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Design of High Order Digital IIR Filter using Heuristic Optimization Technique

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Abstract*— The paper establishes methodology for the stable and robust design of high order infinite impulse response (IIR) digital filters using nature inspired technique known as Particle Swarm Optimization (PSO). PSO has been applied to design the stable digital IIR filter in order to avoid local minima, and to enhance the search capability and provides a fast convergence for calculating the optimal filter coefficients. The magnitude response and the stability of the filter have been determined by using Matlab. The pole-zero plot of the filter, describe the stability of the predesigned filter. A comparison has been made with other design techniques, demonstrating that PSO gives better or at least comparable results for designing digital IIR filters than the existing genetic algorithm based techniques. This technique can also be implemented for the design of HP, BP and BS digital IIR filters.

Keywords*— Digital Infinite-Impulse Response (IIR) filter, Particle Swarm Optimization (PSO) algorithm, Low Pass (LP) filter.

I. INTRODUCTION

In the past few decades, digital signal processing has developed rapidly in the area of science and engineering, both theoretically and technically. Digital Signal Processing is an integrated circuit, designed for manipulations of high speed data and used in applications like video communication, data-acquisition, hard disc drive controllers, biomedicine, audio and other image processing applications. This rapid development is a result of the significant advances in digital computer technology and integrated circuit fabrication. In digital signal processing, the processing of signals is carried by digital means. DSP is characterized by the representation of discrete frequency, discrete time or by a sequence of numbers or symbols. The advantages of DSP are its easy to store and use, more efficient, less expensive, applicability to very low frequency signals, easy to configure and less expensive.

In signal processing, a filter is essentially a network or system that are widely used in signal processing and communication circuit systems to remove unwanted parts of the signal such as random noise, prescribed frequency components or to extract meaningful data of the signal. Filters are frequency selective circuit that allows a certain range of frequency to pass while others are attenuated. This categorizes the filters into 4 groups: i) Low Pass ii) High Pass iii) Band Pass iv) Band Stop Filters. Based on the input signal filters are classified as: analog filters and digital filters. Analog filters use electronic components such as resistors, capacitors and op-amps, and operates on continuous-time signals. On the contrary, digital filter performs mathematical operation on a sampled, discrete time signal with the help of digital signal processor (DSP) chip or a processor used in a general purpose computer.

Digital filters are classified as: infinite impulse response (IIR) Digital filters and finite impulse response (FIR) Digital Filters. The output of FIR filter depends on present and past values of input, so also known as non-recursive filter. On the other hand the output of IIR filter not only depends on previous inputs, but also on the previous outputs. IIR filters are also known as recursive type filters, means feeding the output to the input of the same system for the calculation of the current output. IIR filter distinctly meets the design specification of sharp transition band width, pass band ripple and stop band attenuation with lower order as compared to FIR filter. IIR filter requires a large memory to store the previous outputs. On the other hand, FIR filter realization is easier with the requirement of less memory space and design complexity.

Digital IIR filters can be designed either by transformation or optimization method. In the transformation technique, various types of IIR filters (Butterworth, Chebyshev and Elliptic) have been implemented. Bilinear transformation is used for designing of IIR filter, by converting designed analog IIR filter to digital IIR filter. But the designed digital IIR filters by transformation technique are not good in stability. In optimization technique, the performance is measured by mean-square error and ripple magnitudes of both pass-band and stop-band. Optimization is a technique of making something fully perfect or effective as possible and to minimize the cost of production or to maximize the efficiency of production. Optimization methods are mainly of three types i) Direct search methods ii) Gradient based methods and iii) Nature inspired methods. As IIR digital filters have non-linear and multimodal nature of error surface, Conventional gradient-based design methods get easily stuck in the local minima of error-surface. The main constraints considered during designing of an optimal digital IIR filter are: i) high filter stability ii) determination of the lowest filter order iii) minimizing the ripple magnitudes (tolerances) of both pass-band and stop-band. Genetic algorithms based methods have been proposed by researchers [9, 10, 11] for designing of optimal IIR filter. The GA-based algorithms are able to optimize discontinuous and complex functions. But the results have not been so impressive, as Genetic algorithms have very slow convergence. So, a new hierarchical GA (HGA) [12] was proposed to overcome these shortcomings. In this new scheme, the structure of the filter is not fixed and has the capability of designing the lowest order filter. Another approach to solve the problem of designing stable digital IIR filters was proposed, known as hybrid Taguchi GA (HTGA) [13]. In this approach, the Taguchi method is inserted between crossover and mutation operations. Tsai et al. have proposed a new integrated method named as Taguchi-immune algorithm (TIA), for the design of optimal digital IIR filter. In this, the digital IIR filter structure and its coefficients have been coded separately.

The intent of this paper is to design Low Pass stable digital IIR filter using PSO. The designed low-pass digital IIR filter has the minimum magnitude error & minimum tolerance ripples in pass-band and stop-band. The stability of the designed filter has been observed by drawing pole-zero plot.

This paper is organized as: Section II describes the IIR filter design problem. Section III describes the PSO algorithm used for the designing of optimal high order Low Pass digital IIR filters. The evaluated results are compared with those of the results of Tang et al. [12], Ranjit Kaur et al. [3], Tasi et al. [13] in section IV. The conclusion of the proposed work is described in section V.

II. IIR FILTER DESIGN PROBLEM.

A set of filter coefficients is determined to meet the prescribed performance of pass-band width and gain, stop-band width and its attenuation, frequencies at the band edges, and value of peak ripple in the pass-band and stop-band. The design of recursive type IIR filter is described by the difference equation as:

$$y(n) = \sum_{i=0}^{M} p_i\, x(n-i) - \sum_{k=1}^{N} q_k\, y(n-k) \qquad (1)$$

where p_i and q_j are coefficients of the filter. x(n) and y(n) are the input and output of the filter respectively. N and M gives the order of the filter with N≥M. The transfer function of IIR filter is given as:

$$H(z) = \sum_{i=0}^{M} p_i z^{-i} \Big/ 1 + \sum_{k=1}^{N} q_k\, z^{-k} \qquad (2)$$

To meet the specified desired performance response, we need to specify a set of filter coefficients. The cascaded transfer function of digital IIR filter, involving the filter coefficients like poles and zeroes, is stated as:

$$H(w,x) = A \prod_{j=1}^{M} \frac{1 + a_{1j}e^{-jw}}{1 + b_{1j}e^{-jw}} \times \left(\prod_{k=1}^{N} \frac{1 + c_{1k}e^{-jw} + c_{2k}e^{-2jw}}{1 + d_{1k}e^{-jw} + d_{2k}e^{-2jw}} \right) \qquad (3)$$

where $x = [a_{11}, b_{11}, \ldots, a_{1M}, b_{1M}, c_{11}, c_{21}, d_{11}, d_{21}, \ldots, c_{1N}, c_{2N}, d_{1N}, d_{2N}, A]^T$.

The vector x represents the filter coefficients of dimension V×1 with V = 2M+4N+1 and w represents the discrete frequency. In the IIR filter, the coefficients are optimized such that the approximation error function for magnitude is to be minimized. The magnitude response is specified in pass-band and stop-band at K equally spaced discrete frequency points. For optimizing the filter coefficients, L_p-norm approximation error function for magnitude is minimized.

The L_p-norm approximation error $err_m(x)$ for the magnitude response is defined as:

$$err_m(x) = \left\{ \sum_{i=0}^{K} \left| H_d(w_i) - |H(w_i,x)| \right|^p \right\}^{1/p} \qquad (4)$$

where $H_d(w_i)$ is the magnitude response of the ideal IIR filter and $H(w_i,x)$ is the magnitude response of the desired IIR filter. For p = 1, the magnitude response error denotes the L_1 – norm error and is defined as:

$$e_1(x) = \sum_{i=0}^{K} \left| H_d(w_i) - |H(w_i,x)| \right| \qquad (5)$$

Ideal magnitude response $H_I(w_i)$ of IIR filter is given as:

$$H_d(w_i) = \begin{cases} 1, \text{for } w_i \epsilon \text{ passband} \\ 0, \text{for } w_i \epsilon \text{ stopband} \end{cases} \qquad (6)$$

$e_2(x)$ and $e_3(x)$ are the ripple magnitudes of pass-band and stop-band, respectively.

$$e_2(x) = \max_{w_i}|H(w_i,x)| - \min_{w_i}|H(w_i,x)| \quad \text{for } w_i \epsilon \text{ passband} \qquad (7)$$

$$e_3(x) = \max_{w_i}|H(w_i,x)| \quad \text{for } w_i \epsilon \text{ stopband} \qquad (8)$$

$$\text{Minimize } f(x) = \sum_{i=1}^{3} w_i\, e_i \qquad (9)$$

where w_i is a weight assigned to each error function.

The stability constraints which are obtained by aggregating all objectives and stability constraints for optimization are:

$1 + b_{1j} \geq 0$ (i =1, 2,….,M) (10)

$1 - b_{1j} \geq 0$ (i =1, 2,…., M)

$1 - d_{2k} \geq 0$ (k = 1,2,….,N)
$1 + d_{1k} + d_{2k} \geq 0$ (k = 1,2,….,N)
$1 - d_{1k} + d_{2k} \geq 0$ (k = 1,2,….,N)

III. PARTICLE SWARM OPTIMIZATION (PSO).

In 1995, Eberhart and Kennedy have developed a new evolutionary computation technique PSO, inspired by the social behaviour of bird flocking and fish schooling. PSO is used for solving problem in engineering and computer science, as it has capability to handle non-differential objective function and larger search space. It is a naturally inspired technique that is not affected by non-linearity, size and convergence of the problem

In the optimal design of IIR digital filter, the efficiency and capability of PSO in the minimization of multimodal functions with many local and global minima has been verified. In this global optimization technique, each particle flies through the search space of solutions. For optimization, every particle is updated by its own best value (p best) and the global best value (g best). Now each particle modifies its position using the following points: i) the distance between the current position and the p best. ii) the distance between the current position and the g best.

$$v_f = w * C_1 + 2 * \text{rand}() * (p\ best - x_i) + C_2 * \text{rand}() * (g\ best - x_i) \quad (11)$$

$$x_f = x_i + v_f \quad (12)$$

where,
w is the weight with $w_{max} = 0.3$ and $w_{min} = 0.1$.
v_f is the final velocity of particle.
x_i is the current position of particle and x_f is the final position of particle.
p best and g best are the local best and global best of the particle.
rand () is a random value between (0, 1) and C_1 & C_2 are learning factors usually having value 2.

A. Algorithm: Particle Swarm Optimization.

1. Initialize the swarm as initial population where each particle in swarm is a solution vector.
2. Evaluate the fitness function of each particle.
3. Compare each particle's fitness with the current particle's to obtain p best.
4. Now to obtain g best compare fitness evolution with the population's overall previous best.
5. Finally the position and velocity of each particle is updated using the equations (11) and (12).

IV. DIGITAL IIR HIGH ORDER LOW-PASS FILTER DESIGN AND COMPARISONS.

For the comparison of designing High Order digital IIR filters, firstly the lowest order is set exactly same as that given by Kaur Ranjit et al. [3], Tang et al. [12], Tsai et al. [13], then for high order the value of M & N varies from (1, 1) to (5, 5). Therefore the order of the IIR filter has been varied from 3 to 15. The purpose of designing high order Low Pass IIR digital filter is to minimize the function given by equation (9). The prescribed design conditions for the design of High Order Low Pass IIR filter are stated as below:

Table I: Desired Design Conditions for Low-Pass Filter.

Filter Type	Pass-band	Stop-band	$\lvert H(w,x)\rvert$
Low-Pass	$0 \leq w \leq 0.2\pi$	$0.3\pi \leq w \leq \pi$	1

The designing of high order Low Pass IIR digital filter is done by setting 200 equally spaced points within the frequency domain $[0, \pi]$. The IIR digital filter models for 3^{rd} and 13^{th} order filter are expressed as:

1. The designed model obtained for 3^{rd} order LP digital filter is:

$$H_{LP}(z) = .028882\left(\frac{z+1.000866}{z-.666614}\right)\left(\frac{z^2-.227070z+1.001635}{z^2-1.428422z+.730927}\right)$$

2. The designed model obtained for 13^{th} order LP digital filter is:

$$H_{LP}(z) = .000260\left(\frac{z+.657430}{z-.357430}\right)\left(\frac{z+1.226076}{z-.394340}\right)\left(\frac{z-.381330}{z-.288040}\right)\left(\frac{z+.289120}{z+.318746}\right)\left(\frac{z-.230420}{z-.411474}\right)$$

$$\times\left(\frac{z^2-.670841z+0.817970}{z^2-1.449991z+.903346}\right)\left(\frac{z^2+.042655z+.945630}{z^2-.914385z+.293685}\right)\left(\frac{z^2-.311810z+1.130478}{z^2-1.159116z+.389900}\right)\left(\frac{z^2-.253433z+.374524}{z^2-1.372629z+.693598}\right)$$

Table II: Design Results for High Order Low Pass IIR Filter.

Filter Order	**Magnitude Error**	**Pass-band Performance (Ripple Magnitude)**	**Stop-band Performance (Ripple Magnitude)**
3	3.2763	$0.7876 \leq \lvert H(e^{jw})\rvert \leq 1.0373$ (0.2496)	$\lvert H(e^{jw})\rvert \leq 0.129315$ (0.1293)
4	2.0938	$0.8892 \leq \lvert H(e^{jw})\rvert \leq 1.0321$	$\lvert H(e^{jw})\rvert \leq 0.0987$

		(0.1428)	(0.0987)
5	0.4804	$0.9787 \le \lvert H(e^{jw}) \rvert \le 1.0094$ (0.0306)	$\lvert H(e^{jw}) \rvert \le 0.0375$ (0.0375)
6	0.2714	$0.9856 \le \lvert H(e^{jw}) \rvert \le 1.0163$ (0.0307)	$\lvert H(e^{jw}) \rvert \le 0.0162$ (0.0162)
7	0.2496	$0.98743 \le \lvert H(e^{jw}) \rvert \le 1.0035$ (0.0161)	$\lvert H(e^{jw}) \rvert \le 0.0252$ (0.0252)
8	0.3386	$0.9971 \le \lvert H(e^{jw}) \rvert \le 1.0058$ (0.0087)	$\lvert H(e^{jw}) \rvert \le 0.0337$ (0.0337)
9	0.1526	$0.9854 \le \lvert H(e^{jw}) \rvert \le 1.0021$ (0.0166)	$\lvert H(e^{jw}) \rvert \le 0.0126$ (0.0126)
10	0.2386	$0.9935 \le \lvert H(e^{jw}) \rvert \le 1.0073$ (0.0137)	$\lvert H(e^{jw}) \rvert \le 0.0181$ (0.0181)
11	0.1405	$0.9657 \le \lvert H(e^{jw}) \rvert \le 1.0122$ (0.0465)	$\lvert H(e^{jw}) \rvert \le 0.0055$ (0.0055)
12	0.1741	$0.9943 \le \lvert H(e^{jw}) \rvert \le 1.0040$ (0.0096)	$\lvert H(e^{jw}) \rvert \le 0.0123$ (0.0123)
13	0.1202	$0.9857 \le \lvert H(e^{jw}) \rvert \le 1.0051$ (0.0193)	$\lvert H(e^{jw}) \rvert \le 0.0072$ (0.0072)
14	0.1964	$0.9823 \le \lvert H(e^{jw}) \rvert \le 1.0025$ (0.0201)	$\lvert H(e^{jw}) \rvert \le 0.0116$ (0.0116)
15	0.1965	$0.9845 \le \lvert H(e^{jw}) \rvert \le 1.0031$ (0.0186)	$\lvert H(e^{jw}) \rvert \le 0.0121$ (0.0121)

The results obtained by PSO technique for high order Low Pass IIR filter are given in Table 2. It can be seen that the magnitude error and pass-band & stop-band ripples decreases as the order of IIR digital filter increases. This can be shown by plotting the magnitude response graph between the normalized frequency and the magnitude.

For the stability of digital filters, the poles should lie within the unit circle, whereas zeroes can be anywhere. The magnitude response for 3rd order LP IIR filter and 13th order LP IIR filter is given in Fig 1 and Fig 2.The pole-zero plots for 3rd and 13th order LP IIR filters are shown in Fig 3 and Fig 4. The filters are stable as the poles lie within the unit circle.

Table III: Comparison of Low Pass IIR Filter Results.

Method	**Magnitude error**	**Pass-band Performance (Ripple Magnitude)**	**Stop-band Performance (Ripple Magnitude)**
PSO	3.2763	$0.7876 \le \lvert H(e^{jw}) \rvert \le 1.0373$ (0.2496)	$\lvert H(e^{jw}) \rvert \le 0.1293$ (0.1293)
RCGA	4.0095	$0.9335 \le \lvert H(e^{jw}) \rvert \le 1.016$ (0.0825)	$\lvert H(e^{jw}) \rvert \le 0.1510$ (0.1510)
HTGA	4.2511	$0.8914 \le \lvert H(e^{jw}) \rvert \le 1.000$ (0.1086)	$\lvert H(e^{jw}) \rvert \le 0.1247$ (0.1247)

HGA	4.3395	$0.8870 \le \lvert H(e^{jw}) \rvert \le 1.000$ (0.1139)	$\lvert H(e^{jw}) \rvert \le 0.1802$ (0.1802)

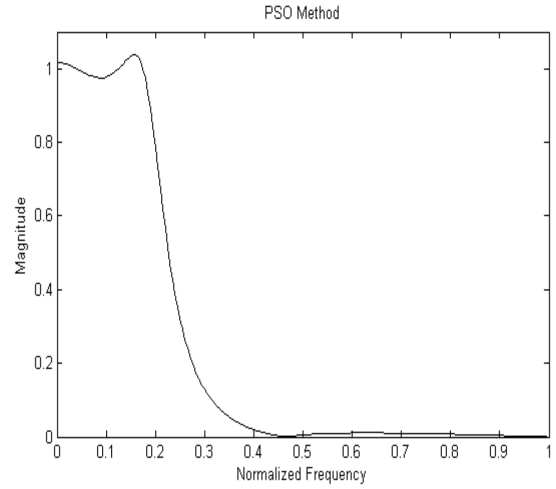

Fig 1: Magnitude Response for 3rd order LP IIR filter.

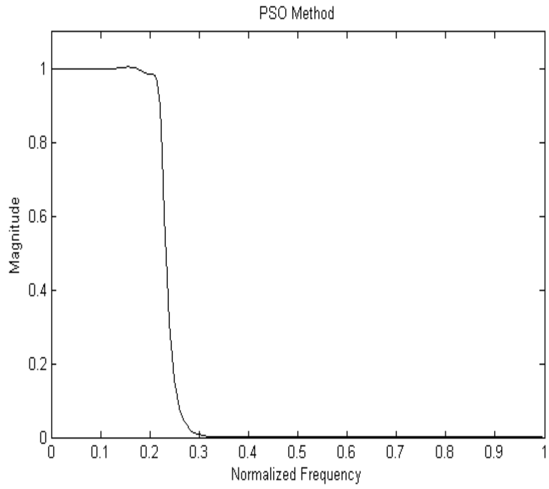

Fig 2: Magnitude Response for 13th order LP IIR filter.

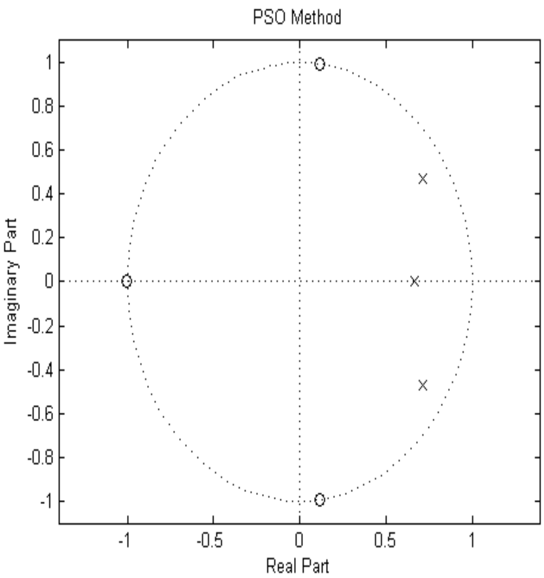

Fig 3: Pole zero plot for 3rd order LP IIR filter.

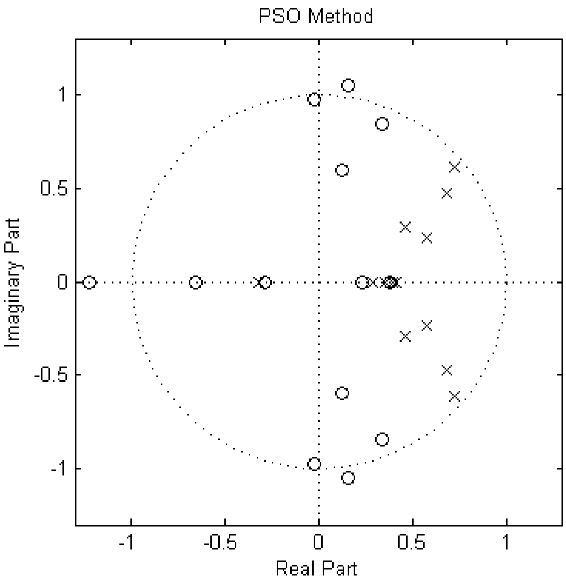

Fig 4: Pole zero plot for 13th order LP IIR filter.

The graph in between number of iterations and magnitude for 3rd and 13th order LP IIR filter is shown in Fig 4 and Fig 5. The magnitude gets stabilized after the best minimum result iteration.

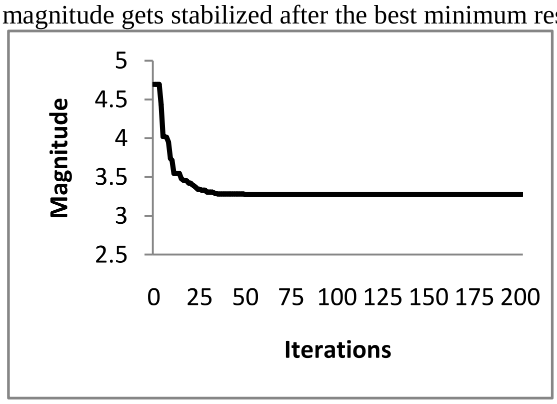

Fig 4: Iterations Vs Magnitude for 3rd order LP IIR filter.

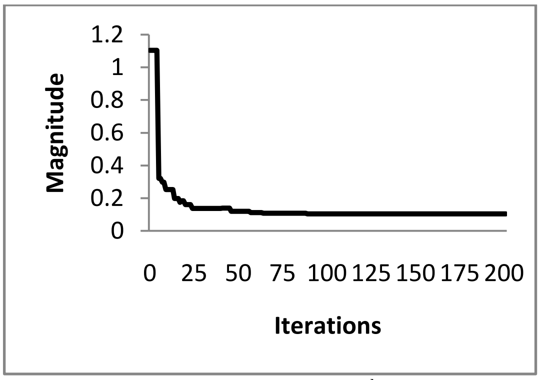

Fig 5: Iterations Vs Magnitude for 13th order LP IIR filter.

The maximum, minimum, average and standard deviation in magnitude error achieved after 200 number of runs for each filter varying values of (M ,N) from (1,1) to (5,5), filter order varying from 3 to 15 is shown in Table 4. The value of standard deviation less than 1 shows that all the IIR digital filters are stable.

Table IV: Maximum, Minimum, Average and Standard Deviation of Magnitude Error.

Filter Order	Maximum	Minimum	Average	Standard Deviation
3	4.8276	3.2763	3.3919	0.1898
4	3.4329	2.0938	2.2702	0.2162
5	5.38941	0.47066	1.350028	0.6301
6	1.4376	0.2381	0.6173	0.3456
7	1.3609	0.2496	0.4643	0.3459
8	1.4987	0.3386	0.8110	0.3171
9	1.3946	0.1526	0.4236	0.1812
10	2.9367	0.2386	0.6313	0.3440
11	2.7461	0.1405	0.3927	0.3550
12	2.6909	0.1741	0.5778	0.3867
13	2.0214	0.1228	0.3663	0.3086
14	1.6445	0.1964	0.3902	0.0929
15	2.9958	0.1965	0.5750	0.4161

V. CONCLUSION

This paper proposes PSO method for the design of high order Low Pass stable digital IIR filter. The order of the filter has been varied from 3 to 15 by varying the values of M & N from (1, 1) to (5, 5). As shown through simulation results, the results shown by PSO are better than that of RCGA and HTGA in terms of magnitude error and ripple magnitudes in pass-band and stop-band. All the designed LP stable IIR digital filters are stable as the poles of all the filters lie within the unit circle. The best results for high order filter are of 13th order filter. So PSO algorithm has better performance over others in terms of magnitude response, convergence speed and stability for the design of high order Low Pass digital filters.

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