1 Estimates for time to collapse of WTC1

1.1 Introduction

This is an attempt to estimate what would be expected for the time of collapse of WTC1 based on observation of the videos of the falling building under an assumption that the collapse was due only to gravity and the description of the collapse found in the NIST report along with some other assumptions. The time of collapse was not analyzed by NIST in their report but I will show that a lower bound for the time of collapse may be estimated from the assumptions mentioned above. I will argue that the building fell faster than it should have. Others have argued that the tower should not have fallen at all. See the paper by Ross in [7] for an argument, based on energy, momentum, and the physical properties of the columns, that the total collapse should not have taken place. Jones [8] has given many other arguments of a different sort which also conclude the official explanation is not adequate. I will focus mostly on the floors in this paper and will not attempt to show the building should not have fallen but that it fell surprisingly fast. These arguments are based on assumptions involving the relative mass of the concrete to the steel in the floors of the building and the claim that the concrete of the floors was essentially all crushed to dust.

Here is a section of the NIST report [6] which was shown to me by Nic Dudzik.

" ...the structure below the level of collapse initiation offered minimal resistance to the falling building mass at and above the impact zone. The potential energy released by the downward movement of the large building mass far exceeded the capacity of the intact structure below to absorb that energy through energy of deformation.

Since the stories below the level of collapse initiation provided little resistance to the tremendous energy released by the falling building mass, the building section above came down essentially in free fall, as seen in videos. As the stories below sequentially failed, the falling mass increased, further increasing the demand on the floors below, which were unable to arrest the moving mass."

The fall of the top of the tower can be seen in the video [14]or [15]. The first of these shows the top floors collapsing in a very complicated manner. After a sufficient amount of dust forms, it is difficult to see what is happening with these upper floors. Also it is very hard to determine whether the above explanation is correct for the collapse of the bottom floors based only on the videos.

Here is some information about what was observed based on what I have gotten from the web. If this information is faulty, it will affect the validity of the estimates presented in this paper. In [1] it says the steel for the floors was 44.5 kg/m^2. Assuming the floors were 207 feet by 207 feet, this yields an estimate for the mass of steel per floor as about 177,146 kg. In [10]it states: "Virtually all the concrete (an estimated 100,000 tons in each tower) on every floor was pulverized into a very fine dust, a phenomenon that requires enormous energy and could not be caused by gravity alone; workers can't even find concrete." The NIST report [5] states that on every floor there were two types of concrete used, normal concrete in the core area and light weight concrete elsewhere. I will assume the floor slab was composed entirely of light weight concrete for the sake of simplicity. In [17] it gives the figure for density of light weight concrete as 1750 kg/m^3. Since the floor was about 207 feet by 207 feet and the slab of concrete was four inches thick, this works out to 707,786 kg for the mass of the concrete in the floor. More generally, if the density is $Den\ kg/m^3$ the mass of the concrete in the floor is

$$404.45 \times Den\ kg.$$

As to the speed of collapse of WTC1, it looks like it came down in about 15 seconds from the time the antenna started to sink. Smaller figures have been put forth, however. The figure given by NIST [6] for the top panels of the building to hit the ground is 11 seconds. They based this on "precise timing of initiation of collapse from video evidence and also on seismic signals recorded at Palisades N.Y." In addition, it has been shown by Legge [4] that the acceleration of the roof of WTC1, if continued, would give a fall time of 10.5 seconds. One of the longest estimates for the time for WTC1 to collapse is in [19] where it is suggested this time was as long as 16 seconds. I could not find a time of collapse for WTC1 in the 9/11 commission report but they claim that WTC2 collapsed in 10 seconds [11]p 322. In [6] they say WTC2 collapsed in 9 seconds.

Greening [3] divides the fall of the towers into two stages, a first stage in which the top floors act like a single mass, and a second stage in which this stack of top floors then collapses when it hits the ground. For WTC1 he uses the figure for the first stage of collapse of this tower as 11.3±1.5 seconds; thus at most 12.8 seconds. Then he argues that it would be about one more second for the top floors to collapse when they hit the ground, yielding a figure of at most 13.8 seconds for the collapse of the entire building. I am basing my figure for the collapse time on the video found in [12]. In this video, it is clear from the beginning of the collapse that the top of the dust cloud is well above the level of the collapsing floors and based on this, one can estimate that it took about 12 seconds, from the time this video started, for the top to reach the ground. This video starts when the collapse of the top floors has been underway for a few seconds as can be seen by viewing the video [14]. From [14] it also appears the top several floors collapse onto an upper floor and then the whole thing crashes down and it is no longer as clear what is happening because of the clouds of dust. However, you can observe there is a large section of floors which begins descending right after the tower dips and then there are two floors which seem to be giving way before clouds of dust obscure what happens. It was not clear to me which upper floor was the "target" of this collapse. However, [2] gives some still pictures which seem to indicate this floor was number 94. I arrived at this by measuring the pictures with a ruler and using the assumption that the width of the building was about 207 feet. Thus there were 16 floors above this one which collapsed onto it and it appears that from the time of this impact till the 94^{th} floor hit the ground was about 12 seconds. The still pictures just mentioned show conclusively that the towers did not fall in two stages, the first in which the top 16 floors move as a lumped mass tamping down the lower floors and the second in which this lumped mass hits the bottom and collapses. The collapse of the top floors appears to have taken place early in the collapse of the building.

1.2 The floating floor model

The towers had columns which supported the floors. These were on the outside and on the core. The idealized problem I will consider is to remove all the columns and consider only the floors, pretending they are floating in the air and do not move till struck. I will consider the collapse of this idealized tower of floating floors under the assumption that all collisions are inelastic. If only gravity is involved in collapse and if both the real tower and the idealized tower fall in the same manner, this idealized tower of floating floors should fall faster than the actual tower because in the idealized tower there is no vertical support whatever, and in the real tower, there were many columns. It is assumed the floors fell in such a way that they encountered a succession of lower stationary floors as described in the above quote from the NIST report.

This consideration of floating floors was used by Greening [3] and has also appeared elsewhere on the web, [13] where it is called: disproving the pancake theory in 110 easy steps and in a paper by Judy Wood, [16]. While there are many differences in the analysis I will present, this idea of considering an idealized building consisting of floating floors in order to obtain some understanding of how fast the real building should have fallen seems to have occurred to many others.

If the tower fell by letting the top floor hit the next floor down and then those two floors combined hitting the floor below that and so on, there would be only one model to consider. However, this is not the case. The tower started to fall from a floor somewhat down from the top. There are two ways of considering the fall of the tower which I will consider in this paper. The first is one employed by Greening [3] in which the top floors above the initiation of collapse fall as a solid block, crushing the floors below and eventually collapsing when it reaches the bottom. This manner of collapse is not supported by videos of the falling building but it is a simple model which is easy to analyse and can be used to gain insight into an appropriate speed of collapse.

The other manner of collapse is more complicated. According to [6] the collapse of WTC1 began with floor 98 moving down to collide with floor 97. More generally, let $M+1$ be the floor which begins moving first, collapsing onto floor M. This other model involves the floors above floor M descending from the force due to gravity until the bottom one, floor $M+1$, collides with floor M. Then the two floors resulting from this collision move downward while floor $M+2$ and above also continue moving downward, these upper floors being unaffected by the collision because, by assumption, the floors are floating with no connection between them. A succession of collisions results in which eventually all the upper floors have collapsed into one lumped mass and then this lump of floors continues crashing down on the floors beneath it, encountering one floor at a time as it goes. I will call the floor immediately below this lump of floors right after it has formed, the target floor. As mentioned above, it appears from the videos that in the actual tower there was a target floor and this was floor 94. The target floor is computed in the floating floor tower since there is no video of the collapse of this imaginary construction.

We will refer to the first method of collapse as the hard top collapse and the second as the soft top collapse.

1.3 A few simple considerations

Here are some simple observations about conservation of momentum and energy which can be used to estimate the time to collapse. Assume for the sake of simplicity that each floor in the tower has the same mass which we can take to equal 1. This discussion pertains to the bottom N floors where N is the target floor.

Denote by q the mass of the top floors above floor N, the target floor. Thus if the target floor is 94, $q = 16$.

For purposes of ease of discussion, re-number these floors from 1 to N from the top down with 1 corresponding to floor N. Thus the floors are being re-numbered in the order in which they will experience a collision from upper floors. It follows j will refer to the floor which is $j-1$ floors down from the target floor. Let h be the spacing between the floors.

Consider a collision between the solid material from the conglomeration of falling floors with floor $j-1$. It is only necessary to consider the solid material above this floor because the dust and other ejected material is either suspended in air or has been thrown out of the way and does not contribute to the collision. Denote by M the mass of this conglomeration of solid material immediately before the collision. Then immediately after the collision, conservation of momentum would require that if L is the speed of the solid right after the collision and u_0 is the speed of the solid immediately before the collision,

$$L\,(M+1) = Mu_0$$

and so

$$L = \frac{Mu_0}{(1+M)}.$$

Assume $(1-r)$ of this mass has been crushed to dust or otherwise ejected so the mass of solid material left for the next collision is

$$r\,(1+M)\,.$$

This solid material falls a distance of h, the spacing between floors at which instant it collides with floor j. The new speed of the solid lump can be computed by conservation of energy. Thus letting u denote this new speed,

$$r\,(M+1)\,gh = \frac{1}{2}r\,(M+1)\,u^2 - \frac{1}{2}r\,(M+1)\,L^2$$

and so

$$u = \sqrt{2gh + L^2}.$$

If there is no loss of mass, then $r = 1$.This is the case considered by Greening [3] in the first part of his paper.

The time taken for the conglomerate to fall from level $j-1$ to level j is

$$\frac{2h}{L+u} \tag{1}$$

because the average speed is $\frac{L+u}{2}$ on this interval due to the constant acceleration of gravity. One can compute the time to fall by adding the times for the solid material to fall between successive floors and obtain a lower bound on the appropriate time for the lower N floors of the actual tower to fall. Note this does not include the time needed for the collisions to take place and there are many other items mentioned a little later which also are neglected which make this a strict lower bound.

The north tower was 1368 feet tall and the floors were 33 inches thick. There were 110 floors so this gives $\frac{1368}{110} = 12.44$ feet per floor. Then you need to take off $\frac{33}{12} = 2.75$ feet per floor to get h in the above. This gives $h = 12.44 - 2.75 = 9.69$ feet. However, this is not really the correct value of h. The floors were supported by metal trusses which were 29 inches high surmounted by a 4 inch thick slab of concrete. After the collapse took place, a pile of floors each 33 inches high was not observed. The trusses collapsed or folded to the side and it seems realistic to assume the result of this collapse would have been about nine inches thick, four inches for the concrete and 5 inches for the steel trusses. Thus a more realistic value of h is $9.69 + 2 = 11.69$ feet. By assigning this value to h in the above argument, I ignore any resistance occurring from the collapse of the trusses and this yields a larger value of u than what would have taken place. Therefore, the length of time in 1 is too short, in keeping with my intent to provide a lower bound for the time to collapse. Other energy sinks which I have ignored in addition to the mechanical deformation of the trusses and floor pans used in the construction of the floors which also have the effect of rendering my computations for the time to collapse too small are those of sound, dissipation from heat, vibrations, seismic energy, and the energy lost to crushing the concrete.Thus the figure I will end up getting for the total time of collapse is too small. The paper by Greening [3] discusses the energy loss to crushing concrete and I will consider this later. It makes a significant difference in the time to collapse.

1.4 Hard top collapse

Here the hard top collapse will be discussed. This has the top floors falling as a solid mass and impacting the target floor after falling a distance of h, the space between floors. Thus the speed, u of this falling unit of floors when it impacts the floor right beneath it is obtained from setting the kinetic energy equal to the potential energy,

$$\frac{1}{2}qu^2 = qgh$$

and solving for the speed which yields

$$u = \sqrt{2gh}.$$

Also the time for this top unit of floors to fall to the target floor is

$$2h/\sqrt{2gh}$$

because the average speed is $\sqrt{2gh}/2$ and the distance to fall is h.

The following maple program in which I have taken $r = 1, q = 16, h = 11.69$, as discussed above, computes the sum of the times to fall between collisions for these values of the parameters. Here is the program. It is the program whose name is wtcA.

```
Digits:=38: r:=1: q:=16.0:Den:=1750:
CM:=110*(177145.6/(404.45*Den+177145.6)):
M:=q: g:=9.8: h:=11.69*.3048: u:=sqrt(2*g*h):
N:=94: sm:=2*h/sqrt(2*g*h): n:=1: while n<N+1
do L:=u*M/(1+M): u:=sqrt(2*g*h+L^2):
sm:=(2*h/(u+L))+sm: M:=r*(M+1):n:=n+1: od: print(n);
print('total_time_bottom_floors'=sm); print('total_mass'=M);
print('speed'=u);
print('total_time_whole_tower'=sm+(2*h*q/(sqrt(u^2+2*g*q*h)+u)));
print('Correct_solid_mass'=CM);
```

In this program "od" signifies the conclusion of the while loop and "Digits" indicates the number of digits used in the computations.

To obtain the time for the top stack of floors to collapse when it reaches the bottom, I am following Greening [3] and am taking this to be the time for the top floor having speed u to reach the bottom under the influence of gravity. Thus this time is

$$\frac{2hq}{\left(\sqrt{u^2 + 2gqh} + u\right)}.$$

because the distance it falls is hq and the average speed is

$$\frac{\sqrt{u^2 + 2gqh} + u}{2}$$

and this figure is added to sm to get the total time to collapse. The other items such as Den and CM will be explained below. It is a simple program and the physics used is all elementary. No advanced knowledge of structural engineering is required.

1.4.1 No loss of mass to dust

First consider the case where there is no loss of concrete to dust. Then in the above program, $r = 1$. When the above program is run, 11.16 seconds is the time for the bottom 94 floors to collapse. Then the total time for the whole tower to collapse, including the collapse of the top unit of 16 floors after it hits the bottom is 12.18 seconds. This is very close to the figures obtained by Greening in [3] although he used 95 as the target floor and also a different spacing between floors. He obtains 11.5 seconds for the collapse of the bottom 95 floors and 12.5 for the collapse of the whole floating floor tower. If I use 12.1 for the spacing between floors, as he did, and let 95 be the target floor as he does, then the above program delivers 11.5 seconds for the collapse of the bottom 95 floors and 12.48 for the collapse of the whole floating floor tower, essentially his results. It is seen that all these figures for collapse of WTC1 are longer than the official NIST figure of 11 seconds and this is without including any other effects such as the crushing of the concrete while totally ignoring the columns and any resistance they may have provided.

1.4.2 Progressive loss of mass to dust

Now consider the case where concrete is lost as dust to the collision process but no allowance is made for loss of energy due to this crushing of concrete. Here I will show that even without considering the loss of energy, the crushing of the concrete must result in a slower time to collapse.

What should the time to collapse be under these idealized assumptions involving magically suspended floors? In the above program, it depends on how r is chosen. Assume none of the steel used in the bottom 94 floors was either crushed or flung out so that all the floor steel was available to impart momentum to the next collision but that all of the concrete was crushed as mentioned above. I will determine r in such a way that the resulting solid mass left after the first stage of collapse of the bottom 94 floors equals the mass of the steel in these floors. Then, running the program with this value of r yields a lower bound on the appropriate time to collapse for WTC1. An assumption a part of the floor steel was lost will result in longer times to collapse and it is obvious from pictures of the collapsing tower that much other than concrete was lost to the collision process. Therefore, the estimates obtained here are biased in favor of faster time to collapse.

Making this simplifying assumption, the total mass left from these floors after all have collapsed should be the total mass of the steel. Thus

$$\begin{aligned} \text{total mass steel} &= \frac{177145.6}{707786 + 177145.6} \times 110 \\ &= 22.019\,799 \end{aligned}$$

More generally, if Den is the density of the concrete in kg/m^3 the total mass of steel is given by

$$\text{total mass steel} = \frac{177145.6}{404.45 \times Den + 177145.6} \times 110$$

Note that I am assuming the top block of floors which collapses at the end is also susceptible to the loss of concrete during the first stage of collapse.

I will adjust the parameter r such that the total amount of solid mass left is approximately 22. This is easy to do by running the program and using a process of trial and error. Corresponding to $r = .957$ one obtains a remaining solid mass of 22.15. Thus a lower bound for the time to collapse for WTC1 is obtained from using this value of r. This yields a time to collapse of the bottom 94 floors of 14.38 seconds and a time to collapse of the whole tower of 15.95 seconds, longer than the NIST claims and also longer than most other claims. These times are for the floating floor model and totally neglect any support from the supporting columns and yet they are comparable or even a little longer than many of the observed times to collapse for the actual buildings.

In viewing the videos of the falling building, it does not seem remarkable that at each collision about 1/10 of the total mass was lost. This will violate the above assumption that none of the floor steel was lost. However, there is also no compelling reason to keep this assumption either. Thus, letting $r = .9$ and running the program yields a total time of collapse for the whole tower of 21.33 seconds with a total remaining solid mass of 9.

1.4.3 Energy loss to crushing of concrete

In this section estimates are obtained which include the loss of energy to crushing the concrete. It shows this can make a substantial difference in the time to collapse. Of course much more was done with the concrete than to crush it. It was blown all over. This took even more energy and I am ignoring this because I do not know how to include it easily. An attempt has been made to consider this question in [22]. For dramatic pictures of these clouds of dust, see either [18] or [22].

I will use a sloppier argument here in obtaining estimates for the sake of simplicity. I will neglect conservation of momentum in the fall of the bottom floors and only consider energy. In fact, not all the energy is available for use in crushing the concrete as I am assuming. Therefore, the predictions for the time to fall will be further biased in favor of rapid collapse. Also, it is worth noting that the concrete was only one of many substances which were crushed. I am totally ignoring the energy needed to crush anything else and this was a substantial amount of material and this gives a further bias in favor of more rapid time to collapse.

As before, denote by M the solid mass of the conglomeration of falling floors immediately before collision with floor $j-1$ and assume its speed is u_0. Also denote by th the thickness of a floor in meters and let h be the spacing of the floors in meters. Thus

$$th + h = 12.44 \times .3048 \text{ meters.}$$

The total energy of this conglomeration along with floor j is then

$$(M+1)\,gh + \frac{1}{2}Mu_0^2$$

and just before the collision with floor j, assuming $(1-r)$ of the mass has been lost as dust, the energy is

$$\frac{1}{2}r\,(M+1)\,u^2$$

where u is the speed of the falling lump of floors immediately before the collision with the next floor. Letting E denote the energy required to crush $(1-r)\,(M+1)$ kg of concrete which is what gets crushed between the collision with floor $j-1$ and the collision with floor j, it follows

$$(M+1)\,gh + \frac{1}{2}Mu_0^2 - \frac{1}{2}r\,(M+1)\,u^2 \geq E. \tag{2}$$

Greening [3] gives an explanation on how to compute the energy to crush a kg of concrete which can be used to get a formula for E. Consider a cube having sides $diam\ \mu m$ in length. Then supposing density is given in kg/m^3, the mass of this cube is

$$density \times diam^3 \times 10^{-18}.$$

It follows one kg has

$$\frac{1}{density \times diam^3 \times 10^{-18}} = \frac{1}{density \times diam^3} \times 10^{18}$$

of these cubes in it. Therefore, the surface area of one kg fractured into cubes having sides of length equal to $diam\ \mu m$ is

$$\begin{aligned} & \frac{1}{density \times diam^3} \times 10^{18} \times 6 \times d \times d \times 10^{-12} \\ = & \frac{1}{density \times diam} \times 6 \times 10^6 \end{aligned}$$

As indicated in [3], the energy to fracture concrete is about 100 $joules$ per m^2. Therefore, the energy needed to fracture one kg of concrete to size $diam\ \mu m$ is

$$\frac{100}{density \times diam} \times 6 \times 10^6.$$

Therefore, from 2,

$$\begin{aligned} & (M+1)\,gh + \frac{1}{2}Mu_0^2 - \frac{1}{2}r\,(M+1)\,u^2 \\ \geq & \left(\frac{100}{density \times diam} \times 6 \times 10^6\right)(1-r)\,(M+1)\,. \end{aligned}$$

Letting

$$\frac{100}{density \times diam} \times 6 \times 10^6 \equiv fac$$

it follows

$$u \leq \sqrt{\frac{1}{r}\left(2gh + \frac{M}{M+1}u_0^2 - 2fac\,(1-r)\right)}.$$

Assigning the u to equal to the formula on the right in the above inequality will result in u becoming larger than it should and so the remaining computations will result in the total time to collapse smaller than it should be. This leads to the following program which is just a small modification of the earlier one. In this program, *diam* is a measure of the size of the dust as explained above and Den referrs to the density in kg/m^3. The program is the one titled wtcB.

```
>Digits:=38:r:=.99:q:=16.0:Den:=1750:diam:=100:
h:=11.69*.3048:th:=12.44*.3048-h:g:=9.8:
CM:=110*(177145.6/(404.45*Den+177145.6)):
fac:=((100/(Den*diam))*6*10^6): M:=q: u:=sqrt(2*g*h):
N:=94: sm:=2*h/sqrt(2*g*h): n:=1: while n<N+1
do print('u'=u); L:=u*M/(1+M): TC:=2*(h+th)/(u+sqrt(2*g*(h+th)+u^2)):
u:=sqrt(max((1/r)*(2*g*h+(M/(M+1))*u^2-2*fac*(1-r)),0)):
sm:=sm+TC: M:=r*(M+1):n:=n+1: od: print(n);
print('total_time_bottom_floors'=sm);print('speed'=u);
print('total_time_whole_tower'=sm+(2*h*q/(sqrt(u^2+2*g*q*h)+u)));
print('Correct_solid_mass'=CM);print('total_mass'=M);print('r'=r);
```

Running this program for the value $r = .99$ and $100\mu m$ dust as indicated above, it follows the remaining mass is 66.73 and the time to collapse of the bottom 94 floors of the floating floor model is at least 35.99 while the time to collapse for the whole floating floor tower is 38.53, substantially more than the official estimates for the actual buildings which are all no larger than 16 seconds and usually less than 15. Note also that the remaining mass, 66.73, is nowhere near 22.01, the mass which would result if all the concrete were crushed. Attempts to achieve this desired mass by adjusting r result in huge times to collapse and are indicative of the fact that eventually there is not enough energy available between collisions to crush the specified amount of concrete and the speed of the falling lump of floors is artificially set to 0 by the program. This was ignored by the earlier formulation which was unaffected by the energy needed to crush the concrete and r could be adjusted to achieve a final solid mass of about 22. This illustrates the conflict between energy requirements to crush the concrete and the requirement to simultaneously collapse in a short time.

It is interesting to consider an extreme situation in which all the top floors have collapsed onto floor 94 before this floor begins to move down. This lump of collapsed floors would have higher speed than that used above in which the top 16 floors falls only a distance of h and moves down as a solid lump till it collapses when it reaches the bottom. It is easy to obtain a very crude upper bound for the speed of this lump of floors. Let its kinetic energy equal the sum of the potential energies of each of the upper 16 floors. Thus a little more generally where there are q upper floors, this speed, u is the solution to

$$\sum_{k=1}^{q} kgh = \frac{1}{2}ghq^2 + \frac{1}{2}ghq = \frac{1}{2}qu^2$$

and so

$$u = \sqrt{gh\,(q+1)}.$$

Placing this initial speed in the above program with r = 0.989492 and finding the time to collapse for the bottom 94 floors yields the time for the bottom 94 floors to collapse is 31.29 seconds and the final speed of the lump is only .818 meters per second. The total remaining solid mass is 65.2 and any attempt to obtain a smaller total mass, closer to the desired figure of 22 by adjusting r very much will only result in the above program assigning $u = 0$ because there is insufficient energy to crush the specified amount of concrete during the collision. By assigning such a large value to the initial speed of the falling lump I have made the time to collapse much smaller than it should be but it is still over twice the value of that which was observed. As mentioned above, the time to collapse of the bottom 94 floors seems to have been around 12 seconds.

I have used the figure of 100 μm for the size of the particles of dust. This seems to be a reasonable approximation which errs on the side of producing a more rapid collapse time based on the study of the dust from the collapsed buildings found in [21]. This study is trying to identify the sorts of substances found in the dust rather than determining the average size of the various kinds of dust particles. However, they consistantly indicate that among the particles which were less than 75 μm in diameter, were found particles of cement and they do not give examples of cement particles larger than this that I noticed. However, the case of coarser dust is considered in the following.

2 Soft top collapse

The soft top collapse is more consistent with videos of the falling building which clearly show the top floors collapsing before most of the rest of the building falls. This soft top model was motivated by correspondence with Gordon Ross, Frank Legge, and Nic Dudzik. They have done a similar computation for the collapse of the top floors. I have appended this type of computation at the beginning of the program used above. It computes a target floor which is right below the lump of mass obtained when all the top floors have collapsed on to each other from knowledge of the floor on which collapse is initiated, as well as the mass of this lump of floors and its speed and the time it has taken to form. This is then used as input for the earlier program to compute the time to collapse of the bottom floors. Then these two times are added to obtain the time to collapse of the whole tower of 110 floors. As before, the actual tower should take longer to collapse than this idealized one.

It is more interesting to consider r as a function of kinetic energy because if the falling mass has lots of energy, it will be better able to crush the concrete. Here is a simple formula in which r depends on kinetic energy.

$$r = \max\left(0, \left(1 - kMu^2\right)\right)$$

It comes from assuming $(1 - r)$ is proportional to the kinetic energy and that $r \in [0, 1]$. Thus the value for r will be adusted floor by floor to account for changes in kinetic energy of the falling mass of floors.

Above the floor where collapse begins, there are A top floors assumed to be moving together under the acceleration of gravity down on the floor on which collapse begins which then begins to move as soon as it is hit by the top floor immediately above it. This yields a lump of two floors moving with half the speed as that of the upper floor which struck it. Then the next floor up in the A top floors hits this lump of two floors yielding a lump of three floors. Perhaps the lump of floors hits the next floor first before being struck by an upper floor and perhaps not. These cases are considered and the time and other quantities are computed till all the upper floors have collapsed into one lump with known speed. Then the floor immediately below this lump becomes the target floor. The speed of the lump when it hits the target floor is then found and the earlier formula is used. Energy loss is ignored in the computation for the top floors.

The following maple program deals with the above situation. It also considers the thickness of the floors in the computations for the fall of the upper floors. This is not particularly important for the lower floors when it is always the bottom of a lump which is falling a fixed distance to hit a stationary floor but in the top floors, sometimes the lump hits a stationary floor below it and sometimes it is struck from above by one of the upper floors. In this latter case, the thickness of the lump would be significant enough to include in the computations. In using the program, if you want to consider no loss of concrete, just let $k = 0$. This will force $r = 1$ and so no loss of concrete.

2.1 Crushing concrete but no loss of energy

Here is the maple program along with explanations which features the soft top model of collapse with crushing of concrete considered but neglecting loss of energy due to the crushing of the concrete. This is the program called wstBST.

```
Digits:=38: k:=0: diam:=100: Den:=1750: fac:=((100/(Den*diam))*6*10^6):
a:=1:'TOP_FLOORS': h:=11.69*.3048: th:=12.44*.3048-h: g:=9.8:
CM:=110*(177145.6/(404.45*Den+177145.6)): p:=1:q:=1: 'number_of_top_floors':
A:=16: bot:=(A*(h+th))-h: UA:=sqrt(2*g*h):UM:=0: u:=UA: v:=UM: m:=1:'total_time':
T:=2*h/sqrt(2*g*h):DM:=A*(h+th)+th:N:=110-A: A:=A-1:
MA:=1:
```

The above are given values. The density of the concrete is 1750 kg/m^3 *and "fac" is the factor described above which you multiply by the mass in kilograms to obtain the energy needed to crush the given mass of concrete to the size equal to "diam". Thus in this program, this equals 100*μ*m. h is the spacing between floors in meters and th is the thickness of the floors in meters while g is the acceleration of gravity in meters per sec*^{2}*. CM is the "correct" value of solid mass which consists of the total mass of the steel used in the floors. p and q are parameters which will vary during the program. In a collision, p is the mass of what is above and q will be the mass of what is below. The initial collision is between the lowest of the upper floors and the top floor of the bottom so p and q are both equal to 1. A gives the number of top floors. bot is the location of the bottom side of the bottom floor of the top collection of floors which has not yet collided with the growing lump of floors. UA is the speed of the very top floor. Thus initially, at the instant of the first collision, UA:=sqrt(2*g*h) as given above. UM is the speed of the lump. Thus initially this speed equals 0 at the beginning of the first collision. u and v are also parameters which are adjusted in the argument. u is the speed of the top and v the speed of the bottom at the instant of a collision. MA is the mass of the lump. Thus it starts out as 1. m is a counter. T is the time taken. Thus it starts out as the time it takes the bottom of the upper floors to fall a distance of h onto the top of the bottom floors. DM is the location of the bottom* *side of the lump. N is the number of floors below the lump. Thus N=110-A as indicated because when this process starts, one collision is about to occur. Then A is changed to A-1 because at the start of this process, there are now only A-1 floors above the lump which is about to result.*

```
while A>0 do j:=-1: while j*(h+th)<DM
or j*(h+th)=DM do j:=j+1: od:
```

*The first part of the while loop is to determine the location of the top side of the floor right below DM, the bottom side of the falling lump. This is what that little while loop doe*s.

```
L:=(p*u+q*v)/(p+q): r:=max(0,1-k*(p+q)(u-v)^2):
TM:=(-L+sqrt(2*g*(j*(h+th)-DM)+L^2))/g:TA:=((DM-((m+1)*th))-bot)/(UA-L):
```

*This part first calculates the speed right after the collision using conservation of momentum. This speed is L. Next the formula for r is given and the other two definitions of TA and TM are to find whether the next upper floor hits the lump before the lump hits the floor right below it. If TA<TM, then this happens. Otherwise the lump hits the floor below it first. TA is the time for the bottom of the next upper floor to strike the top of the lump. TM is the time for the lump to strike the top of the next floor. Note the inclusion of the thickness of the floors in the computation. In the formula for TA, DM-(m+1)*th locates the top side of the lump while bot is the lower side of the bottom floor of the upper floors.*

```
if TA<TM then
bot:=bot+UA*TA+.5*g*(TA^2)-(h+th):DM:=DM+L*TA+.5*g*(TA^2):
UA:=UA+g*TA: UM:=L+g*TA: p:=1: q:=MA: u:=UA: v:=UM:
A:=A-1:T:=T+TA:
```

This tells what to do when TA<TM or in other words when the bottom upper floor strikes the lump before the lump strikes the next floor below it. First a new value for bot, the bottom of the upper floors is determined. Next the location for the bottom of the lump, DM is specified and UA and UM are adusted. Then p is set equal to 1 because it is the top in the collision and q is set equal to MA because this is the mass of the lump. Then u and v are specified and A and T are modified. A becomes A-1 because there is now one fewer upper floors.

```
elif TM<TA or TM=TA then
bot:=bot+UA*TM+.5*g*(TM^2): DM:=DM+L*TM+.5*g*(TM^2)+th:
UA:=UA+g*TM: UM:=L+g*TM: p:=MA: q:=1:
u:=UM: v:=0: N:=N-1: T:=T+TM:
```

This tells what to do when TM<TA or in other words when the lump hits the next floor before it is struck from above by an upper floor. In this case, p=MA and q=1 because now the lump is on top. Also u=UM and v=0 because the speed of the mass on the top is UM and the speed of the mass on the bottom is 0.

```
fi:MA:=r*(p+q): m:=m+1:
print('speed_lump'=UM); od:
```

Now in either case, when p and q have been determined, the new mass of the resulting lump is r(p+q) because it is assumed that (1-r) of the mass taking part in the collision is crushed. Therefore, r*(p+q) is the new mass of the solid part. m is changed to m+1 because there are now m+1 floors in the lump. Then if A>0, the while loop continues and does the same sequence of steps again, stopping when A=0.*

```
print('N'=N); print('time'=T);
print('toplumpmass'=MA); 'final_speed_of_lump':
'print('final_speed_of_lump'=UM)';print('speed_top'=UA);
print('distanceM'=DM); q:=(p+q)*r: M:=q:i:=-1: while
i*(h+th)<DM or i*(h+th)=DM do i:=i+1: od:
u:=sqrt(2*g*(i*(h+th)-DM)+UM^2):
```

Some things which have been computed are printed and the final mass is given as q=r(p+q) which will be used as input for the computation which describes the collapse of the bottom floors below. The while loop* *identifies the floor right below DM and then u is the speed of the lump when it hits this floor. What follows is the same sort of computation done earlier. sm is the time to fall. Finally, to obtain the total time, the program adds sm and T, the time computed above. The initial value of sm is the time taken for the lump to fall to the next floor. There are then various things printed.*

```
print('speed_of_lumped_floors_hitting_top_floor_of_bottom'=u):
N:=N: sm:=2*(i*(h+th)-DM)/(u+UM):n:=1:
while n<N+1 do r:=max(0,1-k*((M*(u^2)))): L:=u*M/(1+M):
u:=sqrt(2*g*h+L^2):sm:=(2*h/(u+L))+sm: M:=r*(M+1):
n:=n+1: od: print(n); print('total_time_bottom_floors'=sm);
print('total_mass'=M); print('speed'=u);
print('total_time_whole_tower'=sm+T);
print('Correct_solid_mass'=CM); print('k'=k);
```

In the hard top model the time to collapse for the whole tower in the case of no loss of mass to dust was 12.3 seconds and the time to collapse of the bottom 94 floors was 11.33 seconds. This is assuming the spacing between floors was 11.69 feet and the target floor was floor 95. Letting A=7, the soft top model yields the same target floor and the time for the collapse of the whole tower is 13.14 seconds while the time to collapse of the bottom 95 floors is 9.81 seconds. This comparison indicates the soft top model leads to slightly longer times to collapse for the whole tower but shorter times to collapse for the bottom N floors.

Adjusting k so that the appropriate amount of solid mass remains at the end, it follows that for $k = 0.0000024$ the total remaining mass is 22.02 and the total time to collapse is 16.08 seconds while the time to collapse of the bottom 95 floors is 12.74 seconds. This is taking $A = 7$.

In [6] it indicates the collapse initiated with floor 98 falling on to floor 97. Letting $A = 13$, the same computation yields for $k = 0.0000024$ a time to collapse for the whole tower of 14.85 seconds with a time to collapse of the bottom floors of 10.1 seconds. As earlier, these times are close to what was observed for the actual collapse of the building even though these computed times involve an idealized floating floor model in which there are no supports between the floors and all considerations of energy loss to crush concrete is ignored. The consideration of energy to crush the concrete is next.

2.2 Soft top collapse and loss of energy

Here the energy to crush the concrete is considered in the model as was done earlier for the hard top collapse. The program is just a minor modification of wtcBST. It has the same considerations applying to the top collection of floors but includes energy considerations in the computations for the bottom floors. The attached program is called wtccollapseD. Here it is.

```
Digits:=38: k:=0.000005: diam:=100: Den:=1750:
fac:=((100/(Den*diam))*6*10^6):
a:=1:'TOP_FLOORS': h:=11.69*.3048:
th:=12.44*.3048-h: g:=9.8: CM:=110*(177145.6/(404.45*Den+177145.6)):
p:=1:q:=1: 'number_of_top_floors': A:=13:
top:=0: bot:=(A*(h+th))-h: UA:=sqrt(2*g*h):
UM:=0: u:=UA: v:=UM: m:=1:'total_time':
T:=2*h/sqrt(2*g*h): DM:=A*(h+th)+th:
N:=110-A: A:=A-1:MA:=1: while A>0 do j:=-1:
while j*(h+th)<DM or j*(h+th)=DM do j:=j+1: od: L:=(p*u+q*v)/(p+q):
r:=max(0,1-k*(p+q)(u-v)^2):TM:=(-L+sqrt(2*g*(j*(h+th)-DM)+L^2))/g:
TA:=((DM-((m+1)*th))-bot)/(UA-L):'print(j*(h+th)-DM)';
if TA<TM then 'print('TA'=TA)';
bot:=bot+UA*TA+.5*g*(TA^2)-(h+th):
DM:=DM+L*TA+.5*g*(TA^2): UA:=UA+g*TA:
UM:=L+g*TA: p:=1: q:=MA: u:=UA: v:=UM:
A:=A-1:T:=T+TA: elif TM<TA or TM=TA then
bot:=bot+UA*TM+.5*g*(TM^2):
DM:=DM+L*TM+.5*g*(TM^2)+th:
UA:=UA+g*TM: UM:=L+g*TM: p:=MA: q:=1: u:=UM: v:=0:
N:=N-1: T:=T+TM: fi:MA:=r*(p+q): m:=m+1:
print('speed_lump'=UM); od: print('N'=N);
print('time'=T); print('toplumpmass'=MA);
```

```
'final_speed_of_lump': 'print('final_speed_of_lump'=UM)';
print('speed_top'=UA); print('distanceM'=DM);
'LOWER_FLOORS': q:=(p+q)*r: M:=q:i:=-1:
while i*(h+th)<DM or i*(h+th)=DM do i:=i+1: od:
u:=sqrt(2*g*(i*(h+th)-DM)+UM^2):
print('speed_of_lumped_floors_hitting_top_floor_of_bottom'=u):
N:=N: sm:=2*(i*(h+th)-DM)/(u+UM): n:=1: while n<N+1 do
r:=max(0,1-k*((M*(u^2)))): L:=u*M/(1+M): 'print('r'=r)'; d:=0:
TC:=2*(h+th)/(u+sqrt(2*g*(h+th)+u^2)):print('u'=u);
u:=sqrt(max((1/r)*(2*g*h+(M/(M+1))*u^2-2*a*fac*(1-r)),0)):
'print('TC'=TC)'; sm:=sm+(max(2*d/(L+u),0))+TC:
M:=r*(M+1): n:=n+1: od:print(n); print('density_concrete'=Den);
print('remaining_mass'=M);print('correct_mass'=CM);
print('total_time_whole_tower'=sm+T);
print('time_for_bottom_floors'=sm); print('k'=k);print('h'=h);
```

It contains a few artifacts of earlier programs from which it evolved. Note that u the speed of the lump is printed for each collision. It is important to keep track of this because when k is adjusted, it is possible for u to become 0 which means the program gave it that value due to insufficient energy to crush the specified mass of concrete to the desired size during a collision. Similar conclusions are obtained to those used on the hard top model.

For $k = 0.0000053$, $A = 13$ and the size of the crushed concrete $100\mu m$, there is a total solid mass at the end of 65.8 and the time to collapse of the whole tower is 40.28 seconds while the time to collapse of the bottom 80 floors is 35.6 seconds. The final speed of the falling floors is about 5.41 meters per second although in the computed values of u one of the values was close to 2 meters per second. It is not easy to obtain a smaller total remaining mass because adjusting k to be even a little larger results in $u = 0$ which indicates there is insufficient energy to crush the specified amount of concrete in at least one of the collisions. Obviously these times to collapse are far greater than what was observed for the actual tower and in addition, there was a failure to crush all the concrete.

Of course one reason is the insistence that the dust have size $100\mu m$. Crushing the concrete to larger sized dust requires less energy. For example, if the size of the dust is 400 μm the time for collapse of the whole tower is 20.2 seconds corresponding to a final solid mass of 23.26 and $A = 13$. In this case, there seem to be no difficulties in achieving enough energy to crush the specified amount of concrete to this size although the time to collapse is still longer than what was observed.

2.3 Variable size of dust

In the above, the computations are done for a fixed size of dust. For convenience this was usually chosen to be 100 μm. However, it is reasonable to allow the size of the dust produced to vary depending on the speed of the falling lump of floors. This is discussed in Greening [3] and he argues that the size of the dust produced from the crushing of concrete in one floor should be proportional to $1/u$ where u is the speed of the descending lump of floors. I will make a similar modification here. Here is the maple program. It is the one named wtccollapseE.

```
Digits:=38: k:=0.0000012: diam:=1000: Den:=1750:
fac:=((100/(Den*diam))*6*10^6): a:=1:'TOP_FLOORS':
h:=11.69*.3048: th:=12.44*.3048-h: g:=9.8:
CM:=110*(177145.6/(404.45*Den+177145.6)):
p:=1:q:=1: 'number_of_top_floors': A:=13:
top:=0: bot:=(A*(h+th))-h: UA:=sqrt(2*g*h):
UM:=0: u:=UA: v:=UM: m:=1:'total_time': T:=2*h/sqrt(2*g*h):
DM:=A*(h+th)+th:N:=110-A: A:=A-1:MA:=1:
while A>0 do j:=-1: while j*(h+th)<DM or j*(h+th)=DM do j:=j+1: od:
L:=(p*u+q*v)/(p+q): r:=max(0,1-k*(p+q)(u-v)^2):
TM:=(-L+sqrt(2*g*(j*(h+th)-DM)+L^2))/g:
TA:=((DM-((m+1)*th))-bot)/(UA-L):'print(j*(h+th)-DM)';
if TA<TM then 'print('TA'=TA)';
bot:=bot+UA*TA+.5*g*(TA^2)-(h+th):
DM:=DM+L*TA+.5*g*(TA^2): UA:=UA+g*TA:
UM:=L+g*TA: p:=1: q:=MA: u:=UA: v:=UM:
```

```
A:=A-1:T:=T+TA: elif TM<TA or TM=TA then
bot:=bot+UA*TM+.5*g*(TM^2):
DM:=DM+L*TM+.5*g*(TM^2)+th:
UA:=UA+g*TM: UM:=L+g*TM:
p:=MA: q:=1: u:=UM: v:=0: N:=N-1:
T:=T+TM: fi:MA:=r*(p+q): m:=m+1: print('speed_lump'=UM);
od: print('N'=N); print('time'=T);
print('toplumpmass'=MA); 'final_speed_of_lump':
'print('final_speed_of_lump'=UM)';print('speed_top'=UA);
print('distanceM'=DM); 'LOWER_FLOORS':
q:=(p+q)*r: M:=q:i:=-1: while i*(h+th)<DM or i*(h+th)=DM do i:=i+1: od:
u:=sqrt(2*g*(i*(h+th)-DM)+UM^2):
print('speed_of_lumped_floors_hitting_top_floor_of_bottom'=u):
N:=N: sm:=2*(i*(h+th)-DM)/(u+UM): n:=1: while n<N+1
do Diam:=diam/max(u,.0001): print('size_dust'=Diam):
fac:=((100/(Den*Diam))*6*10^6):
```

This is the main change. Here Diam is defined as diam divided by u, the speed of the falling lump. I put in max(u,.0001) to prevent division by 0. Then Diam is used in place of diam in the formula for the energy needed to crush the concrete. Earlier in the program I set diam to equal 1000 because in the computation in which no concrete is made into dust the maximum velocity was about 50 feet per second. Thus this will eliminate energy loss to dust which is any smaller than 20μm although there is no upper bound to the size of the dust. A smaller value for diam will produce smaller dust and longer times to collapse due to larger energy demands for crushing the concrete while a larger value will produce shorter times to fall and larger dust. If I assigned diam the value 2000, then energy loss to dust any smaller than 40 μm would not be considered and this would contradict observations on the size of the dust which formed. There was some very small sized dust.

```
r:=max(0,1-k*((M*(u^2)))):
L:=u*M/(1+M): 'print('r'=r)'; d:=0:
TC:=2*(h+th)/(u+sqrt(2*g*(h+th)+u^2)):print('u'=u);
u:=sqrt(max((1/r)*(2*g*h+(M/(M+1))*u^2-2*a*fac*(1-r)),0)):
'print('TC'=TC)'; sm:=sm+(max(2*d/(L+u),0))+TC:
M:=r*(M+1): n:=n+1: od:print(n); print('density_concrete'=Den);
print('remaining_mass'=M);print('correct_mass'=CM);
print('total_time_whole_tower'=sm+T);
print('time_for_bottom_floors'=sm); print('k'=k);print('h'=h);
```

Running this program for $A = 13$ a remaining mass of 69.06 corresponds to a time to collapse of 28.97 seconds and an average size dust of 90.68 μm. Corresponding to a remaining solid mass of 67.43 the time to collapse is 30.30 and the average size of the dust is 97.34. The largest dust size was 170 μm. As before, it is difficult to get the total mass much less than this because of insufficient energy to crush the specified amount of concrete to the required size. The conclusion that the time to fall was longer than it should have been is not essentially different than the earlier one in which a size of dust was specified.

For interest, set the parameter diam to equal 6000. Then the average size of the dust is 358 μm and the time to collapse is 21.8 seconds corresponding to a final solid mass of 25.88 and $A = 13$.

2.4 Inclusion of more than the floors

From the videos of the falling buildings, it does not appear the columns assisted appreciably in crushing the concrete. They show columns being flung away from the collapse and also they show the core columns standing to a considerable height for a short time after the collapse. Nevertheless, it must be admitted that it is hard to see what was happening because of the clouds of dust. Here I will modify the above argument to consider all the steel, including the columns but I will assume they provided essentially no resistance to the collapse. All this does in the above program is change the ratio of the mass of the steel to the mass of the whole floor.

In [3] the total mass of one floor is defined as 1/110 times the total mass of the building which is said to equal 510000000 kg. Thus the mass of one floor is

$$\frac{510000000}{110} = 4.636\,363\,64 \times 10^6 kg.$$

However, from [20] the total mass of the tower was 450000000 kg. Thus the total mass of one floor according to this reference is

$$\frac{450000000}{110} = 4.090\,909\,09 \times 10^6 kg.$$

This reference also states the total mass of steel is 90000000 kg and the total volume of concrete was 160000 m^3. If the density of concrete is the figure given above, then this works out to

$$160000 \times 1750 = 2.8 \times 10^8 kg$$

of concrete. Therefore, it would seem the mass of concrete per floor would be

$$\frac{2.8 \times 10^8}{110} = 2.545\,454\,55 \times 10^6 kg$$

which is much more than the estimates used earlier in this paper. This figure is erroneous because much more concrete was used in the base of the building than in the part which fell. Therefore, I will continue to use the above figure for mass of concrete in one floor,

$$7.077\,875 \times 10^5 kg,$$

although use of the larger figure would greatly strengthen the assertion that the building fell too fast.

I will also use the total mass of steel per floor as

$$\frac{90000000}{110} = 8.181\,818\,18 \times 10^5$$

according to [20]. This assumption that the mass of steel in the floors was the same in the upper floors as in the lower floors is incorrect but errs on the side of creating faster collapse times. Then, assuming the only thing remaining as solid is the steel and that none of the steel from columns or floors is thrown away, an assumption favoring faster time to collapse which is contrary to the videos of the falling buildings, it follows that the total remaining mass would be

$$\begin{aligned} & \frac{8.181\,818\,18 \times 10^5}{7.077\,875 \times 10^5 + 8.181\,818\,18 \times 10^5} \times 110 \qquad (3) \\ = \; & 58.978\,905\,4 \end{aligned}$$

Actually this number should be lower if you accept either figure for the total mass of the building and the assumption that the only solid which remains at the end is steel because

$$\begin{aligned} & 7.077\,875 \times 10^5 + 8.181\,818\,18 \times 10^5 \\ = \; & 1.525\,969\,32 \times 10^6 < \frac{450000000}{110} \\ = \; & 4.090\,909\,09 \times 10^6 \end{aligned}$$

and now the same problems occur as were noted earlier, very long times to collapse and inability to obtain the specified figure for remaining mass because of insufficient energy available.

For example, using wtccollapseD, assuming $A = 7$ so that the target floor ends up being floor 95, the time to collapse of the whole tower is 36.76 seconds, corresponding to a remaining mass of 65.95. In case $A = 13$ the figures are also too long. One obtains a time of collapse of the whole tower equal to 33.92 seconds, corresponding to a remaining mass of 66.05. These computations both involve crushing the concrete to size $100\mu m$. As earlier, it is not possible to realistically obtain a total remaining solid mass equal to 59 without ignoring the insufficiency of energy to crush the specified mass of concrete.

Next consider the case where the energy to crush the concrete is totally ignored. Then running wstBST with $A = 7$, it follows that, corresponding to a total solid remaining mass of 60.14, there is a time to collapse equal to 13.5 seconds and letting $A = 13$ then corresponding to a total remaining solid mass of 60.11 the time to collapse is 12.45. These estimates are not too far from the official estimates of the time to collapse but they are based on unrealistic assumptions which are known to be false. For example, it is known much of the steel did not remain in the collision process. The pictures show it being thrown out to the sides and the core columns were standing a short time after collapse. The assumption all floors are the same mass is not true either. The columns were much less massive near the top of the building than near the bottom. This was not considered in the above and should result in a slower time to collapse for the idealized model. If either the figure for total mass of the tower used in [3] or the one found in [20] were used in the denominator of 3, then the desired solid mass at the end of the collision would be no larger than

$$\frac{8.181\,818\,18 \times 10^5}{450000000} \times 110^2 = 22.0$$

which would lead to approximately the same collapse times as in the case considered in the first part of the paper where only the floors were used in the model. In particular, the time to collapse, even in the case where no energy loss to crushing of concrete is included, leads to longer times to collapse for the idealized model building than most of the observed times for the actual building. A time of 14.79 seconds is obtained corresponding to a remaining mass of 22.35 and $A = 13$. If the higher figure used in [3] for the total mass of the building were considered, the time to collapse for the idealized model would be even longer.

One can also experiment with considering thicker floors which seems realistic in the case where more is included in the definition of a floor. However, this also leads to times to collapse which are longer than observed. In fact, when wtccollapseD is run with a distance between floors equal to 10 feet rather than 11.69, resulting in a larger thickness of the floor, the time to collapse for the floating floor model actually got larger.

2.5 Resisted fall

All of the above has been for an imaginary building in which none of the supporting columns provide any resistence at all. They either are nonexistent or they behave something like toothpicks. Here I will do a computation which attempts to include resistance from the supporting columns. Thus the analysis is linked to WTC1 rather than the idealized tower of floating floors. This computation was suggested to me by Nic Dudzik. Finding the time to collapse from a knowledge of the remaining non crushed mass will not be so easy in this context but this approach will give a means for including the resistance of the columns as well as variations in mass of a given length of column from the top to the bottom of the tower. It is known that the columns were less massive at the top than at the bottom. Here I will let 1 denote the mass of each of the bottom 7 floors. I will assume the mass of the floors from floor 8 up to floor 44 is .9, the mass of the floors from 45 up to 77 is .8, and the mass of the floors from 78 to the top is .7. Instead of the acceleration of gravity for g I will use

$$g = 32 - 2 \times 32 \times .35.$$

The 2 in the above is a safety factor which should be somewhere between 2 and 5. A factor of 2 is the smallest reasonable safety factor. The .35 comes about because the column will not deliver its full force over the whole length of the interval. Thus it should be multiplied by some number between .35 and .5. I have picked the smallest reasonable values in order to maximize the speed of collapse while still including some resisting force. The maple program which will sum the times in this case and also account for the variations in the mass of the floors is below. I have taken $r = 1$ in this formula. Also, in this setting, I am considering the mass of a floor by taking the mass of the whole building and dividing by 110. This results in a figure which is much larger than the one I used for the floating floors model in which I was careful to use only the mass of the floor and differentiate between the part which came from steel and the part which came from concrete. I am assuming in this that the top floors fall onto the 97^{th} floor and then the columns between this floor and floor 96 disappear, to start the collapse of the bottom 96 floors. Thus the acceleration is taken to initially equal 32 feet per second^2. Here is the program.

```
>Digits:=38: r:=1:a:=.7: b:=.8: c:=.9: q:=a*16.0: M:=q/r: h:=11.69:
sm:=2*h/(sqrt(2*32*h)):u:=83: N:=95: n:=1: while n<N do
g:=max(32 − 2 ∗ 32 ∗ .35, 0): if n<8 then L:=u*M*r/(1+r*M): M:=(r*M+1):
u:=sqrt(2*g*h+L^2): elif n<45 and n>7 then L:=u*M*r/(c+r*M): M:=(r*M+c):
u:=sqrt(2*g*h+L^2): elif n>45 and n<78 then L:=u*M*r/(b+r*M): M:=(r*M+b):
u:=sqrt(2*g*h+L^2): elif n>77 then L:=u*M*r/(a+r*M): M:=(r*M+a):
u:=sqrt(2*g*h+L^2): fi: sm:=(2*h/(u+L))+sm; n:=n+1: od:
print(n); print('total_time'=sm);print('total_mass'=M);
```

Running the program for various values of r and assuming the resistance force is as described above yields the following table.

r	1	.99	.98	.97	.96	.95	.9	.8
Total mass	89	54	35	25	18.8	15	7.1	3.5
Time to collapse	16.6	17.7	19	20.4	22	22.6	31.6	45.8

One should also add about one second to these figures to obtain the total time to collapse because the computations do not calculate for the collapse of the top 16 floors. Clearly, fall times of over 25 seconds are expected with reasonable assumptions, yet the observed fall time for the Tower is less than that. For this table, a very conservative safety factor of 2 was used. This factor can be as large as 5 as explained above. You can see that if a larger safety factor were used, the building would not collapse completely because the downward force would be less than the upward force. That result is consistent with the prediction of Gordon Ross in his analysis which concluded that the fall of the North Tower should have been arrested with much of the lower portion of the Tower remaining standing.

Videos of the falling building suggest the columns punched through the floors or else were flung away from the progressive collisions between floors. This is illustrated by pictures of the collapse in which beams are seen falling outside of the building and the fact that the central columns remained standing shortly after the collapse. Therefore, the above analysis may not be entirely pertinent to the manner of collapse of the building but it is nevertheless, another computation which indicates the time to collapse of the actual building was surprisingly short.

2.6 Summary

There are two observations about the collapse of WTC1 which are difficult to harmonize in the context of the official explanation. One is the time with which the collapse took place and the other is the production of large clouds of dust which are seen forming during the entire collapse in the videos of the falling building. The majority of the paper has explored this difficulty by totally ignoring the columns and any resistance they produced. The considerations were based on the need for energy to crush the concrete and momentum conservation. However, if one ignores energy needed to crush the concrete, a reasonable estimate using resistance of the columns also produces surprisingly long times to collapse.

References

[1] Article by Clifton including information on construction of the buildings.

[2] Still pictures of collapse of top floors.

[3] F. R. Greening, Energy transfer in the WTC collapse.

[4] Evidence for Controlled Demolition: a Short List of Observations

[5] (NIST (2005) Final Report of the National Construction Safety Team on the Collapses of the World Trade Center Towers (Draft)), Sept.-Oct. 2005.

[6] Answers to frequently asked questions

[7] Momentum transfer analysis of the collapse of the Upper Storeys of WTC1

[8] Why indeed did the world trade center buildings completely collapse?

[9] An article on temperature in the floors

[10] Why did the trade center sky scrapers collapse?

[11] 9/11 commission report

[12] Fall of the bottom

[13] Disproving the pankcake theory in 110 easy steps.

[14] Collapse of the top floors

[15] Collapse of the top floors

[16] Billiard balls analogy

[17] Density of concrete

[18] Huge clouds of dust

[19] Time for North tower to fall

[20] Some statistics about the mass of steel and concrete

[21] A detailed study of the types of dust found

[22] An attempt to consider energy lost to the formation of clouds of dust