## How precise is Earth's gravity?

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We use gravity ( $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$ or $32.2 \mathrm{ft} / \mathrm{s}^{2}$ ) often in engineering calculations, but since this course deals with accuracy, have you ever stopped to consider how accurate the value is? You might be surprised to read this, but the value you choose for gravity may not always be "correct," depending on the accuracy you need for your application!

First of all, gravity is not a constant value. It varies along the surface of the earth, with elevation above the earth, and even with time. For the sake of simplicity, we usually use standard gravity ${ }^{1}$, defined as $9.80665 \mathrm{~m} / \mathrm{s}^{2}$, or $32.1740 \mathrm{ft} / \mathrm{s}^{2}$. This value is not your local value instead, it represents the mean gravity at $45^{\circ}$ latitude, at mean sea level. This is the value we typically use in engineering calculations, although we usually round it down to one or two decimal places.

The gravity on the surface of the Earth varies due to a number of effects. The major effects are:

1. The rotation of the Earth. The rotation of the Earth reduces the force you feel at your feet (you would therefore feel lighter at the equator than at the poles of the Earth). Also, the rotation causes the Earth to be shaped like an oblate spheroid (flattened sphere - flatter at the poles), which means that the radial distance you are from the center of the Earth (and hence gravity) varies depending on where you are on the surface. Both of these effects vary with latitude. This variation is predictable, and is given by an equation known as the International Gravity Formula: ${ }^{2}$

$$
g_{0}=9.7803267714\left(\frac{1+0.00193185138639 \sin ^{2} \lambda}{\sqrt{1-.00669437999013 \sin ^{2} \lambda}}\right)
$$

where $\lambda$ is the geographic latitude and $g_{0}$ is called normal gravity. The variation due to Earth's rotation is on the order of $\pm 0.03 \mathrm{~m} / \mathrm{s}^{2}$.
2. Elevation above sea level. At higher elevations, you are further from the center of the Earth, so the Earth's pull is less. How big is this effect? At an elevation of about 1000 m , gravity reduces by about $0.0001 \mathrm{~m} / \mathrm{s}^{2}$.
3. Variation in mass. Gravity is a function of mass, and since the mass of the earth is not uniform, the gravity varies as well. The most accurate measurement of local gravity variation was performed by GRACE (Gravity Recovery and Climate Experiment), a satellite-based measurement. These measurements show that the local variation of gravity (relative to normal gravity) is on the order of about $\pm 0.0006 \mathrm{~m} / \mathrm{s}^{2}$ (See Figure 1).

[^0]

Figure 1. Local variation in gravity (relative to normal gravity), as measured by GRACE (Gravity Recovery and Climate Experiment).
4. Tides. Tidal variation (due to the gravitational pull of the sun and moon) contributes to a variation of about $\pm 0.000003 \mathrm{~m} / \mathrm{s}^{2}$.

These effects are summarized below, illustrating the relative effect of these factors:


Bottom line: If you do not account for latitude, a value for $g$ of $9.8 \mathrm{~m} / \mathrm{s}^{2}$ is as accurate as you can claim. Correcting for latitude allows you to claim two (and perhaps three) decimal-place accuracy. Beyond this, you would have to get better information on your local gravity variation.


[^0]:    ${ }^{1}$ Established by the 1901 International Conference on Weights and Measures (in French, the acronym is GCPM).
    ${ }^{2}$ Established by the World Geodetic System 1984. There are earlier versions of this formula.

