Economic Geography, Fertility and Migration

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July 30, 2006

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JEL classification: J13, J61, R10, R12, R23

Keywords: Economic geography; Fertility; Agglomeration economies; Congestion diseconomies, Migration

Abstract

This paper analyzes the relationship among economic geography, fertility and migration. The empirical evidence presented reveals that persistent regional variations in fertility exist within a country and that regional total fertility rates are negatively related to regional population density. A two-period overlapping generations model of endogenous fertility, incorporating n-regions, agglomeration economies, and congestion diseconomies is constructed to explain this negative relationship. While agglomeration economies have both positive income and negative substitution effects on fertility, congestion diseconomies have a negative income effect on it. Combined with the mobility of people, interaction among these effects generates the negative relationship as a steady-state equilibrium outcome. It is also shown that net migration from regions with lower population density to regions with higher population density occurs in an equilibrium, which, in turn, maintains the regional variations in fertility.

1 Introduction

The recent decline in the fertility rate has attracted much attention in many developed countries since it has significant implications for economic growth, public pensions, health care, and labor markets.[1] In order to examine the relationship between fertility and economic activities, the following two types of models have been considered: non-altruistic and altruistic. In the non-altruistic model, material support during the period of old age dependence is a motive for having children, which indicates that rearing children is treated as investment. This type of model is thought to be more applicable in developing countries (e.g., Zhang and Nishimura [21]). In an altruistic model, having children is regarded as consumption. This type of model is considered to be more applicable to developed countries. In this type of model, there are two modes of inter-generational altruism. In one case, as in Eckstein and Wolpin [6] and Eckstein, Stern, and Wolpin [7], parents derive utility from the number of their children. In the other case, as in Razin and Ben-Zion [16], and Caballe [2], parents also derive utility from their children's utility.

While existing studies have intensively analyzed the determinants of fertility at a country level and, especially, its influence on economic growth (see Ehrlich and Lui [8]), analysis of the determinants of regional variations in fertility within a country has not received much attention thus far. However, looking into the data on regional fertility, we see that regional variations are not ones we can ignore. For example, in Japan, the prefectural total fertility rate in 2000 is 1.07 in Tokyo, 1.28 in Kyoto, 1.31 in Osaka, 1.62 in Tottori, 1.65 in Fukushima, and 1.67 in Saga (Vital Statistics (Ministry of Health, Labour and Welfare)).

This paper explores the determinants of regional variations in fertility within a country. This does of course not imply that analysis regarding the interactions between national fertility and economic activities is unimportant or negligible. It is certainly important and significant. Here, we want to say that regional variations in fertility are *also not negligible* (The total fertility rate in Saga is 0.6 points, or over 50 percents larger than that in Tokyo, for example.), and that it is worth analyzing

[1] In fact, during the past decade (1990-2000), the total fertility rate has declined from 1.8 to 1.6 in the United Kingdom, from 1.5 to 1.3 in Germany, and 1.5 to 1.4 in Japan (OECD [15]), for example.

how such variations and economic activities affect each other. For this purpose, this paper does not focus on the dynamics of national fertility but focuses on the regional variations in fertility within a country at one point in time. Therefore, in the model developed in this paper, it is assumed that there is no growth of total factor productivity, and only the steady state is considered.

In the analysis, we focus on regional population density as a key factor. Urban and regional economists have revealed the positive and negative roles of the geographical concentration of economic agents in an economy. The positive role is called "agglomeration economies". Causes of agglomeration economies include knowledge spillover across firms, the presence of a more extensive division of labor, preference for variety in consumption and increasing returns owing to firm-level economies to scale, and heterogeneity of workers and firms. (See Fujita and Thisse [10], and Duranton and Puga [5] for comprehensive surveys on the micro-foundations of agglomeration economies.) Empirical studies such as Ciccone and Hall [3], Ciccone [4], and Tabuchi and Yoshida [20] showed the existence of agglomeration economies by showing that productivity and the wage rate are higher in a region with higher population density.

Traditionally, changes in wage rate have been thought to have positive and negative effects on fertility. The former is such that a rise in wage rate increases disposable income and increases the fertility rate, namely, the positive income effect. The latter is the effect that raises the opportunity cost of rearing children to reduce the fertility rate because parenting is time consuming and individuals must give up some working time in order to have children, namely, the negative substitution effect.[2] Shultz [18] provided, using Swedish data, empirical results indicating the existence of both effects. It showed that while a rise in male real wage rate increases the total fertility rate, an increase in female real wage relative to male real wage contributes to the decline in the total fertility rate. Therefore, it would be safe to state that population concentration can affect fertility via wage rises due to agglomeration economies.

The negative role is called "congestion diseconomies", which is caused mainly by workers' commuting to business districts, and leads to rises in land rent and cost of living (See Kanemoto [13] and

[2] These effects were fully discussed by Becker [1].

Fujita [9], among others.). Shultz [18] showed that even if wages are held constant, advances in urbanization reduces national fertility. This result indicates that there is a different channel than wage changes through which population concentration affects fertility in a negative way. The National Institute of Population and Social Security Research, Japan [14] surveyed the number of children that couples are going to have and the number of children that couples wish to have under ideal conditions. Its table 78 reports the reasons why couples are going to have fewer children than the ideal. The table shows that whereas 16.8 percent of couples who live in densely inhabited districts (DIDs) choose the unaffordability of having a sufficiently spacious house as one of the reasons, this figure is 5.4 percent for those who live in non-DIDs. These figures indicate that high land rent and, thus, congestion diseconomies have an effect of lowering the fertility rate. Therefore, population density is thought to play a significant role in determining regional variations in fertility via agglomeration economies and congestion diseconomies.

Figure 1 plots the relationship between the prefectural population density and the prefectural total fertility rate in Japan for the year 2000. Here, the prefectural population density is represented by the prefectural population per square kilometer of inhabitable land, on which data are calculated based on the data regarding the prefectural population (Population Census (Statistics Bureau, Ministry of Public Management, Home Affairs and Telecommunications)) and those regarding the areas of inhabitable land (System of Social and Demographic Statistics (Statistics Bureau, Ministry of Public Management, Home Affairs and Telecommunications)). The data regarding the prefectural total fertility rate were collected from Vital Statistics (Ministry of Health, Labour and Welfare).

[Please Insert Figure 1 here]

From this figure, we can see that *the regional total fertility rates are lower for more densely populated regions*. Table 1 summarizes the data to confirm this negative relationship. Figures and tables similar to Figure 1 and Table 1 can be obtained with respect to the years 1980 and 1990, implying that this relationship would be *persistent*.

[Please Insert Table 1 here]

This paper aims to present an explanation of this negative relationship by constructing a simple overlapping generations model of endogenous fertility that includes the altruism of parents toward their children a la Eckstein and Wolpin [6], incorporating n-regions, agglomeration economies, and congestion diseconomies. As stated above, we assume that the growth rate of total factor productivity is zero and we focus on the steady state because we are not interested in analyzing the relationship between economic growth and fertility, but rather between regional economic activities and fertility. Agglomeration economies we consider are external spillover benefits, which is formulated in a way that the productivity rises as more workers exist in a region. As in Henderson [12], we assume that each region specializes in one industry and the strength of this spillover differs from region to region. Congestion diseconomies from commuting are considered to reduce disposable income by raising land rent and cost of living.

The analysis shows how agglomeration economies and congestion diseconomies affect fertility in each region. Agglomeration economies have an effect of raising the wage rate in the corresponding region. Since individuals must give up some working time in order to have children in the model described in this paper, a rise in wage rate has both positive income and negative substitution effects on the fertility rate as discussed above. Congestion diseconomies have an effect of decreasing disposable income and thus lower the fertility rate. This is consistent with the empirical findings stated above. An important point here is that, in the model described in this paper, without congestion diseconomies, the positive income effect and the negative substitution effect balance each other out, and a change in wage rate does not affect fertility rate. With congestion diseconomies, the fertility rate is determined jointly by the wage rate and congestion diseconomies, and an increase in regional population decreases the regional fertility rate.

It is then proved that *the negative relationship between regional fertility rates and regional population density can hold as a steady-state equilibrium outcome.* Since regions are assumed to differ only in terms of the strength of agglomeration economies, a region with stronger agglomeration economies attracts more people. In an equilibrium, the congestion diseconomies offset the attractiveness of the region and the utility level is common to all individuals in all regions. Meanwhile, as discussed above, the regional fertility rate is lower for a region with higher population density, and it is shown that the regional fertility rate differs from region to region in a way that reflects differences in the opportunity cost of rearing children. Therefore, an individual in a region with higher population density restrains himself/herself from having many children and increases consumption. It is also proved that *in such an equilibrium, there is net migration from a region with lower population density to a region with higher population density, which, in turn, enables the regional variations in fertility to be persistent.*

Thus far, Zhang [22], and Sato and Yamamoto [17] are the only theoretical studies that focused on the regional differences in fertility. These studies developed models with two-regions: urban and rural, and analyzed the relationship between urbanization and demographic transition in the historical process.[3] However, the geographical structure of these models is somewhat too simple to analyze the cross-regional variations in fertility. In contrast, the model described in this paper has n-regions, all of which are accompanied by agglomeration economies and congestion diseconomies. Hence, the full-fledged geographical structure is incorporated in it, which would be more appropriate to be used to analyze the regional variations in fertility regarding a developed country such as Japan. Put differently, this paper and the two existing studies differ in the purpose and in the geographical structure of the models presented.

This paper is structured as follows: In section 2, we introduce the basic structure of the model. Section 3 shows the existence of a steady-state equilibrium and examines its properties. Concluding remarks are provided in Section 4.

2 Model

2.1 Individuals

Consider an economy that consists of l regions. Time is discrete and each individual lives for two periods; a working ('young') period, and a retirement ('old') period. Let N_{iyt} and N_{iot} denote the numbers of young and old people in region i ($i = 1, 2, \cdots, l$) in period t, respectively. In the working

[3] Whereas Zhang [22] focused on a rural-urban difference in opportunities for earnings and education, Sato and Yamamoto [17] emphasized the role of urban agglomeration economies and congestion diseconomies.

period, individuals choose regions to live in, supply labor there, and decide on lifetime consumption and their number of children. At the end of period t, each young individual in region i has n_{it} children and each old individual exits the economy. This implies that at the end of period t, $\sum_{i=1}^{l} n_{it} N_{iyt}$ children enter the economy and $\sum_{i=1}^{l} N_{iot}$ individuals exit the economy. These children grow to be young individuals in period $t+1$ so that an individual is young when his/her parent is old. In this model, n_{it} represents the total fertility rate. We assume that young people can migrate from one region to another without any cost but that migration cost is prohibitively high for old people so that they cannot migrate.[4] Hence, it must be the case that the number of young people in region i in period t is equal to the number of old people in region i in period $t+1$:

$$N_{iyt} = N_{iot+1}. \tag{1}$$

Since young people are perfectly mobile among regions, the number of children in region i in period t may not coincide with the number of young people in region i in period $t+1$. However, the total number of children in period t must be equal to the total number of young people in period $t+1$:

$$\sum_{i=1}^{l} n_{it} N_{iyt} = \sum_{i=1}^{l} N_{iyt+1}. \tag{2}$$

This condition is referred to as the law of motion of population.

Individuals are assumed to have an identical utility function of the Cobb-Douglas form and the utility of each individual depends on one's own consumption during the working and retirement periods and the number of children:

$$U = c_{yt}^{\alpha} c_{ot+1}^{\beta} n_t^{\gamma},$$

where c_{yt} and c_{ot+1} are consumption in the working period and in retirement, respectively.[5] There is only one kind of goods in this economy, which we treat as a numeraire. α, β and γ are positive constants and satisfy $\alpha + \beta + \gamma = 1$.

In order to have n_t children, each individual must spend bn_t time, where b is a positive constant. We assume that each working individual is endowed with one unit of time. These assumptions require

[4] Empirical studies have shown that a tendency to migrate declines with age (see Greenwood [11] among others).

[5] The Cobb-Douglass or the log-linear utility function, which has the qualitatively same nature as the Cobb-Douglas utility function, is widely used in models of endogenous fertility including Sato and Yamamoto [17].

that the number of children n_t must satisfy $0 \leq n_t \leq 1/b$. The time to work is $1 - bn_t$. The budget constraint in the working period is

$$(1 - bn_t)w_{it} - dN_{it} = c_{yt} + s_t,$$

where w_{it} denotes the wage rate per working hour. Here, local labor markets are assumed, and therefore, as we see later in detail, the wage rates vary from region to region because of differences in the strength of agglomeration economies. s_t is savings, and dN_{it} is the cost of living in region i. Here, d is a positive constant and N_{it} is the total population in region i ($N_{it} = N_{iyt} + N_{iot}$). We assume that all regions each have the same area and normalize it to one. Therefore, in this model, N_{it} represents the population density of a region, and dN_{it} includes land rent, commuting costs to the central business district (CBD), and represents congestion diseconomies. This representation of congestion diseconomies can be obtained from a monocentric city model. For instance, consider a linear city of width one. Land is owned by absentee landlords. There is one CBD and each young individual commutes to the CBD once per unit time to work and buy goods, and each old individual goes to the CBD once per unit time to buy goods. Each young or old individual consumes one unit of land in which to live, irrespective of age and the number of his/her children. Under these assumptions, the cost of living, that is, the sum of commuting cost and land rent, is described by a linear function of the sum of the number of young people and that of old people. (See Kanemoto [13] and Fujita [9] for a comprehensive discussion on monocentric city models.)[6]

The budget constraint in the retirement period is

$$(1 + r)s_t - dN^e_{it+1} = c_{ot+1}.$$

We assume the international capital and asset market and that the interest rate r is determined exogenously as a positive constant. In making decisions, young people make expectations regarding the cost of living in the next period (dN^e_{it+1}). Assuming perfect foresight, the budget constraint in

[6] For a detailed discussion on how this type of congestion diseconomies affect economic geography, see Tabuchi [19], among others.

the retirement period becomes

$$(1+r)s_t - dN_{it+1} = c_{ot+1}.$$

In maximizing the utility function, individuals regard not only prices but also the number of individuals N_i and N_{it+1} as given. The first-order conditions for the maximization of the utility function in region i give

$$\begin{aligned} c_{iyt} &= \alpha I_{it}, \\ c_{iot+1} &= (1+r)\beta I_{it}, \\ n_{it} &= \frac{\gamma I_{it}}{b w_{it}}, \\ s_{it} &= \beta I_{it} + \frac{dN_{it+1}}{1+r}, \end{aligned} \tag{3}$$

where I_{it} represents the potentially disposable income and is defined as

$$I_{it} = w_{it} - dN_{it} - \frac{dN_{it+1}}{1+r}.$$

It is the disposable income if workers use all their time for working. The indirect utility function is then

$$V_{it} = \alpha^{\alpha}\beta^{\beta}\gamma^{\gamma}(bw_{it})^{-\gamma} I_{it} = B w_{it}{}^{-\gamma} I_{it}, \tag{4}$$

where B is defined as $B = \alpha^{\alpha}\beta^{\beta}\gamma^{\gamma}b^{-\gamma}$.

We can see from (3) that a rise in wage has two effects on the total fertility rate n_{it}. One is the positive income effect that is represented in the numerator of the right hand side. The other is the negative substitution effect that raises the opportunity cost of rearing children. This is described by the denominator of the right hand side. In this model, the former dominates the latter and an increase in wage raises n_{it}. The congestion diseconomies have an effect of lowering disposable income and decreasing n_{it}.

Since young individuals are perfectly mobile, migration occurs so as to equate the indirect utility among regions. Then, we obtain

$$V_{it} = V_{jt} = \overline{v}_t, \quad \text{all } i, j. \tag{5}$$

We refer to this condition as the utility-equalization condition. Furthermore, in order for utility-equalization to be migration-stable, it is sufficient that an individual is worse off if he/she migrates to another region.[7] This requires the following condition:

$$\frac{\partial V_{it}}{\partial N_{iyt}} < 0, \quad \text{all } i. \tag{6}$$

2.2 Production

Next, we turn to the production side of the model. For the expositional simplicity, we assume that there is only one firm in each region.[8] A firm uses capital and labor to produce the numeraire that is freely traded among regions without any transportation cost. The output of each firm in region i is given by

$$y_{it} = \delta {N_{iyt}}^{\varepsilon_i} L_{it}^{\mu} K_{it}^{1-\mu},$$

where ε_i and μ are positive constants and do not exceed one ($0 < \varepsilon_i < 1$ and $0 < \mu < 1$), y_{it} is firm output in region i. L_{it} and K_{it} represent labor and capital input, respectively. Here, constant returns to scale in production are assumed at a firm level. Since we are not interested in analyzing the relationship between fertility and the growth rate of productivity, but rather its relative level in different regions and fertility, we set the growth rate of total factor productivity (TFP) to zero (*i.e.*, δ is a positive constant). ${N_{iyt}}^{\varepsilon_i}$ represents the external spillover effect on production of more workers in the same region. This external spillover represents the agglomeration economies in this model. Since the agglomeration economies are external to individual firms, each firm behaves competitively. ε_i represents the strength of external spillover in each region.[9] As in Henderson [12], we assume that each region specializes in one industry and that regions differ in the strength of this spillover ($\varepsilon_i \neq \varepsilon_j$ if $i \neq j$). Without loss of generality, we assume that for $i < j$, the external spillover is stronger for region i than for region j ($\varepsilon_i > \varepsilon_j$), which implies that $\varepsilon_1 > \varepsilon_2 > \cdots > \varepsilon_l$.

[7] These types of definitions of an equilibrium and its stability are very common to models including multiple cities (or regions) and individual's mobility. See Henderson [12] and Kanemoto [13], among others.

[8] As we see below, this assumption is not essential to the analysis since we assume constant returns to scale in production at a firm level.

[9] The determinants and microfoundations of agglomeration economies are enumerated in Duranton and Puga [5].

Each firm maximizes its profit $\pi_{it} = y_{it} - w_{it}L_{it} - rK_{it}$ with respect to L_{it} and K_{it}, taking prices and the number of workers in its region as given. Keeping in mind that the price of products is one (the numeraire) and that they are traded with no cost, we obtain the corresponding first-order conditions for the profit maximization of a firm:

$$\begin{aligned} \mu\delta N_{iyt}^{\varepsilon_i} L_{it}^{\mu-1} K_{it}^{1-\mu} &= w_{it}. \\ (1-\mu)\delta N_{iyt}^{\varepsilon_i} L_{it}^{\mu} K_{it}^{-\mu} &= r \end{aligned} \tag{7}$$

Note here that we assumed the interest rate is exogenous. This implies that the economy is a small open economy with the perfect mobility of capital and assets. For the analytical simplicity, we don't consider the explicit trading process, and assume the capital and asset market gets clear instantaneously. Figure 2 summarizes the structure of the model.

[Please Insert Figure 2 here]

3 Equilibrium analysis

3.1 Equilibrium conditions

In order to determine an equilibrium, we need two more conditions. First, the economy is assumed to be in a steady state. This implies that none of the variables depends on time since no economic growth is involved in this model. Hereafter, we drop the subscripts that represent time.

Second, the labor market clearing condition. Since a labor market is assumed to be local, the labor supply $(1 - bn_i)N_{iy}$ and the demand L_i in region i must balance each other out:

$$(1 - bn_i)N_{iy} = L_i. \tag{8}$$

Hereafter, the relevant variables of the equilibrium are marked with the superscript $*$. Under the steady state assumption, the variables to be determined are c_{iy}, c_{io}, n_i, s_i, L_i, K_i, w_i, N_{iy}, N_{io}, N_i, and $\overline{v}$. Here, c_{iy}, c_{io}, n_i, and s_i are determined by the optimization behavior of an individual (3).

The labor and capital demands L_i and K_i are given by the firm's profit maximization behavior (7). The wage rate w_i is determined by the labor market clearing condition (8). The number of young people, N_{iy}, is given by the utility-equalization condition (5) and must satisfy the migration-stability condition (6). Then, (1) in the steady state yields N_{io} and N_i:

$$N_{io} = N_{iy}, \ N_i = 2N_{iy}. \tag{9}$$

Finally, the equilibrium utility level $\overline{v}$ is determined to satisfy the law of motion of population (2).

3.2 Existence of an equilibrium

Substituting the labor market clearing condition (8) into the latter equation of firm's first order conditions (7), we obtain:

$$K_i = \left\{ \frac{(1-\mu)\delta}{r} \right\}^{1/\mu} (1 - bn_i) N_{iy}^{(\mu+\varepsilon_i)/\mu}. \tag{10}$$

This and the former equation of (7) give:

$$w_i = \phi N_{iy}^{\varepsilon_i/\mu}, \tag{11}$$

where ϕ is defined as

$$\phi = \mu \left(\frac{1-\mu}{r} \right)^{(1-\mu)/\mu} \delta^{1/\mu}.$$

From (3) and (9), we can see that the fertility rate n_i is a function of the wage rate w_i and the number of young people N_{iy}:

$$n_i = \frac{\gamma(w_i - DN_{iy})}{bw_i} = \frac{\gamma}{b} - \frac{\gamma D N_{iy}}{bw_i},$$

where D is defined as $D = 2d(2+r)/(1+r)$. Substituting (11) into this equation, we have:

$$n_i = \frac{\gamma}{b} - \frac{\gamma D N_{iy}^{(\mu-\varepsilon_i)/\mu}}{b\phi}. \tag{12}$$

In showing the existence of an equilibrium, we first regard N_{iy} as fixed and derive the admissible interval of N_{iy} in which n_i and w_i are well determined. Then, we explore under what conditions there is some $N_{iy} > 0$ that is determined endogenously by the remaining equilibrium conditions and lies in the interval.

In an equilibrium, it is necessary that n_i and w_i satisfy (11), (12), $0 \le n_i \le 1/b$, and $w_i \ge 0$. For any given $N_{iy} \ge 0$, the wage rate is well determined by (11). In order for n_i determined by (12) to satisfy $0 \le n_i \le 1/b$, it is sufficient that $\gamma/b \ge (\gamma D N_{iy}^{(\mu-\varepsilon_i)/\mu})/(b\phi)$. This gives an admissible interval of N_{iy}.

Lemma 1 *In order for an equilibrium to exist, N_{iy} must be in the following interval. (i) When $\mu > \varepsilon_i$, $[0, \underline{N}_{iy}]$, where $\underline{N}_{iy}$ is defined as $\underline{N}_{iy} = (\phi/D)^{\mu/(\mu-\varepsilon_i)} > 0$. (ii) When $\mu = \varepsilon_i$, $[0, +\infty)$ if $1 \ge D/\phi$ and there is no admissible interval otherwise. (iii) When $\mu < \varepsilon_i$, $[\underline{N}_{iy}, +\infty)$.*

Note that for any N_{iy} in the admissible interval, c_{iy}, c_{io}, n_i, s_i, L_i, K_i, w_i, m_i, N_{io}, and N_i are well determined since the inequalities $0 \le n_i \le \gamma/b < 1/b$, $w_i \ge 0$ and $I_i = w_i - DN_{iy} \ge 0$ hold.

For any N_{iy} in the admissible interval, we can see the following results.

Proposition 1 *An increase in N_{iy} raises the wage rate w_i, and lowers the total fertility rate n_i when $\mu > \varepsilon_i$, does not affect n_i when $\mu = \varepsilon_i$, and increases n_i when $\mu < \varepsilon_i$.*

Due to the agglomeration economies, an increase in N_{iy} raises the wage rate in the corresponding region. As explained in the previous section, an increase in wage has an effect of raising the number of children. Meanwhile, an increase in N_{iy} aggravates the congestion diseconomies, which have an effect of lowering the number of children. When $\mu > \varepsilon_i$, the agglomeration economies are not strong and the latter effect dominates the former. When $\mu < \varepsilon_i$, the agglomeration economies are so strong that the former effect dominates the latter.

By examining the admissible interval of N_{iy}, we can prove the existence of an equilibrium.

Proposition 2 *When $\mu > \varepsilon_i$, there exists an equilibrium if b is sufficiently small. Furthermore, if δ is sufficiently large, there is an equilibrium in which a region with stronger agglomeration economies (i.e., with higher ε_i) has more young people, and hence, has higher population density (i.e., $N_{1y}^* > N_{2y}^* > \cdots > N_{ly}^*$ and $N_1^* > N_2^* > \cdots > N_l^*$).*

Proof: See Appendix.

b is a parameter that represents the necessary time to rear a child. Hence, sufficiently small b prevents each individual from having no children. δ is a parameter that represents the total factor productivity (TFP).[10] Sufficiently large δ makes it possible for the economy to have large population and for each region to have a labor pool larger than one, which makes the equilibrium stated above consistent with the interpretation of parameter ε_i. Keep in mind that we interpret ε_i as the strength of external spillover and consider that larger ε_i implies stronger agglomeration economies. However, due to the functional form of the production function ($y_i = \delta {N_{iy}}^{\varepsilon_i} L_i^{\mu} K_i^{1-\mu}$), this interpretation makes sense only when the number of workers N_{iy} is larger than 1. When N_{iy} is smaller than 1, larger ε_i implies less output. In the remainder of this paper, we focus on an equilibrium with $N_1^* > N_2^* > \cdots > N_l^*$ stated in Proposition 2. In this equilibrium, the sizes of population in regions are different, whereas total population is constant.

Before moving to the analysis of variations in fertility rate, we will give a brief and informal discussion on the stability of the steady state equilibrium described in Proposition 2. We start from the steady state and consider a once and for all and exogenous increase in the number of young people at time t. Increases in young people are described by Δ. We consider that these newcomers are allocated to regions in a way of gravity models and assume that each region attracts $\Delta N_{iyt}/N_{yt}$ young people out of Δ. Since the number of young people in the next period is given by

$$N_{yt+1} = \sum_{i=1}^{l} n_{it} N_{iyt},$$

changes in N_{yt+1} due to exogenous increase Δ in N_{yt} are given as

$$dN_{yt+1} = \sum_{i=1}^{l} \left(n_{it} - \frac{\mu - \varepsilon_i}{\mu} \frac{\gamma D N_{iyt}^{(\mu-\varepsilon_i)/\mu}}{b\phi} \right) \frac{\Delta N_{iyt}}{N_{yt}}.$$

Note here that we start from a steady state. Therefore, we have $\sum_{i=1}^{l} n_{it} N_{iyt} = N_{yt}$, which yields

$$dN_{yt+1} = \Delta - \sum_{i=1}^{l} \frac{\mu - \varepsilon_i}{\mu} \frac{\gamma D N_{iyt}^{(\mu-\varepsilon_i)/\mu}}{b\phi} \frac{\Delta N_{iyt}}{N_{yt}} < \Delta.$$

[10] Keep in mind that since we are not interested in analyzing the relationship between productivity growth and fertility, but rather its relative level in different regions and fertility, we set the growth rate of TFP to zero.

Hence, the effects of a small exogenous shock shrink in the next period, implying that the steady state equilibrium is thought to be locally stable.

3.3 Regional variations in fertility rate

From (11), we find that for any given number of young people N_c that is larger than 1, the wage rate is higher in a region with stronger agglomeration economies, that is, $w_i|_{N_{iy}=N_c} > w_j|_{N_{jy}=N_c}$ if $\varepsilon_i > \varepsilon_j$. Furthermore, Lemma 1 states that the wage rate in a region gets higher as the number of young people in the region increases. Hence, in an equilibrium described in Proposition 2, i.e., an equilibrium with $N_{1y}^* > N_{2y}^* > \cdots > N_{ly}^* > 1$, the wage rate is higher in the region with stronger agglomeration economies, that is, $w_1^* > w_2^* > \cdots > w_l^*$. From (3) and (4), we can see that in an equilibrium,

$$n_i^* = \frac{\gamma \overline{v}^*}{bBw_i^{*1-\gamma}}. \tag{13}$$

Using this, we have

$$\frac{n_i^*}{n_j^*} = \left(\frac{w_j^*}{w_i^*}\right)^{1-\gamma},$$

which gives the result that an individual in a region with stronger agglomeration economies has fewer children: $n_1^* < n_2^* < \cdots < n_l^*$. The following proposition summarizes the above results.

Proposition 3 *In an equilibrium in which a region with stronger agglomeration economies has higher population density ($N_1^* > N_2^* > \cdots > N_l^*$), the wage rate is higher and the total fertility rate is lower in that region: $w_1^* > w_2^* > \cdots > w_l^*$ and $n_1^* < n_2^* < \cdots < n_l^*$.*

The results in Proposition 3 are consistent with the stylized facts explained in the introduction: the wage rate is high and the total fertility rate is low in a region with higher population density. The intuition behind this result is as follows. As we can see from (13), the equilibrium fertility rate is jointly a function of the utility level and the wage rate. A region with stronger agglomeration economies attracts more young people, which raises the wage rate in the region. In an equilibrium, the congestion diseconomies offset the attractiveness of the region, and the utility level is common to all people in all regions. Since the utility level is common to all regions, the difference in fertility comes from the difference in the wage rate, implying that the equilibrium fertility rate is lower in a region with higher population density.

Proposition 3 states that a region with a larger population has a lower number of children per individual. In order for this situation to be a steady state, some of the children born in a region with lower population density grow up to migrate to a region with higher population density. In fact, From Proposition 3, we can see that $1 - n_1^* > 1 - n_2^* > \cdots > 1 - n_l^*$. Combined with Proposition 2, this gives $(1 - n_1^*)N_{1y}^* > (1 - n_2^*)N_{2y}^* > \cdots > (1 - n_l^*)N_{ly}^*$. Since n_i^* and N_{iy}^* must satisfy the law of motion of population in the steady state $\sum_{i=1}^{l}(1 - n_i)N_{iy} = 0$, there exists some k such that $(1 - n_i^*)N_{iy}^* > 0$ for all $i < k$ and $(1 - n_j^*)N_{jy}^* \leq 0$ for all $j \geq k$. The above arguments prove the proposition.

Proposition 4 *In an equilibrium with $N_1^* > N_2^* > \cdots > N_l^*$, there is net migration from regions with lower population density to regions with higher population density. Furthermore, the amount of net migration is larger for a region with higher population density.*

This result is also consistent with reality. Table 2 summarizes the data regarding the relationship between the prefectural population density and the amount of net migration of Japanese prefectures. The data concerning net migration are obtained from the System of Social and Demographic Statistics (Statistics Bureau, Ministry of Public Management, Home Affairs and Telecommunications) for the years 2000-2002. The table shows evidence of net migration from regions with lower population density to regions with higher population density.

[Please Insert Table 2 here]

4 Concluding remarks

A stylized fact describing the negative relationship between regional population density and regional total fertility rate was presented. In order to explain this fact, this paper constructed a simple overlapping generations model of endogenous fertility, having multiple regions, agglomeration economies, and congestion diseconomies. It was shown that agglomeration economies and congestion diseconomies, combined with net migration from less densely populated to more densely populated regions, generate a steady-state equilibrium in which a region with stronger agglomeration economies has greater population density, a higher wage rate, and a lower total fertility rate. Thus, in the equilibrium, a negative relationship between regional population density and regional total fertility rate is observed. The important point is that a region with stronger agglomeration economies attracts more people to generate strong congestion diseconomies, which reduces the region's fertility rate. It is probably safe to conclude that the above analysis has shed some light on the significant roles of economic geography and migration in determining geographical features of fertility.

Acknowledgments

I thank Jan K. Brueckner, Gilles Duranton, Maryann Feldman, Masahisa Fujita, Katsuhisa Kojima, Tomoya Mori, Yasusada Murata, Dominique Peeters, Masaru Sasaki, Takatoshi Tabuchi, Akira Yakita, Kazuhiro Yamamoto, anonymous referees, and participants at various seminars for their helpful comments and constructive suggestions. Of course, all remaining errors are my own. This research is financially supported by JSPS (the Japan Society for the Promotion of Science) through a Grand-in-Aid for Scientific Research (B) (No. 14330004), a Grant-in-Aid for Young Scientists (B) (No. 18730162), and a Grand-in-Aid for Scientific Research (S) (No. 18103002).

Appendix: Proof of Proposition 2.

Proof: In order to show the existence of an equilibrium, it is sufficient to prove that for all i, there exist some N_{iy} in the admissible interval that satisfies the utility-equalization condition (5) and the migration-stability condition (6), and that the corresponding common utility level $\overline{v}$ satisfies the law of motion of population (2).

First, we consider the case of $\mu > \varepsilon_i$. In this case, the admissible interval of N_{iy} is $[0, \underline{N}_{iy}]$, where $\underline{N}_{iy} = (\phi/D)^{\mu/(\mu-\varepsilon_i)} > 0$ (which is defined in Lemma 1). We will proceed as follows: (i) we construct an interval of $\overline{v}$ such that for any $\overline{v}'$ in the interval, there exists some $N_{iy} \in [0, \underline{N}_{iy}]$ satisfying (5) and (6), and attaining $V_i = \overline{v}'$ for all i. (ii) We then prove that there is some $\overline{v}$ in the interval that satisfies (2).

Keep in mind that the indirect utility is given by (4): $V_i = Bw_i^{-\gamma} I_i = Bw_i^{-\gamma}(w_i - DN_{iy})$. Using (3), this can be rewritten as $V_i = bBw_i^{1-\gamma} n_i/\gamma$. From (11), we can see that $w_i^{1-\gamma} = 0$ when $N_{iy} = 0$. Furthermore, (12) yields that $n_i = \gamma/b$ when $N_{iy} = 0$. Hence, we have

$$V_i|_{N_{iy}=0} = 0.$$

Also, from (11) and (12), $n_i = 0$ and $w_i = \phi(\phi/D)^{\varepsilon_i/(\mu-\varepsilon_i)}$ when $N_{iy} = \underline{N}_{iy}$. Therefore, we obtain

$$V_i|_{N_{iy}=\underline{N}_{iy}} = 0.$$

Meanwhile, from (11) and (12), for any $N_{iy} \in (0, \underline{N}_{iy})$, it must be the case that $V_i > 0$ since $0 < n_i < \gamma/b < 1/b$ and $w_i > 0$. Hence, the continuity of V_i with respect to N_{iy} implies that V_i has its maximum in the interval $[0, \underline{N}_{iy}]$ and the maximal value $V_{i\,\mathrm{max}}$ is positive.

Take some $N_{iy} \in \{N_{iy} | V_i = V_{i\,\mathrm{max}}\}$ and describe it as $\overline{N}_{iy}$. Then, since V_i is obviously differentiable with respect to N_{iy}, we can take an interval $[\widehat{N}_{iy}, \underline{N}_{iy}] \subset [\overline{N}_{iy}, \underline{N}_{iy}]$ such that for any $N_{iy} \in [\widehat{N}_{iy}, \underline{N}_{iy}]$, the inequality $\partial V_i/\partial N_{iy} < 0$ holds (i.e., the migration-stability condition (6) is satisfied).[11]

Let region h be the one in which individuals enjoy the lowest $V_i|_{N_{iy}=\widehat{N}_{iy}}$ among all regions, that is, $V_i|_{N_{iy}=\widehat{N}_{iy}} \geq V_h|_{N_{hy}=\widehat{N}_{hy}} > 0$ for all i. Let $\widehat{v}_h$ denote $V_h|_{N_{hy}=\widehat{N}_{hy}}$. Then, for any $\overline{v} \in [0, \widehat{v}_h]$, there is some $N_{iy} \in [\widehat{N}_{iy}, \underline{N}_{iy}]$ that satisfies $V_i = \overline{v}$ (i.e., the utility-equalization condition (5) is satisfied) for all i. Let us describe such N_{iy} as $N^*_{iy}(\overline{v})$.

An equilibrium exists if there exists some $\overline{v} \in [0, \widehat{v}_h]$ that satisfies the law of motion of population (2). In a steady state, the law of motion of population (2) becomes

$$\sum_{i=1}^{l}(1 - n_i)N_{iy} = 0. \tag{C1}$$

Define $\Omega(\overline{v})$ as

$$\Omega(\overline{v}) = \sum_{i=1}^{l}(1 - n_i)N^*_{iy}(\overline{v}).$$

[11] Note that $\overline{N}_{iy} \in (0, \underline{N}_{iy})$ and $\widehat{N}_{iy} \in (0, \underline{N}_{iy})$ because for any $N_{iy} \in (0, \underline{N}_{iy})$, $V_i > 0$ and $V_i|_{N_{iy}=0} = V_i|_{N_{iy}=\underline{N}_{iy}} = 0$.

Then, an equilibrium exists if there exists some $\overline{v} \in [0, \widehat{v}_h]$ that satisfies $\Omega(\overline{v}) = 0$.

Reminding the fact that the only $N_{iy} \in [\widehat{N}_{iy}, \underline{N}_{iy}]$ that attain $V_i = 0$ is $\underline{N}_{iy}$, we obtain

$$\Omega(0) = \sum_{i=1}^{l} \underline{N}_{iy} > 0. \tag{C2}$$

Furthermore, note that

$$\Omega(\widehat{v}_h) = \sum_{i=1}^{l} (1 - n_i) N_{iy}^*(\widehat{v}_h).$$

From (3) and (4), we can see that

$$n_i = \frac{\gamma \overline{v}}{bBw_i^{1-\gamma}}. \tag{C3}$$

Since for any $\overline{v} \in (0, \widehat{v}_h]$, $N_{iy}^*(\overline{v}) \in [\widehat{N}_{iy}, \underline{N}_{iy})$ by the definition of $N_{iy}^*(\overline{v})$, we can see that $0 < n_i < \gamma/b < 1/b$ from (12). Hence, plugging (11) into (C3) and evaluating it at $\overline{v} = \widehat{v}_h$ yields

$$\begin{aligned} n_i &= \frac{\gamma \widehat{v}_h}{bB \left(\phi N_{iy}^*(\widehat{v}_h)^{\varepsilon_i/\mu} \right)^{1-\gamma}} \\ &\geq \frac{\gamma \widehat{v}_h}{bB \left(\phi \underline{N}_{iy}^{\varepsilon_i/\mu} \right)^{1-\gamma}} \\ &= \frac{\gamma \widehat{v}_h}{\alpha^\alpha \beta^\beta \gamma^\gamma b^{1-\gamma} \left(\phi \underline{N}_{iy}^{\varepsilon_i/\mu} \right)^{1-\gamma}}. \end{aligned}$$

Because $\underline{N}_{iy}$ does not depend on b, the last term exceeds one for sufficiently small b, and so does n_i for all i. Therefore, for sufficiently small b, it must be that

$$\Omega(\widehat{v}_h) < 0. \tag{C4}$$

We can see obviously that $\Omega(\overline{v})$ is continuous in $\overline{v}$. Combining this fact with (C2) and (C4), it is shown that for sufficiently small b, there exists some $\overline{v}^* \in (0, \widehat{v}_h)$ such that $\Omega(\overline{v}^*) = 0$ (i.e., the law of motion of population (2) is satisfied). Therefore, in this case, an equilibrium exists for sufficiently small b.

Second, we consider the case of $\mu = \varepsilon_i$. In this case, $\partial n_i / \partial N_{iy} = 0$ and $\partial w_i / \partial N_{iy} > 0$ from Proposition 1. Since the indirect utility (4) can be rewritten as $V_i = bBw_i^{1-\gamma} n_i / \gamma$, we can show that $\partial V_i / \partial N_{iy} > 0$ for all N_{iy} in the admissible interval. This implies that there is no N_{iy} that satisfies the migration-stability condition (6).

Finally, the case of $\mu < \varepsilon_i$. In this case, $\partial n_i/\partial N_{iy} > 0$ and $\partial w_i/\partial N_{iy} > 0$ from Proposition 1. We can show that $\partial V_i/\partial N_{iy} > 0$ for all N_{iy} in the admissible interval $[\underline{N}_{iy}, +\infty]$. This implies that there is no N_{iy} that satisfies the migration-stability condition (6).

Above arguments prove the existence of an equilibrium. Next, we show that in the equilibrium proved to exist in the case of $\mu > \varepsilon_i$, a region with stronger agglomeration economies (i.e., higher ε_i) has more young people and higher population density if δ is sufficiently large.

When δ is sufficiently large, $\phi = \mu\{(1-\mu)/r\}^{(1-\mu)/\mu}\delta^{1/\mu}$ is large enough to make $\underline{N}_{iy} = (\phi/D)^{\mu/(\mu-\varepsilon_i)}$ larger than one for all i. Here, we can consider a region H in which individuals enjoy the lowest $V_i|_{N_{iy}=1}$ among all regions, that is, $V_i|_{N_{iy}=1} \geq V_H|_{N_{Hy}=1} > 0$ for all i. Let $\widehat{v}_H$ denote $V_H|_{N_{Hy}=1}$. Define further $\widehat{v}$ as $\widehat{v} = \min[\widehat{v}_H, \widehat{v}_h]$ Then, for any $\overline{v} \in [0, \widehat{v}]$, there is some $N_{iy} \in [\widehat{N}_{iy}, \underline{N}_{iy}]$ that satisfies $V_i = \overline{v}$ and $N_{iy} > 1$ for all i. Let us again describe such N_{iy} as $N^*_{iy}(\overline{v})$. Then, we have that $N^*_{iy}(\overline{v})$ is larger than 1. The same argument as the above one shows that there exists some $\overline{v}^* \in (0, \widehat{v})$ that satisfies the law of motion of population (2) and an equilibrium exists when b is sufficiently small.

Furthermore, we can see, from (11) and (12), that for a given $N_c > 1$, the wage rate is higher and the total fertility rate is higher in the region with stronger agglomeration economies: $w_i|_{N_{iy}=N_c} > w_j|_{N_{jy}=N_c}$ and $n_i|_{N_{iy}=N_c} > n_j|_{N_{jy}=N_c}$ if $\varepsilon_i > \varepsilon_j$. Hence, for any given $N_c > 1$, the indirect utility V_i is strictly higher for a region with stronger agglomeration economies since the indirect utility can be rewritten as $V_i = bBw_i^{1-\gamma}n_i/\gamma$. Suppose now that there are some i and j with $\varepsilon_i > \varepsilon_j$ and $N^*_{jy}(\overline{v}^*) \geq N^*_{iy}(\overline{v}^*)$. From the fact that $\overline{v}^* > 0$ and the construction of $N^*_{iy}(\overline{v}^*)$ and $\underline{N}_{iy}$, we can see that $\underline{N}_{iy} > N^*_{iy}(\overline{v}^*)$, $\underline{N}_{jy} > N^*_{jy}(\overline{v}^*)$, $\underline{N}_{iy} > \underline{N}_{jy}$, $\partial V_i/\partial N_{iy} < 0$ for any $N_{iy} \in [N^*_{iy}(\overline{v}^*), \underline{N}_{iy}]$, and $\partial V_j/\partial N_{jy} < 0$ for any $N_{jy} \in [N^*_{jy}(\overline{v}^*), \underline{N}_{jy}]$. These imply that V_i and V_j intersect at least once in $[N^*_{jy}(\overline{v}^*), \underline{N}_{jy}]$. However, this contradicts the fact that for any given $N_c > 1$, V_i is strictly larger than V_j since $\varepsilon_i > \varepsilon_j$.

Therefore, in this equilibrium, $N^*_{iy}(\overline{v}^*) > N^*_{jy}(\overline{v}^*)$ if $\varepsilon_i > \varepsilon_j$, and thus the proposition holds.

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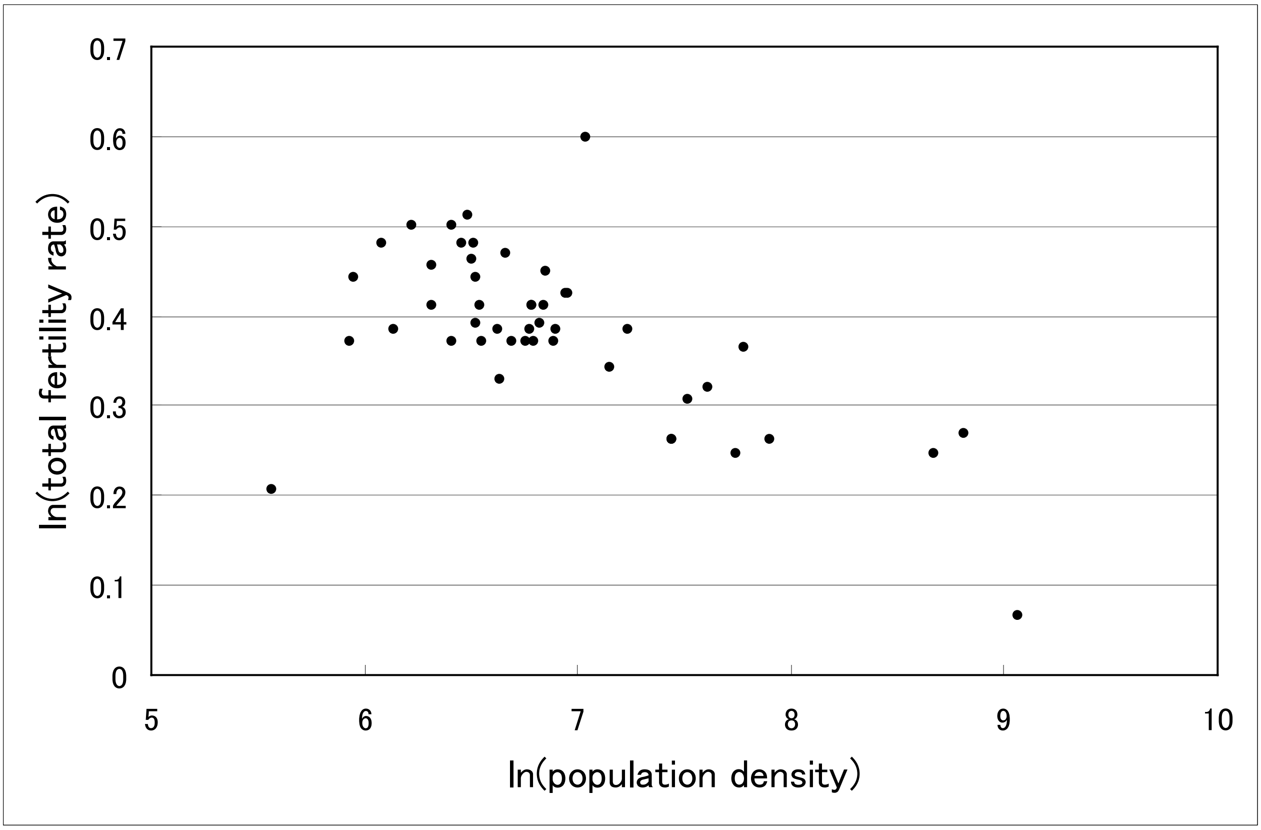

Figure 1. Regional population density and regional total fertility rate in Japan for the year 2000.
Note: the prefectural population density is the prefectural population per square kilometer of inhabitable land.

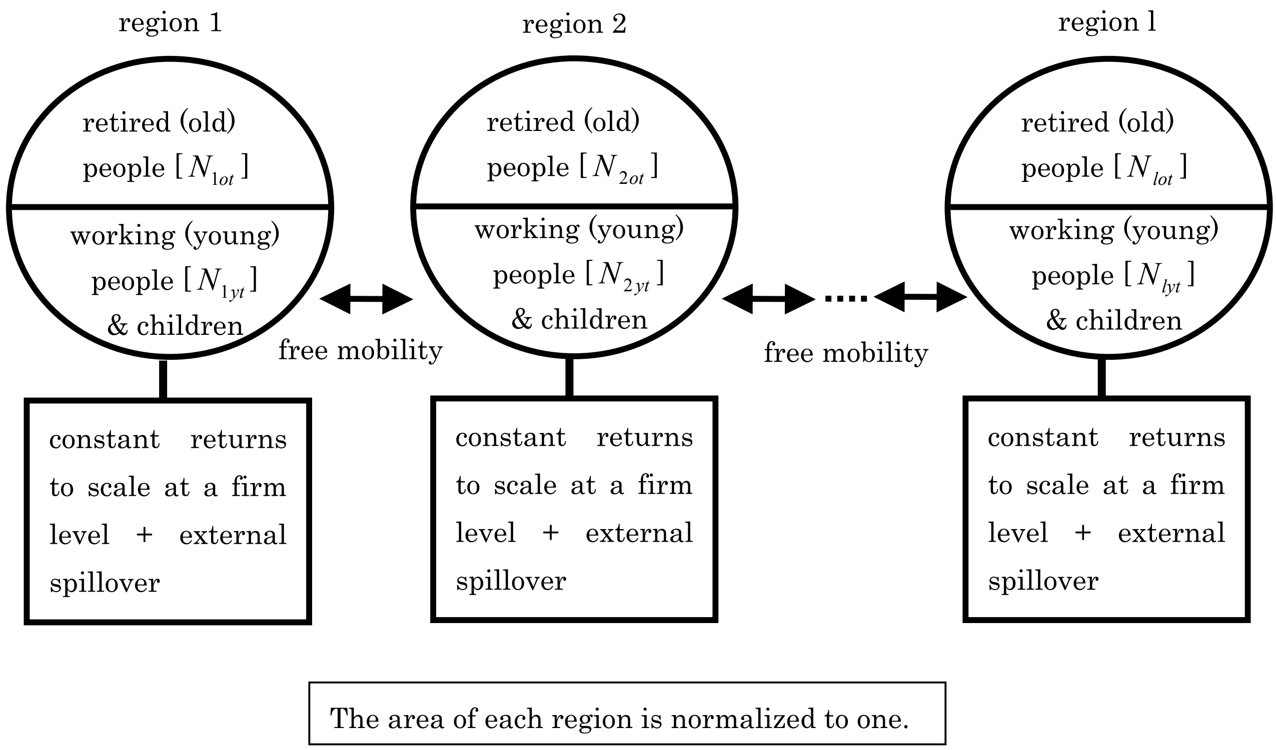

Figure 2. Structure of the model

	Total fertility rate	
	Mean	Median
Low density	1.54	1.56
Moderate density	1.48	1.47
High density	1.39	1.36

Table 1. Regional population density and fertility rate in Japan for the year 2000

Note: Low density and high density regions consist of 17 less densely populated prefectures and 15 more densely populated prefectures, respectively. Moderate density regions consist of the remaining 15 prefectures.

	Average number of net migration	
	Mean	Median
Low density	-2834.37	-2020.33
Moderate density	-2248.93	-1813.67
High density	5461.22	2267

Table 2. Regional population density for the year 2000 and average net migration for the years 2000-2002 in Japan

Note: Low density and high density regions consist of 17 less densely populated prefectures and 15 more densely populated prefectures, respectively. Moderate density regions consist of the remaining 15 prefectures.