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The Fractional Discrete Spline Transform for Signal Processing

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Abstract: ***This paper define a new wavelet transform that is based on basic scaling function , fractional B-spline . This paper also shows the development of fractional discrete spline transform and describes a new family of scaling functions, the (α, τ)-fractional splines , which generate valid multiresolution analyses. These functions are characterized by two real parameters: α, which controls the width of the scaling functions; and τ, which specifies their position with respect to the grid (shift parameter).***

Keywords: ***Wavelet transform , Fractional B-splines , Fractional discrete spline transform , Multiresolution analyses.***

I. INTRODUCTION

Fractional splines have been investigated by Unser and Blu [1], which provide a new direction to spline based signal processing. Fractional discrete cosine transform and fractional sine transform have been proposed by different researchers in recent years [2,3] and have found suitable for different signal processing applications. This has motivated us to develop the fractional discrete spline transform (FRDSPLT). An orthogonal discrete spline transform (DSPLT) has been proposed in [4] for processing of signals using harmonic spline basis functions. In this Section, we introduce the concept of the fractional discrete spline transform (FRDSPLT) and propose an efficient scheme for harmonic analysis of a signal in scale-frequency domain. The proposed FRDSPLT has been considered for discrete-time periodic signals. Classification of harmonic components and percentage harmonic in a discrete-time periodic signal has been considered as an application. Results produced reveal the suitability of the proposed scheme. Although the complexity of the proposed transform is O (N log N), it provide us a precise scale-frequency analysis. In addition, the proposed transform can be used to obtain all levels of an MRA that might justify the expense.

II. DEVELOPMENT OF THE FRDSPLT

Fractional symmetrical B-splines are defined as [1]

$$\beta_*^a = \beta_+^{\frac{a-1}{2}} * \beta_-^{\frac{a-1}{2}}. \qquad (1)$$

where 'a' is the fractional degree and β_+^a is the causal fractional B-spline defined as

$$\beta_+^a(x) = \frac{1}{\Gamma(a+1)} \sum_{j\geq 0} (-1)^j \binom{a+1}{j} (x-j)_+^a. \qquad (2)$$

for all $a > 0$.The noncausal fractional B-spline β_-^a can be obtained by using the relation $\beta_-^a(x) = \beta_+^a(-x)$. Note that $\Gamma(a)$ denotes the gamma function and $\binom{a+1}{j}$ represents the generalized binomial coefficients as explained in [151]. The Fourier transform of the fractional symmetrical B-spline is given by

$$\hat{\beta}_*^a(\omega) = \left(\frac{\sin(\omega/2)}{\omega/2}\right)^{a+1}. \qquad (3)$$

Let us consider the space of periodic functions $f \in L_2[0, N]$. Let $\Im$ be the subspace of N-periodic fractional B-splines of order 'a'. Let S^a denote the corresponding interpolation matrix. Let Λ^a be the diagonal matrix of corresponding eigenvalues $\lambda_k^{(a)}$. Let the family of modified fractional spline basis functions be

$$\eta_k^{(a)}(x) = \frac{1}{\sqrt{\lambda_k^{(2a+1)}}} \sum_{m=0}^{N-1} v_{mk}\, \beta_m^a(x), \quad \text{for} \quad k=0,1,2,\ldots,N-1 \qquad (4)$$

$$\text{where} \quad \beta_m^a(x) = \beta_N^a(x-m).$$

Then the FRDSPLT is defined as:

$$D : L_2\,[0, N] \rightarrow \Re^N$$

$$D\,g(k) = \langle g, \eta_k^a \rangle \quad , \text{ for k=0,1,2,…, N-1} \tag{5}$$

These modified fractional spline harmonic basis functions may also be described in Fourier domain as given below

$$\hat{\eta}_0^a(0) = \alpha_0 \tag{6}$$

$$\hat{\eta}_{N-1}^a(\pi) = \alpha_{N-1}\left(\frac{2}{\pi}\right)^{(a+1)} \tag{7}$$

and for $1 \le n \le (N/2)-1$, we have

$$\hat{\eta}_{2n}^a(\omega_n) = \alpha_{2n}\left(\frac{\sin(\omega_n m/2)}{\omega_n m/2}\right)^{a+1} \tag{8}$$

$$\hat{\eta}_{2n-1}^a(\omega_n) = \alpha_{2n-1}\left(\frac{\sin(\omega_n m/2)}{\omega_n m/2}\right)^{a+1} \tag{9}$$

where ‘ m ’ is the resolution factor and the coefficients α_k are numerical values depending on k, N and the eigenvalues $\lambda_k^{(2a+1)}$.

III. APPLICATION EXAMPLE

An efficient scheme for harmonic analysis of discrete-time periodic signals using the FRDSPLT. From the above discussions (Eqs.6-to 9) , that the spectral information of the proposed transform is finite while keeping modulating sinusoids fixed along the time axis. This property enables the present technique attractive for analyzing harmonic components of discrete-time periodic signals. The proposed technique has been explained below. In this scheme, the resolution factor ‘ m ’has been considered to be an inverse function of the constituent frequency of the discrete-time periodic signal under consideration and is given by $m = (1/f)$. This feature of the proposed scheme makes it attractive for harmonic analysis of a periodic signal. Compute the transform matrix **D**. It is noteworthy to mention here that each row of the transform matrix represents a constituent frequency. The corresponding contour values help us to compute the Standard Deviation, which are used for classification of harmonics (component type and percentage). Simulation results are produced to convince the readers about the power of the present technique.
The proposed scheme can be used for harmonic analysis by using the following algorithm.

Algorithm :

Step 1 **:** Evaluate the Fourier transform of the modified fractional B-spline ..

Step 2 **:** Evaluate the Fourier Transform of the given discrete-time periodic signal {g(k)} by using FFT.

Step 3 **:** Shift the fractional B-spline spectrum with index l and then multiply with the Fourier ` Transform of the given discrete-time periodic signal.

Step 4 **:** Compute the inverse Fourier transform of the product to generate the row of the transform matrix **D** corresponding to a constituent frequency ‘n’.

Step 5 **:** Repeat steps 3 and 4 till one gets all rows of the matrix **D**.

Step 6 **:** Compute the Standard Deviation of the contour corresponding to each row of **D**. The Standard Deviation provides us information about various harmonic components and their percentage harmonics present in the given signal.

Step 7 : Peak values represent the percentage harmonic and their corresponding position in the frequency axis provide us information about the harmonic component (1st, 2nd, 3rd,4th, …etc) present in the signal under consideration.

This algorithm has been implemented in MATLAB to find the transform matrix **D** for different signals. We consider two different signals to highlight the power and usefulness of the proposed scheme. Fig.1 shows analysis of a periodic signal contaminated with 2nd and 5th harmonics with 40% and 12%, respectively. Fig.2 shows analysis of a periodic signal contaminated with 3rd and 4th harmonics with 30% and 18%, respectively. Harmonic components and their corresponding percentage harmonics are clearly observed from figures 1 and 2. Note that the fundamental frequency is $f = 50Hz$ for this example.

IV. RESULT AND DISCUSSION

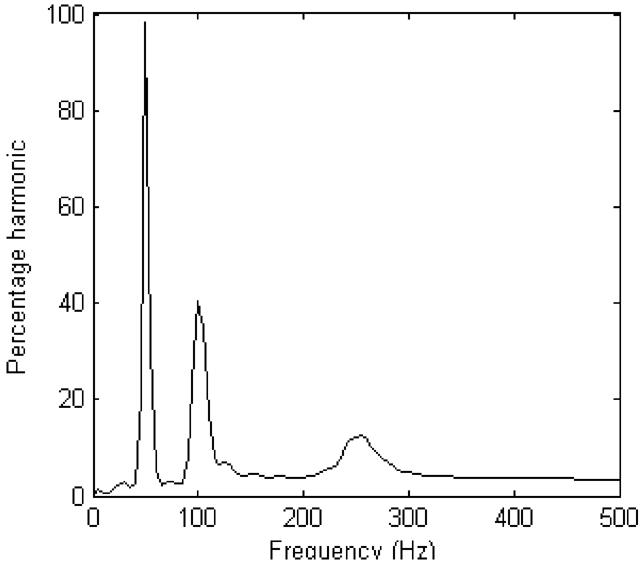

Fig.1. Analysis of a signal containing 2nd and 5th harmonic components with $a = 2.7$

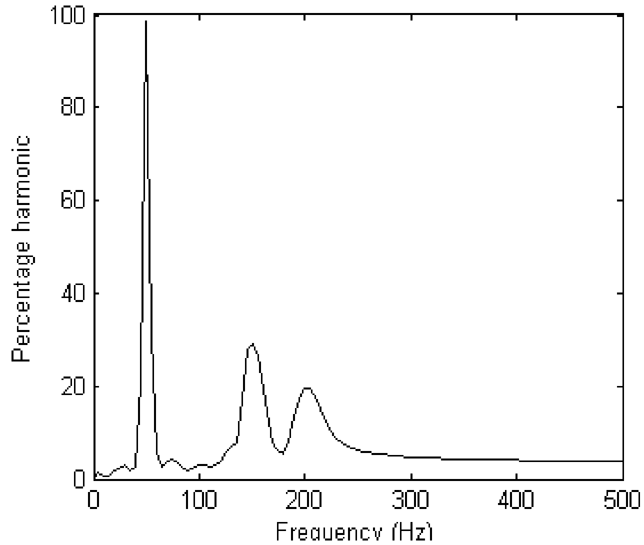

Fig.2. Analysis of a signal containing 3rd and 4th harmonic components with $a = 2.7$

V. CONCLUSIONS

The FRDSPLT has been proposed, which may be used for different signal processing applications . We have briefly outlined one possible application. Results shown in figure 1 and figure 2 reveal the suitability and effectiveness of the proposed technique for harmonic analysis of discrete-time periodic signals [7]. Since the transform derived can be used for multiresolution analysis, we believe that the proposed transform will become useful signal processing tool for different applications in the future.

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