# Series of Monographs in Advanced Mathematics ISTITUTO PER LA RICERCA DI BASE Castello Principe Pignatelli, 86075 Monteroduni (IS), Molise, Italy

## Hyperbolic Geometry and Barbilian Spaces

### Wladimir-George Boskoff

Faculty of Mathematics
University "Ovidius" of Constanta
124 bd. Mamaia, 8700 Constanta, Rumania



HADRONIC PRESS

#### **CONTENTS**

#### **CHAPTER ONE**

1.1. Anharmonic ratio and infinite points for straight lines	1
<b>1.2.</b> Euler's identity and the structure of $\sigma_4$	2
1.3. The number of distinct anharmonic ratii relative to a	
given division	5
1.4. Harmonic and eqiharmonic divisions	7
1.5. The extension of the concept of anharmonic ratio	9
1.6. Projectivity, perspectivity and applications. The infinite	
straight line of the plane. Duality.	11
1.7. Imaginary elements in geometry. Circular points of	
the plane	19
1.8. The Hesse's points	23
1.9. Projectivities on a circle	27
1.10. The real meaning of the imaginary elements; symmetry,	
inversion and similarity	31
1.11. The classification of the projectivities	40
1.12. The composition and the transformation of	
the projectivities	44
1.13. The group of the projectivities of a given circle	48
1.14. The projective model of the Lobacevski's geometry	52
1 15 Rarbilian's distance	55

#### **CHAPTER TWO**

Barbilian Spaces and S-Barbilian Spaces	
2.1. Logarithmic oscillation	57
2.2. Constant Gaussian curvature and influences	66
2.3. S-Riemannian manifolds as oscillant metric spaces	69
2.4. The axiomatic theory of abstract Barbilian Spaces	72
CHAPTER THREE	
Special Barbilian Spaces and	
the Tzitzeica Configuration	
3.1. The Tzitzeica configuration on a sphere	81
3.2. The intrinsec point of view	85
3.3. The characterisation of the special Barbilian Spaces	91
CHAPTER FOUR	
Geodesics in Barbilian Spaces	
<b>4.1.</b> The geodesics of oscillant metric spaces	95
4.2. The axiomatic theory of Apolloniu's lines	10
<b>4.3.</b> The connection between geodesics and	
Apolloniu's lines	103
4.4. Continous Barbilian distance	111
CHAPTER FIVE	
Lagrangean Structures, Convex Geometry and	
Natural Barbilian Spaces	
<b>5.1.</b> Lagrangean structures for I-derivable curves	115

5.2. The generalized Lagrange space induced	
by the Poincaré model of Lobacevski's Geometry	122
5.3. Circular points and central symmetries	130
5.4. gK-Circles, symmetries and L-Barbilian Spaces	134
5.5. The differential point of view for the Axiom I <sub>6</sub>	143
CHAPTER SIX	
Riemannian Structure for Natural Barbilian Spaces	
6.1. The local connection between the pseudosphere	
and 2-dimensional Natural Barbilian Spaces	149
APPENDIX A	154
APPENDIX B	155
INDEX	156
REFERENCES	159