STUDY GUIDE:

Module 7: Rational Numbers, Part 4

In this module we see how rational numbers would have been developed had we elected to use the structure of *place value*. Our study requires that we introduce the reciprocals of the powers of ten. That is, in whole number place value we use such denominations as: 1; 10; 100; 1,000; 10,000; and so on. In this module we include as well the denominations: $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1,000}$, $\frac{1}{10,000}$ and so on. To represent these denominations by place value we use a period called a *decimal point* to separate the whole number denominations from the fractional denominations. The first digit to the *left* of the decimal point names the *ones*. The first digit to the *right* of the decimal point names the *tenths*. That is:

thousands	hundreds	tens	ones	tenths	hundredths	thousandths
2	3	4	5	6	7	8

would be abbreviated as:

2,345.678

We have already used this principle in our monetary system. For example, when we write \$2.37 we mean 2 *dollars*, 3 *dimes*, and 7 *cents*. But a dime is a tenth of a dollar and a cent is a hundredth of a dollar. In other words, the 3 is in the tenths (of a dollar) place and the 7 is in the hundredths (of a dollar) place. So whether or not the denomination is "dollars" we may think of 2.37 as meaning 237 *hudredths*. 2.37 is called a *decimal fraction*.

In this module we study how to add, subtract, and multiply decimal fractions. This arithmetic is very similar to the techniques we use in doing the arithmetic of whole numbers in place value notation.

For example to add, say, 2.37 and 4.51 we may think of the problem as being

$$\begin{array}{r} 237 \textit{ hundredths} \\ +\ 451 \textit{ hundredths} \\ \hline 788 \textit{ hundredths} \end{array}$$

In this way, our adjectives are whole numbers and the fractional parts become the nouns. Since we already know how to use place value to do the arithmetic of whole numbers, it is not a difficult step to translate these results to decimal fractions. The only "complication" that occurs in this module is the placement of the decimal point when we multiply two decimal fractions. Namely while 0.2 + 0.5 means 2 tenths + 5 tenths or 7 tenths; 0.2 X 0.5 means 2 tenths X 5 tenths. As we mentioned in our treatment of common fractions, 2 X 5 is 10 but a tenth times a tenth is a *hundredth*. In any event this problem is readily solved in this module; so that by the end of the module we can add, subtract, and/or multiply any decimal fractions.

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Step 1:

View Videotape Lecture #7.

Step 2:

Read Module 7 of the text.

Step 3:

When you feel that you understand the material presented in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each blank.

Check the Main Ideas

8.23 is called a decimal _____. The period is called a _____. Because the 2 is the first digit to the right of the decimal point it stands for 2 _____. The 3 stands for 3 _____. The first digit to the left of the decimal point always tells us the number of _____.

fraction

decimal point

tenths

hundredths

ones (units)

0 still serves as a place-_____. For example, when we write 8.023 the 0 means that we have _____ tenths. Hence 8.023 stands for: 8 + 2 _____ + 3 _____. Had we written 8.203, the 0 would now mean that we had no _____. That is, 8.203 stands for:

holder

no

hundredths; thousandths

hundredths

8 + 2 _____ + 3 _____.

tenths; thousandths

The fact that 3 + 4 = 7 means that 3 tenths + 4 tenths = 7 _____. Hence in the language of decimal fractions:

tenths

0.3 + 0.4 = _____.

0.7

The same principle holds when we have more digits. For example,

2.345 means 2,345 _____ *thousandths*

and 17.643 means 17,643 _____ *thousandths*

Since 2,345 + 17,643 = 19,988, we conclude that: 2.345 + 17.643 = 19,998 _____, *thousandths*

which as a decimal fraction is _____. *19.998*

Thus, if we use the vertical form, we add two decimal fractions by "lining up" the _____ and proceeding as we would have with whole numbers.

decimal points

If we wanted to subtract 2.34 from 8.67, we could think of 2.34 as being 234 _____ and 8.67 as being 867 _____. That is:

hundredths

hundredths

8.67 - 2.34

may be viewed as:

867 _____ *hundredths*
- 234 _____ *hundredths*
633 _____. *hundredths*

So in decimal fraction form:

8.67 - 2.34 = _____ *6.33*

If we wanted to translate 8.67 into a common fraction we would ignore the decimal point and get _____ as the numerator. Since 8.67 has ___ digits to the right of the decimal point, the denominator would be a 1 followed by _____ 0's. That is, 8.67 is equivalent to the common fraction, _____. In a similar way we see that 2.34 is equivalent to the common fraction, _____. So in the language of common fractions, 8.67 - 2.34 becomes _____ - _____, or $\frac{867 - 234}{100}$

867; two

two

$\frac{867}{100}$

$\frac{234}{100}$

$\frac{867}{100}$; $\frac{234}{100}$

Now suppose we wanted to multiply 0.3 by 0.7 In the language of common fractions, we'd have:

$\frac{3}{10} \times \_\_\_ = \frac{3 \times 7}{10 \times 10}$ *$\frac{7}{10}$*

Hence as a common fraction 0.3 X 0.7 = _____ $\frac{21}{100}$

but as a decimal fraction we write 21 hundredths as _____. Thus in the language of decimal fractions: *0.21*

0.3 X 0.7 = _____ *0.21*

The "recipe" for getting the product is that we ignore the decimal points to get _____ as the product. We then count the total number of digits to the right of the decimal points in the factors, which in this example is _____. We then move the decimal point in 21 (which is assumed to be to the right of ____) 2 places to the left to get 0.21. We use the 0 here only to emphasize the decimal point. That is, 0.21 tells us that we have _____ ones.

21

two

(the) 1

no

We can avoid careless errors in the placement of the decimal point when we multiply if we round off. For example, to the nearest whole number 5.13 rounds off to _____ and 6.94 rounds off to _____. Hence if we round off each factor to the nearest whole number, we get that 5.13 X 6.94 is approximately 5 X ____ or 35. Since 513 X 694 is 356,022 the decimal point must go between the ___ and the ____ if the product is to be "about 35".

5

7

7

5; 6

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Step 4:

Do the Mastery Review on the next page.

Mastery Review

ANSWERS:

1. What digit names the ones-place in 873.1259? 1. _____

2. What place is held by the 5 in 8,503.278? 2. _____

3. What place does the 7 hold in the decimal fraction 240.076? 3. _____

4. Write 0.573 as an equivalent common fraction in lowest terms. 4. _____

5. Write 0.805 as an equivalent common fraction in lowest terms. 5. _____

6. Write 0.85 as an equivalent common fraction in lowest terms. 6. _____

7. Write 0.850 as an equivalent common fraction in lowest terms. 7. _____

8. Write 0.00007 as an equivalent common fraction in lowest terms. 8. _____

9. Write 0.0000007 as an equivalent common fraction in lowest terms. 9. _____

10. What digit names the ones-place in 8,934? 10. _____

11. In the numeral 45,982 where may we place a decimal point without changing the value of the number? 11. _____

12. Find the sum of 4.56 and 8.79. 12. _____

13. Write 4.35 + 17.8 + 8.437 as a decimal fraction. 13. _____

14. Express 7.46 - 2.98 as a decimal fraction. 14. _____

15. What must we add to 2.98 to get 7.46 as sum? 15. _____

16. Express 7.46 - 2.984 as a decimal fraction. 16. _____

17. At $3.57 each, how much will 10 items cost? 17. _____

Mastery Review (cont)

18. Write 0.076843 X 10,000 as a decimal fraction. 18. _____

19. Write 45,867.97 ÷ 1,000 as a decimal fraction. 19. _____

20. Write 0.08 X 0.0006 as a decimal fraction. 20. _____

21. Write 4.213 X 2.1 as a decimal fraction. 21. _____

Answers to Mastery Review

1. 3 2. hundreds (100's) 3. hundredths (1/100 ths)

4. 573/1,000 5. 161/200 6. 17/20 7. 17/20

8. 7/100,000 9. 7/10,000,000 10. 4 11. to the right of the 2

12. 13.35 13. 30.587 14. 4.48 15. 4.48

16. 4.476 17. $35.70 18. 768.43 19. 45.86797

20. 0.000048 21. 8.8473

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Step 5:

Do Self-Test 7, Form A on the next page.

Again, notice to take particular care in reading exercises 6 through 10. Make sure you understand what the problem is asking you to do. Even with a calculator you will get a wrong answer if you don't understand the problem. Remember, if there is anything worse than getting the wrong answer to the right problem, it's getting the *right* answer to the *wrong* problem.

Self-Test 7, Form A

1. Compute: 0.0703 + 0.98452 + 0.0083

2. Compute: (a) (9.41 - 5.863) - 2.97
   (b) 9.41 - (5.863 - 2.97)

3. Compute: (a) 6.17 X 2.03 X 0.3
   (b) (6.17 X 2.03) + 7.97
   (c) 6.17 X (2.03 + 7.97)

4. By rounding off each factor to the nearest whole number, estimate the product:
   5.9856547 X 4.0998763

5. Given that 3,416 X 276 = 942,816; find the value of:
   (a) 3.416 X 2.76
   (b) 0.0003416 X 0.0276

6. At a price of $19.95 each, how much will 6 shirts cost?

7. A gallon of paint is enough to paint 6 walls. You want to buy enough paint to paint 20 walls. The paint comes only in a 1-gallon size and costs $7.95 per gallon. How much must you pay for the paint?

8. There is $54.32 in your checking account. You then deposit your $182.79 pay check into the account. After this you go shopping and pay for your purchases with checks of $34.62, $28.73, and $42.19. How much is left in your checking account after these three checks are written?

9. You have $200 for the purchase of some clothes. You buy 5 shirts for $14.85 each; 2 pairs of slacks for $24.99 each; and 3 ties for $7.49 each. How much do you still have left to spend?

10. The sticker on your car tells you to expect that it will travel 19.7 miles on each gallon of gasoline. Based on this estimate; to the nearest mile, how far will your car travel on 8 1/2 gallons of gasoline?

ANSWERS:

1. _____
2. (a) _____
   (b) _____
3. (a) _____
   (b) _____
   (c) _____
4. _____
5. (a) _____
   (b) _____
6. _____
7. _____
8. _____
9. _____
10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 7, Form A

1. *1.06312*
2. *(a) 0.577 (b) 6.517*
3. *(a) 3.75753 (b) 20.4951 (c) 61.7*
4. *24*
5. *(a) 9.42816 (b) 0.00000942816*
6. *$119.70*
7. *$31.80*
8. *$131.57*
9. *$53.30*
10. *167 miles*

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If you did each problem in Self-Test 7, Form A correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 6.

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Step 6:

Study the solutions for Self-Test 7, Form A on the following pages, giving special emphasis to any problems you failed to answer correctly.

Solutions for Self-Test 7, Form A

1.

If we use the vertical form, we need only line up the decimal points, and then add as we have added whole numbers. So:

$$\begin{array}{r} 0.0703 \\ +\ 0.98452 \\ +\ 0.0083 \end{array}$$

If you feel more comfortable having each number with the same number of decimal digits, you may rewrite the problem as:

$$\begin{array}{r} 0.07030 \\ +\ 0.98452 \\ +\ 0.00830 \\ \hline 1.06312 \end{array}$$

The key point is that by annexing a 0 to 0.0703 we do not change the places represented by the non-zero digits. However if we place a 0 between the 7 and 0 in 0.0703 to get 0.07003, the 3 now names hundred-thousandths rather than ten-thousandths.

If you want to visualize this problem in terms of whole-number adjectives modifying fractional nouns, the above method is an abbreviation for:

$$\begin{array}{r} 7{,}030 \text{ hundred-thousandths} \\ +\ 98{,}452 \text{ hundred-thousandths} \\ +\ \quad 830 \text{ hundred-thousandths} \\ \hline 106{,}312 \text{ hundred-thousandths} \end{array}$$

In the language of common fractions we have:

$$\frac{703}{10{,}000} + \frac{98{,}452}{100{,}000} + \frac{83}{10{,}000} =$$

$$\frac{7{,}030}{100{,}000} + \frac{98{,}452}{100{,}000} + \frac{830}{100{,}000} =$$

$$\frac{7{,}030 + 98{,}452 + 830}{100{,}000} =$$

$$\frac{106{,}312}{100{,}000} =$$

1.06313

Remember the shortcut for translating a decimal fraction into a common fraction. Omit the decimal point to get the numerator. The denominator is a 1 followed by as many 0's as there are digits to the right of the decimal point.

This tells us to divide 106,312 by 100,000. In decimal form we move the decimal point 5 places to the left to get 1.06312

1. (cont)

As a final note, if you have trouble visualizing how to move the decimal point, use the language of mixed numbers. Namely:

$$\begin{array}{r} 1 \\ 100{,}000\overline{)106{,}312} \\ -100{,}000 \\ \hline 6{,}312 \end{array} \quad R\ 6{,}312 = 1\frac{6{,}312}{100{,}000}$$

If you want to rewrite 6,312/100,000 in decimal form, DON'T reduce to lowest terms first. Remember that in decimal form, all denominations must be powers of ten.

This tells us, among other things, that the answer must be a little more than 1.

2.

This exercise emphasizes that we can subtract a series of more than two decimal fractions by working on two at a time. However, since subtraction is not associative, how we group the numbers is important. Remembering to the do the arithmetic within the parentheses first, we have:

(a)

$$9.41 - 5.863 = \begin{array}{r} 9.410 \\ -5.863 \\ \hline 3.547 \end{array}$$

In the previous modules we showed that subtraction of rational numbers did not have the associative property. The fact that decimal fractions are simply another language for rational numbers, means that subtraction using decimal fractions is automatically non-associative.

Make sure you subtract 3 from 10. That is, we have:

9,410 thousandths
- 5,863 thousandths
3,547 thousandths

Hence:

$$\underbrace{(9.41 - 5.863)}_{3.547} - 2.97 = 3.547 - 2.97 = \begin{array}{r} 3.547 \\ -\ 2.970 \\ \hline 0.577 \end{array}$$

Since we're subtracting 0 from 7, we could simply have "brought down" the 7 here. Make sure you see the difference between 7 - 0 and 0 - 7.

(b) Here we deal with the same 3 decimal fractions in the same order, but subtracted with a different grouping. Starting within the parentheses we have:

2 (b). (cont)

$$5.863 - 2.97 = \begin{array}{r} 5.863 \\ -\ 2.97 \\ \hline 2.893 \end{array}$$

We can bring down the 3 because nothing is being subtracted from it.

Therefore:

$$9.41 - (5.863 - 2.97) =$$

$$9.41 - \quad 2.893 = \begin{array}{r} 9.410 \\ -2.893 \\ \hline 6.517 \end{array}$$

Here we annexed a 0 to 9.41 to remind us to subtract 3 from 0. But once the decimal points and the digits are properly lined up, the problem is purely one of whole-number arithmetic.

So while (a) and (b) look exactly the same when the parentheses are omitted, the placement of the parentheses makes a decided difference in the final answer.

3.

Part (a) shows us how the associative property for multiplication allows us to find the product of three or more decimal fractions. Parts (b) and (c) show us how to handle two operations (addition and multiplication) in the same problem--and why grouping is important.

(a)

We may think of 6.17 X 2.03 X 0.3 as meaning (6.17 X 2.03) X 0.3. To find the product of 6.17 and 2.03 we do the problem as if the decimal points were not present. This results in the whol number problem:

$$\begin{array}{r} 617 \\ \times\ 203 \\ \hline 1851 \\ 000 \\ 1234 \\ \hline 125251 \end{array}$$

This is simply the associative property for multiplication. If you wanted to, you could write the problem as: 6.17 X (2.03 X 0.3). We'll soon verify that this would give us the same answer.

If this part of the problem gives you trouble you should review the appropriate parts of Module 3.

3 (a). (cont)

In 6.17 there are 2 digits to the right of the decimal point. In 2.03 there are also 2 digits to the right of the decimal point. So altogether, there are 4 digits to the right of the decimal points. Hence we move the decimal point in 125251 4 places to the left to get 12.5251.

The decimal point in a whole number is assumed to be to the right of the digit in the ones-place. That is, 125,251 is the same as:

125,251.
decimal point
(Not a period)

Therefore:

$(6.17 \times 2.03) \times 0.3 =$

$12.5251 \times 0.3 =$

(1) $\begin{array}{r} 125251 \\ \times \quad 3 \\ \hline 375753. \end{array}$

(2) Now move the decimal point 5 places to the left to get:

3.75753
54321

12.5251 X 0.3
1234 5

To see why this method works, we can convert the problem into common fractions. That is:

$(6.17 \times 2.03) \times 0.3 =$

$\left(\frac{617}{100} \times \frac{203}{100}\right) \times \frac{3}{10} =$

$\left(\frac{617 \times 203}{100 \times 100}\right) \times \frac{3}{10} =$

$\left(\frac{125,251}{10,000}\right) \times \frac{3}{10} =$

$\frac{125,251 \times 3}{10,000 \times 10} =$

$\frac{375,753}{100,000}$ 3R 75,753 $= 3\frac{75,753}{100,000}$

You can also round off to get a rough check. For example 6.17 is about 6 and 2.03 is about 2. So 6.17 X 2.03 is about 12 (which is consistent with 12.5251) and 12 X 0.3 is 3.6 which is reasonably close to 3.75753

Once we have 375,753/100,000 we move the decimal point in 375,753 five places to the left to get the answer in decimal form.

3 (a). (concluded)

Had you elected to do the problem as 6.17 X (2.03 X 0.3), you get:

6.17 X (2.03 X 0.3) =

6.17 X 0.609 =

3.75753

203 X 3 = 609 so 2.03 X 0.3 = 6.09

617 X 609 = 5553 + 000 + 3702 = 375753.

6.17 X 0.609 = 3.75753.

54321

(b)

Based on the placement of the parentheses, we first want to multiply 6.17 by 2.03; which we've already done in part (a). Hence:

(6.17 X 2.03) + 7.97 =

12.5251 + 7.97 =

$$\begin{array}{r} 12.5251 \\ +\ 7.97 \\ \hline 20.4951 \end{array}$$

From here we have the same type of problem as in Exercise 1.

(c) In (b) we added the 7.97 after we multiplied by 2.03. Now we want to add 7.97 to 2.03 and then multiply 6.17 by this sum.

See? (6.17 X 2.03) + 7.97 and 6.17 X (2.03 + 7.97) are quite different sets of instructions.

That is:

$$2.03 + 7.97 = \begin{array}{r} 2.03 \\ +\ 7.97 \\ \hline 10.00 \end{array} = 10$$

203 hundredths + 797 hundredths = 1,000 hundredths = 1,000/100 = 10

Therefore:

6.17 X (2.03 + 7.97) =

6.17 X 10 =

61.7

Remember that to multiply a decimal fraction by 10, we move the decimal point 1 place to the right.

3 (c). (cont)

If the parentheses are omitted in parts (b) and (c) both parts read: 6.17 X 2.03 + 7.97. If for some reason you wanted to do part (c) by multiplying 6.17 by 2.03 first, you'd have to use the distributive property. That is:

If all you did was multiply 6.17 by 2.03 and then added 7.97 you'd be doing part (b not part (c).

$$6.17 \times (2.03 + 7.97) =$$

$$(6.17 \times 2.03) + (6.17 \times 7.97)$$

$$12.5251 + 49.1749 = \begin{array}{r} 12.5251 \\ +\ 49.1749 \\ \hline 61.0000 \end{array} = 61.7$$

617
X 797
4319
5553
4319
491749

4.

As we've indicated before, rounding off is often a good way to check the plausibility of an answer. Even if you have a calculator, you want to guard against careless errors in entering the digits. Moreover, it is possible (as in this exercise) that the number of digits exceeds the capacity of your calculator. So if we had to calculate 5.9856547 X 4.0998763 by hand and wanted to guard against serious errors, we might round off each factor to the nearest whole number to get:

$$5.9856547 \doteq 6$$

$$4.0998763 \doteq 4$$

Hence: $5.9856547 \times 4.0998763 \doteq$

$$6 \times 4$$

That is, 5.9856547 is between 5 and 6, but closer to 6. The shortcut is to notice that the first digit to the right of the whole numbers is 9. Since this is more than 5, we increase the whole number part (5) by 1 (6) and drop the fractional part, etc.

which tells us that we want to make sure that the placement of the decimal point gives us a number that's approximately 24.

5.

This exercise is designed simply to reinforce your knowledge of how to place the decimal point in a multiplication problem.

So in parts (a) and (b) we'll assume that we already know that 3,416 X 276 = 942,816.

(a)

3.416 X 2.76 =
123 45

9.4 2 8 1 6
5 4 3 2 1

If you compare parts (a) and (b) you'll notice that the factors have the same sequence of non-zero digits. So the products will have the same sequence of digits but a different placement of the decimal point.

3.416 ≐ 3 and 2.76 ≐ 3. Hence 3.416 X 2.76 ≐ 3 X 3 or 9. This verifies that the decimal point in 9.421816 is correctly placed.

In terms of common fractions we have:

$$3.416 \times 2.76 = \frac{3,416}{1,000} \times \frac{276}{100}$$

$$= \frac{3,416 \times 276}{1,000 \times 100}$$

$$= \frac{942,816}{100,000}$$

$$= 9.42816$$

Don't bother reducing to lowest terms. We want our final answer to have a denominator that is a power of 10.

See why we move the decimal point 5 places to the left?

(b)

0.0003416 X 0.0276 =
1234567 1234

0. 0 0 0 0 0 9 4 2 8 1 6 .
11 10 9 8 7 6 5 4 3 2 1

The 0's to the right of the decimal point must be counted as digits. So in all we have 7 + 4 or 11 digits to the right of the decimal points.

We had to move the decimal point 11 places to the left. But since 942,816 has only 6 digits to the left of the decimal point, we have to annex 11 - 6 or 5 zeroes.

As a check on the proper number of 0's, notice that $0.0003416 \doteq 0.0003$

$0.0276 \doteq 0.03$

So the product should be approximately:

0.0003 X 0.03 or 0.000009
1234 56 654321

6.

Since each shirt costs \$19.95 and we're buying 6 shirts, we're paying \$19.95 six times. So the cost of the 6 shirts is:

$$\$19.95 \times 6 = \begin{array}{r} \$19.95 \\ \times \quad 6 \\ \hline \$119.70 \end{array}$$

Up to now we've been asking such questions as "Find the product of 19.95 and 6". But in this problem, we have to read the question and decide that we have to multiply 19.95 (dollars) by 6.

Note 1:

If you prefer to think in terms of whole numbers, view the problem as spending 1,995 cents six times. In this case, the cost would be:

(1,995 X 6) cents =

11,970 cents =

\$119.70

Note 2:

It is easier to multiply by 20 than by 19.95. So think of \$19.95 as being

\$20 - 5 cents.

Now to compute 6 X \$19.95, we can write:

6 X (\$20 - 5 cents)

and use the distributive property to get:

(6 X \$20) - (6 X 5 cents) =

\$120 - 30 cents =

$$\$120 - \$0.30 = \begin{array}{r} \$120.00 \\ - \ 0.30 \\ \hline \$119.70 \end{array}$$

This is even a good way to find an approximate answer. That is, \$19.95 is a little less than \$20, so 6 X \$19.95 is a little less than 6 X \$20

In terms of a common denomination, we have:
6 X (\$20 - \$0.05)

On an adding machine we would most likely just find the sum of six \$19.95's

7.

Here's an example in which we have to perform both multiplication and addition in the same problem. We're told that each 1-gallon can of paint will allow us to paint 6 walls. At this rate, then, it takes:

one 1-gallon can of paint to paint *6* walls;
two 1-gallon cans of paint to paint *12* walls;
three 1-gallon cans of paint to paint *18* walls;
four 1-gallon cans of paint to paint *24* walls.

This may seem like a "baby" way to do this problem, but it allows you to see exactly what's going on. A quicker way would be to divide 20 (walls) by 6 (walls) to get 3 1/3. Since we can't buy 3 1/3 cans we have to buy 4 cans.

So 3 cans of paint isn't enough; while 4 cans is more than enough. But since the cans only come in one size, we can't buy a fractional number of gallons. Hence to get the job done you have to buy four 1-gallon cans of paint.

That is, 20 is between the 3rd multiple of 6 and the 4th multiple of 6.

Now that we know we need 4 cans, we use the same principle we used in Exercise 6, to conclude that since we have to pay $7.95 four times, the total cost for the paint is:

$$4 \times \$7.95 =$$

$$4 \times (\$8 - \$0.05) =$$

$$(4 \times \$8) - (4 \times \$0.05) =$$

$$\$32 - \$0.20 =$$

$$\$31.80$$

In terms of accounting the left-over paint is worth money. You bought 4 cans and used only 3 1/3 cans. So you still have 2/3 of a can left. 1/3 of $7.95 is $2.65, so 2/3 of $7.95 is 2 X $2.65 or $5.30. We usually say in this case that our paint inventory is $5.30

Of course we could have computed this more directly:

$7.95 X 4 = $31.80

The main difference between this exercise and the previous one is that in this exercise you have to figure out how many cans of paint to buy.

In Exercise 1, we were told that we were buying 6 shirts.

8.

Here we have to know when to add and when to subtract. In terms of banking, we add to our account when we deposit money, and we subtract from our account when we withdraw money.

Writing checks is a form of withdrawing (taking out) money from your checking account.

When this problem starts we have \$54.32 in our checking account. We deposit our pay check for \$182.79. To find out how much we now have in our account we add \$182.79 to our \$54.32 balance to get:

$$\begin{array}{r} \$\ 54.32 \\ +\ 182.79 \\ \hline \$237.11 \end{array}$$

After we write the first check, we subtract that amount from \$237.11 to see how much we have left. So after paying with the check for \$34.62, we find that our new balance is:

$$\begin{array}{r} \$237.11 \\ -34.62 \\ \hline \$202.49 \end{array}$$

In terms of your checkbook, the series of transactions might look like this:

\$ 54.32 Balance
+ 182.79 Deposit
\$237.11 New Balance
-34.62
\$202.49 New Balance
-28.73
\$173.76 New Balance
-42.19
\$131.57 New Balance

You now have \$202.49 in the account (This is your new balance) and because your next check is for \$28.73, you subtract \$28.73 from \$202.49 to get:

$$\begin{array}{r} \$202.49 \\ -\ 28.73 \\ \hline \$173.76 \end{array}$$

Be careful when you check your balance with the bank. The bank doesn't know whether you've written a check until the check is returned to the bank. You have to keep track of the balance in order to make sure you don't spend more than you have.

Now we have \$173.76 in our checking accounts and we write a check for \$42.19. So we subtract \$42.19 from \$173.76 to find that we have:

8. (cont)

$$\begin{array}{r} \$173.76 \\ -\ 42.19 \\ \hline \$131.57 \end{array}$$

as your balance.

As usual there are more ways than one to get an answer. Once we know that we have $237.11 in our account before we go shopping, we can *add* the amounts we paid and subtract this sum from our $237.11 deposit. That is:

Balance before shopping:	Amount spent shopping:
$ 54.32	$34.62
182.79	28.73
	42.19
$237.11	$105.54

$$\begin{array}{rl} \$237.11 & \text{Balance before shopping} \\ -\ 105.54 & \text{Amount spent shopping} \\ \hline \$131.57 & \text{Balance after shopping} \end{array}$$

This method is probably better for helping you remember how much money you've spent shopping.

A good way of double checking your balance is to work it out both ways and making sure that both ways give you the same answer.

Again, the key point is to make sure that you know when to add and when to subtract. The actual operations of adding and subtracting decimal fractions was taken care of in Exercises 1 and 2.

We have to keep repeating this point, but this is what arithmetic is all about. We have to be able to use it or else there isn't too much sense in being able to "do" it.

9.

This problem combines the principles used in Exercises 6 and 8. We find the cost of each item; then find the sum to give us the total we spent; and then we subtract this from the amount we started with (in this case, $200) to find out how much money we have left. So working item by item we have:

Solutions for Self-Test 7, Form A (cont)

9. (cont)

Amount spent for shirts:

5 shirts @ $14.85 = 5 X $14.85

= $74.25

"@" means the price of each

5 X $14.85 =
5 X ($15 - 15 cents) =
$75 - 75 cents = $15.25

Amount spent for slacks:

2 pair @ $24.95 = 2 X $24.99

= $49.98

$24.99 = $25 - 1 cent; so
2 X $24.99 = 2 X ($25 - 1¢)
= $50 - 2¢

Amount spent for ties:

3 ties @ $7.49 = 3 X $7.49

= $22.47

Obviously you could enter the cost of these items in any order you wanted to.

Total Amount Spent So Far:

$74.25 (shirts)
\+ 49.98 (slacks)
\+ 22.47 (ties)
$146.70

We could also have proceeded the way we would in keeping a checkbook. Namely:

$200.00 original balance
- 74.25 cost of shirts
$125.75 new balance
- 49.98 cost of slacks
$ 75.77 new balance
- 22.47 cost of ties
$ 53.30 new balance

Amount Left To Spend:

$200.00 (the original amount)
\- 146.70 (the amount already spent)
$ 53.30

Do the problem any way you understand best. The important thing is to get the correct answer.

10.

This problem is designed to remind you that we work with decimal fractions other than in monetary situations. We know that we are supposed to get 19.7 miles for each gallon of gasoline we use.

So at this rate, on 8 gallons we'd get:

19.7 X 8 or 157.6 miles

and on 9 gallons we'd get:

19.7 X 9 or 177.3 miles

Make sure to include the denomination so you know what you have. Write 157.6 miles not just 157.6 and so on.

10. (cont)

Since $8\frac{1}{2}$ is between 8 and 9, we now know that we'll get between 157.6 miles and 177.3 miles.

To get the exact answer we want to multiply 19.7 by $8\frac{1}{2}$. We can now proceed in several directions. For example:

This is a nice way to work if fractions still bother you. Every fraction is between two whole numbers and we can use this to get upper and lower bounds for our answer. That is, we can find an answer that's too small and another that's too big. This gives us the range that the correct answer must be in.

Method 1: CONVERTING TO COMMON FRACTIONS

$$19.7 = \frac{197}{10} \text{ and } 8\frac{1}{2} = \frac{17}{2}$$

Therefore:

$$19.7 \times 8\frac{1}{2} = \frac{197}{10} \times \frac{17}{2}$$

$$= \frac{197 \times 17}{10 \times 2}$$

$$= \frac{3,349}{20} = 167 \text{ R}9 = 167\frac{9}{20}$$

$$\begin{array}{r} 167 \\ 20\overline{)3349} \\ -20 \\ \hline 134 \\ -120 \\ \hline 149 \\ -140 \\ \hline 9 \end{array}$$

Since 9/20 is less than half, we're closer to 167 than to 168. So to the nearest mile we can expect to travel 167 miles on 8 1/2 gallons of gasoline.

Method 2: CONVERTING TO DECIMAL FRACTIONS

$\frac{1}{2} = \frac{5}{10} = 5$ tenths $= 0.5$

Therefore $8\frac{1}{2} = 8.5$. So:

$$19.7 \times 8\frac{1}{2} = 19.7 \times 8.5$$

$$= 167.45$$

which to the nearest mile is 167.

Notice that 9/20 = 45/100. Hence 167 and 9/20 is the same as 167.45

Notice that our answer falls in the required range of between 157.6 miles and 177.3 miles.

Solutions for Self-Test 7, Form A (concluded)

10. (concluded)

In completing our discussion of this exercise, we'd like to point out an important aspect of rounding off. If all we wanted was a reasonable estimate for how far we could go on $8\frac{1}{2}$ gallons of gasoline, we might have rounded 19.7 off to 20 and 8.5 to either 8 or 9. So we'd get as our estimate either:

20 X 8 = 160 miles

or

20 X 9 = 180 miles

But neither of these two answers is within the nearest mile of the correct answer (167 miles).

In general, the earlier and the more often you round off the less accurate your answer will be.

So:

KEY POINT

If you want an accurate answer, rather than just a plausible estimate, don't round off until the last step of the computation.

****

Step 7:

Do Self-Test 7, Form B on the next page.

****

Self-Test 7, Form B

ANSWERS:

1. Compute: 0.0806 + 0.83252 − 0.0083.

1. _____

2. Compute: (a) (8.23 − 4.971) − 1.89

   (b) 8.23 − (4.971 − 1.89)

2. (a) _____

   (b) _____

3. Compute: (a) 8.23 X 3.01 X 0.04

   (b) (8.23 X 3.01) + 6.99

   (c) 8.23 X (3.01 + 6.99)

3. (a) _____

   (b) _____

   (c) _____

4. By rounding off each factor to the nearest whole number, estimate the product:

   6.2345687 X 7.89765204

4. _____

5. Given that 452 X 314 = 141,928; find the value of:

   (a) 4.52 X 31.4

   (b) 0.000452 X 0.0314

5. (a) _____

   (b) _____

6. At a price of $29.99 each, how much will 8 shirts cost?

6. _____

7. A gallon of paint is enough to paint 8 walls. You want to buy enough paint to paint 28 walls. The paint comes only in a 1-gallon size and costs $10.16 per gallon. How much must you pay for the paint?

7. _____

8. There is $62.37 in your checking account. You then deposit your $172.97 pay check into the account. After this you go shopping and pay for your purchases with checks of $28.37, $46.27, and $53.69. How much is left in your checking account after these 3 checks are written?

8. _____

9. You have $225 for the purchase of clothes. You buy 6 shirts for $12.75 each; 3 pairs of slacks for $14.99 each; and 4 ties for $8.78 each. How much do you still have left to spend?

9. _____

10. The sticker on your car tells you to expect that it will travel 21.4 miles on each gallon of gasoline. Based on this estimate; to the nearest mile, how far will your car travel on 7 1/3 gallons of gasoline?

10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 7, Form B

1. *0.92142*
2. *(a) 1.1369 (b) 5.149*
3. *(a) 0.990892 (b) 31.7623 (c) 82.3*
4. *48*
5. *(a) 141.928 (b) 0.0000141928*
6. *$239.92*
7. *$40.64*
8. *$107.01*
9. *$68.41*
10. *157 miles*

****

If you did each problem in Self-Test 7, Form B correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 8.

****

Step 8:

View the solutions for Self-Test 7, Form B on Videotape Lecture 7S. Pay special attention to the solutions of those problems for which you failed to get the correct answers.

Step 9:

Do Self-Test 7, Form C on the next page.

Self-Test 7, Form C

1. Compute: 0.0903 + 0.76837 + 0.0046

2. Compute: (a) (7.45 - 2.684) - 1.93
   (b) 7.45 - (2.684 - 1.93)

3. Compute: (a) 9.45 X 2.17 X 0.6
   (b) (9.45 X 2.17) + 7.83
   (c) 9.45 X (2.17 + 7.83)

4. By rounding off each factor to the nearest whole number, estimate the product:
   3.89763289 X 4.00157899976

5. Given that 812 X 213 = 172,956; find the value of:
   (a) 81.2 X 2.13
   (b) 0.00812 X 0.00213

6. At a price of $24.95 each, how much will 7 shirts cost?

7. A gallon of paint is enough to paint 7 walls. You want to buy enough paint to paint 20 walls. The paint comes only in a 1-gallon size and costs $9.79 per gallon. How much must you pay for the paint?

8. There is $73.27 in your checking account. You then deposit your $164.89 pay check into the account. After this you go shopping and pay for your purchases with checks of $37.24, $15.94, and $28.63. How much is left in your checking account after these 3 checks are written?

9. You have $250 for the purchase of clothes. You buy 4 shirts for $13.79 each; 3 pairs of slacks for $23.98 each; and 5 ties for $8.27 each. How much do you still have left to spend for clothes?

10. The sticker on your car tells you to expect that it will travel 22.7 miles on each gallon of gasoline. Based on this estimate; to the nearest mile, how far will your car travel on 9 3/4 gallons of gasoline?

(ANSWERS ARE ON THE NEXT PAGE)

ANSWERS:

1. _____
2. (a) _____
   (b) _____
3. (a) _____
   (b) _____
   (c) _____
4. _____
5. (a) _____
   (b) _____
6. _____
7. _____
8. _____
9. _____
10. _____

Answers for Self-Test 7, Form C

1. *0.86327*

2. *(a) 2.836* *(b) 6.696*

3. *(a) 12.3039* *(b) 28.3365* *(c) 94.5*

4. *16*

5. *(a) 172.956* *(b) 0.0000172956*

6. *$174.65*

7. *$29.37*

8. *$156.35*

9. *$81.55*

10. *221 miles*

****

THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #7.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN THE STUDY OF MODULE #8.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE, YOU SHOULD CONSULT WITH A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

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