 90° in advance of the voltage xy, and its size is determined by dividing the magnitude of the arbitrarily chosen vector xy by the given x.

By combining the vectors y and y_o, the vector $\frac{y_1}{n}$ is obtained and, n being given, also the vector y_1 which is n times larger than $\frac{y_1}{n}$ and of opposite direction.

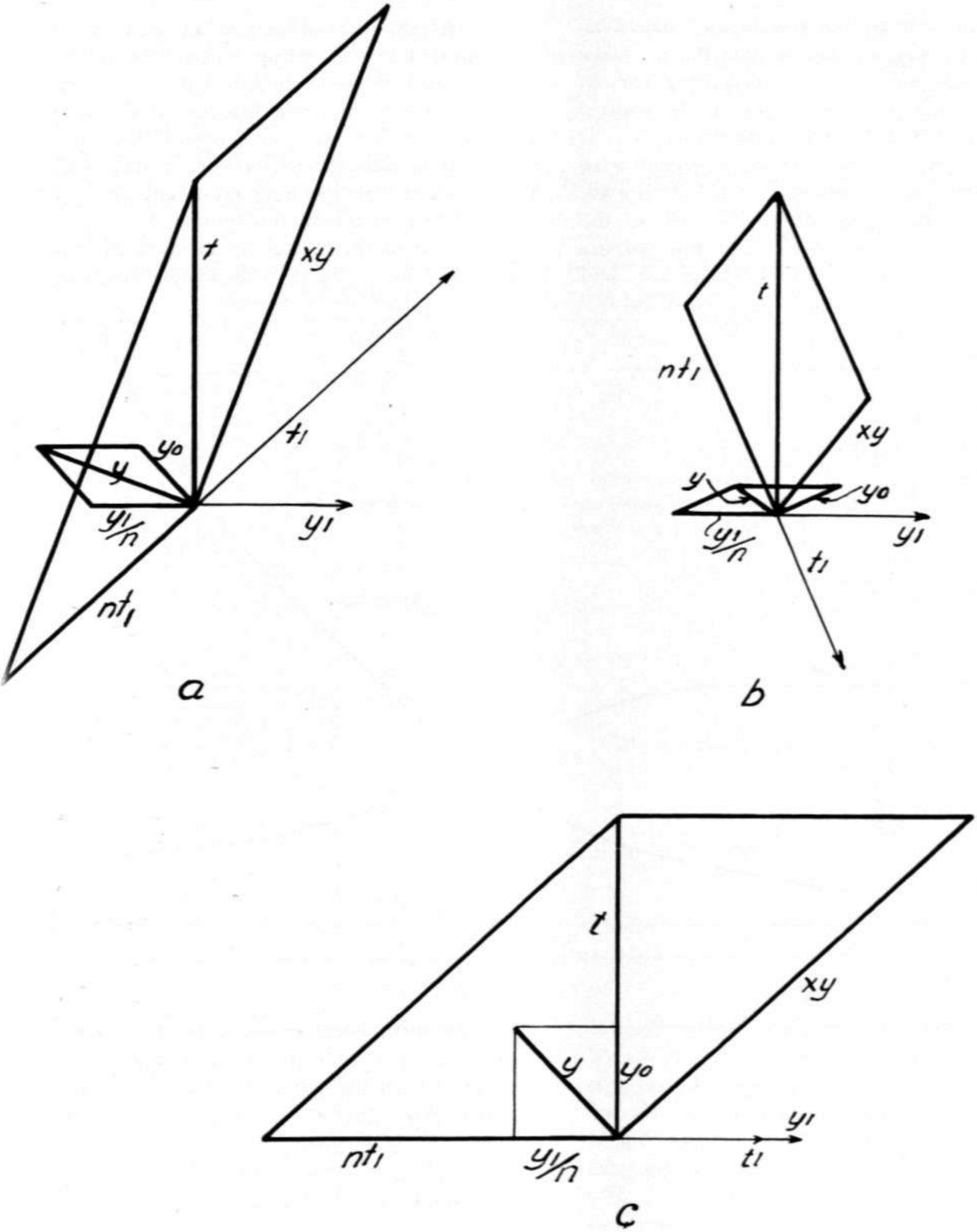
As $\frac{xy}{y} = \frac{nt}{y_o} = x$, and the vectors y and y_o respectively are at right angles to xy and nt_1 respectively, the vector $\frac{y}{n}$, composed of y and y_o, must form an angle of 90° with t, and its magnitude bear the same relation to the magnitude of t as y to xy.

Hence
$$\frac{y_1}{nt} = \frac{y}{xy}$$

$$y_1 = \frac{nt}{x}$$

The magnitude and phase of y_1 are thus constant for given values of the impressed voltage, the condenser reactance, and the ratio of transformation.

On account of unavoidable losses by the ohmic resistance in leads and transformer windings,



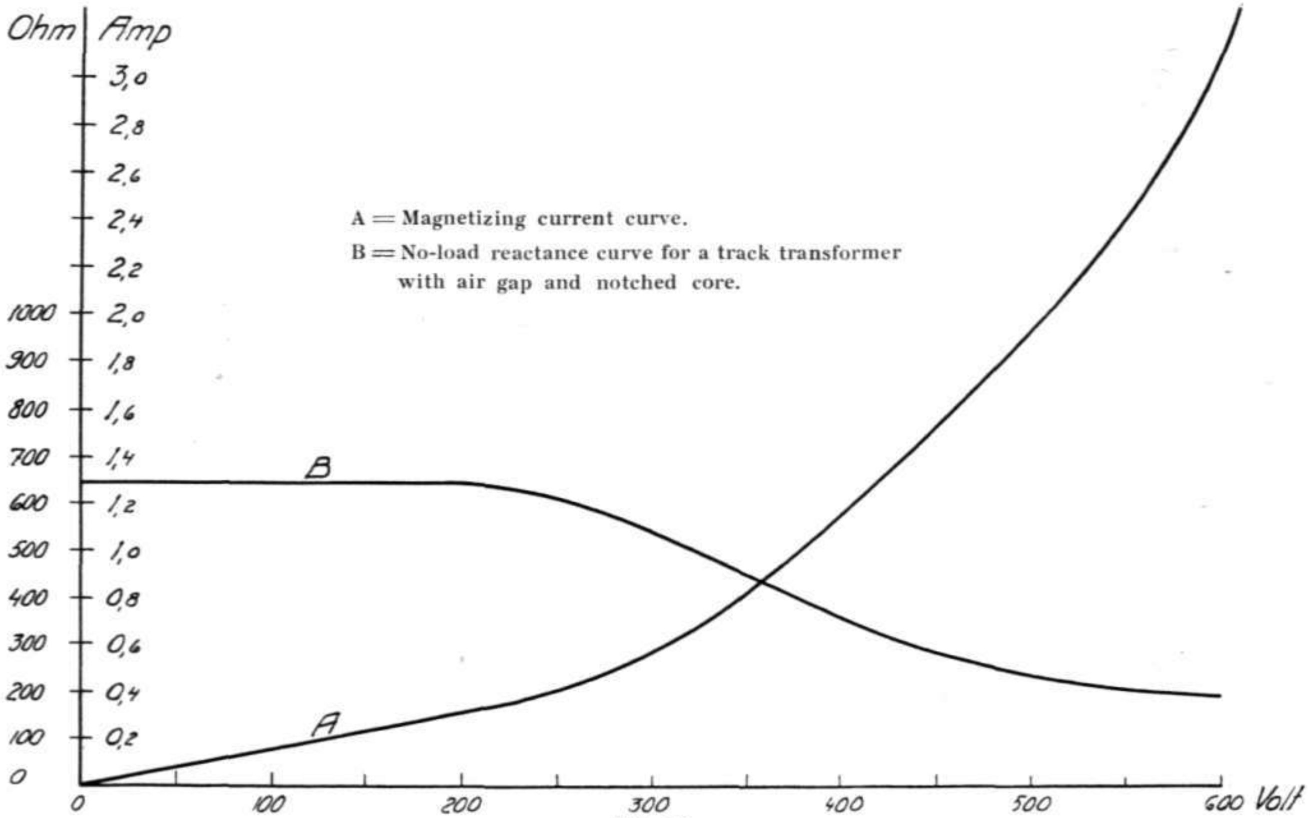
R 2098 Fig. 12.

magnetic leakage in the transformer, etc., absolutely constant current y_1 cannot be obtained in practice. The calculations, however, indicate how the device should be arranged to attain the best possible result.

The condenser should have a high reactance in order to neutralize the influence of the ohmic resistances on x. A high impressed voltage t is therefore an advantage, as a smaller condenser may then be used.

The iron core of the transformer should be provided with an air gap, so that the reactance will be constant, and the magnetizing current thus proportional to the voltage of the primary winding within sufficiently wide limits.

The air gap also retains the sinusoidal wave form of the current, which is of importance for the functioning of the device, in view of the demand for series resonance. For this purpose the iron losses should also be kept low, and magnetic saturation be avoided within the voltage limits for which constant current is desired.



R 2099

Fig. 13.

Due to the resonance between the capacity of the condenser and the inductance of the primary winding, however, high voltages will be produced at the condenser and the transformer when the secondary circuit is open. This rise of the voltage is limited by the transformer not being capable of retaining its reactance when the voltage becomes too high. In order to limit the rise of the voltage more exactly, the iron cores of the transformers designed for this purpose are equipped with a notch, so arranged that the iron in the notched section becomes saturated when the primary voltage reaches such values that constant current will no longer be required. After the iron in the notched section has become saturated, the section will function as an additional air gap, and the reactance of the transformer is rapidly altered. The resonance with the condenser will therefore cease, and the rise in the voltage is automatically limited.

The curve marked *A* in fig. 13 illustrates the values at various voltages of the magnetizing current in a transformer designed on the above mentioned principle. The curve marked *B* shows the alterations of the no-load reactance at various voltages. The diagram indicates that a rapid increase of the magnetizing current and a reduction of the reactance starts at about 250 volts in the primary winding.

Fig. 14 shows the same track circuit as fig. 1, provided with constant current feed.

A condenser of 5 microfarad is used. At 50 cycles we then get:

$$x = \frac{1}{2 \times 3.14 \times 50 \times 5} \times 10^6 = 640 \text{ ohms.}$$

An impressed voltage $t = 220$ volts gives

$$\frac{y_1}{n} = \frac{220}{640} = 0.35 \text{ amp.}$$

To obtain the necessary amount of current $y_1' = u = 5.2$ amps., a transformer ratio of

$$n = \frac{y_1}{0.35} = \frac{5.2}{0.35} \sim 15.$$

is required.

The diagram gives us the vector

$$OC = t_1 = 12.7 \text{ volts.}$$

The vector

$$nt_1 = 15 \times 12.7 = 190 \text{ volts}$$

is marked off in the opposite direction to OC.

$$\frac{y_1}{n} = \frac{5.2}{15} = 0.35 \text{ amp.}$$

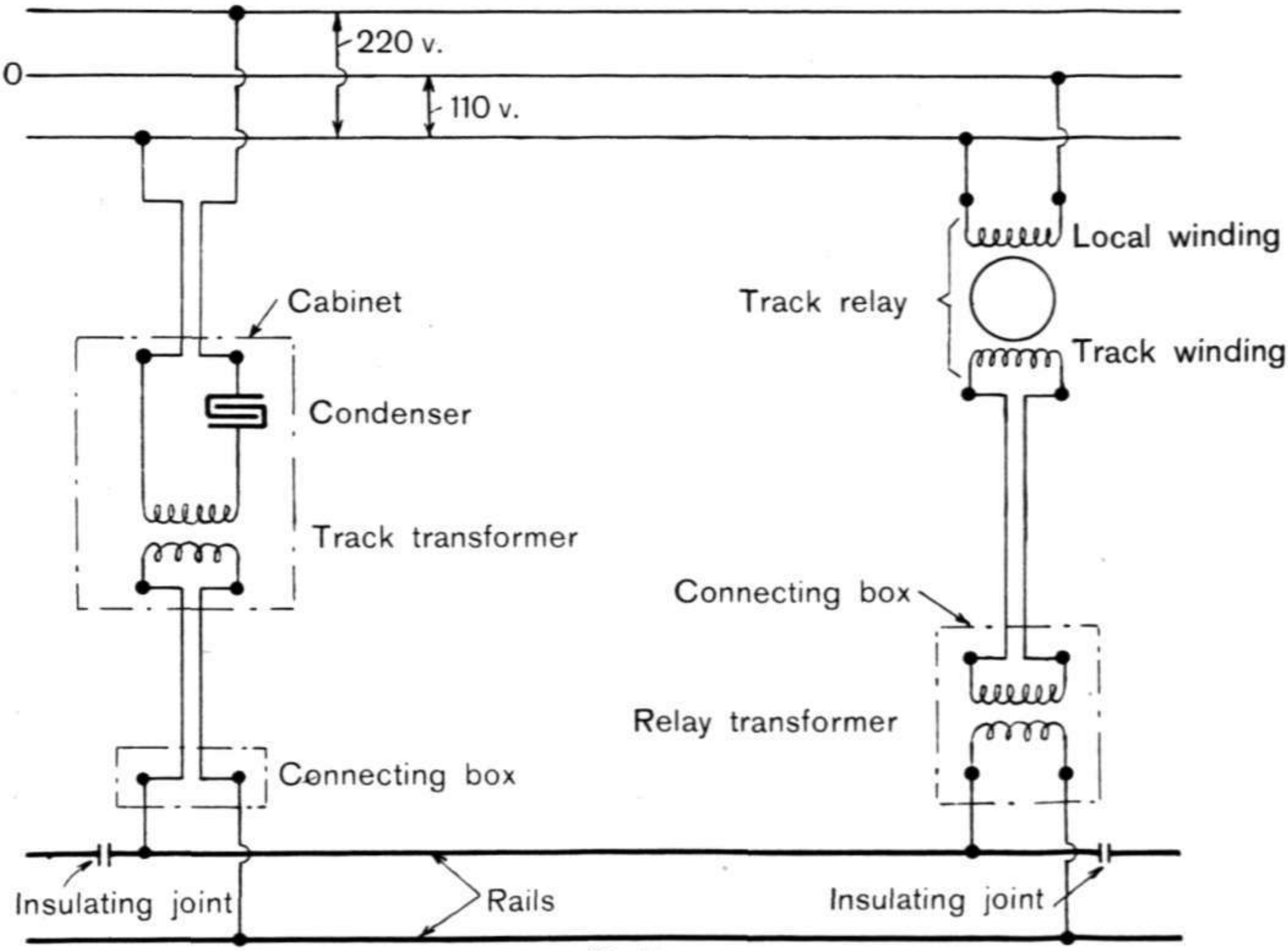


Fig. 14.

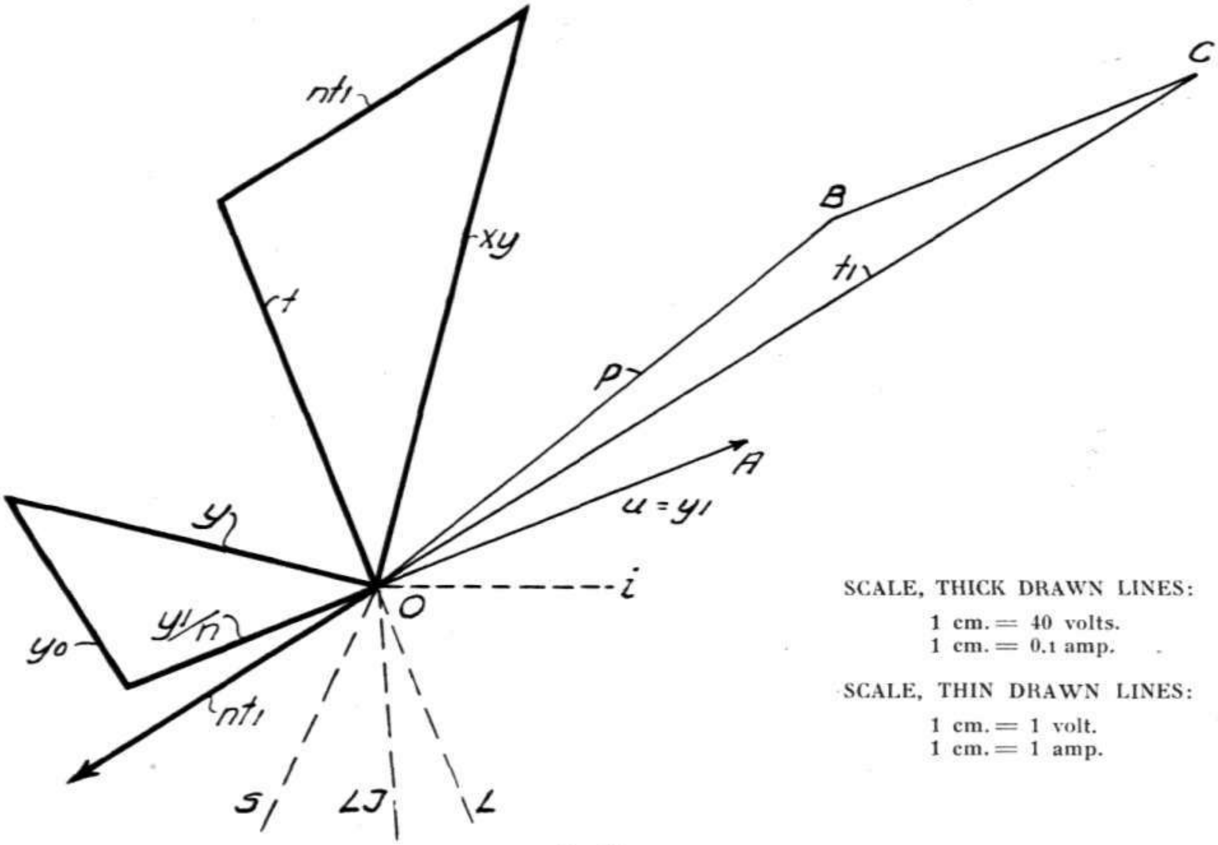
Fig. 15 shows the vector diagram of the track circuit, using the values of u and p already calculated for fig. 2. OA and BA are representing the vectors of u and p, with the relay transformer current i as a reference axis. BC is the voltage drop in the secondary of the track transformer including the leads to the rails, and is therefore plotted in phase with the current. If the resistance in the winding and the leads is assumed to be 1 ohm, we get:

$$BC = 1 \times 5.2 = 5.2 \text{ volts.}$$

The vector of the magnetizing current, $y_o = \frac{nt_1}{640} = \frac{190}{640} = 0.30$ amp., is then plotted 90° behind the voltage vector nt_1.

The diagram now gives us $y = 0.5$ amp., and the voltage drop $xy = 640 \times 0.5 = 320$ volts is marked off 90° behind the current y.

By combining the vector of the condenser voltage xy with the vector of the transformer voltage nt_1, the vector for the input voltage t is obtained. As expected, this last vector will be



SCALE, THICK DRAWN LINES:

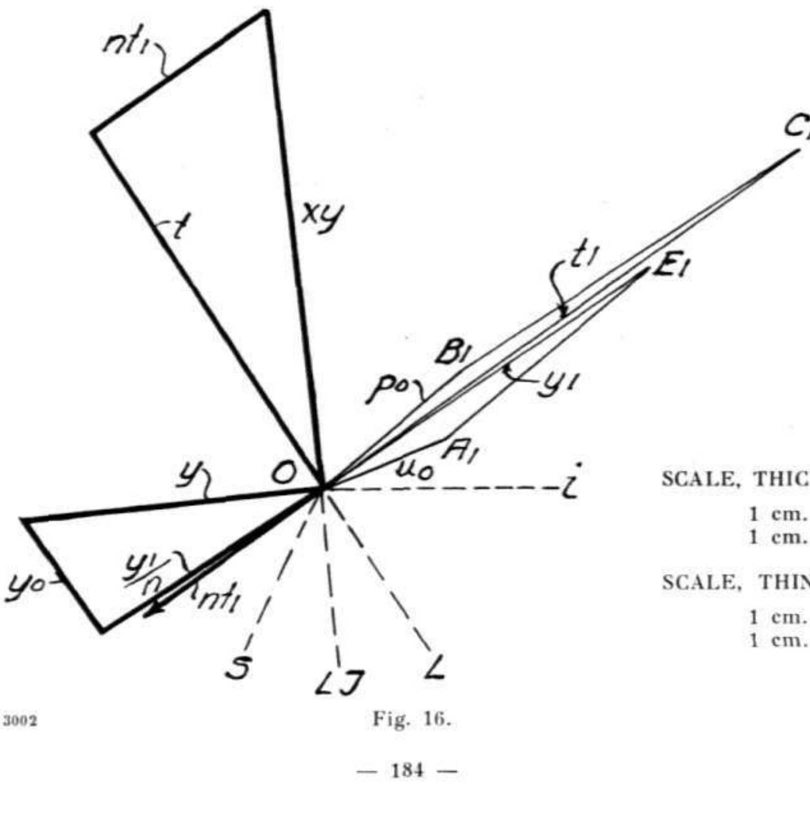
1 cm. = 40 volts.
1 cm. = 0,1 amp.

SCALE, THIN DRAWN LINES:

1 cm. = 1 volt.
1 cm. = 1 amp.

R 3001

Fig. 15.



SCALE, THICK DRAWN LINES:

1 cm. = 40 volts.
1 cm. = 0,1 amp.

SCALE, THIN DRAWN LINES:

1 cm. = 1 volt.
1 cm. = 1 amp.

R 3002

Fig. 16.

220 volt, its phase being 90° in advance of the current y_1.

Fig. 16 shows the appearance of the vector diagram when the track is shunted at the transformer end so that the track relay is released, i. e.

$$p_o = 2.5 \text{ volts. } \underline{|40°}.$$
$$u_o = 1.7 \text{ amps. } \underline{|22°}.$$

We draw the vectors $OA_1 = u_o = 1.7$, and $OB = p_o = 2.5$, with the current i as reference axis. The shunt current $A_1\,E_1$ is then plotted in phase with p_o, and so that $OE_1 = y_1 = 5.2$.

The voltage drop in the leads and in the track transformer secondary, $B_1\,C_1 = 1 \times 5.2 = 5.2$ volt, is plotted in phase with OE_1.

The diagram gives $OC_1 = t_1 = 7.7$ volts.

We now draw the vectors of

$$nt_1 = 15 \times 7.7 = 115 \text{ volts.}$$

$$y_1 = \frac{5.2}{15} = 0.35 \text{ amp., and}$$

$$y_o = \frac{115}{640} = 0.18 \text{ amp.}$$

The diagram gives us the vector $y = 0.40$ amp., and $xy = 640 \times 0.40 = 256$ volts is now plotted at right angles to y.

t is now obtained by combining nt_1 and xy. As expected, we get $t = 220$ volts and its phase is 90° in advanee of y_1.

If we now assume that the track transformer and the local relay winding are fed from the same power mains, as is the case when a single phase A. C. supply is used, the voltage vector of the local winding will have a direction opposite to that of the vector OC in figs. 4 and 6 and to the vector t in figs. 15 and 16. From fig. 4 we find that the phase angle between L and the ideal phase position LI is 60° à 70°. A more suitable phase deflection can of course be obtained by connecting the track circuit and the local winding to different phases, e. g. two different feed lines of a 3-phase power supply.

From the diagrams of figs. 6 and 15, it is evident that a suitable phase deflection between the voltages of the local and the track windings of the relay is also obtained when a single phase supply is connected.

The vector diagram of fig. 15 shows that the reference axis i is leading the local voltage. If the condenser is short-circuited, the voltage of the track winding of the relay rotates forward about 90°. This will cause a reversal of the torques acting in the relay, and will consequently release the track relay. Defective insulation causing a short-circuit in the condenser can thus not cause an improper attraction of the track relay, but is immediately announced by the release of the relay.

A two element relay receiving power in two separate windings being used, it is important that a proper phase angle is obtained between the currents in the two relay windings, designated "local" and "track" in figs. 1 and 14. For the type of relay used in the previous examples, the most suitable phase deflection between the currents is obtained when the track voltage is about 30° behind the local voltage. The efficiency of the relay is then highest. When the phase deflection is altered, this deflection from the most efficient angle must be compensated by increasing the supply of power.

With the aid of the diagram in fig. 2, we have indicated in the vector diagrams of figs. 4, 6, 15, and 16 the phase position of the relay track voltage S, as well as the ideal phase position LI of the relay local voltage. The difference in phase between S and LI is 30°.

We are now going to compare the power consumption of a track circuit using a constant voltage transformer with the power consumpation of the same circuit when fed from a current transformer.

According to fig. 4, $5.2 \times \frac{16}{110} = 0.76$ amp. at 110 volts, $\cos \varphi_S = 0.99$, is used for the track feed.

The relay local winding takes 0.6 amp. at $\cos \varphi_L = 0.78$.

Consequently the total current from the mains

$$\sqrt{(0.6 \cos \varphi_L + 0.76 \cos \varphi_S)^2 + (0.6 \sin \varphi_L + 0.76 \sin \varphi_S)^2} = \sqrt{1.49 + 0.23} = 1.31 \text{ amps.}$$

$$\cos \varphi_N = \frac{1.22}{1.31} = 0.93.$$

Volt-amps. consumed $= 1.31 \times 110 = 145$.
Watts consumed $= 0.93 \times 145 = 135$.

According to fig. 6, $5.2 \times \frac{16}{110} = 0.76$ amp. at 110 volts, $\cos \varphi_S = 0.64$, is used for feeding the track.

Thus the total consumption from the mains

$$= \sqrt{(0.6 \cos \varphi_L + 0.76 \cos \varphi_S)^2 + (0.6 \sin \varphi_L + 0.76 \sin \varphi_S)^2} = \sqrt{0.92 + 0.90} = \sqrt{1.82} = 1.35$$

$$\cos \varphi_N = \frac{0.96}{1.35} = 0.71.$$

Volt-amps. consumed $= 1.35 \times 110 = 149$.

Watts consumed $= 0.71 \times 149 = 106$.

According to fig. 15, 2×0.5 amps. $= 1.0$ amp. at 110 volts, $\cos \varphi_S = 0.57$, is used.

The total consumption from the mains

$$= \sqrt{(0.6 \cos \varphi_L + 1.0 \cos \varphi_S)^2 + (0.6 \sin \varphi_L - 1.0 \sin \varphi_S)^2}$$

$$= \sqrt{1.08 + 0.20} = \sqrt{1.28} = 1.13 \text{ amps.}$$

$\cos \varphi_N = \frac{1.04}{1.13} = 0.92$ (the current is leading the voltage).

Volt-amps. consumed $= 1.13 \times 110 = 124$.

Watts consumed $= 0.92 \cdot 124 = 114$.

The above examples indicate that it is possible by employing constant current to attain considerable reduction in the consumption of volt-amps., obviously because of the interaction between the condensive load in the track transformer and the inductive load in the local winding of the track relay. Should the condensive current, as in the above example, be predominant, this is as a rule no drawback, as the excess is absorbed by other inductive loads connected to the same mains, such as relays, choke coils, etc., the result being an improved power factor for the whole supply system.

An example of the advantages of using constant current was obtained from the plant at the Stockholm South Station, where 14 track circuits with frequency-selective track relays type L were provided with devices for constant current feed. The track circuits were previously provided with inductive series resistances according to fig. 1. The alteration was made in order to raise the release values of the track circuits with the object of increasing the margin of safety.

To obtain fully comparable measurements before and after the alteration, the release value for one of the longest track circuits was measured immediately before and after the change-over, with only about a 10 minute interval between the two measurements, so that the ballast leakage and the line voltage should be exactly the same on both occasions. Before the change, i. e. with a constant voltage transformer and inductive series resistance, the voltage at the track winding of the track relay was measured to be 7.6 volts, and at the local winding 107 volts. The front contacts of the relay opened 1.5 à 2 mm when the rails were shunted with 0.30 ohm. After the change over to current feed, the relay voltages were 8.1 and 107 volts respectively, but the release of the relay now occurred when the rails were shunted with 0.65 ohm.

By the alteration, the power consumption of the whole plant, including daylight signals as well as point indication relays and other relays, was reduced from 2 450 volt-amps., $\cos \varphi = 0.76$, to 2 020 volt-amps., $\cos \varphi = 0.90$. The difference would have been larger if the track relays had not previously been adjusted to receive the least possible power from the track. After the change-over to constant current, the power in the track windings could be increased by about 10 per cent without inconvenience to the shunting qualities, whereby a desirable increase of the attraction and contact pressure of the relay armatures was obtained.

Track circuits with constant current input had

Fig. 17. Track relay, relay transformer, track transformer, and condenser.

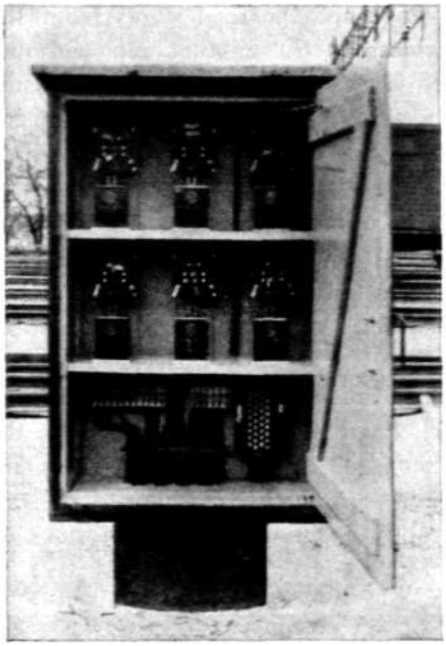


Fig. 18. Cabinet with track transformers and condensers.

earlier been put in use at the Gothenburg Central with 70 track circuits, at Hallsberg with 45, at Abisko with 8, and at Kiruna with 7 track circuits. The total number of such track circuits is consequently today 139, including the Stockholm South, the circuits varying in length from a few tens of metres up to about 500 metres. Frequency-selective relays type L with 6 front and 2 back contacts are used in all cases. Only one rail of the track circuit is insulated, as the other rail of the track must be continuous, to serve as return for the electric traction which uses alternating current of a frequency of 15 or $16^2/_3$ cycles.

The shunt values are found to vary between 0.65 and 1.1 ohm, which result would not have been possible to obtain with these relays with the former supply system using constant voltage and series limiting resistance.

The power factor of the Gothenburg plant has proved to be practically 1 and for Hallsberg 0.98, in both cases for the whole plant, i. e. including daylight signals, point-indication relays, etc. For comparison may be mentioned that the power factor in other similar plants, but with the track feed according to fig. 1, has proved to be 0.70—0.80.

The majority of the track circuits in use are short, and do not require as much current as has been assumed in the examples above. For the convenient supply of the proper amount of current to the track circuit, the transformers have been provided with several taps on the secondary side, so that the ratios 15, 11, 9, and 7.7 can be obtained. A tap for fine adjustments has also been arranged, so that these ratios can be altered to 12.5, 10, 8.5, and 7.2. When an impressed voltage of 220 volts and a 5 microfarad condenser is used, the current can thus be varied from $\frac{15 \times 220}{640} = 5.2$ amps. at a ratio of 15, to $\frac{7.2 \times 220}{640} = 2.25$ amps. at the lowest ratio.

The track transformers and their condensers are mounted in ordinary wooden cabinets placed

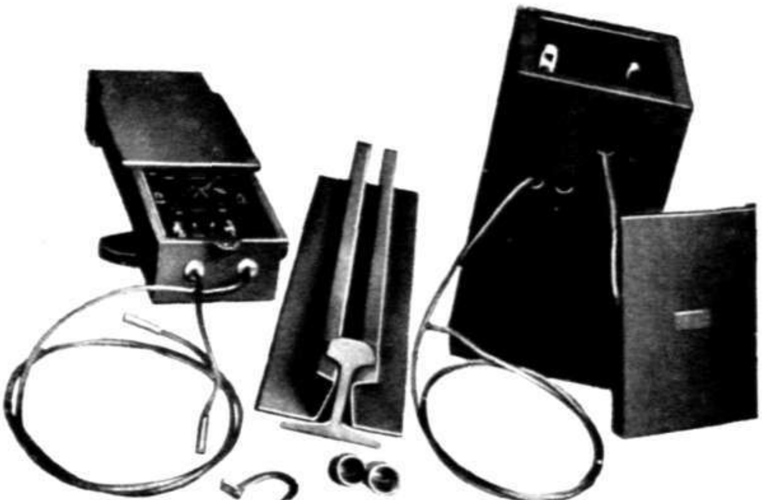


Fig. 19. Cast iron box with relay transformer and leads, fibre parts for an insulating joint, rail bond for welding, and cable connecting box with leads.

in the open air. For a number of adjacent track circuits a common cabinet is used, in which the supply mains are available. A 2-wire armoured underground cable for each track circuit is laid down from this cabinet to a connecting box placed close to the track, where the cable ends and the cable wires are connected each to its rail by a bare copper wire welded or soldered to the rail. The relay transformers are fitted in cast iron boxes close to the track. The primary winding is connected directly to the rails by bare copper wires. From the secondary winding two wires are then first led to a cable distribution box common to several track circuits, from which a joint multicore cable leads to the signal cabin.

The length of the feed wires from the track transformers to the cabinet can without inconvenience vary considerably when constant current is used, as the current required in each case can be regulated within wide limits, both by selecting a suitable tap on the track transformer secondary and by using different impressed voltages. It has therefore met with no difficulty to assemble all the track relays in the signal cabin, which makes the whole arrangement simpler and more easily surveyed, and gives a higher efficiency and a quicker functioning of the track circuits than is possible with the track relays mounted in detached cabinets near the track and repeated by separate relays in the signal cabin.

Investigations regarding mutual induction in parallel conductors earthed at the ends.

By *G. Swedenborg.*

The problem of the mutual induction in parallel conductors earthed at the ends is of great theoretical and practical interest. It is not an infrequent occurrence that induced voltages of considerable magnitude may be caused in communication lines by earthing faults in power lines. As an example we may mention that a case recently occurred in Sweden where tensions of 2 000 à 3 000 volt were repeatedly generated from a power line in a bunch of parallel telephone lines. The cause was flash-overs to earth in the power line. The risk of such voltages occurring in lines of communication is obvious. The consequences in this case were fortunately limited to the blowing of some safety fuses and more or less protracted interruptions of communications. In any case it is of course important to be able to determine fairly accurately the effect of a certain inducing current, so that the risk can be ascertained and suitable steps taken. The mathematical treatment of the problem is rather complex, as the earth enters as part of the electromagnetically connected circuits. The secondary current phenomena appearing in the earth have thus a considerable effect, and are in their turn dependent on the electrical properties of the earth, of which little is known, at least at the depths here in question. To arrive at approximate formulæ of practical use for the computations, it is therefore necessary to have recourse to simplifying hypotheses.

Earlier, the return currents in the earth used to be considered concentrated to a line underneath the inducing line. If the depth of this line was chosen equal to the heigth of the power line above the earth's surface, far too small values of the mutual induction were obtained. When the return currents were assumed to be concentrated to a greater depth, however, it was established that this was not independent of the distance between the inducing conductor and the line in which the induction arose.

The results obtained from measurements in various parts of Sweden have indicated that an equivalent depth of several km. must be selected for the return current to obtain agreement between the measured and the computed values. We see thus that the electrical properties of the earth's crust at considerable depths should be of importance.

The formula given by Breisig in his wellknown work "Theoretische Telegraphie"

$$m = 2\ (\log \frac{2\,l}{a} - q) \cdot 10^{-4}\ H/\text{km},$$

where $l =$ the parallel length
$a =$ the distance between the conductors
$q =$ a constant > 1,

was at one time used as a standard for the computations. The constant q was intended to replace the figure 1 in the expression for the mutual induction between two parallel single conductors without return conductor:

$m = 2\ (\log \frac{2\,l}{a} - 1) \cdot 10^{-4}\ H/\text{km}$; ($l$ is supposed to be large in relation to a)

The object of introducing q was thus to make allowance for the effect of the return currents in the earth.

In practice, however, it was found that a very different q had to be selected in different cases in order to obtain agreement with the measured results. In Sweden the best agreement was generally obtained with $q = 2.5$ to 3.5.

In Germany it was found that very varying q-values had to be chosen in different places. In some instances a value as large as 7 had to be used. But the size of q affects the result of the calculation considerably as subtraction has to be made from a logarithmic term, the numerical value of which is fairly small. The formula could therefore hardly be considered practical for the pre-determination of the induced effect.

When the International Committee for long distance telephony (CCI) was formed in 1924, the problem of disturbances in communication lines from power lines was made a special group among the questions to be dealt with. For computing the induction from a conductor earthed at its end, the formula

$$m = \frac{0.004}{\sqrt{a}} \ H/\text{km}$$

was suggested, where $a =$ the distance between the conductors in metres (the maximum value of the distance at which the formula should be valid was fixed to 1 000 metres).

The measuring results in Sweden as a rule indicated 3 to 4 times larger induction than that given by this formula. The method could therefore not be considered reliable. As regards the dependence on the distance between the conductors, the formula further gave an incorrect picture of the actual conditions, as, when the conductors are close together, the mutual induction is not in reality reduced as rapidly with the distance between the lines as indicated by the root expression, while at great distances conditions are reversed.

In the measuring method so far described, no consideration has been given to the dependence on the frequency. According to recent researches, however, the mutual induction is a function of the frequency also. A better basis for computations had to be found by other means. In some investigations published by Breisig in 1925, the induction effect from a conductor of finite length was computed on the assumption that the conductor was totally surrounded by a homogeneous medium of a certain electrical conductivity. The case presents a certain analogy with that of a conductor, surrounded by a metal cover, for which it is desired to compute the compensating effect of the current in the cover in case of induction effects from the outside. The problem of the compensating effect of cable-cover currents is as a matter of fact supposed to have suggested Breisig's new method of computation. Pleijel has also discussed the problem of compensation in cables in the September number of "Teknisk Tidskrift" 1923 and in a separate detailed treatise printed in 1925.

An investigation of the subject by Rüdenberg was published at about the same time as Breisig's paper. The inducing conductor is here assumed to be of infinite length and lying in the plane of the earth surface surrounded by a ditch of semicircular section, the radius of which is supposed to imitate the height above ground of the conductor. This arrangement is assumed in order to simplify the calculations relatively to the case of the inducing conductor being at a certain height above the ground. Assuming the magnetic field to be concentric to the conductor, Rüdenberg computes the earth currents, and from those the EMF:s arising at various distances from the inducing conductor.

The results obtained by Breisig and Rüdenberg indicate that the mutual induction ought to be highly dependent on the frequency. The formulæ show that the induction can be expressed as a function of a parameter $a \sqrt{\sigma \nu}$, where a is the distance between the conductors, σ the earth's conductivity, and ν the frequency.

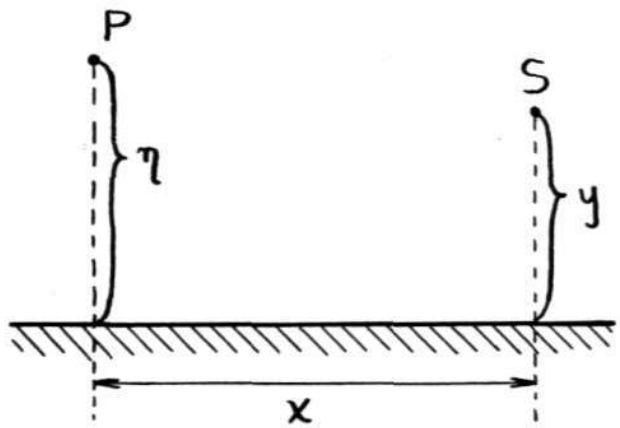
The mutual induction obviously decreases with increasing distance between the conductors. The nature of the parameter immediately shows that an increase of either conductivity or frequency should also have a reducing effect on the induction.

That the induction coefficient actually is highly dependent on the frequency was experimentally proved by a series of systematic measurements undertaken in 1925 by Siemens & Halske at Döberitz, the military training ground in the neighbourhood of Berlin. A number of test lines, 5 km. long and various distances apart (maximum 1 km.) were built here, and the induction was determined at frequencies varying from $16^2/_3$ to 2 000.

The calculations of Rüdenberg give a slightly higher induction effect than Breisig's. Both methods, particularly Breisig's, give a too rapidly diminishing induction when the distances between the conductors are increased, and have in recent years been superseded in the CCI by calculations proposed in two papers by Pollaczek, published in 1926 and 1927.

In these calculations, the inducing conductor is assumed to be of infinite length, and the attenuation in the line is disregarded. In a recently published paper, Pollaczek has shown that to disregard the changes of amplitude and phase angle along the line has no practical effect on the result at low and speech frequencies.

The respective media above and below ground are assumed to be homogeneous. The magnetic field generated by the current in the inducing conductor will cause eddy currents in the earth, originating secondary magnetic fields. Pollaczek



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Symbols: $0, \eta$ = coordinates of the primary line (P).
x, y = coordinates of the secondary line (S).

$k = e^{\frac{3j\pi}{4}}\sqrt{4\pi\sigma\omega}$; σ = conductivity of the earth; $\gamma = 1{,}7811$ (The Bessel constant).

a) for small distances:

$$m = \left\{2\ \log\frac{2}{\gamma\,|k|\,\sqrt{x^2+(y-\eta)^2}} + 1 - j\frac{\pi}{2} - \frac{4\,jk\,(y+\eta)}{3}\right\}10^{-9}\,H/\text{cm};\ \text{Conditions: } |k|\sqrt{x^2+(y+\eta)^2} < 0{,}5;$$

b) for medium distances:

$$m = \left\{-\frac{4}{k^2x^2} + 2\,j\pi\,\frac{H_1^{(1)}(|kx|)}{|kx|}\right\}\cdot 10^{-9} = \left\{-\frac{4}{k^2x^2} + 4\,\frac{kei^1(|kx|) - j\cdot ker^1(|kx|)}{|kx|}\right\}10^{-9}\,H/\text{cm};\ \text{Conditions: } \left\{\begin{array}{l} |kx| \leq 3; \\ \dfrac{y+\eta}{x} < 0{,}05; \end{array}\right.$$

c) for large distances:

$$m = \left\{2\log\sqrt{\frac{x^2+(y+\eta)^2}{x^2+(y-\eta)^2}} - \frac{4}{k^2}\cdot\frac{x^2-(y+\eta)^2}{[x^2+(y+\eta)^2]^2} + 4\,jk\,(y+\eta)\left(\frac{1}{k^2(x^2+(y+\eta)^2)} + \frac{3\,x^2-(y+\eta)^2}{k^4(x^2+(y+\eta)^2)^3}\right)\right\}10^{-9}\,H/\text{cm};$$

$$\text{Conditions: } |k|\sqrt{x^2+(y+\eta)^2} > 3{,}5;$$

Fig. 1. Computation, according to Pollaczek, of the mutual coefficient in two parallel single conductors earthed at the ends.

gives differential equations for the electric and magnetic fields and, making certain simplified assumptions, finds by the solution of these equations an expression for the inducing EMF as a function of various influencing quantities. From this the mutual induction coefficient is computed, which is a complex quantity dependent on the frequency, the distance between the conductors, and the conductivity of the earth. In fig. 1, we give some approximate formulæ deduced by Pollaczek for the mutual induction in two parallel air lines.

The expression kx thus proves to be an essentially determinant parameter (analogous to the conclusions of Breisig and Rüdenberg). To facilitate computation, m might therefore conveniently be graphically given as a function of the distance, multiplied by the square root of the product of conductivity and frequency.

At large distances the induction will diminish, according to the Pollaczek computations, in inverse ratio to the square of the distance. When the inducing line is closer, however, the induction will diminish considerably more slowly with the distance.

At about the same time as Pollaczek, Carson also dealt with this problem with similar results. The theory is therefore frequently called the Pollaczek-Carson method.

Computations have since been made by other investigators, some of whom (e. g. Mayr) have assumed that only a certain layer of the earth's crust is conducting.

A practical verification of the Pollaczek computations was considered of great interest. Two extensive series of measurements were carried out in Germany in 1928, one in an Oldenburg fen district, the other in a limestone district at Münsingen near Ulm in Würtenberg. In both places, 5 km. long test lines were built at different distances apart (up to 3 km.). As at Döberitz, measurements were taken at frequencies varying from

$16^2/_3$ to 2 000. Both amplitude and phase angle of the mutual induction were measured by means of excellent instruments and apparatuses fitted in special measuring cars. The measuring equipment had been arranged by the German Telephone Administration in conjunction with Siemens & Halske. The same appliances were later used for the Swedish Skillingaryd tests and are described in detail below.

The results obtained in Germany showed that if, to verify the Pollaczek formulæ, the conductivity according to the observed induction values were computed, constant values were not obtained. Although the conductivity proved to be approximately the same for a given measuring frequency and varying distances between the lines, the conductivity was considerably reduced with rising frequency when varying frequencies and constant distance between the lines were used. The explanation was thought to be that at higher frequencies the earth currents are forced to go nearer the surface, where the conductivity may conceivably be worse than at greater depths. To allow for this, it was suggested that the conductivity should be assumed to be a function of the frequency, and the best agreement was then obtained by assuming inverse proportionality to the square root of the frequency:

$$\sigma = \frac{c}{\sqrt{\nu}},$$

The CCI has recently choosen the function

$$\sigma = 1{,}5 \cdot 10^{-12} \cdot \frac{1}{\sqrt{\nu}} \text{ c. g. s.}$$

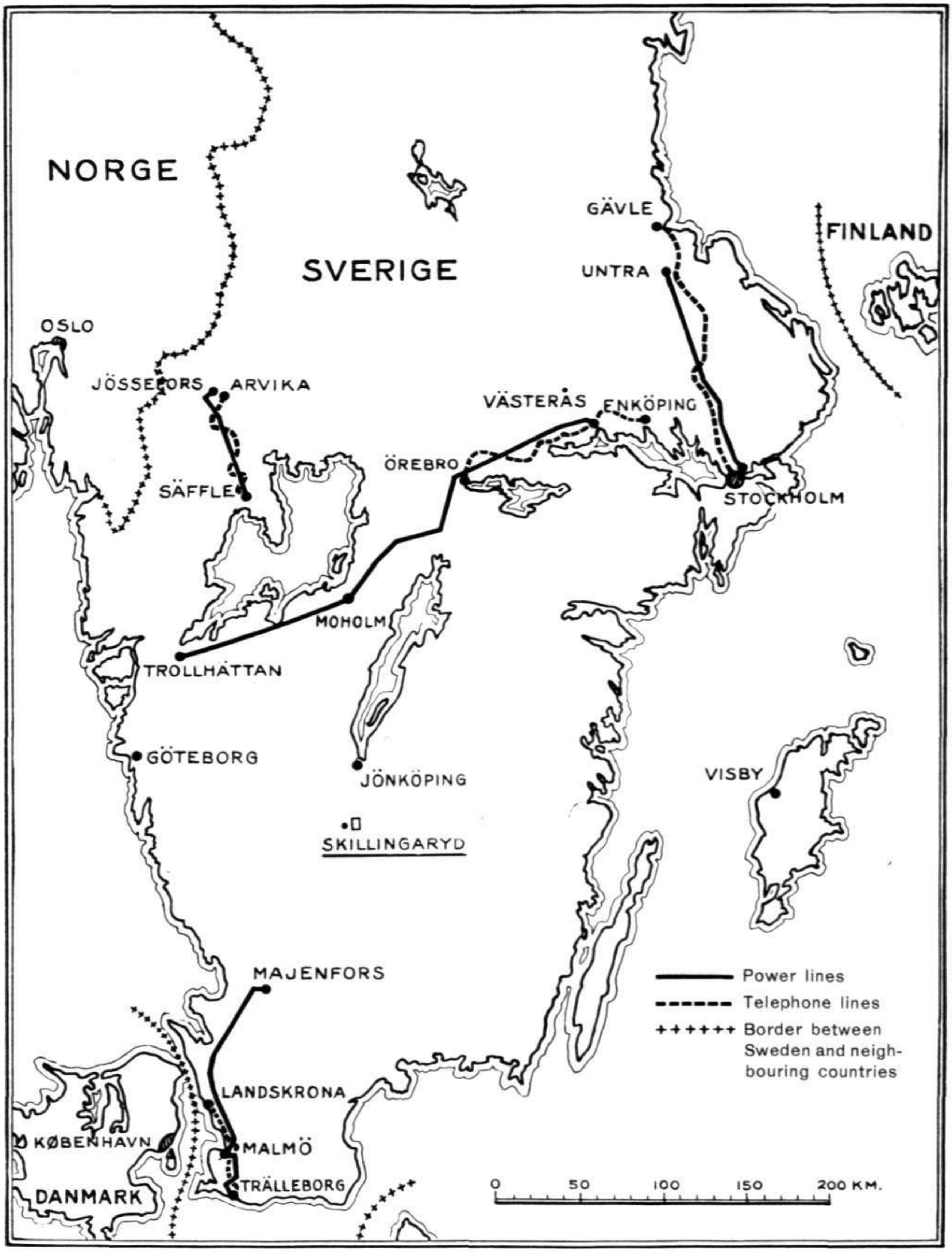
for the conductivity, and has thus arrived at the standard curves for mutual induction given in fig. 17 (the fulldrawn curves). The coefficient in this expression has been determined on the basis of the results of the German measurements.

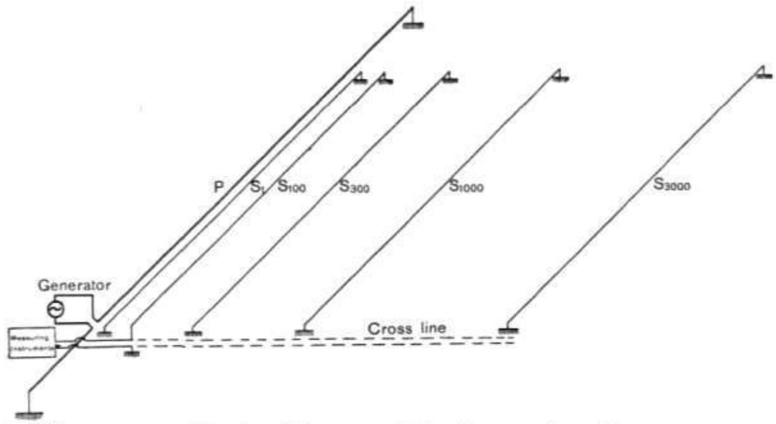
When checking the results of measurements in Sweden by the Pollaczek formulas, the computed conductivity has proved notably low. The mutual induction in this country has in most cases proved to be several times larger than that computed by the CCI curves. The differences are so apparent that a closer systematic investigation was considered justified. It is supposed that the general rocky nature of this country causes a particularly high induction effect. In the spring of 1930 it was decided to make systematic measurements in some convenient place in Sweden under the auspices of CMI, and with test lines and measuring appliances identical with those used in the German measurements in 1928.

A locality had then to be found where test lines might conveniently be put up at such a distance from any power lines that no disturbing influences sufficient to interfere with the measurements need be feared. Attention was directed to the shooting range at Skillingaryd, a place situated 40 km. south of the town of Jönköping. The Skillingaryd range at first proved to be exposed to strong influence from a 3-phase net, although there was only a local 3-kV direct generator-fed power line in the neighbourhood, the strength of which was only a few hundred kW. Fortunately it proved possible practically to eliminate the disturbances by such simple means as breaking the earthing resistance of the generators' zero point in the power station. An example of the significance of the zero-point connexion in the power lines for the disturbing effects on parallel telephone lines was thus provided.

Fig. 2. The lay-out of the test lines at Skillingaryd.

The test lines were put up across the field as shown in fig. 2. The five parallel rows of poles were made perfectly straight. The primary inducing line was put up furthest to the west on ordinary telephone poles. The line consisted of two 3 mm. copper wires placed close together and attached to insulators on hooks. The object


NORGE
SVERIGE
FINLAND
DANMARK
OSLO
JÖSSEFORS
ARVIKA
SÄFFLE
GÄVLE
UNTRA
VÄSTERÅS
ENKÖPING
ÖREBRO
STOCKHOLM
MOHOLM
TROLLHÄTTAN
GÖTEBORG
JÖNKÖPING
SKILLINGARYD
VISBY
MAJENFORS
LANDSKRONA
KØBENHAVN
MALMÖ
TRÄLLEBORG
Power lines
Telephone lines
Border between Sweden and neighbouring countries
0 50 100 150 200 KM.



R 2076 Fig. 3. Diagram of the lines and earths.

of selecting two parallel-connected wires was to reduce the resistance. One metre below this line the first secondary line of 1.5 mm. bronze wire was put up. At various distances (100, 300, 1 000 and 3 000 metres respectively) from the primary line, four 1.5 mm. bronze wires were put up on pole material, kindly supplied by the Swedish Field Telegraph Corps. The primary line was 7 km. and all the secondary lines 5 km. in length. This was done in order to avoid, if possible, the voltage drop of the earth return current of the primary line influencing the measurements of the induced voltages in the secondary line. The two extra lengths of line (1 km. at each end), however, subsequently proved too short for this purpose. The return currents in the earth caused potential differences between the earths of the secondary lines, which were particularly noticeable in the results at low frequencies. The only way to avoid this would have been to make the primary line considerably longer which, however, would have involved considerable expenditure, particularly as very heavily wooded ground beyond the shooting range would then have been entered. When judging the measuring results, it now became necessary to make allowances for the effect of the earth potentials caused by the return currents at low measuring frequencies.

A cross line was put up at right angles to both the primary and secondary lines, as shown in fig. 2. The place for the measurements was situated at the point where this cross line reached the primary line. The cross line, and ordinary twisted 2-wire telephone line of 1.5 mm. bronze wire, served as an auxiliary line between the measuring place and the secondary circuits. A diagram of the connexions used for this purpose is given in fig. 3.

The appearance of the primary pole line is seen in fig. 4. Fig. 5 shows the southern end of the secondary line 100 m. away from the primary line. The picture also shows one of the poles of the cross line. Special wires were arranged

R 2077 Fig. 4. The primary pole line.

R 2078 Fig. 5. The southern end of the 100 m. line. On the cross-line pole seen in the foreground, taps and leads were arranged to facilitate connexions.

on such poles in order to facilitate the different connexions desired.

A resistance of less than 25 ohms was from the outset considered desirable in each of the primary line earths, and of 50 ohms in each secondary line earth. All the earths consisted of star-shaped buried hoop iron (6-rayed star). For a 25 ohm earth about 600 m. of hoop iron were needed, for a 50-ohm earth about half that quantity. A depth of 25 or 50 cm. proved to give approximately the same result, and they were therefore buried only 25 cm. deep. As an experiment, two lengths of hoop iron were placed some distance apart in the same trench, to reduce if possible the length of the star rays to half, but this method did not have the desired effect on account of the diminished area occupied by the star, and was therefore abandoned.

Hoop iron was also used for earths in the earlier German measurements. This was placed in a circular ring with radiating arms. For a given earth resistance, only about $^{1}/_{10}$ of the length of hoop iron needed at Skillingaryd was required. This may be regarded as an indirect sign of the difference in earth conductivity.

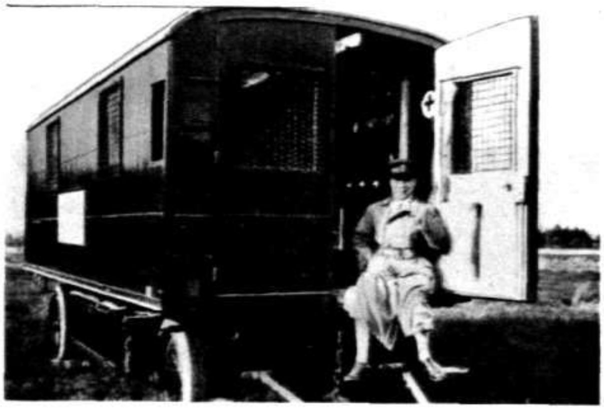
Fig. 5 shows the hoop iron earth at the southern end of the 100 m. line projecting from the ground.

Limitation of the earth resistance was necessary on account of the exactitude required in the measurements.

In connexion with the earth resistances, we might mention that the surface soil at Skillingaryd consists chiefly of sand. Its depth is not known, but is probably comparatively small, as the rocky ground is visible here and there on the range. The rock is gneiss. Skillingaryd lies a few miles west on the boundary line between the granite of eastern Sweden and the gneiss of western Sweden. The geological conditions are of course important, as according to the theories the conductivity of the ground plays an important role in the mutual induction.

As mentioned above, the measuring equipment was the same that was used for the 1928 measurements in Germany. The instruments were mounted in two cars specially adapted for this purpose, and exterior views of these are shown in figs. 6, 7 and 8. In one car (the measuring car), the actual measuring instruments were mounted, and

R 2081 Fig. 6 Exterior view of the measuring car.

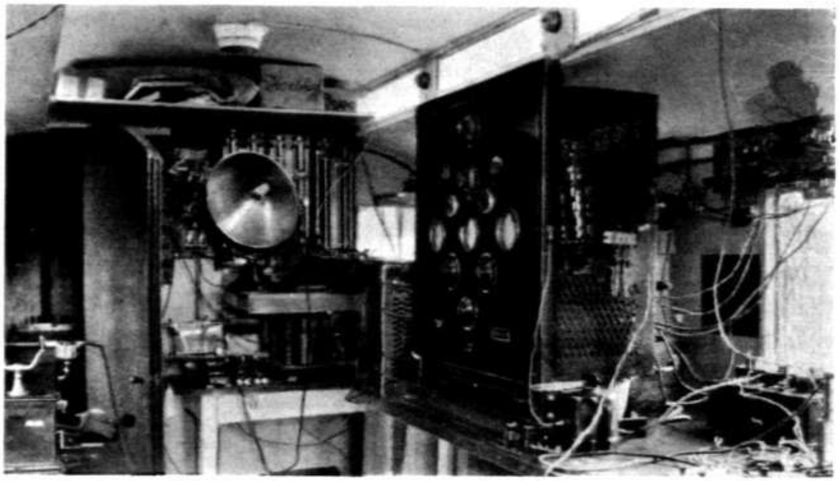


R 2080 Fig. 7. Exterior view of the machine car. Mr. Sterner, who had charge of the building of the test lines, is sitting on the step.

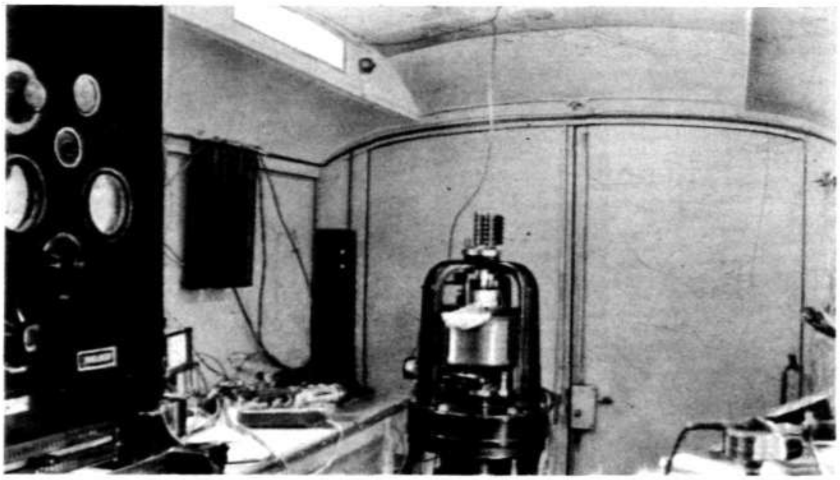
R 2079 Fig. 8. Transport of a car from the railway station to the place for the measurements.

a powerful amplifier was also fitted here to give great amplitude to the inducing voice-frequent currents. The amplifier, with a push-pull coupling, could give a maximum output of c. 200 watt. Figs. 9 and 10 show the interior of this car.

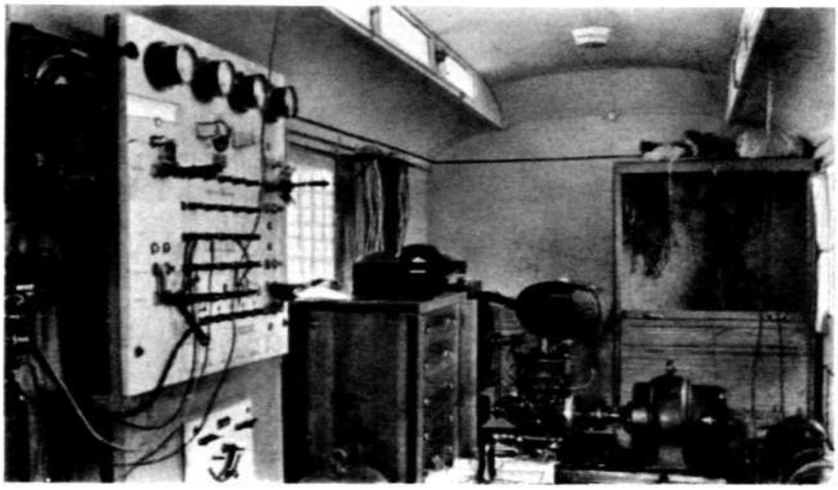
In fig. 10 a Franke-machine, used for measurements at frequencies in the 150—2 000 range, is seen in the centre in front of the door.



R 2082 Fig. 9. Interior view of the measuring car. To the left is the entrance to the dark room, and in the centre the 200 watt amplifier. The oscillograph is placed just behind the electrical radiator.



R 2083 Fig. 10. Interior of the measuring car. The Franke-machine is standing in front of the doors.



R 2084 Fig. 11. Interior of the machine car. In the background the charging set for the 230 volt battery can be seen.

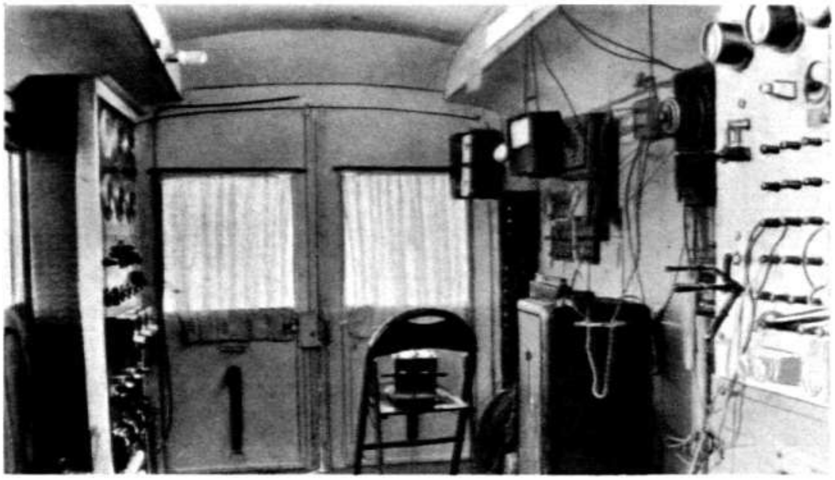
The various sources of current and the instrument board of the amplifier were mounted in the second car (the machine car). The primary current supply consisted of a 220 volt battery of c. 100 amp. hours' capacity, placed in two large boxes underneath the car. The battery was charged by a set comprising a generator coupled to an internal combustion motor. This set is seen in fig. 11. The amplifier instrument board is visible on the left of fig. 12. The amplifier filament current consumption was c. 30 amp., and the anode voltage 1 500 volt. Filament current and anode voltage were obtained from two special motor generators fed by the storage battery.

The inducing alternating currents within the $16\,^2/_3$—300 frequency range were taken from a pair of generators, coupled to motor generator to obtain a convenient regulation of the speed. The one A. C. generator was used for the $16\,^2/_3$—100 cycle range, the other for the 100—300 cycle range.

No external electrical power supply was thus required, which was an obvious advantage considering the risk of disturbing influences.

The two cars were placed 50 m. apart so that the noise and vibration from the machine car should not disturb the observations in the measuring car.

All measurements were made as compensation measurements. For the lower frequencies (up to 300 cycles) two different methods were used. In the one Geyger's compensator (fig. 13) was used. Two vibration galvanometers were alternatively used as zero instruments, one of which was a

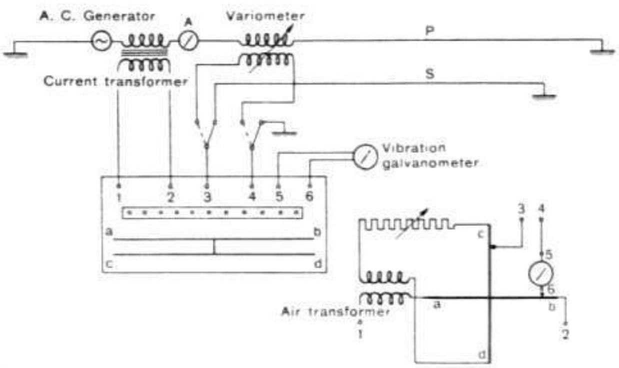


R 2085 Fig. 12. Interior of the machine car. To the left is the instrument board of the two motor generators belonging to the amplifier equipment.

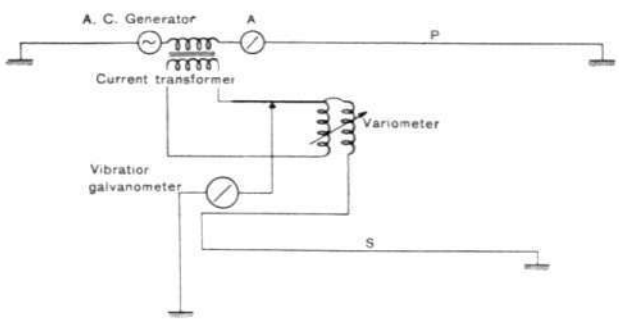
needle galvanometer, electromagnetically tuned according to Schering and Schmidt (for the lowest frequencies up to and including 140 cycles), the other a string galvanometer, mechanically tuned according to Moll (for the remaining frequencies up to and including 300 cycles).

The diagram shows how by means of an air transformer, the secondary circuit of which contained an ohmic resistance of such magnitude that the transformer will act with practically no load, two currents are obtained in the two bridge-wires, the phases of which are mutually deflected 90°. The constants are such that at a current of 0.5 amp. in the primary winding of the air transformer a voltage drop of 1 mV per cm. is obtained in each of the two bridge wires, which are united in the centre. In compensation measurements this arrangement may thus be regarded as a kind of voltmeter by which the voltage sought is conveniently obtained, divided into its components. Each bridge wire is 40 cm. long, and the amplitude of the voltage which can be compensated will obviously be rather limited. The various secondary lines were therefore connected to a potentiometer, by which only a portion of the voltage sought was taken out for compensation. The ratio of the current transformer was selected so that half an ampere was obtained in the secondary circuit from one ampere in the primary line. On account of the low impedance of the compensator arrangements, the transformer is practically shortcircuited, and the phase angle between the currents will therefore be almost 180°. To check this, the voltage from a variometer connected as shown in fig. 13 was also compensated.

In the second method, the Larsen compensation appliance was used, fig. 14. The voltage, adjustable as to amplitude and phase angle, opposing which the voltage sought is connected, is here obtained by means of a bridge wire and a variometer. The same galvanometers as before were used as zero instruments. The use of two



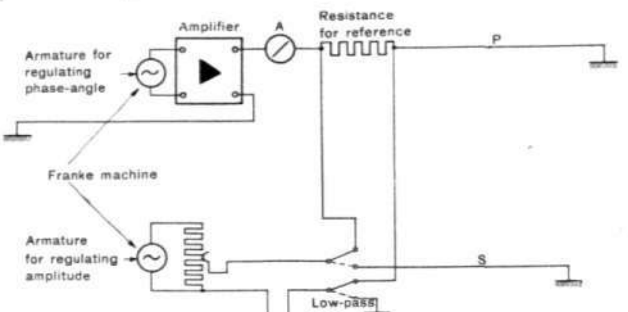
R 2086 Fig. 13. Measuring the mutual induction coefficient by a Geyger Compensator.



R 2087 Fig. 14. Measuring the mutual induction coefficient by a Larsen compensator.

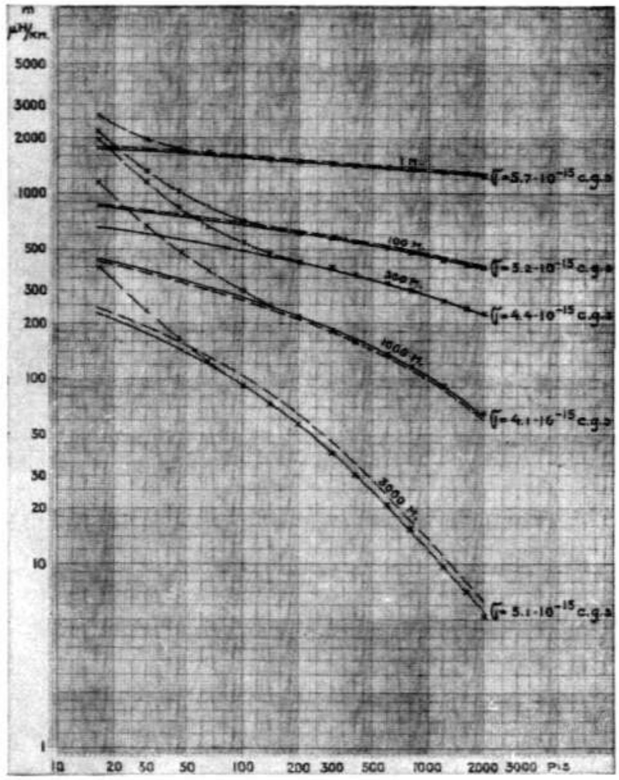
measuring methods for one and the same frequency range gave good control of the results.

For audio-frequencies, when an ordinary telephone receiver could be used as zero instrument, the connexions shown in fig. 15 were used. A Franke-machine supplies the current here. This machine has two mutually alike armatures, the "amplitude" armature and the "phase" armature, driven by a D. C. motor and revolving at an adjustable speed in a multi-polar field. By means of a graduated dial, the "phase" armature can be adjusted relatively to the "amplitude" armature, so that any arbitrary phase angle between the generated voltages can be obtained. The amplitude armature may be raised out of or lowered into the magnetic field, by which means the voltage amplitude can be regulated. An arrangement of this description should obviously be particularly suitable for these measurements. The primary line was fed from the phase armature during the test and the amplifier — indicated in the diagram on fig. 15 — was connected for measurements on the 1000-m. and 3000-m. lines (a powerful inducing current being essential in this case for the exactitude of the measurements). The compensating voltage was taken out from the potentiometer connected to the amplitude armature. In front of the telephone receiver, which was used as a zero indicator, a low-pass filter was introduced to remove the machine harmonics, which would otherwise have disturbed the listening. The phase angle of the induced voltages relatively to the inducing current could not be determined from that angle alone to which the two armatures were mutually adjusted, as the angle of the primary line current relatively to the voltage generated in the phase armature is not exactly known. For this reason a standard



R 2048 Fig. 15. Measuring the mutual induction coefficient by a Franke-machine.

The mutual induction coefficient in parallel conductors at Skillingaryd.



R 3014 Fig. 16. Results of the Skillingaryd measurements, compared with the values computed according to the Pollaczek formulæ.

× Measured values

——— Values computed on the Pollaczek formulæ, with earth conductivity selected to give best agreement with the mesured values.

– – – – Values computed acc. to the Pollaczek formulæ, with earth conductivity fixed at 4.4.10—15 c. g. s.

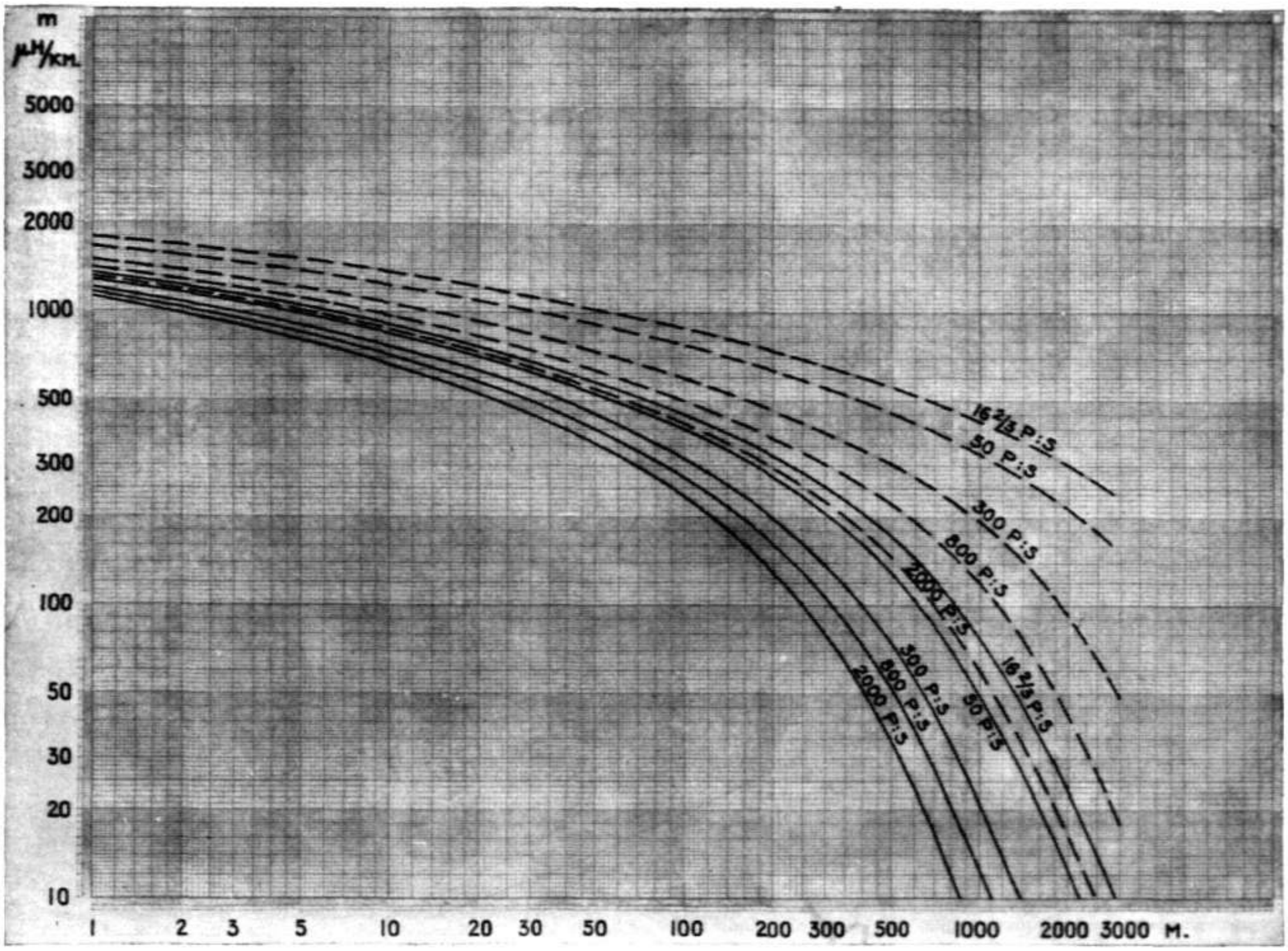
The distance between the lines is given at each curve.

Table 1.

Measured mutual induction values, in μ H/km at various line distances and pequencies.

Frequencies	Mutual inductance at the distance				
	1 m	100 m	300 m	1000 m	3000 m
16²/₃	2617	2147	1959	1186	396
30	1950	1317	1160	692,5	231
45	1740	1007	850	495,5	166
65	1643	842	671	382,5	125
100	1583	728	549	300,0	92,5
140	1554	674	487	253,8	74,0
200	1514	626	439	219,8	58,2
300	1483	581	395	185,0	41,0
400	1453	556	366,4	162,6	30,4
600	1418	519	331	136,6	20,5
800	1382	487	299,7	117,6	15,4
1200	1348	447	261,7	92,8	9,73
1600	1327	423	237,9	77,2	7,06
2000	1316	407	218,2	65,3	5,24

Comparison between the results of the Skillingaryd measurements and the C. C. I's values.



R 3013

Fig. 17. Comparison between the results of the Skillingaryd measurements and the CCI values.
— — — — Skillingaryd measurement. ———— CCI-values.

Table 2.
The quotient between the Skillingaryd results and the CCI-values at various periodicities and line distances.

	1 m.	10 m.	100 m.	300 m.	1000 m.	3000 m.
$16^3/_4$ p:s	1,32	1,49	1,93	2,62	5,54	25,7
50 »	1,29	1,45	1,92	2,67	6,47	28,9
300 »	1,24	1,40	1,87	2,80	8,50	22,2
800 »	1,21	1,34	1,83	2,81	9,00	13,2
2000 »	1,10	1,29	1,78	2,78	8,10	7,6

resistance was connected in series with the primary line, and the voltage over this resistance was separately compensated. The angle sought was then obtained as the difference between the angles observed in the two compensations.

The full series of measurements comprised the frequencies $16\,^2/_3$, 30, 45, 65, 100, 140, 200, 300, 400, 600, 800, 1200, 1600 and 2000. As in the German measurement series, both 50- and 150-cycle frequencies were avoided on account of the risk of interference with possible disturbances from power lines in the neighbourhood.

The results of the measurements are given in the diagrams figs. 16 and 17, and in the Tables.

On the curve sheet, fig. 16, the observed values have been marked by crosses. For every line distance, that earth conductivity has been computed which gives the best agreement between the values measured and the voltages obtained by the Pollaczek formulas. It was then noted that excellent agreement could be obtained at the higher frequencies, and that the values which had to be used for earth conductivity did not differ much in the 5 line distances. The computed values are given in full-drawn curves. At lower frequencies the measured values are dis-

Table 3.

Measured values of induced voltages in the secondary lines, expressed in volt per 1 amp. in the primary line.

Frequency	Induced voltage at distance				
	1 m	100 m	300 m	1000 m	3000 m
$16^2/_3$	1,37	1,12	1,03	0,62	0,199
30	1,84	1,24	1,09	0,65	0,218
45	2,46	1,43	1,20	0,70	0,235
65	3,36	1,72	1,37	0,78	0,255
100	4,97	2,29	1,73	0,94	0,291
140	6,82	2,96	2,14	1,12	0,325
200	9,51	3,93	2,76	1,38	0,365
300	14,0	5,47	3,73	1,74	0,385
400	18,3	6,99	4,61	2,05	0,382
600	26,7	9,77	6,24	2,58	0,386
800	34,7	12,24	7,52	2,96	0,387
1200	50,8	16,85	9,87	3,50	0,367
1600	66,7	23,27	11,95	3,88	0,355
2000	82,7	25,55	13,72	4,10	0,329
	1,23	1,21	1,20	0,675	0,202

Table 4.

Values of the phase angle observed between the inducing primary line current and the induced voltages in the secondary lines.

Frequency	Line distance				
	1 m	100 m	300 m	1000 m	3000 m
$16^2/_3$	144° 35′	163° 35′	168° 40′	171° 10′	174° 0′
30	127° 50′	152° 20′	159° 30′	164° 10′	169° 0′
45	118° 25′	142° 20′	151° 10′	158° 0′	166° 0′
65	111° 20′	133° 30′	143° 20′	152° 0′	164° 40′
100	105° 25′	124° 55′	134° 40′	145° 25′	164° 0′
140	102° 45′	119° 45′	129° 15′	142° 40′	163° 30′
200	100° 45′	116° 15′	125° 30′	140° 0′	166° 15′
300	99° 25′	113° 55′	124° 10′	140° 0′	173° 50′
400	99° 0′	113° 25′	123° 55′	141° 40′	177° 40′
600	98° 40′	113° 30′	125° 10′	144° 50′	183° 40′
800	98° 35′	114° 25′	127° 35′	150° 30′	186° 40′
1200	98° 45′	116° 0′	131° 40′	159° 35′	192° 0′
1600	98° 50′	117° 45′	135° 25′	166° 50′	195° 0′
2000	99° 10′	119° 5′	139° 20′	173° 10′	196° 40′

tinctly higher than the computed ones. The reason for this must be that at low frequencies the earth potentials of the return current, in addition to the pure induction voltages, play an important role. That this is so is distinctly indicated by Table 3, giving the phase angles measured between induced voltage and inducing current. Instead of approaching 90° as in Table 6

Table 5.

Earth conductivity, computed from the mutual induction observed, and expressed in 10^{-15} c. g. s.

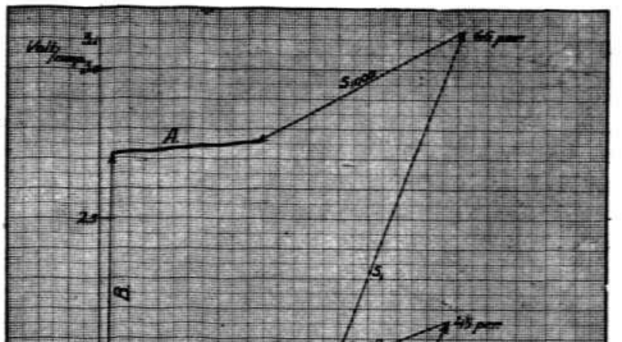
Frequency	Line distance				
	1 m	100 m	300 m	1000 m	3000 m
$16^2/_3$	—	—	—	—	0,76
30	0,5	—	—	0,17	2,7
45	2,9	0,46	0,28	0,83	4,0
65	5,5	1,7	1,1	2,0	5,0
100	6,4	3,6	2,5	3,2	5,4
140	5,6	4,4	3,5	3,9	5,3
200	6,2	5,0	4,1	4,1	5,2
300	5,8	5,2	4,3	4,3	5,2
400	5,6	5,1	4,4	4,3	5,2
600	5,4	5,2	4,3	4,1	5,0
800	5,8	5,5	4,5	4,0	4,9
1200	5,6	5,5	4,5	4,0	4,9
1600	5,2	5,4	4.6	3,9	5,0
2000	4,8	5,2	4,7	4,0	5,3

Table 6.

Mutual induction in μ H/km, computed by the use of an average value of earth conductivity suitable to each line-distance (selected to give the best agrement with the values observed at higher frequencies).

Frequency	Line distance				
	1 m	100 m	300 m	1000 m	3000 m
	Earth conductivity value used in the computation. In 10^{-15} c. g. s.				
	5,7	5,2	4,4	4,1	5,1
$16^2/_3$	1770	863	669	446	226
30	1710	805	610	390	180
45	1670	768	570	351	149
65	1635	731	536	319	123
100	1595	691	495	279	95,5
140	1560	660	465	250	76,2
200	1500	620	431	221	59
300	1480	581	393	189	42
400	1450	555	366	166	31,0
600	1415	519	329	136	20,2
800	1380	490	301	117	14,7
1200	1345	453	265	91,0	9,36
1600	1320	427	241	75,0	6,83
2000	1300	405	222	64,2	5,40

(which is computed according to Pollaczek's theory), the phase angle rises considerably with reduced frequency. To obtain proof that the earth potential of the return current actually has this effect, a special measurement was made, when the induced voltage was measured in a



R 2089 Fig. 18. Vector diagram of the measured voltages in the 1-metre line, the 1000-metre line and the loop.

S_1 = 1-m line values; S_{1000} = 1000-m line values; B = loop values; A = the resultant of the three values observed.

Table 7.

Computed phase angle values between inducing current and induced voltages, when the uniform earth conductivity value $4{,}4 \cdot 10^{-15}$ c. g. s. is used.

Frequency	Line distance				
	1 m	100 m	300 m	1000 m	3000 m
16²/₃	95° 0′	100° 15′	103° 35′	110° 20′	124° 25′
30	95° 10′	101° 0′	104° 55′	113° 15′	130° 10′
45	95° 20′	101° 35′	105° 50′	115° 5′	135° 10′
65	95° 25′	102° 5′	106° 45′	117° 30′	139° 55′
100	95° 30′	102° 45′	108° 10′	121° 0′	147° 0′
140	95° 40′	103° 20′	109° 25′	123° 55′	152° 30′
200	95° 50′	104° 10′	111° 10′	126° 50′	158° 10′
300	96° 0′	105° 0′	112° 35′	131° 30′	165° 30′
400	96° 5′	105° 50′	113° 50′	134° 50′	171° 10′
600	96° 15′	106° 45′	116° 50′	140° 30′	177° 20′
800	96° 25′	107° 50′	118° 30′	145° 0′	180° 5′
1200	96° 35′	109° 5′	121° 40′	151° 45′	181° 45′
1600	96° 45′	110° 15′	124° 5′	156° 35′	181° 20′
2000	96° 50′	111° 20′	126° 10′	160° 15′	180° 40′

rectangular loop consisting of the 1 metre line (S_1) and the 1000 m. line (S_{1000}) and cross lines put up at right angles to these lines to close the loop at the ends, (for this special purpose a 1 km. long tarred wire was laid out across country at the north end.) This loop was thus insulated

Table 8.

Observed induction and resistance values in the 7 km primary line, earthed at the ends.

Frequency	Induction (μ H)	Resistance (ohm)
100	—	55,0
140	—	55,1
300	17,00	55,4
300	16,90	55,9
400	16,85	56,3
600	16,80	58,1
800	16,70	59,5
1200	16,50	63,6
1600	16,55	67,9
2000	16,50	71,2

Table 9.

Observed mutual induction values, in μ H/km, between the primary line and a rectangular loop consisting of the 1-m line, the 1000-m line and cross lines between their ends.

Frequency	Mutual induction	Angle
16²/₃	1331	90° 43′
20	1329	90° 44′
45	1329	90° 44′
65	1326	90° 50′
100	1330	91° 0′
140	1330	91° 30′
200	1325	92° 0′
300	1310	92° 0′
400	1305	92° 0′
600	1305	92° 15′
800	1302	93° 15
1200	1304	94° 15′
1600	1304	94° 25′
2000	1312	95° 20′

from earth. If the potential difference between the earths of either line S_1 and S_{1000} were without significance, a voltage vector diagram of the values observed in S_1, in S_{1000} and in the loop, would form a triangle. But this was not the case, as we see in fig. 18, where vector diagrams are given for the voltages at the four lowest measuring frequencies. The three vectors give a resultant which must be caused by the difference in earth potential. It is very striking that the resultants are of almost the same size and direction as the total potential difference, measured by direct current, of the two cross lines.

In view of this we may assume that the full-drawn curves of fig. 16 give a correct picture of the induction even at low frequencies. When drawing a vector diagram of the voltages thus obtained — with phase angles computed according to the theory (Table 6) — the three vectors of S_1, S_{1000} and the loop respectively, actually form closed triangles, which confirms that the extrapolated curves may represent the true values.

An average value of the earth conductivity was computed for the five line distances. In doing this, the values for the 3000 m. line were first corrected, as the distance between the lines in this case is not small in comparison to the length of line. Pollaczek has given the necessary correction as the quotient of the diagonal of the rectangle of parallelism and the length of the paralellism. After making this correction, the average conductivity value for the 3000 m. line was again computed, and subsequently the final average value of all the five lines was determined. This value was $4{,}4 \cdot 10^{-15}$ c. g. s. In fig. 16, dotted curves indicate the induction coefficients obtained for the several distances when using this uniform conductivity. It will be noticed that these dotted curves are very close to the full-drawn.

When plotting the dotted curves in fig. 17, the uniform value of the earth conductivity has been used. The C C I standard curves have also been plotted in the same diagram for comparison.

The great divergence between the Skillingaryd curves and the C C I curves is very striking. At $16\,^2/_3$ cycles and 3 km. line distance, for instance, the induction observed at Skillingaryd is about 20 times larger than in central Europe. This in its turn means that in Sweden far greater disturbing effects must be expected from power lines in parallel communication lines than in the plains of the Continent. To attain an equal freedom from disturbances, more rigorous steps are consequently required in Sweden. This circumstance is of course economically unfavourable.

The Skillingaryd measurements further show that Pollaczek's theory at least in one instance has been verified without giving the earth conductivity as a function of the frequency. As appears from the above, the earth conductivity then proved to be several times smaller than the values observed in the German measurements. In the same connexion it is of interest that analogous systematic induction measurements have in recent years been made at different places in the United States. Using Carson's computations, values of earth conductivity slightly exceeding the C C I values were then observed. The American measurements indicate that, at least in the places where tests have so far been made, the conditions are similar to those in Germany.

The question might possibly be raised whether the measurements at Skillingaryd may be considered representative of Swedish conditions, or

Date	Power line	Telephone line	Part of Sweden, where the parallelism is	Length of the parallelism	Inducing current	Measured voltage	Computed voltage	
							According to the CCI-curves	According to the Skillingaryd tests
Febr. 1929	Stockholm—Untra	Stockholm—Gävle	Eastern Sweden	132 km	5.4 amp.	29 volt	5.5 volt	34 volt
March 1929	Majenfors—Malmö—Trälleborg	Malmö—Landskrona	Southern Sweden	31 km	30 amp.	48 volt	83 volt	180 volt
March 1929	Majenfors—Malmö—Trälleborg	Malmö—Landskrona	Southern Sweden	27 km	30 amp.	11 volt	26 volt	91 volt
May 1930	Jössefors—Säffle	Arvika—Säffle	Western Sweden	60 km	8 amp.	91 volt	19 volt	65 volt
March 1931	Trollhättan—Västerås	Enköping—Örebro	Central Sweden	87 km	97 amp. at Västerås * 51 amp. at Örebro *	340 volt	20 volt	235 volt

* Capacity current at an arranged earth fault in one phase at Västerås. The physical circuit was then set for a break at Moholm (204 km. from Västerås).

R 2085 Fig. 19. Part of the laboratory compartment ni the State Railway dynamometer car.

The abovementioned measurement of induced voltage from the power line Trollhättan—Västerås was made on March 15th this year in connexion with certain experiments arranged by the Board of Waterfalls to test the quenching of arcs when using Petersen coils in the power line. Artificial flash-overs from one phase to earth were then arranged at Västerås. During the fraction of a second while the arc lasted, a powerful capacity current flowed in the faultless phases. This current caused strong induction in the parallel telephone and telegraph lines, although the average distance between these lines and the power line is several kilometres. The results showed that the induced voltage in the Enköping—Örebro communication lines was no less than 340 volt. On account of its brief duration, this voltage could not be observed by ordinary A. C. instruments, and it was therefore necessary to use an oscillograph. The State Railway Administration "Dynamometer car", the equipment of which includes an oscillograph, was kindly lent for this purpose. The car was placed near the Örebro station, close to a test pole where communication lines suitable for the measurements were available.

Figs. 19 and 20 are photographs of the laboratory compartment of the dynamometer car, taken during the measurements on March 15th.

whether the Skillingaryd field possibly is particularly unfavourable from an induction point of view. Measurements with 25- and 50-cycle currents have in the course of years been made at different places where 3-phase power lines and communication lines run parallel. The induction has thus been measured from the power lines Trollhättan—Västerås, Västerås—Enköping, Uppsala—Enköping, Enköping—Strängnäs, Stockholm—Värtan, Hammarforsen—Sundsvall, Arvika—Säffle and Majenfors—Malmö—Trelleborg. In every case, except the last named, the induction proved to be several times larger than that computed by the C C I curves. To show the divergences, and to what extent they agree with the Skillingaryd results, the computed and observed values for some of the parallelisms are given below. The computation has been done by dividing the parallel stretches into sections, generally 1 km. in length. At crossings a more exact division has been made, the lengths of the sections being then only 100 or 200 m.

These values show that the induction effect in Central Sweden obviously is somewhat larger than in the Skillingaryd district. In south-western Skåne, on the other hand, the induction effect proved strikingly low, and according to the theory the explanation should be that the earth conductivity in that district is particularly good.

R 2086 Fig. 20. The laboratory compartment in the State Railway dynamometer car.

The measurements on the lines Hammarforsen—Sundsvall and Untra—Stockholm mentioned above were carried out both in winter and summer. The results from the first named line were very interesting, in that the mutual induction proved to be c. 20 per cent. larger in winter than in summer. The latter line, however, showed no trace of a seasonal difference. It has been suggested that the particularly high induction in Sweden can be explained by the "more or less frozen state" of the ground. This supposition is of course absurd, as any frozen layer must be infinitely thin compared to the depths that are of importance in this case. The electrical differences at the parallelism Hammarforsen—Sundsvall in winter and summer may possibly be explained by the seasonal difference in water level in the great river near the power line.

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Laying and Fitting of Lead-sheathed Cables.

By *E. Olson.*

The rapidity with which cable manufacturing has developed during the last 15 years has caused quite different demands to be made on the men using the cables than was formerly the case. Outwardly, there is not much difference in the cables, but their internal quality has changed considerably. A careful study of working conditions, by which a complete knowledge of what is required of a good cable has been gained, comprehensive investigations of materials, and improved manufacturing, measuring, and control methods, have made the cables better, more durable, and more adapted to their purpose.

To get the full benefit of all the advantages of a modern cable, the first essential is recognition of the factors having a detrimental effect on the cables, and how to counteract these, a knowledge of which cables and fittings give the best working result in each instance, and how the cables should be laid and fitted to retain their good qualities.

All planning work aims at producing a plant all the parts of which are equally reliable, or, in technical terms: the plant must be of uniform strength. Electrical power lines should also be of uniform strength, but another aspect must be considered here, which is particularly prominent in a power line consisting of both aerial wires and cable. In durability and reliability cables are superior to aerial wires, but any faults occurring in a cable line are more difficult to repair. The following practical rule should be followed when planning a line: The more difficult a repair will be, and the more time it will take, the greater security must be provided against any faults occurring! A submarine cable cannot be repaired in winter in very cold weather, and the odds against faults occurring should therefore be the largest possible.

A well planned and well laid cable line has long been considered the most reliable part of an electrical plant, and if modern materials are used, there is consequently no difficulty in obtaining the desired durability. This applies not only to the cables but also to fittings, i. e. joint and end boxes and distribution heads. The old idea that the boxes are always the weakest points of a cable line is out of date. In this sphere also the labours of recent years have led to considerable progress.

Choice of cables. The copper core is usually dimensioned according to the load, with due allowance for both ohmic and inductive voltage drop. For high powers, one should always examine whether a larger number of small cables would not be more advantageous than a smaller number of heavy ones. The heavier a cable, the worse the heat conductivity, the more troublesome the laying, and the harder the fitting. If the choice is between one heavy and two smaller cables, the latter alternative is usually better, as it generally offers reserve opportunities.

Single phase cables are often preferable to 3-phase in cable lines for from 20 kV upwards. By special methods of armouring and manufacturing it is possible to reduce the losses in armouring and lead sheath to only a small fraction of the copper losses. The single phase cable is more easily cooled, easier to handle in laying, and needs fewer joints. The greatest advantage, however, is that a reserve is obtainable at considerably lower cost than if a 3-phase cable were used. Four single phase cables provide practically the same reserve as two 3-phase cables.

The difficulties of transport usually determine the length of underground cables. A cable drum of a given weight takes a single phase cable three times longer than a 3-phase cable, and the number of joints will therefore be correspondingly reduced. Submarine cables, which should preferably be made in one length, are the only ones where the length is limited by the equipment of the cable factory.

The old method of increasing the reliability

R 1972 Fig. 1.

of a cable line by using cables for higher voltage is nowadays quite unnecessary. The only result is a more expensive plant, where the cables, on account of greater difficulties in cooling, will not stand as large a load.

The protection of the cables outside the lead sheath is selected with due regard to the chemical or mechanical stresses to which the cables will be exposed when laid. Bare lead cable may be used where the cable will be exposed to neither chemical nor mechanical injury. Unarmoured but asphalted cable is laid where chemical but no mechanical damage is expected. Armoured cable is used everywhere else. Band armouring is used when the cable will not be exposed to tensile stresses. If such occur, the cable is armoured with round, flat, or z-shaped iron wire.

When planning a cable line, due allowance should always be made for the fact that heat is produced in the cable, and for the convenient conduction of this heat. When cables are laid in the ground, this should be firm and clean, and the cable should preferably be embedded in a layer of clean gravel. Indoor cable conduits should be roomy and easily accessible for cleaning and inspection.

Laying. Underground cables are either unrolled from a cart or lorry driving along the cable trench, or else they are drawn out from a fixed cable drum. The latter method of laying is the most usual and most costly, and requires more care to avoid damage to the cable. For long and heavy cables, an ample number of laying-rollers should always be provided, and the workmen be given rope nooses to be fixed on the cable as shown in fig. 1.*)

*) This method has been worked out by Mr Laurell of the Stockholm Electricity Works.

The laying should be done so that the cable is bent as little as possible while being put into position. Each time the cable is bent, the lead sheath will stretch, and vacuum blisters will form between the lead cover and the insulation. Ionization and incandescent phenomena, with consequent destruction of the insulating layer, are liable to occur in these.

The temperature of a cable must be at least 5°C. if it is to be bent without risk. This practically precludes the taking up of long cables in the winter, and makes the laying considerably more difficult.

Submarine cables are usually laid from a vessel, on which the cable drum is fixed and well stayed. Powerful braking devices must be provided, so that the running out of the cable can be conveniently controlled. The vessel is moved along the line where the cable has to be laid either by towing, by warping along a rope, or by kedging. These methods of laying are illustrated in figs. 2, 3 and 4. Fig. 5 shows a very long cable stowed in a lighter for laying in a lake, on the same principle as the transatlantic cables. This method of laying is not quite suitable for heavy high tension cables. When loaded in the ship, the cable will be given a turn in each coil, which may damage the comparatively thick insulation of a high tension cable. Rotary drums of the kind shown in fig. 6 are therefore better for such cables. A drum of this kind was used when laying the Öland cable, and it proved excellent, although the total weight of this cable

R 1973 Fig. 2.

R 1974 Fig. 3.

was 180 tons. The drum may rest on either rollers or balls, or else it may be built into a tank, as shown in fig. 7, in which the whole drum with the cable floats, allowing the bearing pressure to be conveniently regulated by raising or lowering the level of the liquid.

Choice of Fittings. Certain general rules may be given for cable fittings. A joint-box must be of the same mechanical and electrical strength as the cable itself. The connexion between the conductors must not alter the ohmic resistance of the conductor, and the boxes must be easily fitted. The insulation must be as like that of the cable itself as possible, so that no "sliding voltages" can arise. The distance between the conductors must be sufficiently strongly fixed to neutralize the impact effect of occurring short circuits. The boxes must be tight, and provided with expansion room inside the box, so that changes of volume of the compound in the box consequent on changes of temperature cannot cause variations of pressure large enough to bring about direct connexion between the insulation in the box and the surroundings.

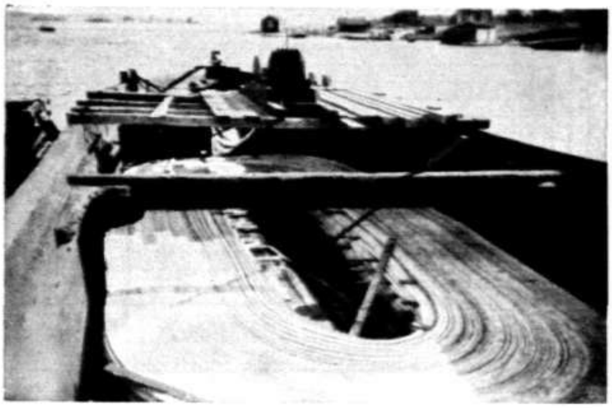
What has been said of joint-boxes applies in pertinent parts also to distribution heads, with the additional condition that the leading-in insulators must have a higher flash-over voltage than the insulators which carry the bare conductors to which the cable is to be connected.

Fig. 9 illustrates a joint-box, filling the above requirements well, for cables up to 6 600 volt. The insulation consists of paper sleeves impregnated by an oil compound, and held together by a strong wrapping of similarly impregnated paper. Between the two halves of the box a bitumen packing is placed, which, when screwed up while hot, easily enters all inequalities and fills the space between the flanges completely. The upper part is provided with room for expansion. The filling hole is placed so that this cannot be filled with compound.

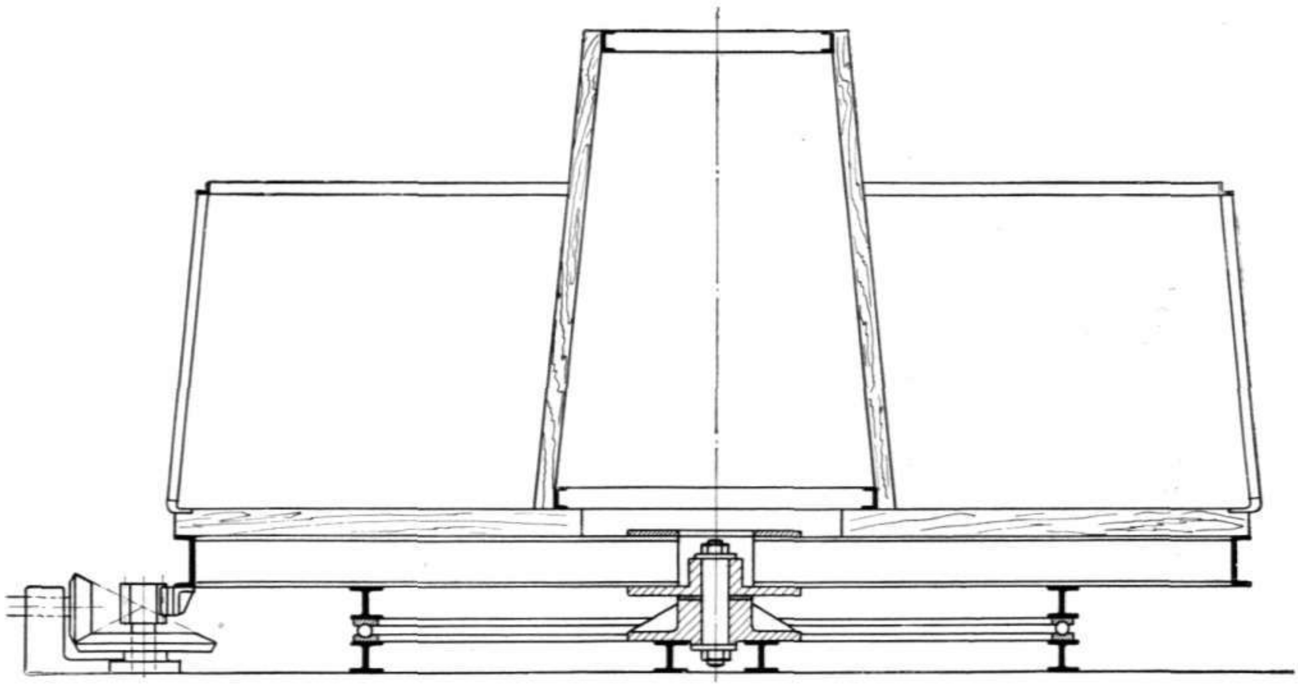
Joints for high voltage cables are most reliable if the boxes are filled with oil compound instead of ordinary compound. Fig. 8 shows such a box for a 33 kV 3-phase cable with each phase lead covered. The insulation consists of an oil-impregnated paper sleeve placed in a thin sheet iron tube. To make the voltage drop between the joint sleeve and the lead sheath less sudden, a serving of asbestos yarn is put next to the lead sheath, gradually diminishing in thickness from the lead sheath. Fig. 10 shows the effect on the voltage distribution of this serving.

The three single phase boxes are connected to a common expansion vessel, and the whole is

R 1975 Fig. 4.

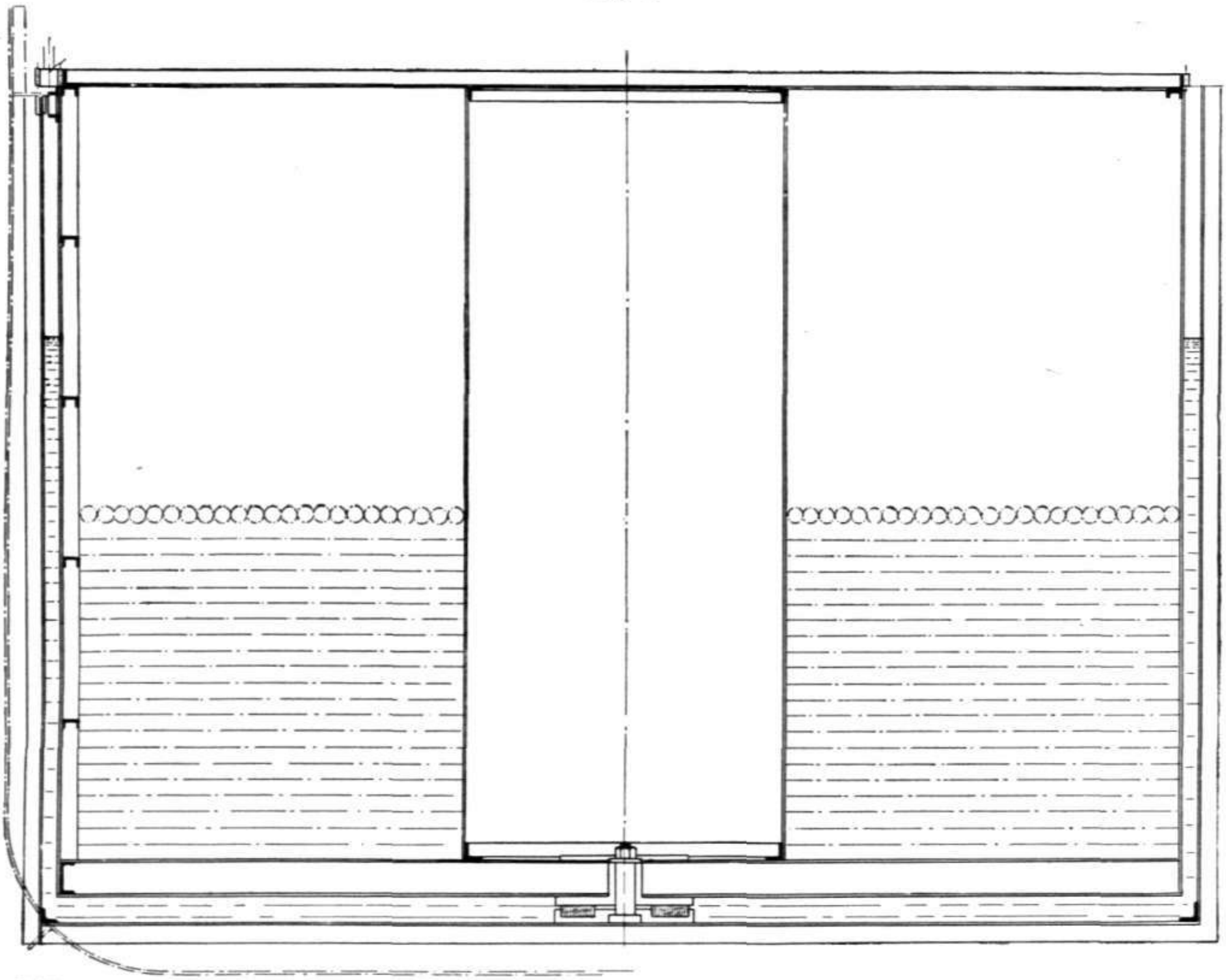


R 1976 Fig. 5.



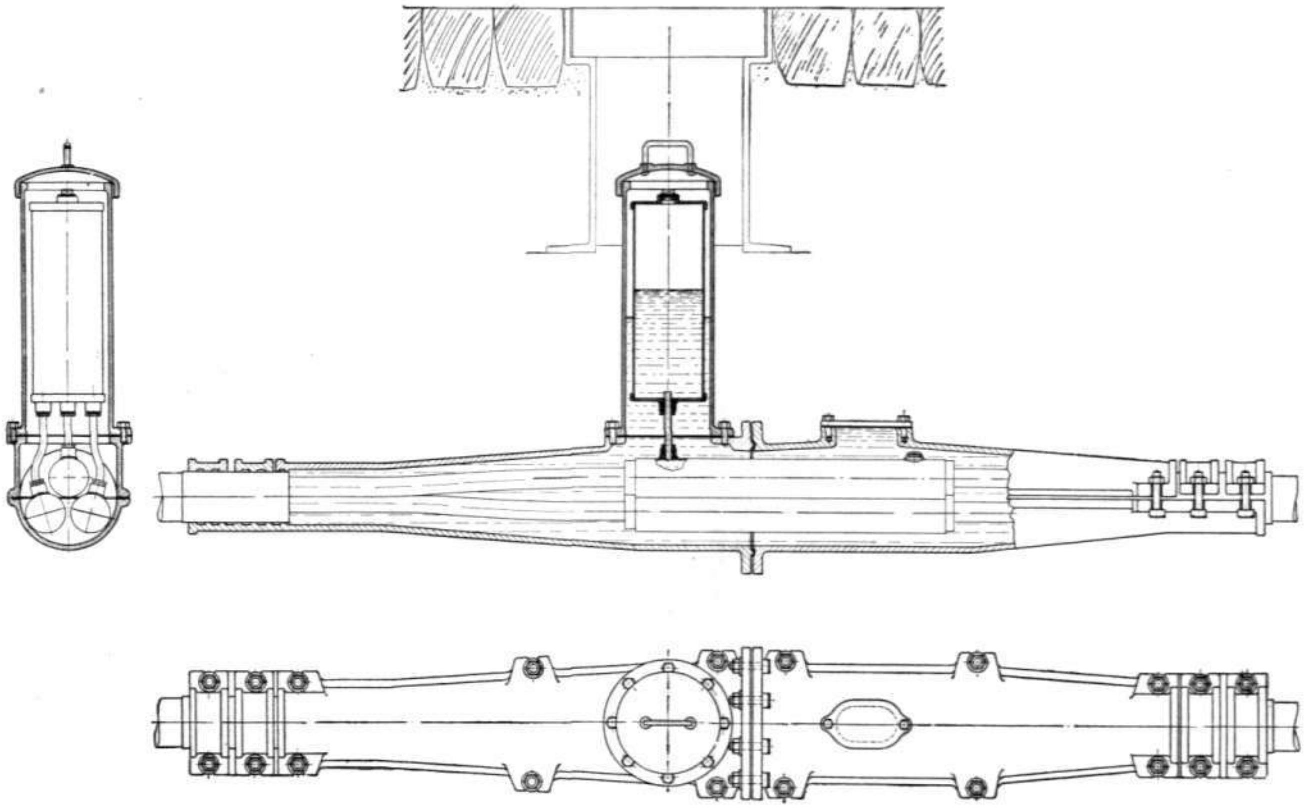
R 1987

Fig. 6.



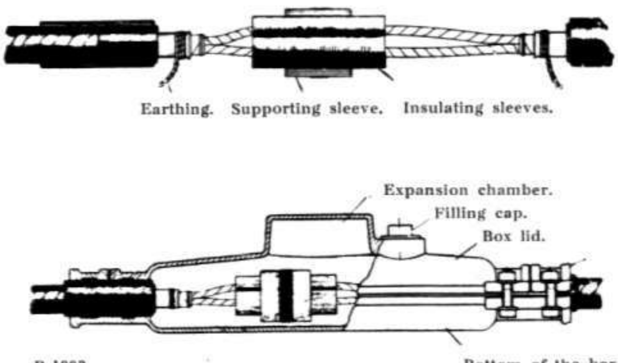
R 1985

Fig. 7.



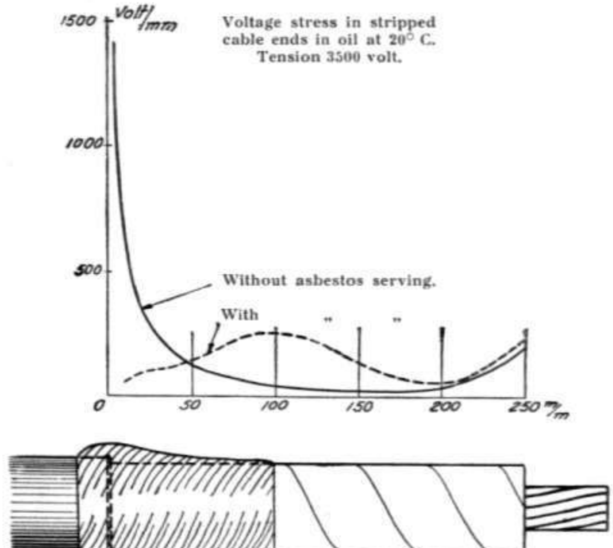
R 1988

Fig. 8.



R 1983

Fig. 9.



R 1984

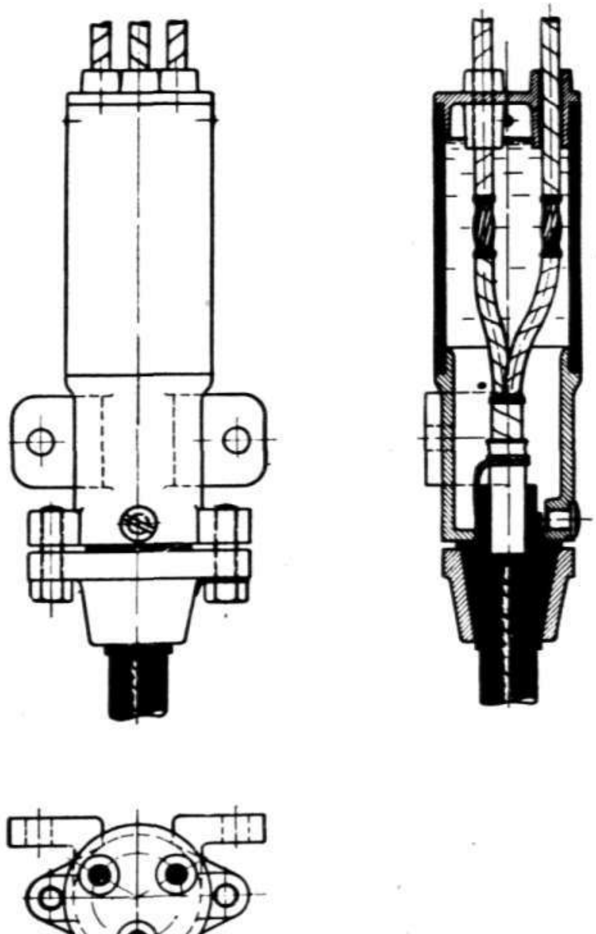
Fig. 10.

placed in a powerful cast iron protective casing, in its turn filled with asphalt.

Single phase boxes are as a rule made of unmagnetic material, and should be provided with expansion vessels and may also conveniently be fitted with a vacuum gauge, so that the internal pressure may be easily observed.

An indoor cable termination for up to 11 000 volt needs no box if the room where the connexion is made is perfectly dry and has a fairly even temperature, and if the cable end is not under pressure.

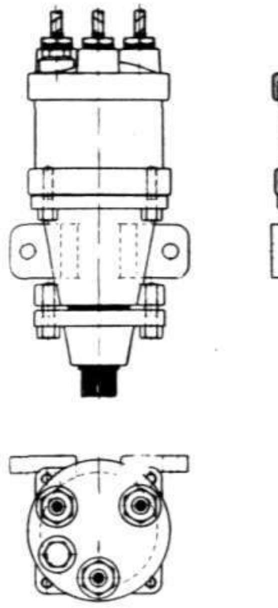
A perfectly reliable cable termination for up to 3 300 volt can be obtained with boxes of the design shown in fig. 11, if the temperature does not vary too much and no moisture ordinarily occurs, and if the slope of the cable is not more than 15: 100. The lower part, with a mechanical clamping device, is of cast iron, but the upper part is wholly of bakelite. The insulation of the cable conductors continues through the box to the place where they are connected, and is above



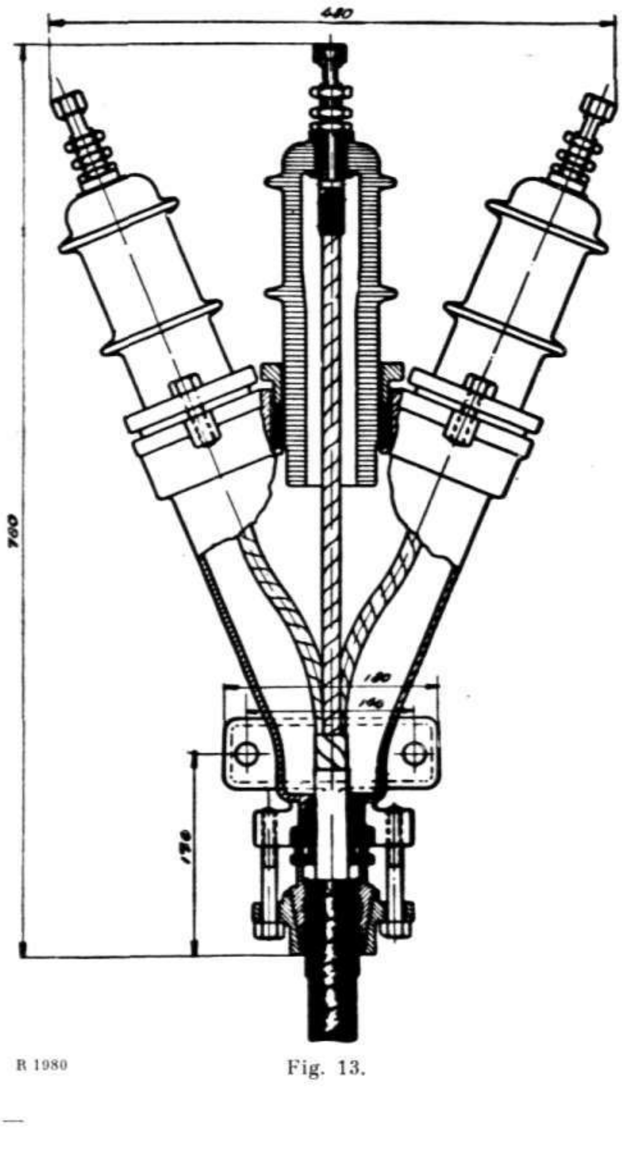
R 1978 Fig. 11.

the box wrapped with oilcloth tape painted with shellac or insulating lacquer. This design requires small dimensions, yet gives long distances between the phases and earth.

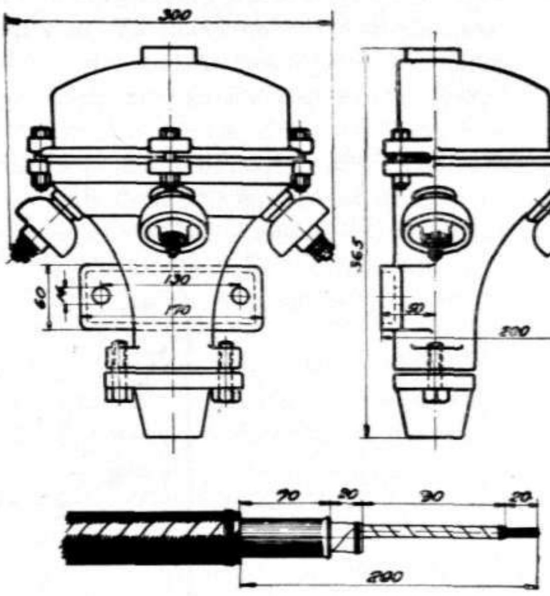
When the room where the cable terminates is very damp, or if the cable is under pressure, entirely closed boxes are used, effectively caulked where the wires are taken out. This necessitates the use of leading-in bolts to carry the conductors through the insulators. A simple pressure box which can be used for cables up to 3 300 volt is shown in fig. 12. Here also the upper part



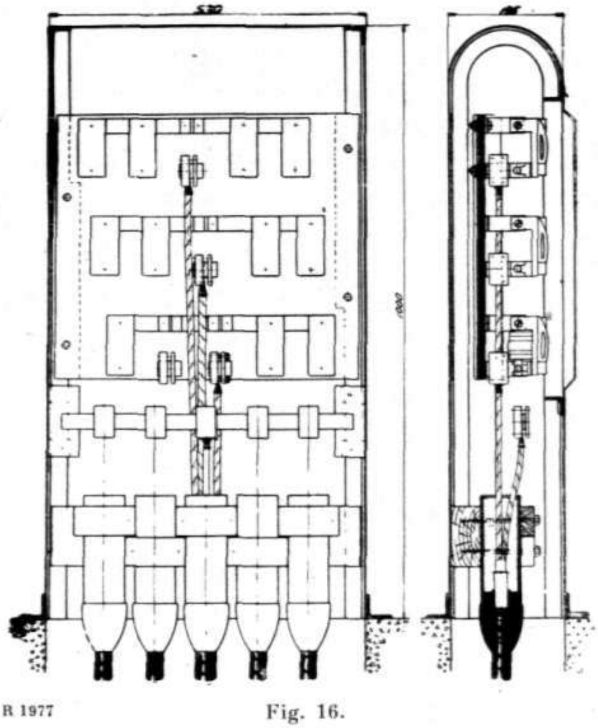
R 1979 Fig. 12.



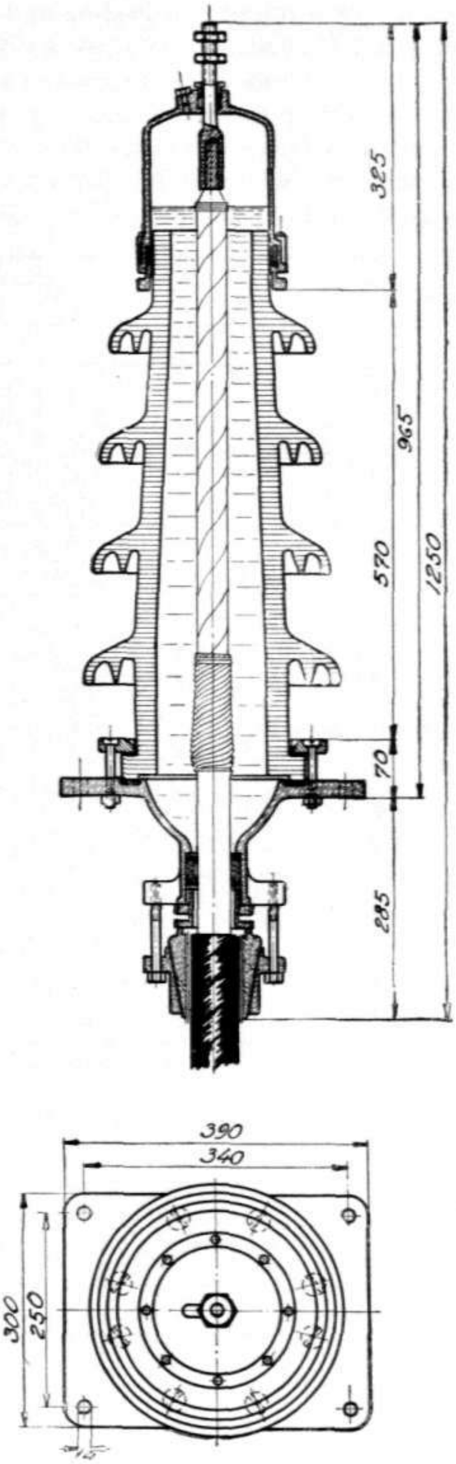
R 1980 Fig. 13.



R 1981 Fig. 14.



R 1977 Fig. 16.



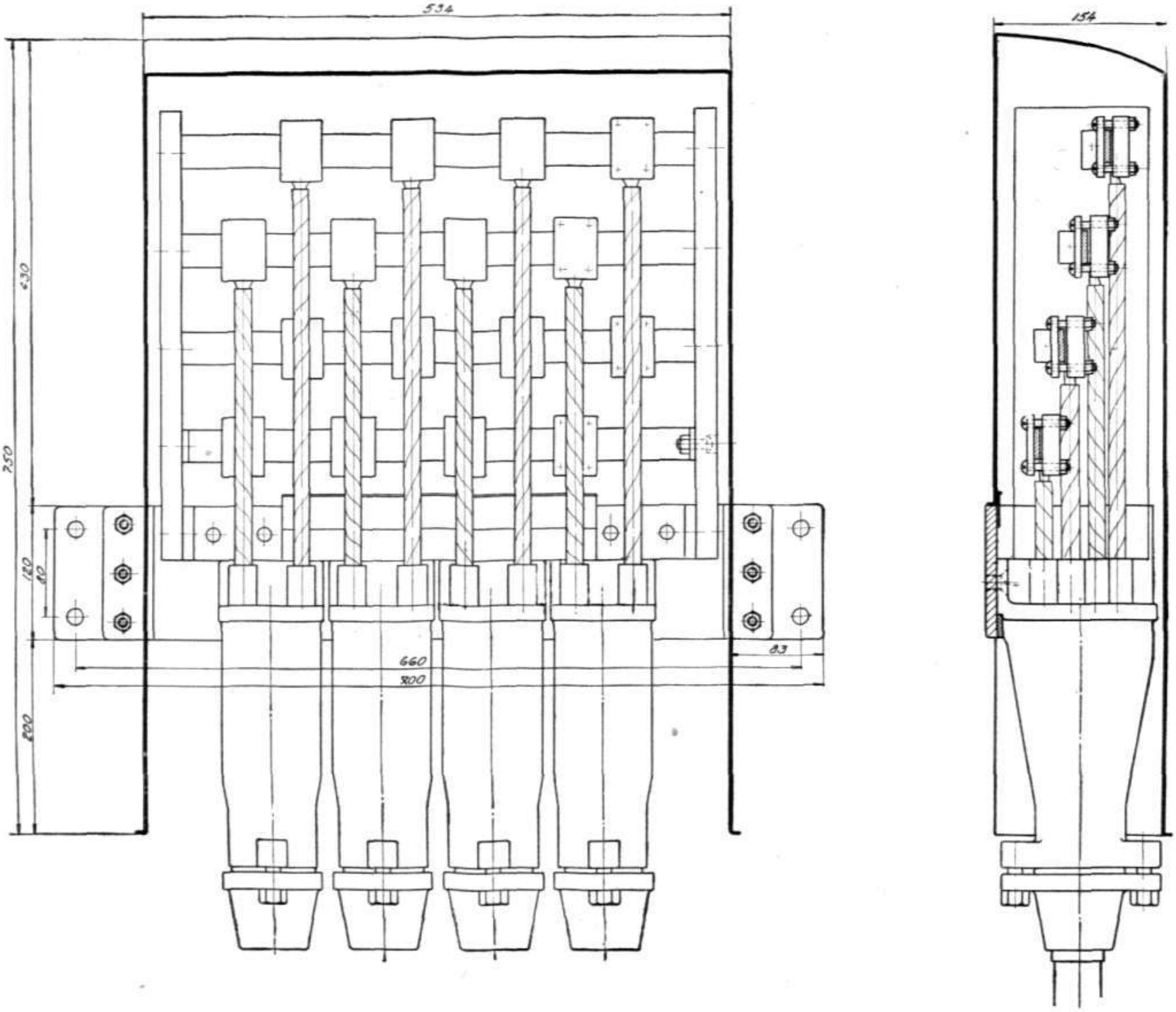
R 1986 Fig. 15.

is made of bakelite. The conductors of the cable are joined to another insulated cable, e. g. a rubber insulated one, by a smooth jointing sleeve which passes through a packing box with rubber-lead packing. The upper portion of the box serves as an expansion chamber.

Other indoor cable terminations, where the estimated pressure does not exceed 5 kg per cm², are best made as in fig. 13, illustrating a standard 11 kV indoor 3-phase box. Rubber-lead packing is used both for the insulators and for the leading-in of the cable. Tow steeped in red lead is wrapped round the leading-in bolt—an old and well tried packing. The box is filled with oil compound, and room for expansion is either obtained by filling the box completely with oil compound at a temperature of 150°, which in cooling will give sufficient room for expansion in the upper part of the box, or by screwing an expansion chamber to the middle leading-in bolt.

Figs. 14 and 15 illustrate a couple of outdoor cable boxes, the first for up to 3.3 kV, and the latter a single phase termination box for 55 kV. Both are made according to the rules given above, and have proved absolutely reliable in use.

Boxes placed above ground have in recent years been increasingly used for branching the cables, instead of the formerly used underground



R 1989

Fig. 17.

Outdoor boxes are usually exposed to greater variations of temperature than indoor ones, and as they have to do their work in any wheather it is important that the leading-in insulators should be designed to provide an ample flash-over voltage under any conditions. Outdoor cable boxes should therefore never be purchased without making sure that the porcelain is suitable.

joint boxes. Such boxes offer several advantages, and as they are no more expensive, there is every reason to use them. Fig. 16 shows a test hut for five 4-conductor cables for maximum 200 amp. The measurements given indicate that the size is relatively small, and there should be no difficulty in finding a suitable place for it in any town or village.

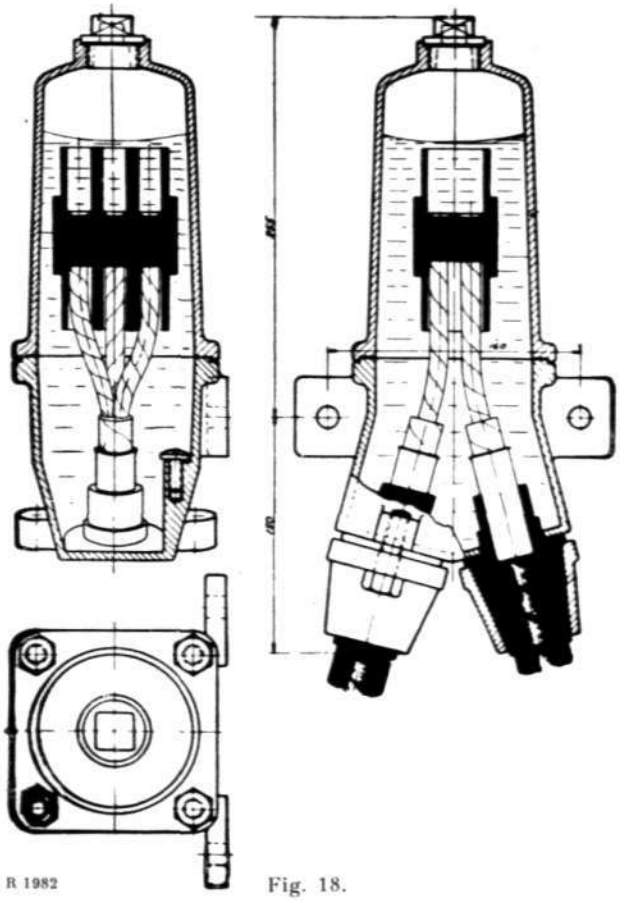
If cables are branched without fuses, and if they are not required to be easily separated when live, cable boxes of the type shown in fig. 17 are used. This box is made to be fixed directly on a wall or on a couple of poles fixed in the ground. The cables terminate in boxes which are put into the large box from the front, and the conductors are connected to the collecting bars by clamps without soldering.

In such parts of a country where the ground is covered by snow, and frozen during a large part of the year, cable fittings of the type shown in fig. 18 may suitably be used for both joints and branchings. Such boxes should preferably be placed above ground, where they are always accessible. The principle "no underground boxes in a new cable line" is nowadays frequently adopted, and though it is acknowledged that this principle gives increased reliability in working, it is perhaps not generally known that it is also frequently both simpler and cheaper than the older system.

Fitting. A good fitter who knows his job is of paramount importance to get a good cable installation. The cost of putting up modern, easily fixed cable fittings will always be only a small fraction of the total cost of the installation, but as the whole result will depend on the care with which it is fitted, too much stress cannot be laid on having this work done by none but really first class workmen. The training of the men is also of great importance and can hardly be overestimated. It pays to let the men who are going to do cable work learn this thoroughly, either in some large electricity works or in a cable works.

Above all, the work requires cleanliness and precision. The distance between conductors of different voltages is very small in a cable in comparison to what it is in other parts of an electrical plant, and consequently an apparently insignificant mistake in the fitting may easily spoil the result. An absolutely essential condition is that the boxes are kept free of every trace of moisture. Even perspiring hands are dangerous, and a person afflicted in that way must remember to wash his hands frequently with benzine when occupied with cable fittings. For work on high voltage cables, the fitter must wear rubber gloves whenever he touches the cable insulation directly. The fittings must be kept perfectly clean, and the work should preferably be done in a tent, which in damp weather must be heated.

The most suitable season for cable labour is the early summer, when the relative moisture of the air is low, and the most unsuitable is the autumn, when the relative moisture often approaches saturation point. This is a point which electricity works in particular would do well to remember when ordering their cables.



R 1982 Fig. 18.

Cable compound and oil should be carefully heated, so that they are not burnt, and the temperature must always be checked by a thermometer. Too hot compound is as bad as too cold. For cables impregnated with resin oil, too hot compound is particularly dangerous. Such oil contains substances which are vaporized at 150—200° C., and the compound will then become porous and sensitive to moisture. The old rule, that the temperature of the compound must be 150° C. when poured into the box, is made to be followed!

The laying and fitting of the oil-cellulose insulated high tension cable is a special branch of electrotechnics, and what has been said above is only intended as a few brief but necessary hints on the subject for engineers and executives engaged in power transmission work.

On Transmission Levels and Level Measurements in Long Distance Telephone Lines.

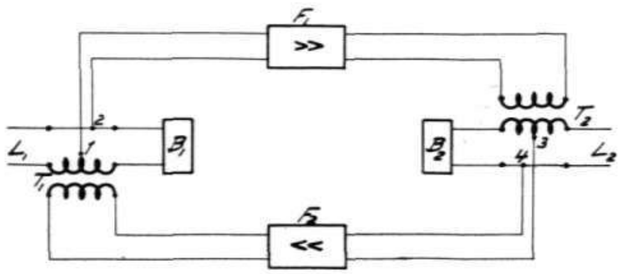
By *I. I:son Svedberg.*

The present paper gives a brief account of the principles of telephone communication by two- and four-wire systems. The conception transmission level is defined, and the level values recommended by the CCI are quoted. A method for measuring transmission levels elaborated by Svenska Radioaktiebolaget is described, and full details are given of the instruments for transmission measurements built on this principle. The method is based on the use of an amplifier instead of an auxiliary generator for level measurements. A correct frequency and curve shape is automatically obtained when the instrument is calibrated, and measurement errors caused by asymmetry of harmonics are eliminated. This latter problem is dealt with, and devices for avoiding errors of measurement caused by its influence in attenuation measurements are described. The mode of employing the new instrument for measurements according to the old method is finally also described.

Long distance communication is a conception which has been steadily expanding in telephony. The thermionic valves have provided a means of overcoming the line attenuation, and the distance at which telephone communication is possible has continuously grown with their perfection. This has, however, given rise to a number of new problems regarding the choice of the most suitable repeaters to use, where to connect them in the line, and how to supervise their working.

The Principles of Telephone Repeaters.

A thermionic valve being able to amplify an impulse in one direction only, some special circuits must be used to obtain amplification in both directions. For this purpose, a bridge circuit



R 2058 Fig. 1. Circuit diagram of 2-wire repeater.

with a differential transformer as in fig. 1 is employed to lead the currents of the two directions of speech each over its repeater.

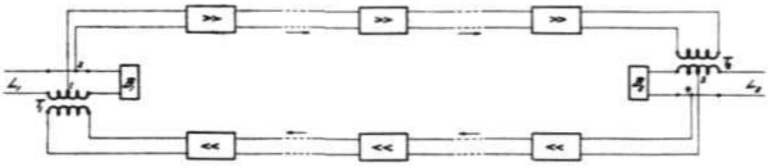
A current from the line L_1 is diverted in such a manner that half the power goes to the input side of the repeater F_1 and the other half to the output side of repeater F_2. As the repeaters are active in one direction only, the latter part of the current is of no importance. The first half, on the other hand, is amplified by F_1 and led to one winding of the transformer T_2. The other winding of the same transformer is connected on the one side to the line L_2 and on the other to the *line balance* B_2, a device designed to give the same impedance as the line. The centre tap of this winding goes to the input side of the repeater F_2. The amplified current from F_1 will therefore, as B_2 and L_2 offer equal resistances, be equally divided between them.

The current from L_1 will therefore continue to L_2, amplified by F_1. The repeater F_2 will not then be affected, as no potential difference occurs between the points 3 and 4, the voltage drops over B_2 and L_2 being equal.

Similarly, the repeater F_2 will act in case of speech in the direction from L_2 to L_1, while F_1 will not be affected if the impedances of L_1 and B_1 are equal.

The condition requisite for a satisfactory working of this arrangement is obviously that line balances can be made to imitate the lines sufficiently accurately. If the agreement is not complete, part of the amplified current will go through the repeater serving the other direction, causing echoes, distortion, or even oscillations in the repeater.

The greater the number of repeaters in a line of communication, the more insistent must be the demand that each functions as perfectly as possible. To avoid the necessity of balancing every single repeater, the speech currents in each direction can be led in separate pairs of lines, and one-direction repeaters be connected in each pair according to fig. 2. This may be regarded as arising out of fig. 1 by a division of the repeaters F_1 and F_2 into several stages, suitably placed along the line. The real difference in the systems thus lies in the number of lines required for a conversation, and they are therefore designated *two-* and *four-wire repeating systems respectively.*



R 2059 Fig. 2. Circuit diagram of 4-wire repeater.

Transmission Level.

When a line is to be equipped with repeaters, these should naturally be as few as possible. The number will be determined by the line attenuation, and by the amplification of each repeater. The conception *transmission level* is used in order to obtain a suitable expression for these quantities and their mutual relationship.

By the transmission level of a power, current, or voltage, we understand the ratio of either of these to a quantity of the same kind selected as a standard of comparison.

For reasons of expediency, the natural logarithms of this ratio are used for expressing current and voltage levels, and half the natural logarithms of the ratio for defining the power level.

If for instance the power P obtains in a point of the line, and P_o, which is regarded as a standard of comparison, in another, the power level in the first point is

$$p = {}^1/_2\, {}^e\log \frac{P}{P_o} \text{ nepers} \quad \ldots\ldots\ldots \quad (1)$$

When $P = P_o$, p will be $= 0$, and hence the power P_o is said to correspond to *zero level.*

By international agreement, *1 milliwatt* has been established as zero power level.

If over a *resistance of 600 ohms, phase angle $=0$,* we have the power corresponding to zero level, the voltage over this resistance is further regarded as the zero voltage level. and the current through it as the zero current level. If we symbolize these by V_o and I_o respectively, we thus get

$$\frac{V_o^2}{600} = 10^{-3} \quad \ldots\ldots\ldots \quad (2)$$

or $\quad V_o = 0.775$ volt (R. M. S. value)

and $\quad 600 \cdot I_o^2 = 10^{-3} \quad \ldots\ldots\ldots \quad (3)$

or $\quad I_o = 1.29 \times 10^{-3}$ amp. (R. M. S. value)

We will now assume the voltage V_1 between the branches at a certain point of a line of infinite length. If the line attenuation per kilometre is β, the voltage at a point s kilometres from the first point in the direction of the transmission will be

$$V = V_1 \cdot e^{-\beta s} \quad \ldots\ldots\ldots \quad (4)$$

We substitute the transmission levels

$$v_1 = {}^e\log \frac{V_1}{V_0}$$

$$v_2 = {}^e\log \frac{V}{V_0}$$

and divide equ. (4) by V_0

$$\therefore \frac{V}{V_0} = \frac{V_1}{V_0} \cdot e^{-\beta s}$$

Taking the logarithms in this expression, we get

$$v = V_1 - \beta s \quad \ldots\ldots\ldots \quad (5)$$

The voltage level is thus a rectilinear function of the length of the line. The difference between the levels at two points is equal to the total attenuation for the length of line between them.

The same is applicable to power levels, for with analogous symbols we get

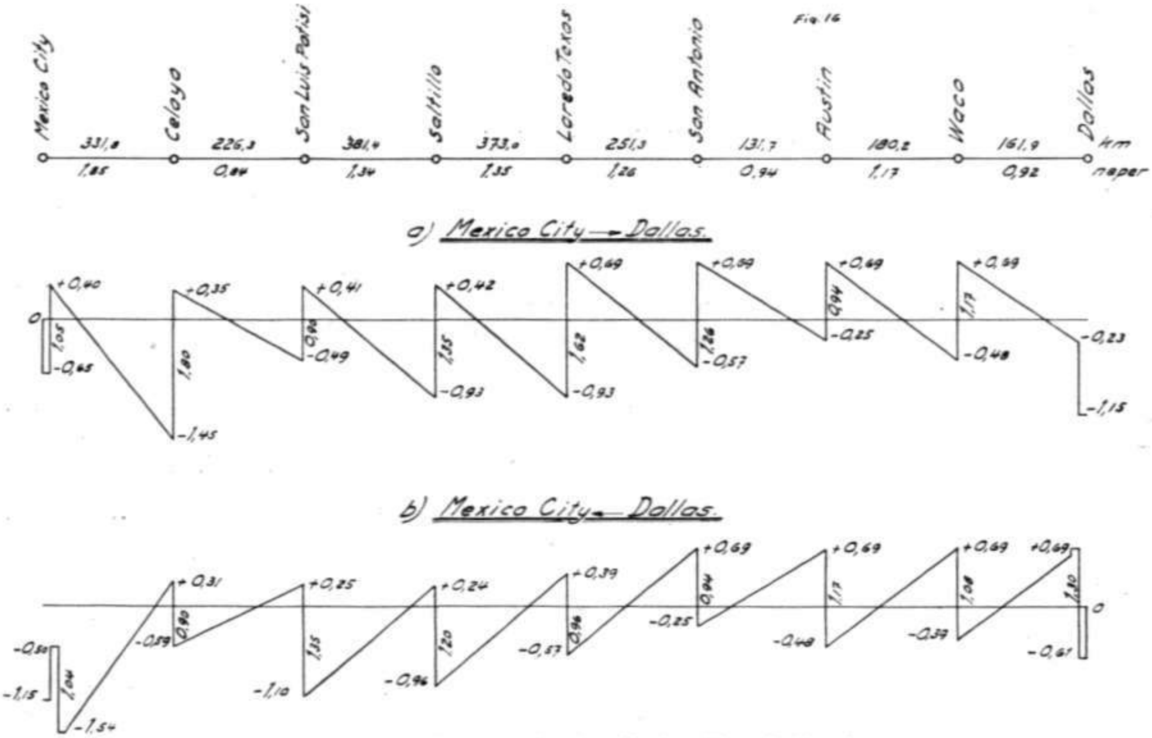
$$\left.\begin{aligned} P &= P_1 \cdot e^{-2\beta s} \\ p &= {}^1/_2\,{}^e\!\log \frac{V}{V_0} \\ p_1 &= {}^1/_2\,{}^e\!\log \frac{V_1}{V_2} \end{aligned}\right\} \qquad \text{.............................. (6)}$$

and

$$p = p_1 - \beta s \text{ .... (7)}$$

will be —0.23 nepers. Various exchange equipment reduces it by 0.92 nepers to —1.15 nepers. The *over-all transmission loss* for the distance Mexico City—Dallas, 2037.6 kilometres (1266 miles), is thus 1.15 nepers.

For the opposite direction a diagram as in fig. 3b is obtained.



R 2060

Fig. 3. Level diagrams for the Mexico City—Dallas line.

Level Diagrams.

Fig. 3 is a *diagram of voltage levels* for both directions of speech in a two-wire communication between Mexico City and Dallas, measured with 800-cycle current.

If, in the Mexico City exchange, voltage of zero level is fed from a generator to the terminals of a subscriber's line, the level in the exchange equipment will drop to —0.65 nepers (fig. 3a). The voltage is increased in a two-wire repeater by 1.05 nepers, and at the beginning of the Celaya line the voltage level is consequently +0.40 nepers. The line attenuation for this line is 1.85 nepers, and the level will consequently drop, according to the above, to —1.45 nepers at Celaya. The repeater in this exchange raises the level to +0.35 nepers, and so on. At Dallas, the incoming level

Limits of Amplification.

For reasons of cost, it would of course be desirable for each repeater to have large amplifying power so as to reduce the number of repeater stations in a given line. The risk of disturbances, however, imposes certain limits. The lowest permissible input level is determined by the consideration that speech must be loud enough to dominate completely any cross talk from neighbouring lines or other disturbances. The output level is similarly limited, as no cross talk must be produced in other lines.

In 2-wire repeaters the difficulties of balancing the lines must also be considered. The greater the amplification, the more accurate must obviously the balance be.

In practice, the most suitable level limits for cable lines have been found to be (CCI, "White Book", 1926):

	For 2-wire repeaters	For 4-wire repeaters
Input level:	−1.6 nepers	−3.0 nepers
Output level:	+0.6 ,,	+1.1 ,,

The greatest expedient attenuation between two repeaters will accordingly be 2.2 nepers for a 2-wire system and 4.1 nepers for a 4-wire system.

The values are generally kept well within these limits, and the problem will then be to distribute the amplification in the most effective manner to the various stations along the line.

For 2-wire repeaters the CCI recommends the following procedure, which gives an over-all transmission loss of 1.30 nepers.

Assume for instance a line to be divided into five sections with an attenuation of b_1, b_2, b_3, b_4, and b_5 respectively. Let the amplification in the first repeater be f_1, in the intermediate repeater stations f_2 and f_3, and finally in the terminal station f_4. We then select

$$\left.\begin{aligned} f_1 &= b_1 + \frac{b}{2} - 0.65 \\ f_n &= {}^2/_2\, b_n + {}^1/_2\, b_n + 1 \\ f_4 &= b_5\, \frac{b}{2}\, 4 - 0.65 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots (n = 2 \text{ or } 3)$$

Each intermediate repeater thus compensates the attenuation in half a section on either side of it. The terminal repeaters deal with half a section on the line side and all but 0.65 nepers on the exchange side.

Over-all Transmission Loss and Transmission Levels in International Telephony.

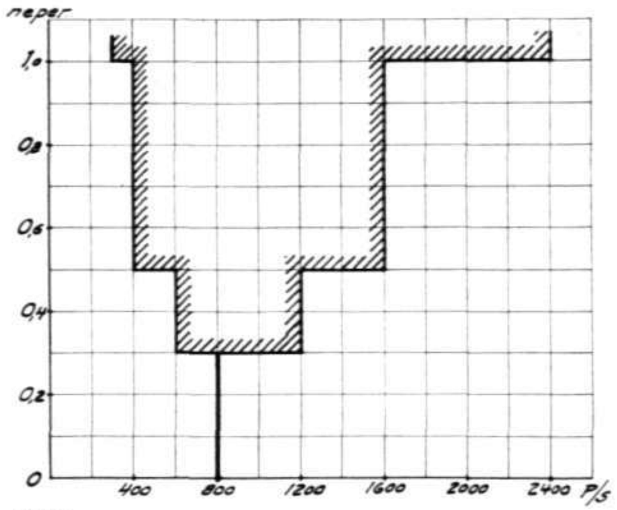
At the CCI conference in Brussels in June 1930, certain standards for the selection of over-all loss values in International communications, and level values at frontier exchanges, were established, and for the frequency dependence of the two.

The maximum over-all transmission loss in a 2-wire system was thus fixed at 1.3 nepers, and in a 4-wire system at 1.1 nepers at a frequency of 800 cycles. The conference, however, considered it desirable that these values be kept within 1.0 and 0.8 nepers respectively, provided that the stability of the line or cross talk attenuation in the line were not thereby reduced below the limit values set by the CCI. Nor must the noise voltage reach larger maximum values than those normally permitted.

The over-all transmission loss at other frequencies may exceed the *loss actually measured at 800 cycles* by the following amounts, in 2-wire as well as in 4-wire systems:

Frequency range	Permissible increase of overall transmission loss in relation to the loss at 800 cycles
600—1200 cycles	0.3 nepers
400—600 ,,	0.5 ,,
1200—1600 ,,	0.5 ,,
<400 ,,	1.0 ,,
>1600 ,,	1.0 ,,

These limits for the variation with the frequency of the over-all loss is graphically illustrated in fig. 4.



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Over-all loss at 800 cycles.	Max. Recom.	Absolute limit.
2-wire system.	1.0 neper.	1.3 neper.
4-wire system.	0.8 neper.	1.1 neper.

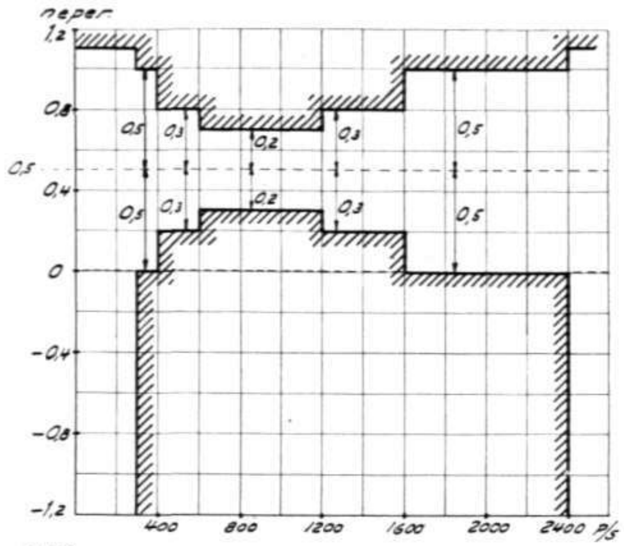
Conditions of stability.

2-wire system.	Stability at least 0.4 neper.
4-wire system.	Over-all loss for all frequencies > 0.5.

Fig. 4. Permissible increase of over-all transmission loss in international lines, in relation to the loss at 800 cycles.

The lower limit of over-all loss is determined with due regard to the stability of the system. In 4-wire systems the over-all loss must not fall below 0.5 nepers at any frequency, and in 2-wire systems the margin of safety for stability must be at least 0.4 nepers, i. e. an increase of amplification in a repeater by 0.4 nepers must not cause the system to oscillate.

According to the CCI rules, the transmission level at frontier exchanges of international 4-wire systems must be kept within the bounds shown



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All level values must be between the shaded areas. Normal level 0.5 neper.

Fig. 5. Permissible power level at frontier stations of international 4-wire lines, measured on the output side in the direction of the frontier.

in fig .5. The normal power level on the output side of the repeater, measured in the direction of the frontier, shall be +0.5 nepers at 800 cycles, and the following deviations from this level are allowed:

for the frequency range	600—1200	cycles	± 0.2 nep.
" " " "	400—600	"	± 0.3 "
" " " "	1200—1600	"	
" " " "	300—400	"	± 0.5 "
" " " "	1600—1400	"	

Nowhere in the frequency range used must the power level exceed +1.1 nepers.

Measurement of Transmission Levels.

When the transmission levels at each of the repeater stations of a certain line have been decided upon, provision must also be made for ensuring good transmission by checking that these values are kept within the tolerances allowed.

The principle of determining the transmission level is to send out a current of known frequency from an audio-frequency generator *G* at one terminal station (*A* in fig. 6). The current is measured by thermo-couple and galvanometer (*T*), and the outgoing level is adjusted by a suitable attenuator to zero level at the beginning of the line. At the other end of the line, *C*, a resistance corresponding to the characteristic impedance of the line is connected, and the voltage over this resistance is measured by means of a voltmeter *V*, which gives the value of the voltage attenuation in the whole line. The voltage level behind a repeater is determined in the same manner by measuring it with a voltmeter *V*. From the voltage-level values obtained, the power level may also be computed as described below.

Special voltmeters must be used for these measurements. Their measuring range must correspond to voltage levels from —3.5 nepers to +2 nepers (i. e. from 0.024 to 5.74 volt, *R. M. S.* value) at all audio-frequencies (200—5000 or even 10 000 cycles). For measurements at an intermediate station, they must further possess an

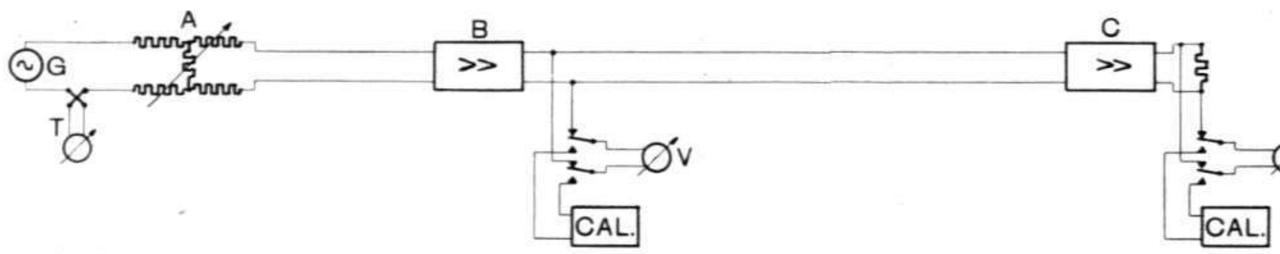

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Fig. 6. Diagram of level measurements.

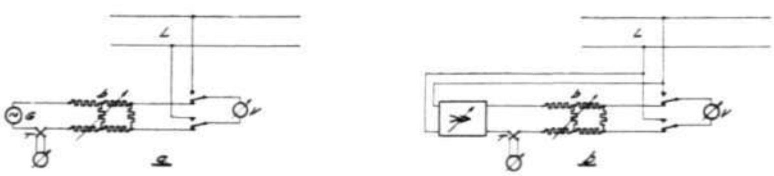

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Fig. 7. Old and new method for calibrating a level-measuring instrument.

input impedance large enough (100 000 ohms) to prevent their affecting the level of the line when connected. Valve voltmeters of one kind or another must therefore be used.

In actual measuring practice (cf. fig. 6) the line voltage is first observed on the voltmeter *V*, which is then switched to a calibrating circuit, where the proportion of a known voltage required to give the same reading over again is ascertained.

This known auxiliary voltage must be of the same frequency as the line voltage. To make the comparison in the valve voltmeter correct, it should also be of the same curve form. So far, a separate local generator *G* (Fig. 7a) has been used for the generation of this voltage, which is fed to the artificial line *b* over a thermocouple with a galvanometer *T*. Each time the measuring frequency is changed, the generator *G* must consequently be adjusted to the same frequency, and the current adjusted until a certain reading is obtained on the galvanometer *T*. The valve voltmeter reading is then adjusted by means of *b* until it coincides with that obtained from the line voltage. The voltage level at the input terminals of the valve voltmeter is then directly read on the artificial line *b*.

The method has some drawbacks, however. Setting the generator to each new measuring frequency means a loss of time. There is also some risk of one station misunderstanding the sending station when the frequency to be used for measuring is announced. These drawbacks are automatically avoided by the method used by Svenska Radioaktiebolaget in its Transmission Measuring Set type TRM. In this apparatus, the auxiliary voltage is obtained, as shown in fig. 7b, from an amplifier *F* fed by the actual line voltage, which thus positively gives the correct frequency for comparison, without being specially set to it. The same curve shape of the voltages compared is also obtained, which is not invariably the case with the older method. A rather important advantage is also that when portable sets are used for measurements, heavy and bulky generators need not be carried.

For practical reasons the measuring apparatus should preferably use approximately constant input voltage to the valve voltmeter, and an adjustable, calibrated voltage divider is therefore employed, permitting a known part of the line voltage, sufficiently large to give a suitable deflection to the valve voltmeter, to be taken out.

The high input impedance necessary requires this potentiometer to be of the special design set forth in the following description of the Measuring Set.

The principle of measuring is shown in the circuit diagram, fig. 8. Let the voltage level in the line *L*, and with that over the *primary potentiometer F*, be *p* nepers, which, according to the definition on page 00, corresponds to a voltage

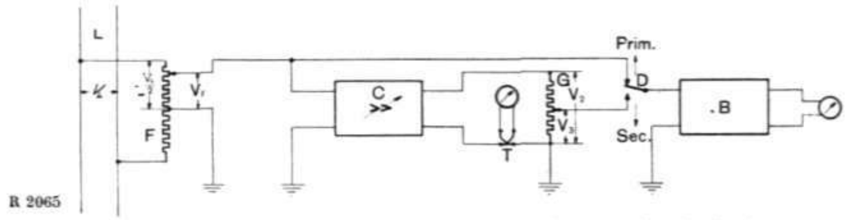


Fig. 8. The principle of measurement used in the Svenska Radioaktiebolaget set.

$$V_1 = 0.775 \cdot e^p \text{ volt.}$$

At a certain setting of the primary potentiometer, corresponding to the reading *m* nepers, we have over its contact a voltage V_F and the scale is made such that

$$\frac{V_F}{V_1} = k_1 e^{-m},$$

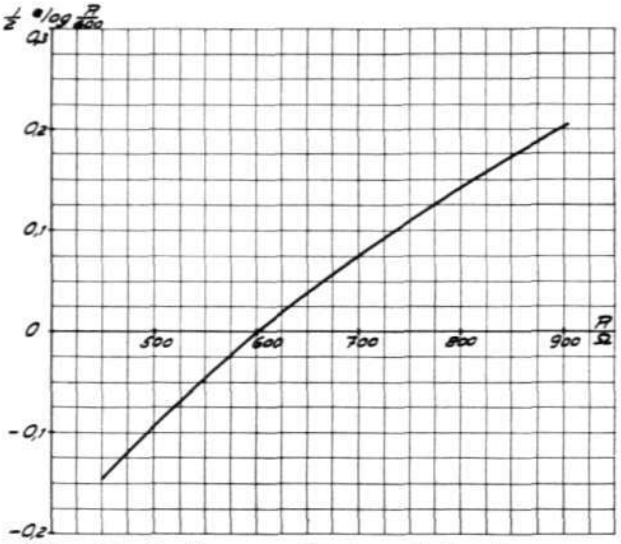
where k_1 is a constant and *m* may be varied in steps of 0.5 nepers.

We consequently have

$$V_F = V_1 k_1 e^{-m} = 0.775\ k_1 e^{p-m} \text{ volt.}$$

When the switch *D* is in position "Prim.", this voltage is fed to the valve voltmeter, which is designed to be deflected to about midway on the scale by $0.775 \cdot k_1$ volt, i e. when $p - m = 0$.

The first step in the measuring procedure will therefore be to select, with the switch *D* in the "Prim." position, a step on the primary potentiometer which will give a deflection approximately midway on the scale, and to observe this exactly. The sensitivity of the valve voltmeter should preferably be adjusted so that this deflection is always the same, e. g. midway on the scale.



R 2066 Fig. 9. The correcting term $^1/_2\ {}^e\log \frac{R}{600}$

The voltage V_F, and consequently the level, is then finally determined by comparing it to a known voltage V_3 taken out from the *secondary potentiometer* G in the output circuit of the amplifier C. By the previous adjustment of the primary potentiometer, the voltage V_F has been adjusted to a value suitable for the amplifier. The output of this is regulated by a variable resistance so that the thermo-galvanometer T gives a certain reading. The current through G is thereby fixed, as well as the voltage V_2.

The switch D is now set in the "Sec." position, and the sliding contact on G is moved until the same reading is obtained on the valve voltmeter as in the "Prim." position. We then have

$$V_3 = V_F.$$

The scale of D is calibrated to give directly a value n nepers, determined by

$$\frac{V_3}{V_2} \text{ being} = k_2 \cdot e^n,$$

where k_2 is a constant, and n varies continuously from $n = -0.20$ to $n = +0.7$.

We thus have

$$k_2 V_2 \cdot e^n = 0.775\, k_1 - e^{p-m}.$$

If the design is such that

$$k_2 V_2 = 0.775\, k_1$$

e^n will thus be $= e^{p-m}$, or $p = m + n$.

The voltage level will consequently be the sum of the scale readings on the two potentiometers F and G.

Calculation of Power Level from Voltage Level.

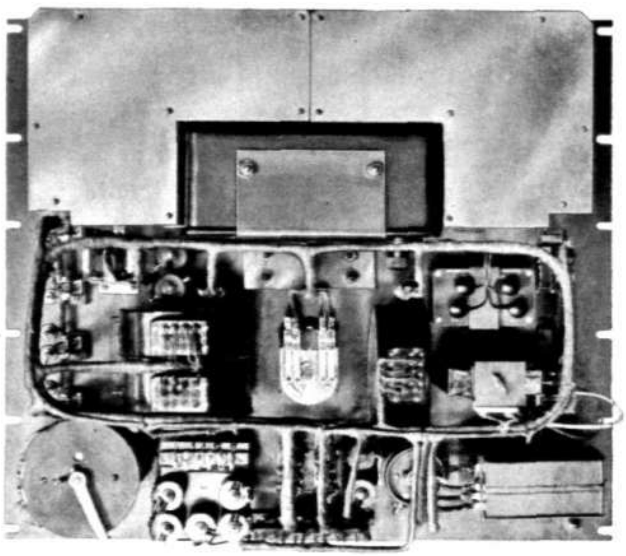
If the voltage V is measured over a resistance R, the power level will, according to the definition, be

$$p = {}^1/_2\ {}^e\log \frac{\frac{V^2}{R}}{P_0} = {}^1/_2\ {}^e\log \frac{\frac{V^2}{R}}{\frac{V_0^2}{600}}$$

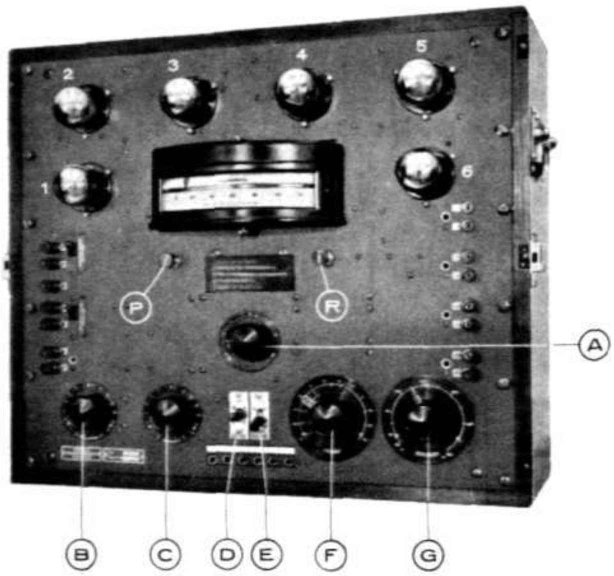
or

$$p = e\log \frac{V}{V_0} - e\log \sqrt{\frac{R}{600}} \text{ nepers,}$$

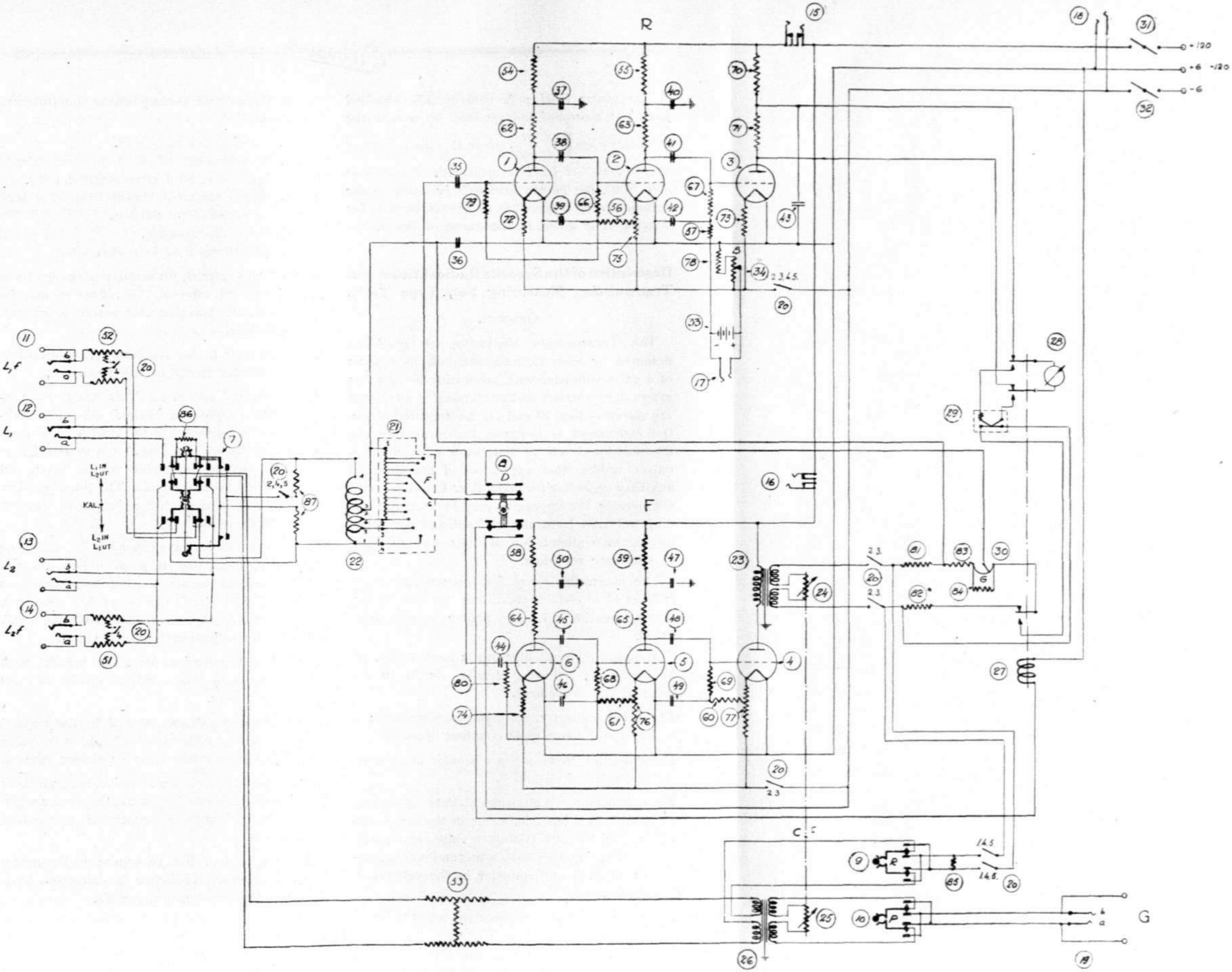
where $e.\log \frac{V}{V_0}$ obviously is the voltage level.



R 2067 Fig. 10. Transmission Measuring Set, Type TRM. Back view.



R 2068 Fig. 11. Transmission Measuring Set, type TRM. Front view.



The contacts marked (20) all belong to the main switch A, and the figures at the respective contacts signify that they are closed for the corresponding position of the switch A.

Fig. 12. Circuit diagram of the Svenska Radioaktiebolaget Transmission Measuring Set.

The power level p is consequently obtained from the measured voltage level by subtracting from this e log $\sqrt{\frac{R}{600}}$, *where R is the resistance over which the level is measured. When $R =$ 600 ohms, the power level will be equal to the voltage level.* The value of the correction for varying R is given in the curve of fig. 9.

Description of the Svenska Radioaktiebolaget Transmission Measuring Set, Type TRM.

General.

The Transmission Measuring Set type TRM designed by Svenska Radioaktiebolaget consists of a valve voltmeter with calibrating devices. Its external appearance and mechanical construction are shown in figs. 10 and 11. As indicated above, the instrument is designed for measuring the transmission levels of telephone lines, and requires, unlike other appliances of the kind, no auxiliary generator for doing this. Combined with a generator, the apparatus may be used for sending out zero transmission level and for determining loop attenuation of lines, and the gain of telephone repeaters.

The measuring range for transmission levels is from +2 to —3.5 nepers. Gain up to 5 nepers, and attenuation up to 3.5 nepers, can be measured.

A complete circuit diagram is given in fig. 12. We note the following details, visible on the front of the instrument.

The valve voltmeter, consisting of the valves 1, 2, and 3, with their coupling units.

The amplifier, made up in a similar manner by the valves 4, 5, and 6.

The galvanometer (Cambridge Pattern *R*), serving partly as a D. C. indicator in the anode circuit of the last voltmeter valve, and partly —in conjunction with a thermal converter— as an A. C. milliammeter in the calibrating circuit.

The main switch A, by which the set is adjusted for the various measurements which can be made. It comprises all the contact devices marked 20 in the diagram. The figures against each contact group indicate that this is closed in the corresponding main switch positions, and correspond to the following uses.

1. Sending out zero level.
2. Measurement of incoming transmission level (i. e. level at terminal station).
3. Measurement of transit level (i. e. level at intermediate station).
4. Gain measurement.
5. Measurement of loop attenuation.

The potentiometer B, for regulating the grid bias of the end valve of the voltmeter, and for setting the galvanometer needle to suitable deflection.

The resistance C, for regulating the current in the normal circuit of the instrument.

The measuring switch D. This has three positions. In position "Prim." the valve voltmeter is connected to the voltage to be measured. In position "Sec." it is switched over to a known, adjustable voltage taken out from the normal circuit. The third position, "Norm.", is used when adjusting the current in this circuit.

The line switch E, for change-over of incoming and outgoing lines in gain- or loop attenuation measurements, so that both directions of a 2-wire circuit can be quickly measured. It has also an intermediate position, "Cal.", used for adjusting the normal level.

The primary potentiometer F, in parallel with the incoming line, a voltage divider in steps of 0.5 nepers.

The secondary potentiometer G in the normal circuit. The calibration voltage for the valve voltmeter is taken from its sliding contact.

The push-button switches P and R, between them forming a device for eliminating errors when measuring with asymmetrical alternating current.

Other details of the Transmission Measuring Set will be described below in connexion with their use in various measurements.

The two first positions of the main switch are the most important, and are therefore dealt with more fully. They serve for adjusting the normal circuit and the valve voltmeter of the instrument. All other measurements are a combination of these two.

1. Sending out Zero Transmission Level.

An audio-frequency generator is connected to the "Generator" terminals of the instrument, and the line to one of the terminals L_1 or L_2. The measuring switch D is set to the "Norm." position, which sets the connexions as in the simplified diagram fig. 13. Let us first assume the line switch E to be in the central position as in will of course be zero level. The whole sending device may be regarded as a kind of "normal generator". The internal resistance of such a generator must be constant at 600 ohm. This object is attained in the *TRM* by making the attenuator 53 so large that alterations of resistance 25 will be imperceptible in the internal resistance of the generator.

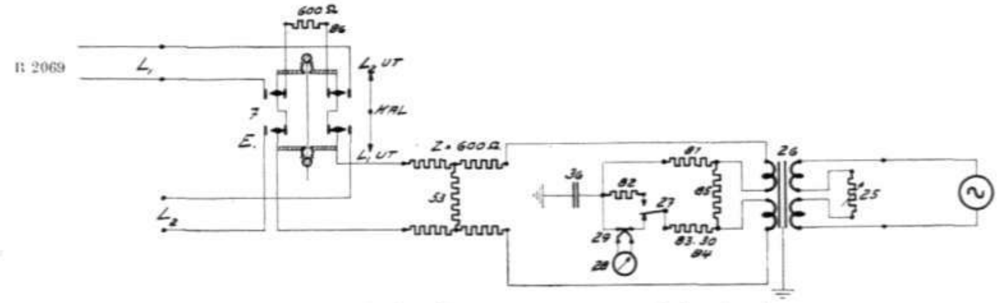


Fig. 13. Sending out zero transmission level.

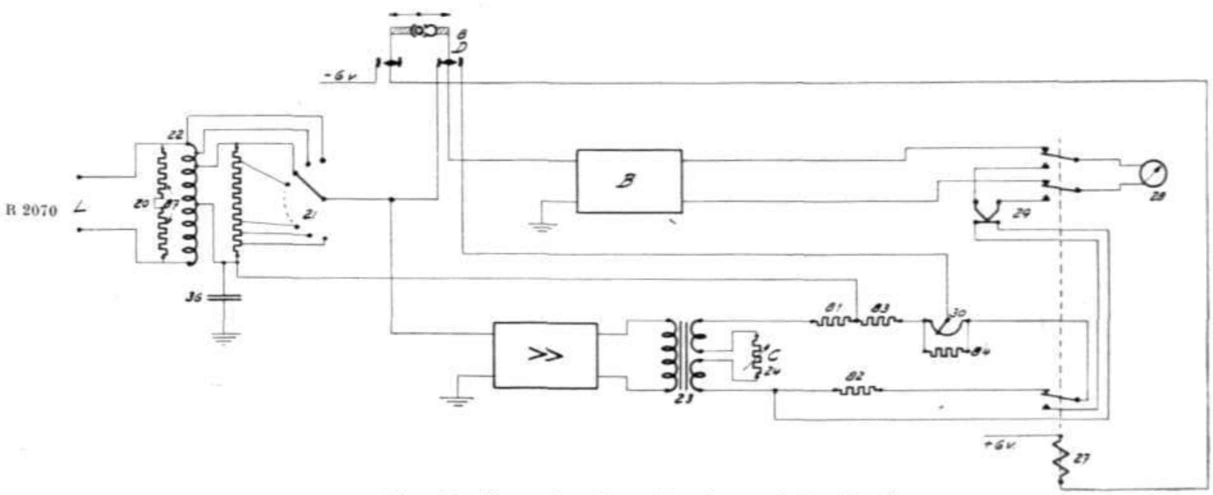


Fig. 14. Measuring incoming transmission level.

the figure. The current from the audio-frequency generator will then go over the transformer 26 through some resistances to the attenuator 53, which ends in resistance 86. The current is measured by the thermo-couple 29 and the galvanometer 28. The circuit is earthed over the condenser 36, and the resistance 81 is connected in for the sake of symmetry. The instrument is designed so that when the current is now regulated by the resistance C until the galvanometer needle takes up a central position on the scale, zero voltage level will be obtained over the resistance 86. By changing the line switch E to the position L_1 *OUT* or L_2 *OUT*, the line connected to the corresponding terminals will take the place of resistance 86, and if the line characteristic is 600 ohms, the voltage level there

2. Measuring an Incoming Transmission Level (level at terminal station).

A zero level is assumed to be sent out from one end of the line as described above. The other end of the line is connected to the line terminals L_1 of a Transmission Measuring Set, the main switch of which is set to position 2. The connexions in this case are shown in fig. 14. The line switch is assumed to be set to L_1 *IN*.

A terminal resistance 87 of 600 ohm, and a choke coil 22, the centre tap of which is earthed through condenser 36, are connected in parallel with the incoming line. This arrangement gives perfect symmetry of currents in the two line branches. The choke coil also serves as an auto-transformer, being tapped at every 0.5 neper ratio difference. Between the centre point and the nearest tap, a resistance is placed, also divided into steps of 0.5 nepers. All the taps are taken to a switch *F*.

The choke coil, the resistance, and this switch between them form *the primary potentiometer*. The *amplifier*, feeding the normal circuit, is connected in parallel with this.

The measuring is done as follows, in accordance with the principle stated above:

a. With the measuring switch *D* in "Prim." position, the primary potentiometer dial *F* is turned to the setting which brings the galvanometer needle nearest to the centre of the scale. (This is marked by a red line and the number 60.)

b. By means of dial *B*, the galvanometer needle is adjusted exactly to the 60-mark on the scale.

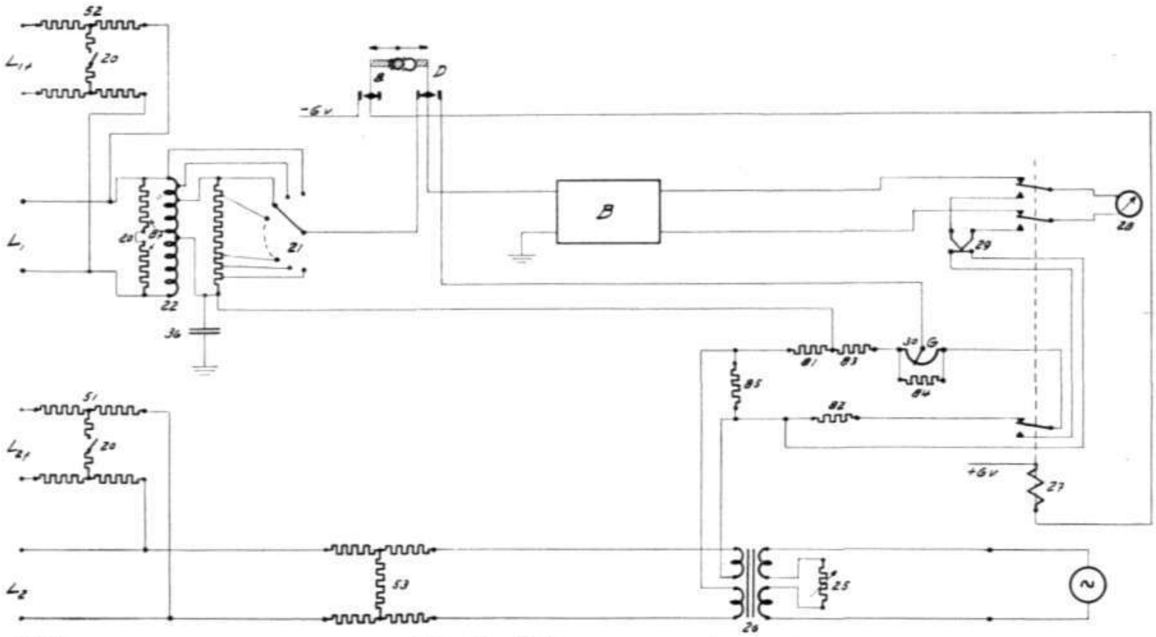
c. The measuring switch is depressed in "Norm." position. Relay 27 is then energized, and galvanometer 28 is connected to thermocouple 29 in the normal circuit. The galvanometer needle is brought to the centre of the scale by dial *C* (normal circuit).

d. The measuring switch *D* is set to the "Sec." position, and dial *G* (the secondary potentiometer) is turned until the central reading is again obtained on the galvanometer scale.

e. The sum of the readings *G* and *F*, obtained on the *inner* scale of the potentiometer *G*, will be the incoming level. The scale graduations are made so that the readings always have to be added, whether the levels be positive or negative.

3. Measuring a Transit Transmission Level
(level at an intermediate station).

The main switch *A* is set in position 3. This disconnects resistance 87 over the "incoming line" (fig. 12), which in this instance is a *parallel connexion* to the line measured, and the measurement is taken in exactly the same way as described for 2. The input impedance of the apparatus is in this case large enough not to affect the line level appreciably.



R 2071

Fig. 15. Gain measurements.

4. Gain Measurements.

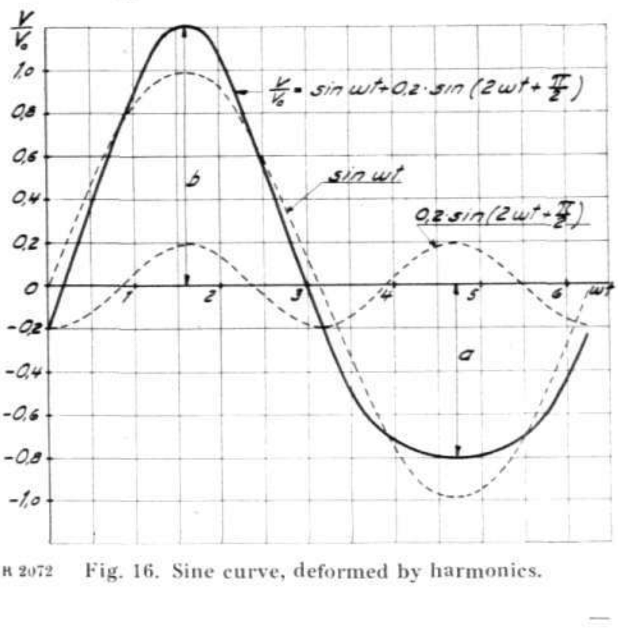
As the Transmission Measuring Set contains devices both for sending out a certain normal level and for measuring an unknown level, these may be readily combined for measuring the attenuation in a network, e. g. a cable loop or a repeater, which latter may be regarded as introducing a negative attenuation. We will assume that the degree of amplification of for instance a 2-wire repeater is to be determined.

The two line terminals of the repeater are then connected to the L_1f and L_2f terminals of the Transmission Measuring Set, and 600-ohm resistances are connected to the repeater as balances. The gain adjustment for the direction to be examined is set to the position desired, while the gain in the opposite direction is completely supressed by setting the potentiometer to zero. We assume that the gain in a direction from "West" to "East" is to be measured, and that the "W" side of the repeater is connected to L_2f, and its "E" side to L_1f. The procedure is as follows:

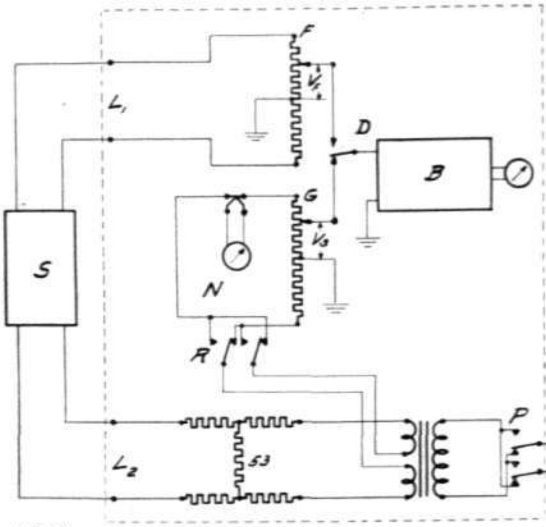
a. The main switch A is set in position 4. The connexions will then be as in fig. 15, where, however, the line switch E is left out to simplify the diagram.

b. Zero voltage level of the desired frequency is set at the line terminals as described in 1 above. As the attenuation of the artificial line 51 connected in parallel to this tap is 1.5 nepers, the "W" side of the repeater connected to L_2f will thus always receive a voltage of —1.5 neper level. Increased by a certain amount in the repeater, this is fed over the artificial line 52 (1.5 nepers) to the input choke coil of the Measuring Set, where it is measured as described in 2 above.

c. The gain sought is the sum of the *outer* scale readings (c-c) on F and the graduation marked in the same manner on G. To measure the amplification of the repeater in the opposite direction, the line switch E is reversed and the procedure repeated. The potentiometers of the repeater must of course also be re-set.



R 2072 Fig. 16. Sine curve, deformed by harmonics.



R 2073
Fig. 17. Circuit diagram for elimination of errors of measurement caused by asymmetry of harmonics.

5. Loop Attenuation Measurements.

The two ends of the line the attenuation of which is to be measured are connected to the L_1 and L_2 terminals.

The main switch A is set in position 5, and the measurements are taken as described in 4 above.

The readings are in this case (as in 1 and 2) taken on the *inner* of the double level-scales (a-a, b-b) of the primary potentiometer and on the corresponding scale of the secondary potentiometer. This, it will be readily understood, will give us the value of the attenuation with *reversed sign.*

Elimination of Errors of Measurement caused by Asymmetry of the Measuring Current.

When the attenuation of a network, e. g. a loop, is measured, the results obtained are sometimes different if the branches are exchanged, even if these are perfectly symmetrical. This happens if the measuring current used has so-called asymmetric harmonics, i. e. its positive and negative half-waves are not of the same amplitude, which may be caused by the generator producing harmonics which do not pass through zero simultaneously with the fundamental frequency. Fig. 16 is an extreme instance of a sine curve deformed in this way, of the equation

$$V = V_0 \sin \omega\tau + 0.20\ V_0 \sin\left(2\ \omega\tau + \frac{\pi}{2}\right),$$

where the difference in the peak values of the two half-waves *a* and *b* is very distinct. Such asymmetry may arise even if the generator used gives off a pure sine voltage. If the line contains sources of distortion (iron cores, repeaters) these may generate harmonics which arrive at the measuring point after different times of propagation, and there combine with the fundamental frequency into an asymmetric voltage curve.

We will now consider the connexions for loop attenuation measurements, as illustrated in fig. 17. Current is fed from the generator *G* to the normal circuit *N* and to the loop *S* measured, which latter terminates in the potentiometer *F*. The voltage V_F is determined by equalizing it to the adjustable calibrated voltage Vg at the secondary potentiometer *G* by means of the valve voltmeter *B*. To retain the symmetry of the line, only *half* the input voltage can be measured, as the tap of the voltage divider connected to the cathode of the valve voltmeter is thereby also earthed. The last stage of the valve voltmeter being designed as a half-wave rectifier, and thus only affected by one of the half-waves of the input voltage, it is obviously essential that *the same* half-wave of the current is measured at *F* and at *G*, if there is any asymmetry of harmonics. If we assume, for instance, that the amplitude corresponding to *a* in fig. 16 is measured at *F*, and the one corresponding to *b* at *G*, and that *a* is smaller than *b*, the value obtained for the attenuation in *S* will be too large, as the voltage will appear to be reduced more than what corresponds to the actual attenuation. If the switch *P* is changed over, the direction of the current will be altered at both measuring points, and the half-wave of greater amplitude will then be measured at *F* and the smaller at *G*. In that case too small a value for the attenuation will obviously be obtained. If the switch *R* is now also changed over, the direction of the current is reversed only in the normal circuit, and the same half-wave (viz. *b*) will then be measured at both potentiometers, and a correct attenuation value will be obtained. If, finally, *P* is changed while *R* remains in the latter position, a correct result will also be obtained, as in this case also *the same* half-wave (*a*) will be compared. The four possible combinations of the switches *P* and *R* will thus give two correct, one too small, and one too large value.

In the Transmission Measuring Set, type TRM, *P* and *R* are designed as push-button switches. The rule for their use is that *one measurement is made for each position of P. If the same result is obtained, R is in the correct position. But if different results are obtained, R should be changed over and new measurements made.*

If asymmetry is generated by the object measured, as sometimes happens for example in a repeater, agreement in values will not always be obtained with any of the four combinations of *P* and *R*. The combination giving a value nearest to the average value is then selected, and the observations are completed with the switches in that position.

It should be noted that in level measurements according to 2 and 3 above, the effect of asymmetric harmonics is automatically eliminated, as the connexion between the primary and the secondary potentiometers via the amplifier will always cause the same half-wave to be compared by the valve voltmeter.

Measurement of Transmission Levels by Means of a Local Auxiliary Generator.

If a series of measurements is to be made of different levels, but of the same frequency, it may be expedient to employ the older method of feeding the normal circuit from a local auxiliary generator. The procedure will then be the same as in *loop attenuation measurements.* The main switch is set in position 5 and the incoming line is connected to L_1. The L_2 terminals remain empty. Otherwise the operation is carried out as in 5 above.

If the *incoming level* (over-all transmission loss) is to be determined, the line switch *E* is put in position L_1 *IN*. The line will then be terminated by resistance 87 in the instrument.

For measurement of a *transit level,* the same switch is set to "Cal.". This disconnects the load resistance, and the high input impedance required for the Transmission Measuring Set is obtained.

In either case the level values are read on the scales *G* and *F* in the same way as for measurements according to 2 and 3 above.

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