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Fuzzy Sets and Artificial Intelligence: A Survey

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Abstract*— *The theory of fuzzy has advanced in a variety of ways and in many disciplines. Applications can be found, for example, in artificial intelligence, computer science, medicine, control engineering, decision theory, expert systems, logic, management science, operations research, pattern recognition, and robotics. Mathematical developments have advanced to a very high standard and are still forthcoming today. In this review, the basic mathematical framework of fuzzy set theory will be described, as well as the most important applications of this theory to other theories and techniques. Fuzzy set theory, the theory of neural nets and the area of evolutionary programming have become known under the name of 'computational intelligence or 'soft computing'. The relationship between these areas has naturally become particularly close. In this review, however, we will focus primarily on fuzzy set theory. Applications of fuzzy set theory to real problems are abound. Some references will be given. Classification with Fuzzy Logic is generated more number of rules. Since the RST is utilized in our work, the classification using Fuzzy can be done with less amount of complexity. The performance analysis is based on Sensitivity, Specificity and Accuracy with different cluster numbers. To describe even a part of them would certainly exceed the scope of this review

Keywords*— *Fuzzy Sets, Artificial Intelligence, Fuzzy Logic, Computational Intelligence, Soft Computing.

I. INTRODUCTION

Fuzzy mathematics is the study of fuzzy structures, or structures that involve *fuzziness* i.e., such mathematical structures that at some points replace the two classical truth values 0 and 1 with a larger structure of degrees. Most of our traditional tools for formal modelling, reasoning, and computing are crisp, deterministic, and precise in character. Crisp means dichotomous, that is, yes-or-no type rather than more-or-less type [1]. In traditional dual logic, for instance, a statement can be true or false—and nothing in between. In set theory, an element can either belong to a set or not; in optimization a solution can be feasible or not. Precision assumes that the parameters of a model represent exactly the real system that has been modelled. This, generally, also implies that the model is unequivocal, that is, that it contains no ambiguities. Certainty eventually indicates that we assume the structures and parameters of the model to be definitely known and that there are no doubts about their values or their occurrence. Unluckily these assumptions and beliefs are not justified if it is important, that the model describes well reality (which is neither crisp nor certain). In addition, the complete description of a real system would often require far more detailed data than a human being could ever recognize simultaneously, process, and understand. This situation has already been recognized by thinkers in the past. In 1923, the philosopher B. Russell referred to the first point when he wrote: 'All traditional logic habitually [2].

The development up to the recent state of the art is surveyed. However, all the systems proved to be algebraizable in the technical, which means that they also have algebraic semantics, provided by a class and usually a variety of algebraic structures and which sometimes are denied immediately via an algebraic semantics. It is an old approach, dating back to the early days of fuzzy set theory, to identify the membership degrees of fuzzy sets with truth degrees of a suitable many-valued logic. In different forms, this idea has been ordered and explained [3]. This point of view toward fuzzy set theory has been one of the motivations behind the development of mathematical fuzzy logics. Therefore one may expect that the recent results in this field of mathematical fuzzy logics give rise to a return to this starting point to use the new insights e.g. for a coherent development of a (formalized) fuzzy set theory [4]. But more is of interest here because, in classical logic, there is a reach field of extensions of classical first order logic which all lead to systems in between first order and second order logic. Such systems are e.g. determined by generalized quantifiers, particularly by cardinality quantifiers. So there is the parallel problem for the fuzzy case, to explore also in this context the space of logics in between first and second order. Thus classical logic can be viewed as a fuzzy logic for those special occasions when *by chance* the realizations of all predicates are crisp [1-5].

To demonstrate the usefulness of this definition we show that it is equivalent to some important properties studied in fuzzy literature. For finitary logics, the following are equivalent:

- L is weakly implicative fuzzy logic.
- Each L-matrix is a subdirect product of linear ones. *(Subdirect representation property)*
- Each theory in L can be extended to one whose Lindenbaum-Tarski matrix is linear.

(Linear extension property)

The large freedom in defining fuzzy mathematical notions correlates with freedom in interpreting the informal meaning of membership degrees. Depending on the intended interpretation, various structures of membership degrees and various definitions of fuzzy mathematical notions are appropriate [6]. Vice versa, particular structures of membership degrees and particular definitions of fuzzy mathematical notions admit only some of all possible informal interpretations and applications of the fuzzified theory. Unfortunately, this fact is seldom reflected in the practice of the fuzzy community. The omission of such considerations can result in arbitrariness of definitions, inappropriateness of applications, and completely unclear methodology, for all of which fuzzy mathematics has often (and in many cases quite justly) been reproached and disrespected by the mainstream mathematical community [6][7]. *Deductive* (or *formal, symbolic, mathematical)* fuzzy logic follows the modus operandi of classical logic. Without necessarily claiming that the philosophical notion of truth as such is (or is not) many-valued, it employs semantically models that assign intermediary truth degrees to propositions. In deductive fuzzy logic, like in fuzzy mathematics in general, a richer structure of truth degrees enables to model gradual change between truth and falsity, which seems appropriate in many real-life situations. For a long time, probability theory and statistics have been the predominant theories and tools to model uncertainties of reality [8][9][10]. They are based—as all formal theories— on certain axiomatic assumptions, which are hardly ever tested, when these theories are applied to reality. In the meantime more than other 'uncertainty theories' have been developed, which partly contradict each other and partly complement each other. Fuzzy set theory—formally speaking—is one of these theories, which was initially intended to be an extension of dual logic and/or classical set theory. During the last decades, it has been developed in the direction of a powerful 'fuzzy' mathematics. When it is used, however, as a tool to model reality better than traditional theories, then an empirical validation is very desirable [9][11-15].

II. LITERATURE SURVEY AND GENERALIZED REVIEW OF FUZZY LOGIC

Siegfried Gottwald and University at Leipzig discussed some open problems in the field of mathematical fuzzy logic which may have a decisive influence for the future development of fuzzy logic within the next decade [3][4]. Matthias Baaz and Richard Zach explained that there is a rich structure of infinite-valued Godel logics, only one of which is compact. It is also shown that the compact infinite-valued G¨odel logic is the only one which interpolates, and the only one with an r.e. entailment relation [14]. Petr Cintula introduced that A theory of fuzzy partitions is an important part of any theory meant to provide a formal frame- work for fuzzy mathematics. In Henkin-style higher-order fuzzy logic is introduced and proposed as a foundational theory for fuzzy mathematics. Here we investigate the properties of fuzzy partitions within its formal framework [15]. Each cluster from the clustering phase is classified in this second phase using a Fuzzy Logic. Fuzzy Inference is a method of generating a mapping from a given input to an output using fuzzy logic. Then, the mapping gives a basis, from which decisions can be generated or patterns discerned. Membership Functions, Logical Operations, and If-Then Rules are used in the Fuzzy Inference Process. The Stages of Fuzzy Inference Systems are,

a) Fuzzification
b) Fuzzy Rules Generation
c) Defuzzification

The Structure of the Fuzzy Inference System is given in the fig. 1. The three stages are also illustrated in the figure with the cluster as the input and the classification result as the output.

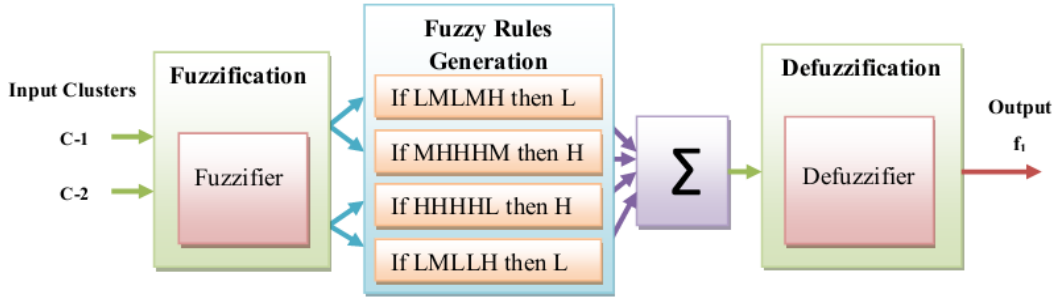

Fig 1: Structure of Fuzzy Inference System

A. Fuzzification

During the Fuzzification process, the crusty quantities are converted into fuzzy. For the Fuzzification process, the input is the two clusters, *C*-1 *and C* -2, that are the output of clustering algorithm using Rough Set Theory [4][5]. After that, the minimum and maximum value of each cluster is calculated from the input features. The process of Fuzzification is computed by applying the following equations:

$$ML^{(c-1)} = min + \left(\frac{max - min}{3}\right) \quad (5)$$

$$XL^{(c-1)} = ML + \left(\frac{max - min}{3}\right) \quad (6)$$

$(ML^{(c-1)})$ - Minimum limit values of the feature *M* .
$(XL^{(c-1)})$ - Maximum limit values of the feature *M* .
Use these equations (5) and (6), for calculating the minimum and maximum limit values for other cluster *C* - 2 also. And also, three conditions are provided to generate the fuzzy values by using these equations.

Conditions

1. All the "Cluster 1 (C -1)" values are compared with "Minimum Limit Value ($(ML^{(c-1)})$) ". If any values of Cluster 1 values are less than the value ($(ML^{(c-1)})$), then those values are set as L.
2. All the "Cluster 1 (C -1)" values are compared with "Maximum Limit Value ($(XL^{(c-1)})$) ". If any values of Cluster 1 values are less than the value($XL^{(c-1)}$) , then those values are set as H.
3. If any values of "Cluster 1 (C -1)" values are greater than the value($ML^{(c-1)}$), and less than the value ($XL^{(c-1)}$) then those values are set as M . Similarly, make the conditions for other cluster C-2 also for generating fuzzy values.

B. Fuzzy Rules Generation

According to the fuzzy values for each feature that are generated in the Fuzzification process, the Fuzzy Rules are also generated.

C. General form of Fuzzy Rule

"IF A THEN B"

The "IF" part of the Fuzzy Rule is called as "antecedent" and also the "THEN" part of the rule is called as "conclusion". The output values between L and H of the FIS is trained for generating the Fuzzy Rules.

D. Defuzzification

The input given for the Defuzzification process is the fuzzy set and the output obtained is a single number. As much as fuzziness supports the Rule Evaluation during the intermediate steps and the final output for every variable is usually a single number. The single number output is a value L ,M or H . This value of output $f1$, represents whether the given input dataset is in the Low range, Medium range or in the High range. The FIS is trained with the use of the Fuzzy Rules and the testing process is done with the help of datasets [5][6][7].

Basic Definitions and Operations

Definition 1 If X is a collection of objects denoted generically by x, then a fuzzy set A˜ in X is a set of ordered pairs:

$$\tilde{A}=(x,\mu_{\tilde{A}}(x)|x \in X) \quad (1)$$

$\mu_{\tilde{A}}(x)$ is called the membership function (generalized characteristic function) which maps X to the membership space M. Its range is the subset of nonnegative real numbers whose supremum is finite. For sup $\mu_{\tilde{A}}(x) = 1$: normalized fuzzy set.

In Definition 1, the membership function of the fuzzy set is a crisp (real-valued) function. Zadeh also defined fuzzy sets in which the membership functions themselves are fuzzy sets [7][8][9]. Those sets can be defined as follows:

Definition 2 A type m fuzzy set is a fuzzy set whose membership values are type m − 1, m > 1, fuzzy sets on [0, 1]. Because the termination of the fuzzification on stage $r \leq m$ seems arbitrary or difficult to justify, Hirota[17] defined a fuzzy set the membership function of which is pointwise a probability distribution: the probabilistic set.

Definition 3 A probabilistic set A on X is defined by defining function μA,

$$\mu A : X \times (x, \omega) \rightarrow \mu A(x, \omega) \in \_C \quad (2)$$

and $(_C,B_C) = [0, 1]$ are Borel sets.

Definition 4 A linguistic variable is characterized by a quintuple (x,T(x),U,G, ˜M), in which x is the name of the variable, T(x) (or simply T) denotes the term set of x, that is, the set of names of linguistic values of x. Each of these values is a fuzzy variable, denoted generically by X and ranging over a universe of discourse U, which is associated with the base variable u; G is a syntactic rule (which usually has the form of a grammar) for generating the name, X, of values of x. M is a semantic rule for associating with each X its meaning. ˜M(X) is a fuzzy subset of U. A particular X, that is, a name generated by G, is called a term (e.g., Figure 1).

E. Fuzzy Analysis

A fuzzy function is a generalization of the concept of a classical function. A classical function f is a mapping (correspondence) from the domain D of definition of the function into a space S; $f(D) \subseteq S$ is called the range of f. Different features of the classical concept of a function can be considered fuzzy rather than crisp. Therefore, different 'degrees' of Fuzzification of the classical notion of a function are conceivable:

1. There can be a crisp mapping from a fuzzy set that carries along the fuzziness of the domain and, therefore, generates a fuzzy set. The image of a crisp argument would again be crisp.

2. The mapping itself can be fuzzy, thus blurring the image of a crisp argument. This is normally called a *fuzzy function*. Dubois and Prade call this 'fuzzifying function'.

3. Ordinary functions can have properties or be constrained by fuzzy constraints. Particularly for Case 2, definitions for classical notions of analysis, such as, extrema of fuzzy functions, integration of fuzzy functions, integration of fuzzy functions aver a crisp interval, integration of a crisp function over a fuzzy interval, fuzzy differentiation, and so on have been suggested. It would exceed the scope of this survey to describe even a larger part of them in more detail [10].

F. Empirical Evidence

So far fuzzy sets and their extensions and operations were considered as formal concepts that need no proof by reality. If, however, fuzzy concepts are used, for example, to model human language, then one has to make sure that the concepts really model, what a human says or thinks. One has, with other words, to 'extract' thoughts from the human brain and

compare them with the modeling tools and concepts. This is a problem of psycho-linguistics Bellman and Zadeh suggested in their paper that the 'and' by which in a decision objective functions and constraints are combined can be modeled by the intersection of the respective fuzzy sets and mathematically modeled by 'min' or by the product. The interpretation of a decision as the intersection of fuzzy sets implies no positive compensation (trade-off) between the degrees of membership of the fuzzy sets in question, if either the minimum or the product is used as an operator. This is also true if any other t-norm is used to model the intersection or operators, which are no t-norms but map below the min-operator. It should be noted that, in fact, there are decision situations in which such a 'negative compensation' (i.e., mapping below the minimum) is appropriate. The interpretation of a decision as the union of fuzzy sets, using the max-operator, leads to the maximum degree of membership achieved by any of the fuzzy sets representing objectives or constraints. This amounts to a full compensation of lower degrees of membership by the maximum degree of membership. No membership will result, however, which is larger than the largest degree of membership of any of the fuzzy sets involved. Observing managerial decisions, one finds that there are hardly any decisions with no compensation between either different degrees of goal achievement or the degrees to which restrictions are limiting the scope of decisions. It may be argued that compensatory tendencies in human aggregation are responsible for the failure of some classical operators (min, product, max) in empirical investigations. The following conclusions can probably be drawn: neither t-norms nor t-conorms can alone cover the scope of human aggregating behavior. It is very unlikely that a single nonparametric operator can model appropriately the meaning of 'and' or 'or' context independently, that is, for all persons, at any time and in each context.

There seem to be three ways to remedy this weakness of t-norms and t-conorms: one can either define parameter dependent t-norms or t-conorms that cover with their parameters the scope of some of the nonparametric norms and can, therefore, be adapted to the context. A second way is to combine t-norms and their respective t-conorms and such cover also the range between t-norms and t-conorms (which may be called the range of partial positive compensation). The disadvantage is that generally some of the useful properties of t- and s-norms get lost. The third way is, eventually, to design operators, which are neither t-norms nor t-conorms, but which model specific contexts well. Empirical validations of suggested operators are very scarce. The 'min', 'product', 'geometric mean', and the γ -operator have been tested empirically and it has turned out, the γ -operator models the 'linguistic and', which lies between the 'logical and' and the 'logical exclusive or', best and context dependently. It is the convex combination of the product (as a t-norm) and the generalized algebraic sum (as a t-conorm). In addition, it could also be shown that this operator is pointwise injective, continuous, monotonous, commutative and in accordance with the truth tables of dual logic. Limited empirical tests have also been executed for shapes of membership functions, linguistic approximation, and hedges.

G. Applications

It shall be stressed that 'applications' in this review mean applications of fuzzy set theory to other formal theories or techniques and not to real problems.

Fuzzy Logic, Approximate Reasoning, and Plausible Reasoning

Logics as bases for reasoning can be distinguished essentially by three topic-neutral items: truth values, vocabulary (operators), and reasoning procedures (tautologies, syllogisms). In dual logic, truth values can be 'true' (1) or 'false' (0) and operators are defined via truth tables (e.g., Table 2). *A* and *B* represent two sentences or statements which can be true (1) or false (0). These statements can be combined by operators. The truth values of the combined statements are shown in the columns under the respective operators 'and', 'inclusive or', 'exclusive or', implication, and so on. Hence, the truth values in the columns define the respective operators.

Considering the *modus ponens* as one tautology:

$$(A \wedge (A \Rightarrow B)) \Rightarrow B \quad (40)$$

or:

Premise: *A* is true
Implication: If *A* then *B*
Conclusion: *B* is true.

Here four assumptions are being made:

1. *A* and *B* are crisp.
2. *A* in premise is identical to *A* in implication.
3. True = absolutely true
False = absolutely false.
4. There exist only two quantifiers: 'All' and 'There exists at least one case'.

In *fuzzy logic*, the truth values are no longer restricted to the two values 'true' and 'false' but are expressed by the linguistic variables 'true' and 'false'. In approximate reasoning additionally the statements *A* and/or *B* can be fuzzy sets. In plausible reasoning, the *A* in the premise does not have to be identical (but similar) to the *A* in the implication. In all forms of fuzzy reasoning, the implications can be modelled in many different ways. Which one is the most appropriate can be evaluated either empirically or axiomatically. Of course, the models for implications can also be chosen with respect to their computational efficiency (which does not guaranty that the proper model has been chosen for a certain context).

G. Fuzzy Rule-Based Systems (Fuzzy Expert Systems and Fuzzy Control)

Knowledge-based systems are computer-based systems, normally to support decisions in which mathematical algorithms are replaced by a knowledge base and an inference engine. The knowledge base contains expert knowledge. There are different ways to acquire and store expert knowledge. The most frequently used way to store this knowledge are if–then rules. These are then considered as 'logical' statements, which are processed in the inference engine to derive a conclusion or decision. Generally, these systems are called 'expert systems'. Classical expert systems processed the truth values of the statements. Hence, they were actually not processing knowledge but symbols. In the 1970s, crisp rules in the knowledge base were substituted by fuzzy statements, which semantically contained the context of the rules. Naturally the inference engine had to be substituted by a system that was able to infer from fuzzy statements. It shall be called here 'computational unit'. The first, very successful, applications of these systems were in control engineering. They were, therefore, called 'fuzzy controllers'. The input to these systems was numerical (measures of the process output) and had to be transformed into fuzzy statements (fuzzy sets), which was called 'fuzzification'. This input, together with the fuzzy statements of the rule base, was then processed in the computational unit, which delivered again fuzzy sets as output. Because the output was to control processes, it had to be transformed again into real numbers. This process was called 'defuzzification'. This is one difference to fuzzy expert systems, which are supposed to replace or support human experts. Hence, the output should be linguistic and, rather than having 'defuzzification' at the end, these systems use 'linguistic approximation' to provide user-friendly output. Several methods have been developed for fuzzification, inference, and defuzzification, and at the beginning of the 1980s the first commercial systems were put on the market (control of video cameras, cement kilns, cameras, washing machines, etc.). This development started primarily in Japan and then spread to Europe and the United States. It boosted the interest in fuzzy set theory tremendously, such that in Europe one was talking of a 'fuzzy boom' around 1990. Research and development in the area of fuzzy control is, however, still ongoing today [1, 3, 7].

III. CONCLUSION

Since its inception in 1965 as a generalization of dual logic and/or classical set theory, fuzzy set theory has been advanced to a powerful mathematical theory. In more than 30,000 publications, it has been applied to many mathematical areas, such as algebra, analysis, clustering, control theory, graph theory, measure theory, optimization, operations research, topology, and so on. In addition, alone or in combination with classical approaches it has been applied in practice in various disciplines, such as control, data processing, decision support, engineering, management, logistics, medicine, and others. It is particularly well suited as a 'bridge' between natural language and formal models and for the modelling of non stochastic uncertainties.

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