Study Guide

Module 2: Addition and Subtraction of Whole Numbers

In Module 1 we showed how numerals were invented to represent numbers. In this module, we want to show how we can use our numerals to find how to combine two or more tally counts into a single count. This process is known as *addition*. For example, to combine a group of five tally marks with a group of three tally marks to find the total number of tally marks, all we had to do was "amalgamate" the two groups of tally marks. We simply would write them side by side. That is, we could write: ||||| |||. But as counts became greater, the tally marks system became increasingly more awkward. So people turned to place value. While place value was more efficient it required new vocabulary. To indicate that we had combined counts of five and three to get a total of eight we would write:

$$5 + 3 = 8$$

If we were combining counts of fifty and thirty into a single count (at best a tedious procedure with tally marks), we'd simply write:

$$50 + 30 = 80$$

(that is 5 tens + 3 tens = 8 tens)

We continue this process in this module to explain the structure of addition using the language of place value. *Place value affords a very convenient system for adding whole numbers, no matter how great or how many.*

Addition, in a very natural way, brings with it the study of *subtraction*. Suppose that we have eight tally marks and that we want to subtract five of them. That is, we want to take away five tally marks from a collection of eight. One way of doing this is to write eight tally marks and cross out five of them. That is: ~~|||||~~||. *But this can be interpreted as an addition problem.* Namely, the number we have to add to five (the number of crossed out tally marks) to get eight (the total number of tally marks) is three (the number of tally marks that aren't crossed out).

In terms of fill-in-the-blanks,

(1) $5 + 3 = \_\_\_$

is an *addition* problem; while

(2) $5 + \_\_ = 8$

is a *subtraction* problem.

It is traditional to rewrite (2) in the form:

(3) $8 - 5 = \_\_\_$

This module is concerned with the processes of addition and subtraction as they appear in place value notation. The module concludes with a discussion of estimating answers--a technique that is at least as relevant in the modern age of electronic calculators as it was in the "old days".

Step 1:

View Videotape Lecture #2.

Step 2:

Read Module 2 of the Text.

Step 3:

When you feel that you understand the material presented in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each blank.

Check The Main Ideas

The process of combing two or more separate counts into a single total count is called _____. For example, to add 7 and 8 using tally marks we could write seven tally marks followed by _____ more tally marks. If we did this we'd get a total of _____ tally marks. In more modern notation we'd write this as 7 + ___ = 15. "7 + 8 = 15" is read as: "seven ____ eight is (or, equals) fifteen". We call 15 the _____ of 7 and 8; while 7 and 8 are called the "terms" or "_____".

addition
eight
fifteen
8
plus
sum
summands (addends)

Place value is an improvement over tally marks. For example if we want to add 52 and 43, we may add 2 and 3 to get 5 _____; and 5 and 4 to get 9 _____. In place value notation we write 9 tens plus 5 ones as _____. Hence 52 + 43 = _____. This is a lot more pleasant than having to write a total of _____ tally marks.

ones
tens
95, 95
ninety five (95)

The main problem with place value is that we can't have more than ____digit in any place. For example if had 2 tens and 13 ones, the sum would be _____. But if we wrote it as 213 we'd be more likely to read it as _____.

one
33 (thirty three)
two hundred thirteen

The point is that in addition if we use place value we must always exchange ten of any power of ten for ___ of the next greater power of ten. This is known as "_____". For example in adding

one
carrying

$$\begin{array}{r} 56 \\ +\ 78 \\ \hline \end{array}$$

we'd say: "6 + 8 = 14, bring down the ____ — *4*

and 'carry' the _____". This simply means that — *1*

we may view 14 as 4 ones and __ ten. Since 5 — *1*

tens and 7 tens are 12 tens, after we carried we'd

have a total of _____ tens. In other words the sum — *thirteen*

of 56 and 78 is _____. — *134*

Turning to subtraction, change-making is a good

example for showing why subtraction is really a

form of _____. For example, we may think of — *addition*

23 - 9 as meaning the number which we must add to ___ — *9*

to get _____ as the sum. "-" is called a _____ — *23, minus*

sign. Accordingly we read 23 - 9 as "23 _____9" — *minus*

At any rate since 14 + 9 = 23, 23 - 9 = _____. — *14*

The idea of "take away" or "subtract" probably came

from the tally mark interpretation. For example,

we could start with _____ tally marks; then cross — *23*

out (or subtract) _____ of them. This would — *9*

leave us with _____ tally marks. — *14*

When we use place value to subtract, we use the

word "borrow" to indicate that we've exchanged one

of any denomination for _____ of the next lower — *ten*

denomination. For example if we write $\not{3}^{2}\,{}^{1}4$, it

means that we've exchanged a ten for _____ones. — *ten*

That is, we mean that 34 is another way of writing

2 _____ and 14 _____. Thus if we wanted to — *tens, ones*

subtract 18 from 34, we could write:

$$\begin{array}{r} \not{3}^{2}\,{}^{1}4 \\ -\ 1\ 9 \\ \hline 1\ 5 \end{array}$$

In terms of addition 34 - 19 = 15 means that

15 + 19 = _____. *34*

Sometimes when we deal with addition or subtraction and many digits are involved we prefer to have a quick estimate of the answer. Suppose, for example, that we want to estimate the sum: 2,837 + 7,296. In terms of consecutive multiples of a thousand, 2,837 is more than _____ but less than _____. *2,000; 3,000*

Since 3,000 - 2,837 = 163 and 2,837 - 2,000 = 837, 2,837 is closer to _____ than to _____. So *3,000; 2,000*

to the nearest thousand, 2,837 is _____. *3,000*

In a similar way, 7,296 is between 7 _____ *thousand*

and 8 _____. Since the digit immediately to the *thousand*

right of 7 is 2; 7,296 is closer to ___ thousand *7*

than to _____ thousand. Hence to the nearest *8*

thousand 7,296 is _____. Therefore, if we round *7, 000*

off each summand to the nearest thousand, the problem 2,837 + 7,296 may be replaced by the simpler problem:

3,000 + _____. Using this estimate we'd say that *7,000*

2,837 + 7,296 is approximately _____. *10,000*

If we needed a more accurate estimate, we could round off each term to, say, the nearest hundred.

In this case 2,837 is between 2,800 and _____ while *2,900*

7,296 is between 7,200 and _____. The digit to *7,300*

the right of 8 in 2,837 is 3. Hence to the nearest

hundred 2,837 rounds off to _____. 7,296 *2,800*

to the nearest hundred rounds off to _____. *7,300*

Since 28 + 73 = 101,

2,800 + 7,300 = _____. Hence a more accurate approximation for the sum of 2,837 and 7,296 is _____. In most real-life situations we would have to decide on the degree of accuracy that was required. But even if we required the exact answer, knowing an approximate answer would help us guard against making preposterous errors.

10,100

10,100

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Step 4:

Do the Mastery Review.

Mastery Review	Answers:
1. Show how a shepherd would use tally marks to indicate the total number of sheep if there were 6 sheep in one part of the pasture and 8 sheep in another part of the pasture.	1. _____
2. What is the sum of 6 and 8?	2. _____
3. Find the sum of 58 and 4?	3. _____
4. Fill in the blank: 4 + 58 = _____	4. _____
5. How much is 4 + 9?	5. _____
6. What is the sum of 592 and 0?	6. _____
7. What is the sum of 3,000,000,000 and 4,000,000,000?	7. _____
8. What is the sum of 425 and 361?	8. _____
9. Find the sum of 3 thousand and 4 hundred?	9. _____
10. Find the sum of 89 and 74.	10. _____

11. A shepherd has six sheep in one part of the pasture, three sheep in another part of the pasture and seven sheep in a third part of the pasture. Show how the shepherd could indicate the total number of sheep by using tally marks.

11. _____

12. How much is 8 + 9 + 4?

12. _____

13. How much is 51 + 34 + 12?

13. _____

14. Find the sum of 45,876 and 39,457.

14. _____

15. What number must we add to 7 to get 13 as the sum?

15. _____

16. What number must be added to 531 to get 986 as the sum?

16. _____

17. Subtract 17 from 52.

17. _____

18. How much is 823 - 374?

18. _____

19. Use borrowing to subtract 3,685 from 8,301.

19. _____

20. What is the 35th multiple of ten?

20. _____

21. What is the 35th multiple of a hundred?

21. _____

22. What is the 9th multiple of 10^7?

22. _____

23. Is 37,200 a multiple of:

23.

(a) a thousand? (a) _____

(b) a hundred? (b) _____

(c) ten? (c) _____

24. Between what two consecutive multiples of a thousand is 37,200?

24. _____

25. Round off 681 to the nearest hundred.

25. _____

26. Round offf 681 to the nearest ten.

26. _____

27. Round off 48,390 to the nearest hundred.

27. _____

28. Round off each of the following numbers to the nearest billion:

28.

(a) 3,234,899,988 (a) _____

(b) 5,879,001,123 (b) _____

29. By rounding off each number to the nearest billion, estimate the sum of 3,234,899,988 and 5,879,001,123.

29. _____

Answers to Mastery Review

1. |||||| |||||||| 2. 14 3. 62 4. 62 5. 13

6. 592 7. 7,000,000,000 8. 786 9. 3,400

10. 163 11. |||||| ||| ||||||| 12. 21 13. 97

14. 85,333 15. 6 16. 455 17. 35 18. 449

19. 4,616 20. 350 21. 3,500 22. 90,000,000

23 (a) No (b) Yes (c) No

24. 37,000 and 38,000 25. 700 26. 680 27. 48,400

28. (a) 3,000,000,000 (b) 6,000,000,000

29. 9,000,000,000

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Step 5:

Do Self-Test 2: Form A (on the next page)

Self-Test 2: Form A

ANSWERS:

1. Add:

$$\begin{array}{r} 345{,}086 \\ 122{,}903 \\ 22{,}011 \\ \hline \end{array}$$

1. _____

2. Find the sum:

$23{,}837 + 147{,}236 + 8{,}405$

2. _____

3. Round off 87,297,345 to the nearest

3.

(a) thousand (a) _____

(b) million (b) _____

(c) ten million (c) _____

4. By rounding off each term to the nearest thousand, estimate the sum:

$44{,}905 + 19{,}844 + 20{,}099$

4. _____

5. By rounding off each term to the nearest trillion, estimate the sum:

$$\begin{array}{r} 4{,}786{,}237{,}942{,}854 \\ 9{,}237{,}421{,}837{,}078 \\ 2{,}900{,}001{,}989{,}976 \\ 2{,}199{,}873{,}956{,}837 \\ \hline \end{array}$$

5. _____

6. Write 32,023 - 8,946 as a place value numeral.

6. _____

7. Subtract 8,946 from 32,023

7. _____

8. What must we add on to 8,946 to get 32,023 as the sum?

8. _____

9. Find the difference:

$$\begin{array}{r} 80{,}002 \\ -\ 24{,}817 \\ \hline \end{array}$$

9. _____

10. Find the value of:

10.

(a) $(9{,}234 - 4{,}958) - 2{,}367$ (a) _____

(b) $9{,}234 - (4{,}958 - 2{,}367)$ (b) _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 2: Form A

1. *490,000*
2. *179,478*
3. *(a) 87,297,000*

   (b) 87,000,000

   (c) 90,000,000
4. *85,000*
5. *19 trillion (19,000,000,000,000)*
6. *23,077*
7. *23,077*
8. *23,077*
9. *55,185*
10. *(a) 1,909*

    (b) 6,643

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If you did each problem in Form A correctly, you may if you wish proceed to the next module. Otherwise continue with Step 6.

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Step 6:

Study the solutions to Self-Test 2: Form A on the following pages, giving special emphasis to any problems you failed to answer correctly.

Solutions for Self-Test 2: Form A

1.

If we use vertical form with the powers of ten present, we have:

	hund-thou.	*ten-thou.*	*thou.*	*hund.*	*ten*	*one*
	3	4	5		8	6
(+)	1	2	2	9		3
(+)		2	2		1	1
	4	8	9	9	9	10
	4	8	9	9	10	
	4	8	9	10		
	4	8	10			
	4	9				

When the powers of ten are present, we do not 0 as a place holder.

In going from row-to-row we are exchanging 10 of a power of ten for one of the next greater power. When we have only 10, none are left after the exchange.

Now to omit the powers of ten, we have to use 0's as place holders and we obtain as our answer:

490,000

Put in the 0's first to get 4 9 0 0 0 0; then place the comma to get 490,000

Note that we do not actually have to name the different powers of ten, *as long as we keep track of the places.* That is we could have written:

3	4	5		8	6
1	2	2	9		3
	2	2		1	1
4	8	9	9	9	10

We then finish the problem as we did above; after which we can read the answer by the placement of the commas.

If we want to use the "carrying" system, we proceed as we just did above, except that we carry the 1 and bring down only the 0. That is, since 10 has two digits it cannot appear in a place value column. We'd get:

```
  11 11
 345,086
 122,903
  22,011
---------
 490,000
```

Now the 0's are necessary to ensure proper place value

We could write 22,011 as 02211 to make sure there's a matchup of digits:

```
 345,086
 122,903
 022,011
```

but omitting this 0 causes no confusion in place value. Namely, the 3 in 345,086 already holds this place.

"Carrying" merely simplifies the amount of work that we had to do when the powers of ten were present.

Solutions for Self-Test 2: Form A (cont)

2.

The horizontal format of this problem is most convenient when we use a calculator. Since we aren't using a calculator, the best strategy is to transform the problem into the vertical format of the previous problem. To ensure that the denominations match up, we align the digits in the ones place. That is:

```
  23,837
 147,236
   8,405
```

and we can then use "carrying" to obtain:

```
  11  1
  23,837
 147,236
   8,405
 -------
 179,478
```

Notice how we never have to add more than two digits at a time. For example, in the thousands place we have: 8 + 7 = 15; 15 + 3 = 18, and 18 + 1 = 19. So we brought down the 9 and carried the 1.

Suppose we wanted to add 174 and 82. It would be wrong to write:

```
  174
+  82
 ----
  994
```

Namely 7 + 2 = 9 only if 7 and 2 modify the same noun; but in this problem we have 7 tens and 2 ones.

By the commutative property we could have started at the top to add. We'd get: 1 + 3 = 4; 4 + 7 = 11; and 11 + 8 = 19.

3.

(a) To round off to the nearest thousand, we replace the last three digits by 0's. That is:

```
87,297,345
       ↓↓↓
87,297,000
```

So 87,297,345 is between 87,297,000 and 87,298,000.

That is, it is between the 87,297th multiple of a thousand and the 87,298th multiple of a thousand.

To see which of the two multiples we round off to, we look at the digit immediately to the right of the thousands digit. It's a 3 and this is less than 5. So the answer is 87,297,000. More "mechanically":

3. (a) (cont)

To round off to a given power of ten (in this case, 10^3, or 1,000)

Step 1

Draw a vertical line immediately to the right of the place to which we're rounding off. In this example:..... *87,297,|345*

Step 2

Mark, either physically or mentally, the digit immediately to the right of the vertical line..... *87,297,|345*

Step 3

Replace each digit to the right of the vertical line by 0..... *87,297,|000*

Step 4

Look at the marked digit (in this case, 3). If it is less than 5, simply remove the vertical line and what's left is the answer. If it is 5 or more, increase the number to the left of the vertical line by 1, remove the vertical line, and then what's left is the answer..... *87,297,000*

The key point is that 7,345 is closer in value to 7 thousand than to 8 thousand. *8,000 - 7,345 = 655*
7,345 - 7,000 = 345

(b) Using the mechanical recipe, place a vertical line after the digit in the millions-place and mark the digit immediately to the right of this line..... *87,|297,345 (The commas may be omitted)*

Replace each digit to the right of the vertical line by 0..... *87,|000,000*

Then, since the marked digit is 2 (which is less than five), simply remove the vertical line..... *87, 000,000*

The main point is that 87,297,345 is between 87 million and 88 million, but closer in value to 87 million.

3.

(c) This time we place the vertical line immediately to the right of the ten-millions digit and mark the digit immediately to the right.....

8|7,297,345
↑

Next replace each digit to the right of the vertical line by 0.....

8|0,000,000
↓

Since the marked digit (7) is greater than 5, increase the number to the left of the vertical line by 1 and then remove the vertical line.....

↓

↓

9 0,000,000

So the answer is 90,000,000. Again, the key is that counting by ten-millions, 87,297,345 is between the 8th multiple of ten million (80,000,000) and the 9th multiple of ten million (90,000,000); but closer in value to 90,000,000.

Special Case:

If you happen to be exactly half way between the two consecutive multiples, use your own discretion. For example to the nearest thousand, 8,500 is either 8,000 or 9,000.

4.

Using ≐ to indicate that we've rounded off to the nearest thousand, we have:

44,905 ≐ 45 thousand or 45,000

44,|905 becomes 45,000
↑

19,844 ≐ 20 thousand or 20,000

19,|844 becomes 20,000
↑

20,009 ≐ 20 thousand or 20,000

20,|009 becomes 20,000
↑

Hence:

44,905 + 19,844 + 20,099 ≐
↓ ↓ ↓
45,000 + 20,000 + 20,000 [= 85 thousand]

45
+ 20
+ 20
85

5.

This has too many digits for the calculator, so we have to do it the "long" way. But the rounding-off procedure helps us to guard

5. (cont)

against "preposterous" errors. This time we'll let ≐ mean that we've rounded off to the nearest trillion. We get:

$$\begin{array}{r} 4{,}786{,}237{,}942{,}854 \doteq 5{,}000{,}000{,}000{,}000 \\ 9{,}237{,}421{,}837{,}078 \doteq 9{,}000{,}000{,}000{,}000 \\ 2{,}900{,}001{,}989{,}976 \doteq 3{,}000{,}000{,}000{,}000 \\ \underline{2{,}199{,}873{,}956{,}837 \doteq 2{,}000{,}000{,}000{,}000} \\ \doteq 19{,}000{,}000{,}000{,}000 \end{array}$$

4,|786,237,942,854 becomes
↑
5, 000,000,000,000
and so on.

This tells us that whatever the correct answer is, it should be "reasonably" close in value to 19 trillion.

6.

In vertical form with the powers of ten present, the problem looks like:

	ten-thousands	thousands	hundreds	tens	ones
	3	2		2	3
(-)		8	9	4	6

Don't be confused by the wording of the problem. 32,023 and 8,946 are each numbers. Their difference is one number. We want that We want to know what that number is, expressed in the language of place value.

Since we can't take 6 ones) from 3 (ones), or 4(tens) from 2 (tens), or 9 (hundred) from 0 (hundred) or 8 (thousand) from 2 (thousand), we have to "borrow". That is, we exchange a ten for 10 ones; a hundred for 10 tens; a thousand for 10 hundreds, and a ten-thousand for 10 thousands. This sequence of steps leads to:

	ten-thousands	thousands	hundreds	tens	ones
	3	2		2	3
	2	12		2	3
	2	11	10	2	3
	2	11	9	12	3
	2	11	9	11	13
(-)		8	9	4	6
	2	3	0	7	7

The first five rows are paraphrases of the minuend.

and we see that the answer is 23,077.

Again, we don't have to write the 0 in the hundreds-place. But in the next step we do--so we might as well include it as a reminder here.

If the place values are not present, then the

6. (cont)

previous steps take on the form:

$$\begin{array}{r} {}^{1}2\ {}^{1}\!\not{3}\ \not{2}\ {}^{1}\!\not{0}\ {}^{1}1 \\ \not{3}\ \not{2}\ \not{0}\ \not{2}\ {}^{1}3 \\ -\quad 8\ 9\ 4\ 6 \\ \hline 2\ 3,\ 0\ 7\ 7 \end{array}$$

In either form we've rewritten 32,023 as 20,000 + 11,000 + 900 + 110 + 13; that is, 2 ten-thousands, 11 thousands, 9 hundreds, 11 tens, and 13 ones.

7.

This is simply another way of stating the previous problem. Namely, 32,023 - 8,946 means the same thing as saying to subtract 8,946 from 32,023.

8.

This, too, is another way of restating Problem 6. But this emphasizes the connection between addition and subtraction.

In fact, if you did problems 6 and 7 correctly but got problem 8 wrong, it means that you can perform the "subtraction operation" but that you don't fully understand what it means.

The main point here is that 32,023 - 8,946 means the number we must add to 8,946 to get 32,023 as the sum. If we use the change-making method we get:

Start with.....	8,946
Add.....	7
to get.....	8,953
Add.....	70
to get.....	9,023
Add.....	3,000
to get.....	12,023
Add.....	20,000
to get.....	32,023

You should feel free to use whatever method of subtraction you prefer. Ideally, you should try to understand each method.

So altogether, we've added:

20,000 + 3,000 + 70 + 7, or:

23,077

9.

We may perform this subtraction by any of the previously discussed methods. For example:

(i)

8				2	=
7	10			2	=
7	9	10		2	=
7	9	9	10	2	=
7	9	9	9	12	
(-) 2	4	8	1	7	
5	5	1	8	5	= 55,185

Here we've used the vertical lines so that we would not have to write the name of each power of ten.

(ii) We could do the same as we did in (i) except that we'll use the borrowing notation:

$$\begin{array}{r} 7\ 9\ 9\ 9 \\ \not{8}\ \not{10}\ \not{10}\ \not{10}\ 12 \\ -\ 2\ 4\ 8\ 1\ 7 \\ \hline 5\ 5\ 1\ 8\ 5 \end{array}$$

In both (i) and (ii) we're rewriting 80,002 as: 70,000 + 9,000 + 900 + 90 + 12

(iii) Next we use the change-making method:

Start with	24,817
Add.....	5
to get.....	24,822
Add	80
to get.....	24,902
Add.....	100
to get.....	25,002
Add	5,000
to get.....	30,002
Add	50,000
to get.....	80,002

So all in all we added:

5
80
100
5,000
50,000
55,185

Note that regardless of the method we used to do this problem, rounding off to the nearest thousand will at least check whether our answer is reasonable

To this end:

$$80{,}002 - 24{,}817 \doteq$$
$$80{,}000 - 25{,}000$$

and 80,000 - 25,000 = 55,000 which seems reasonably close in value to 55,185.

If we had obtained 65,185 as as answer, our estimate should warn us that we've made a substantial error.

10.

The main thrust of this problem is to help you (i) better understand how parentheses are used as grouping symbols and (ii) realize that, unlike in addition, grouping of terms makes a big difference when we subtract.

(a) Recall that everything within parentheses is treated as a single number. *From a practical point of view, this means to do all the arithmetic within the parentheses before we do the rest of the problem.*

Since 9,234 - 4,958 = 4,276, we have:

$$(9{,}234 - 4{,}958) - 2{,}367 =$$
$$\downarrow$$
$$4{,}276 \qquad - 2{,}367$$

and since 4,276 - 2,367 = 1,909; we see that the answer to part (a) is 1,909.

Computational Checks

$$\begin{array}{r} 8\ {}^{1}1\ {}^{1}2 \\ \not{9}\ \not{2}\ \not{7}\ {}^{11}4 \\ -\ 4\ 9\ 5\ 8 \\ \hline 4\ 2\ 7\ 6 \end{array} \qquad \begin{array}{r} 3\ \ 6 \\ \not{4}\ {}^{1}\not{2}\ 7\ {}^{1}6 \\ -\ 2\ 3\ 6\ 7 \\ \hline 1\ 9\ 0\ 9 \end{array}$$

(b) Performing the subtraction within the parentheses first we get 4,958 - 2,367 = 2,591. Hence:

$$9{,}234 - (4{,}958 - 2{,}367) =$$
$$\downarrow$$
$$9{,}234 - \quad 2{,}591$$

Then since 9,234 - 2,591 = 6,643, the answer to part (b) is 6,643.

Before you start this problem it might be beneficial to notice that is the parentheses are omitted in (a) and (b) both parts read the same. Namely:

9,234 - 4,958 - 2,367

This grouping represents what would happen if you saw "9,234 - 4,958 - 2,367" and did the subtraction in the order the terms appeared (for example, on a calculator). You'd first subtract 4,958 from 9,234 and then subtract 2,367 from that answer.

Computational Checks

$$\begin{array}{r} \ \ 8 \\ 4\ \not{9}\ {}'5\ 8 \\ -\ 2\ 3\ \ 6\ 7 \\ \hline 2\ 5\ \ 9\ 1 \end{array}$$

$$\begin{array}{r} 8\ {}'1 \\ \not{9}\ \not{2}\ {}'3\ 4 \\ -\ 2\ 5\ \ 9\ 1 \\ \hline 6\ 6\ 4\ 3 \end{array}$$

10. (continued)

It is not our purpose until later in the course to apply arithmetic to practical problems. However, a practical situation may help us distinguish why order is important.

An Application of Part (a)

You have \$9,234 in the bank and you pay a bill for \$4,958. After this you pay another bill for \$2,367. To find how much money you have left, you first subtract 4,958 from \$9,234, and then from this amount (your balance), you subtract \$2,367. That is, you're subtracting \$2,367 from the answer you got when you subtracted \$4,958 from \$9,234.

That is: \$9,234 - \$4,958

Symbolically:
(\$9,234 - \$4,958) - \$2,367

An Application of Part (b)

Your goal is to build your bank account up to \$9,234. You already have \$4,958 in the account. An unexpected expense of \$2,367 occurs and you pay it. To find out how much money you now need to fulfill your goal, you first subtract \$2,367 from \$4,958 to find out how much money you still have in your account.

Then you subtract this amount from \$9,234. That is, you want the amount that must be added to (\$4,958 - \$2,367) to give \$9,234 as the total (sum).

In summary, you've computed \$9,234 - (\$4,958 - \$2,367) (i.e, first subtract \$2,367 from \$4,958. Then subtract this answer from \$9,234.)

Comparing these two applications we discover a very important point that will play a major role

Solutions for Self-Test 2: Form A (concluded)

10. (concluded)

in the rest of our course. Namely:

While we're not saying that it is easy for everyone to compute either

$$(9{,}234 - 4{,}958) - 2{,}367 \qquad (i)$$

or

$$9{,}234 - (4{,}958 - 2{,}367) \qquad (ii)$$

the fact remains that these are still only techniques that test a mechanical skill.

On the other hand, to be able to analyze a situation and decide that we have to compute either (i) or (ii) to get the correct answer is quite another matter. Such applications require that we be able to read and interpret properly; and once that's done, we have to be able to translate the results into the language of mathematics.

That's why we feel it is crucial that you learn to understand what we're doing rather than to memorize it.

For example, in the real-world no one will ever come up to you and say: "Hey, how much is: 9,234 - (4,958 - 2,367)?"

But they may well describe a real situation (such as we did with our two applications) in which you'll have to formulate the arithmetic problem that has to be solved in order for us to solve the problem that was described.

****

Step 7:

Do Self-Test 2: Form B.

Self-Test 2: Form B

ANSWERS:

1. Add:

$$\begin{array}{r} 461{,}704 \\ 206{,}813 \\ \underline{\ 21{,}483} \end{array}$$

1. _____

2. Find the sum:

$$43{,}234 + 123{,}806 + 5{,}082$$

2. _____

3. Round off 92,983,840 to the nearest

(a) thousand

(b) million

(c) ten million

3.

(a) _____

(b) _____

(c) _____

4. By rounding off each term to the nearest thousand, estimate the sum:

$$56{,}897 + 18{,}023 + 21{,}989$$

4. _____

5. By rounding off each term to the nearest trillion, estimate the sum:

$$\begin{array}{r} 5{,}299{,}760{,}998{,}778 \\ 3{,}911{,}345{,}878{,}993 \\ 4{,}877{,}003{,}994{,}345 \\ \underline{2{,}099{,}998{,}777{,}659} \end{array}$$

5. _____

6. Write 84,015 - 9,867 as a place value numeral.

6. _____

7. Subtract 9,867 from 84,015.

7. _____

8. What must we add to 9,867 to get 84,015 as the sum:

8. _____

9. Find the difference:

$$\begin{array}{r} 90{,}004 \\ -\ \underline{32{,}276} \end{array}$$

9. _____

10. Find the value of:

(a) $(8{,}256 - 3{,}768) - 1{,}273$

(b) $8{,}256 - (3{,}768 - 1{,}273)$

10.

(a) _____

(b) _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 2: Form B

1. *690,000*
2. *172,122*
3. *(a) 92,984,000*

   (b) 93,000,000

   (c) 90,000,000
4. *97 thousand (97,000)*
5. *16 trillion (16,000,000,000,000)*
6. *74,148*
7. *74,148*
8. *74,148*
9. *57,728*
10. *(a) 3,215*

    (b) 5,761

****

If you did each problem in Form B correctly, you may, if you wish, proceed to the next module. Otherwise continue with Step 8.

****

Step 8:

View the solutions for Self-Test 2: Form B on Videotape Lecture 2S. Pay special attention to the solutions of those problems for which you failed to get the correct answers. *Feel free to rewind the tape at any time to restudy any problems that give you difficulty.*

Step 9:

Do Self-Test 2: Form C

Self-Test 2: Form C

ANSWERS:

1. Add:

$$\begin{array}{r} 521,236 \\ 203,421 \\ \underline{45,343} \end{array}$$

1. _____

2. Find the sum:

$$52,347 + 236,144 + 9,062$$

2. _____

3. Round off 78,123,925 to the nearest:

(a) thousand

(b) million

(c) ten million

3.

(a) _____

(b) _____

(c) _____

4. By rounding off each term to the nearest thousand, estimate the sum:

$$21,086 + 31,944 + 43,802$$

4. _____

5. By rounding off each term to the nearest trillion estimate the sum:

$$\begin{array}{r} 6,945,234,876,256 \\ 7,823,999,564,954 \\ 5,099,998,678,679 \\ \underline{3,234,566,987,365} \end{array}$$

5. _____

6. Write 72,011 - 6,789 as a place value numeral.

6. _____

7. Subtract 6,789 from 72,011.

7. _____

8. What must we add to 6,789 to get 72,011 as the sum?

8. _____

9. Find the difference:

$$\begin{array}{r} 70,006 \\ -\ \underline{13,259} \end{array}$$

9. _____

10. Find the value of:

(a) $(6,213 - 3,837) - 1,256$

(b) $6,213 - (3,837 - 1,256)$

10.

(a) _____

(b) _____

Answers for Self-Test 2: Form C

1. *770,000*
2. *297,553*
3. *(a) 78,124,000*

   (b) 78,000,000

   (c) 80,000,000
4. *97 thousand (97,000)*
5. *23 trillion (23,000,000,000,000)*
6. *65,222*
7. *65,222*
8. *65,222*
9. *56,747*
10. *(a) 1,120*

    (b) 3,632

****

THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE # 2.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE #3.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE YOU SHOULD CONSULT A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

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