

STUDY GUIDE:

Module 1: The Development of Place Value

In this module we shall see how people first learned to count. The most primitive way was to draw a picture of the object being counted and to reproduce the picture once for every object that was present.

Later, a simple mark (called a tally mark) was used to replace the picture of the object being counted.

But as the number of objects being counted became greater and greater, newer and better innovations had to be made so that we could recognize the meaning of the numbers we were talking about.

In a step by step manner going from tally marks to Roman numerals; from Roman numerals to the abacus; and from the abacus to our present place value system, the saga of modern counting is unfolded.

The module ends with the message that the search for knowledge is on-going and that no matter how advanced we become, new problems always seem to manage to arise. In this way, we find that many of the problems that bothered ancient peoples come back to haunt us even in the present century.

Step 1:

View Videotape Lecture #1

Step 2:

Read Module 1 of the text.

Step 3:

When you feel you understand the material presented in Steps 1 and 2, complete the following "Check-The-Main-Ideas" Self-quiz by correctly filling in each blank.

Check the Main Ideas:

Arithmetic, like any other language uses symbols.
Symbols that represent numbers are called _____.

numerals

In the earliest sign languages it was clear that
people viewed numbers as _____ rather than as nouns.
For example, to represent three horses, people would
draw _____ horses. And if they wanted to represent
three people they would draw _____ people.

adjectives

three (3)

three (3)

Later they realized that they could just as
easily draw three _____ whether they were counting
people or horses. For example, twelve sheep would
be represented as _____.

tally marks

|||||

But after awhile it
became difficult to distinguish one large number of
sheep from another large number of sheep. So people
invented special symbols or _____ to stand for
"ten", "hundred", and "thousand". The Romans used
_____ to stand for "ten"; C to stand for _____ and
M to stand for _____. They also used _____ to
stand for "one". "one", "ten", "hundred" and
"thousand" are called _____ of ten.

numerals

X; "hundred"

"thousand"; I

powers

The problem with Roman numerals is that they
require a different letter of the alphabet for
each _____. To help avoid this problem
people drew vertical lines in a horizontal row.
Each line represented a different _____.

power of ten

power of ten

For example if a marker was placed on the line furthest to the right it stood for 1 _____. But if it was placed on the next line to the left, the same marker stood for 1 _____. If no marker appeared on, say, the hundreds-line, it meant that there were _____ hundreds.

one

ten

no

To avoid having to draw lines, people invented special numerals; 0,1,2,3,4,5,6,7,8, and 9; that were called _____. The digit 0 was called a _____. For example by writing 407, the 0 told us that there were no _____. 0 wasn't necessary in Roman numerals because the absence of _____'s told us there were no tens. To help make

digits

place-holder

tens

X

it even easier to read and represent large numbers, the digits were grouped from right-to-left in clusters of _____. The first group named the number of _____. The next group named the number of _____; and the next group named the number of _____. So the numeral 435,856,207 meant 207 _____, 856 _____, and _____ million. In this situation 0 was still a place holder. For example 345,000,234 meant that we had 345 _____. But if we wrote 345,234 it meant that we had 345 _____.

three; units

thousands

millions

units; thousands

435

million(s)

thousands(s)

Scientific

_____ -notation was introduced to handle situations in which a great many digits appeared. To indicate that we had a 1 followed by 37 zeroes we would write _____. Conversely, if we saw 10^{37} it would tell us that we had the numeral consisting of a _____ followed by _____ 0's.

10^{37}

1

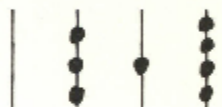

thirty seven (37)

Step 4:

Do the Mastery Review.

Mastery Review

Answers:

1. Use tally marks to show how a shepherd might keep track of the fact that he had nine sheep. 1. _____
2. How would the shepherd have indicated that he had thirteen sheep? 2. _____
3. Using Roman numerals, how would the shepherd have indicated that he had thirteen sheep? 3. _____
4. Using no more than nine each of the X's and I's, use Roman numerals to represent the number forty three (43). 4. _____
5. How would we express the number three thousand two hundred fourteen (3,214) using Roman numerals? 5. _____
6. What number is shown on the sand-reckoner below? 6. _____

7. In the place value numeral 567 what noun does 5 modify? 7. _____
8. In the place value numeral 675 what noun does 5 modify? 8. _____
9. What number is named by CCCI? 9. _____
10. What number is named by: 10. _____

11. What number is named by 301? 11. _____
12. What number is named by 310? 12. _____
13. What number is named by 6540? 13. _____
14. How do we read the number: 14. _____
6,403,697,492,184 ?

- | | |
|---|-----------|
| 15. What number is named by 4,000,000,000 ? | 15. _____ |
| 16. What number is named by 4,700,000 ? | 16. _____ |
| 17. What number is named by 4,070,000 ? | 17. _____ |
| 18. Write nine hundred sixty three as a place value numeral. | 18. _____ |
| 19. Write nine hundred sixty three thousand as a place value numeral. | 19. _____ |
| 20. Write nine hundred sixty three billion four hundred three million eight hundred thirty two thousand four hundred twelve as a place value numeral. | 20. _____ |
| 21. Write fifteen billion four hundred thousand as a place value numeral. | 21. _____ |
| 22. Write 100,000,000,000,000,000,000,000 in scientific notation. | 22. _____ |
| 23. Write 100,000,000,000,000,000,000,000,000 in scientific notation. | 23. _____ |
| 24. Write 10^{16} as a place value numeral. | 24. _____ |

Answers:

1. ||||| 2. ||||| 3. XIII 4. XXXXIII 5. MMMCCXIIII
6. three hundred fourteen (314) 7. hundred(s) 8. one(s)
9. three hundred one 10. three hundred one 11. three hundred one
12. three hundred ten 13. six thousand five hundred forty
14. 6 trillion 403 billion 697 million 492 thousand 184 (units)
15. 4 billion 16. 4 million 700 thousand 17. 4 million 70 thousand
18. 963 19. 963,000 20. 963,403,832,412 21. 15,000,400,000
22. 10^{23} 23. 10^{26} 24. 10,000,000,000,000,000

Step 5:

Do Self-Test 1: Form A

Self-Test 1: Form A

In problems 1 through 5; E stands for one, F stands for ten, G stands for a hundred, and H stands for a thousand.

Answers:

1. What number is named by HHGFFFFEEEE ?
2. What number is named by HHHFFFFFF ?
3. Which numeral names the greater number; GGGGGGFFFFFF or H ?
4. Without using more than nine of any one use E,F,G,and H to write the number four thousand three hundred fifty two.
5. Rewrite GGGGGGGGGFFFFFFFFFFFFFFFFEEEEEEEEEEEEE so that none of the letter E,F,G,or H appears more than nine times.
6. What number is shown on the sand reckoner below?

•	•	•	•	•										
•	•	•	•	•										
•	•	•	•	•										
•	•	•	•	•										
•	•	•	•	•										
- a. At \$1 per object, what is the cost of 100 objects?

b. At \$1 per digit, what is the cost of the numeral, 100?
- Read the number represented by the numeral:
23,807,027,000,000,000 ?
- Write as a place value numeral: eighty four quadrillion two hundred thirty seven billion.
- True or false: 10^4 is the number 10 followed by 4 zeroes.

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 1: Form A

1. 2,134 (two thousand one hundred thirty four)
2. 3,040 (three thousand forty)
3. H
4. HHHHGGGGFFFFFEE
5. HFFFFEE
6. 21,453,000,000,000 (twentyone trillion four hundred fifty three billion)
7. a. \$100
b. \$3
8. twenty three quadrillion eight hundred seven trillion twenty seven billion
9. 84,000,237,000,000,000
10. False

If you did each problem in Form A correctly, you may if you wish proceed to the next module. Otherwise continue with Step 6.

Step 6:

Study the Solutions to Self-Test 1: Form A, with special emphasis on any problems you failed to answer correctly.

Solutions for Self-Test 1: Form A

1.

If we count the number of letters we have:

two H's or 2 thousands

one G or 1 hundred

three F's or 3 tens

four E's or 4 ones.

That is:

thousands	hundreds	tens	ones
(H)	(G)	(F)	(E)
2	1	3	4

To get the adjectives (digits) that modify the powers of ten we have to count the number of times each letter appears

So the answer is 2,134

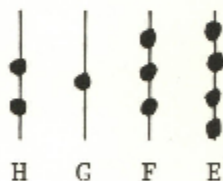
Alternative Solutions

(a) The symbol that names the power of ten is not the important thing. So, for example, if you feel more comfortable with Roman numerals, simply replace each E by an I, each F by an X, each G by a C and each H by an M. That is:

HHGFFFFEEE
↑↑↑↑↑↑↑↑
MMCXXXIIII

That is, both I and E are symbols for one; X and F are both symbols for ten etc.

(b) In terms of the sand reckoner, label the lines E, F, G, and H as shown below; placing a marker on each line for each time that letter appears.



In this diagram the letter names the line while the number of markers give us the digit in each place.

Note the importance of order. Since the line furthest to the right names the ones-place, it must be labeled "E" because E represents "one".

Solutions for Self-Test 1: Form A (cont)

2.

This is essentially the same problem as Problem 1 but emphasis on the place holder is introduced.

Again we count the number of times each letter appears and we obtain:

three H's or 3 thousands

four F's or 4 tens.

Therefore:

thousands (H)	hundreds (G)	tens (F)	ones (E)
3		4	

If we now leave out the nouns and switch to place value we must enter 0 as a place holder in each column where we had none. That is:

3	0	4	0
---	---	---	---

3.

This problem tries to emphasize the importance of viewing numbers as adjectives. For example, when we say that 1 is less than 6 we assume that 1 and 6 are adjectives that modify the same noun. For example 1 thousand is not less than 6 hundred. In this problem H means a thousand while GGGGGGFFFFFF means 6 hundreds and 5 tens, or 650. Since one thousand is more than six hundred fifty, H names the greater number.

In terms of the abacus or sand reckoner, we have:



GGGGGGFFFFFF contains more symbols than H but H names the greater number. It's analogous to 11 pennies being more coins than 1 quarter, but that 1 quarter has greater purchasing power.

Solutions for Self-Test 1: Form A (cont)

4.

This is the "reverse" of the first three problems. Now we start with our present numerals and construct the "EFGH" numeral. To this end we observe that four thousand three hundred fifty two means:

4 thousands or four H's
3 hundreds or three G's
5 tens or five F's
and
2 ones or two E's

That is, H still stands for a thousand, G stands for a hundred and so on.

Writing four H's, three G's, five F's, and two E's we get:

HHHHGGGFFFFFEE

5.

The main aim of this problem is to emphasize the power-of-ten property that ten of any denomination is equal to one of the next greater denomination.

*That is: ten 1's = 1 ten,
ten 10's = 1 hundred,
ten 100's = 1 thousand etc.*

In this case, we can exchange ten E's for an F; ten F's for a G; ten G's for an H. Or, in other words, we can "cross out" ten of any letter and replace them by one more of the next letter. Therefore, starting with GGGGGGGGCGFFFFFFFFFFFEEEEEEEEEE

(a) Cross out ten E's and annex an additional F:

GGGGGGGGCGFFFFFFFFFFF~~EEEEEEEEEE~~
F

(b) Cross out ten F's and annex another G:

GGGGGGGGCG~~FFFFFFFFFF~~FF~~EEEEEEEEEE~~
G F

If we had more than nine E's still left, we'd repeat step (a).

(c) Cross out ten G's and annex an H.

H ~~GGGGGGGGCG~~FFFFFFFFFFF~~EEEEEEEEEE~~
H F

If we didn't have ten G's the process would end.

(cont)

Solutions for Self-Test 1: Form A (cont)

5. (cont)

Now we simply copy over what hasn't been crossed out to obtain HFFFFEE

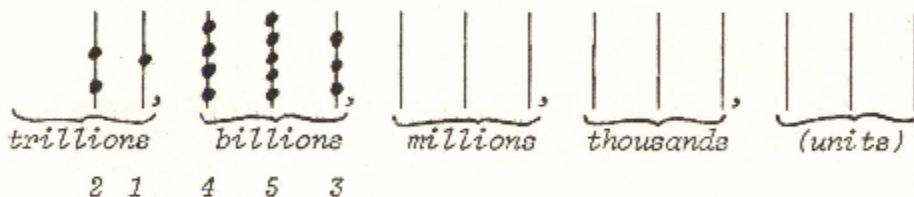
Note that we can "revisit" this problem in more of a place-value format. Namely, we have 9 hundreds, 13 tens, and 12 ones. In a step-by-step way we can exchange ten of one denomination for one of the next higher denomination to obtain:

thousands	hundreds	tens	ones
	9	13	12
	9	14	2
	10	4	2
1		4	2

which corresponds to 1 H, 4 F's, and 2 E's.

6.

This problem shows that even with the sand-reckoner it is often difficult to keep track of denominations. So in terms of thousands, millions, and billions we have:



So the answer is 21 trillion 453 billion and if we want to write the answer in place-value notation we must use 0's to indicate the absence of millions, thousands, and units. That is, the answer is:

21,453,000,000,000

Notice that we have no need for a place holder. We know there are no hundreds because no G's appear.

That is, we have nine G's, thirteen F's and twelve E's

Each row is a different way of expressing the same number. In terms of money, if you had nine \$100-bills, thirteen \$10-bills and twelve \$1-bills, you'd have a total of \$1,042.

The commas were added for convenience. Remember that we start at the right and "count off" in three's.

But as long as the lines are present, we do not need 0. The line itself serves as the place holder

Solutions for Self-Test 1: Form A (cont)

7.

The purpose of this problem is again to help you distinguish between digits and the number represented by the digits.

(a)

Here we are buying 100 objects. Since each object costs \$1, we are paying \$1 a hundred times or a total of \$100.

(b)

Here we are buying only *three* objects. Namely the numeral 100 consists of three digits--a 1 and two 0's. Since each digit costs \$1, the three digits combined cost \$3.

In other words, in place-value notation a hundred is represented by the 3-digit numeral 100.

For example, they could be house numerals or jersey numerals. Notice, in this, case that 0 is indeed a digit. You must pay \$1 for it the same as for the 1

8.

Here we have another example in which even using place value there are many nouns to keep track of.

Recall how the nouns have been chosen:

$\begin{array}{ccccccc} \underline{2} & \underline{3} & , & \underline{8} & \underline{0} & \underline{7} & , & \underline{0} & \underline{2} & \underline{7} & , & \underline{0} & \underline{0} & \underline{0} & , & \underline{0} & \underline{0} & \underline{0} & , & \underline{0} & \underline{0} & \underline{0} \\ \text{quad-} & \text{trillions} & & \text{billions} & & \text{millions} & & \text{thousands} & & \text{units} & & & & & & & & & & & & & \\ \text{rillions} & \end{array}$

Again notice in this system of nouns, the biggest numerical adjective we need is nine hundred ninety nine (999)

If we want to translate everything into words, we have twenty three quadrillion eight hundred seven trillion twenty seven billion.

Solutions for Self-Test 1: Form A (concluded)

9.

This is the "reverse" of the previous problem.

This time we "key" on the nouns "thousand", "million", "billion" and so on. "Eighty Four" modifies "quadrillions" and "two hundred thirty seven" modifies "billions". In other words, we have:

84 quadrillion and 237 billions. Putting these in their proper "places" we have:

$\overline{\text{quad-}} \overline{\text{trillions}} \overline{\text{billions}} \overline{\text{millions}} \overline{\text{thousands}} \overline{\text{units}}$
8 4, 2 3 7,
rillions

Now replacing each dash by the place-holder 0 we have:

84,000,237,000,000,000

10.

Nothing is more frustrating than a false statement that is "almost" true. Yet in using scientific notation it is important to understand that in 10^4 , the 4 counts the total number of 0's, including the 0 in 10. That is, 10^4 is a 1 followed by four 0's—not a 10 followed by four 0's.

In general if n is any whole number 10^n names the place value numeral that consists of a 1 followed by n zeroes.

Remember to put the 237 in the proper place. It modifies "billions"

The dashes themselves could serve as the place-holders. The symbol 0 isn't important. The important thing is to hold the place.

10 followed by four 0's would be 100000 and this is 10^5 . 10^5 is 100,000 while 10^4 is 10,000. There is quite a difference between 10,000 and 100,000.

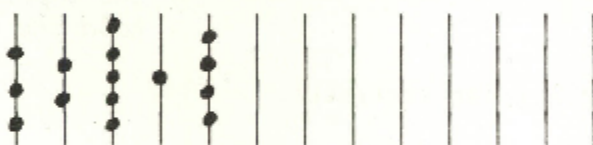
Step 7:

Do Self-Test 1: Form B

Self-Test 1: Form B

Answers:

In problems 1 through 5; N stands for one, O stands for ten, P stands for a hundred, and Q stands for a thousand.

1. What number is named by QQQPPPOON? 1. _____
2. What number is named by QQNNNN? 2. _____
3. Which numeral names the greater number:
P or OOOOOOONNNNN ? 3. _____
4. Without using more than nine of any one
letter use N,O,P,and Q to write the the
number five thousand six hundred thirty two. 4. _____
5. Rewrite QQQPPPPPPPPPOOOOOOONNNNNNNNNNNNNNN
so that none of the letters N, O, P, or Q
appear more than nine times. 5. _____
6. What number is shown on the sand-reckoner below? 6. _____

7. a. At \$1 per object what is the cost of 300 objects? 7. a. _____
b. At \$1 per digit, what is the cost of the numeral 300? b. _____
8. Read the number represented by the numeral:
187,813,005,000,000,000 8. _____
9. Write as a place value numeral:
Eight hundred four quadrillion five hundred
thirty seven million. 9. _____
10. True or False: 10^7 is 10 followed by 7 zeroes? 10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 1: Form B

1. 3,421 (three thousand four hundred twenty one)
2. 2,004 (two thousand four)
3. P
4. QQQQPPPPPPPOOONN
5. QQQPNNNN
6. 3,251,400,000,000 (three trillion two hundred fifty one billion four hundred million)
7. a. \$300
b. \$3
8. one hundred eighty seven quadrillion eight hundred thirteen trillion five billion
9. 804,000,000,037,000,000
10. False

If you did each problem in Form B correctly, you may, if you wish, proceed to the next module. Otherwise continue with Step 8.

Step 8:

View the solutions for Self-Test 1: Form B on Videotape Lecture 1S.

Pay special attention to the solutions of those problems for which you failed to get the correct answers. *Feel free to rewind the tape at any time to restudy problems that give you difficulty.*


* * * * *

Step 9:

Do Self-Test 1: Form C

Self-Test 1: Form CAnswers:

In problems 1 through 5; e stands for a thousand, f stands for a hundred, g stands for ten, and h stands for one.

1. What number is named by hhgggfeeee ? 1. _____
2. What number is named by eeggg ? 2. _____
3. Which numeral names the greater number:
h or ggg ? 3. _____
4. Without using any more than nine of each of
letters e,f,g, or h write the number six
thousand five hundred forty three. 4. _____
5. Rewrite eeeeeeeeeffggggggggghhhhhhhhhhhhhh
so that none of the letters e,f,g, or h appear
more than nine times. 5. _____
6. What number is shown on the sand-reckoner below? 6. _____

7. a. At \$1 per digit, what is the cost of the
numeral 400? 7. a. _____
b. At \$1 per object, what is the cost of
400 objects? 7. b. _____
8. Read the number represented by the numeral:
46,830,050,000,000,000 8. _____
9. Write as a place value numeral:
Seventy quadrillion eight hundred thirty
nine thousand. 9. _____
10. True or false: 10^9 is a 1 followed by 9 zeroes? 10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 1: Form C

1. 4,132 (four thousand one hundred thirty two)
2. 2,030 (two thousand thirty)
3. ggg
4. eeeeeefffffgggghhh
5. eeeghhhh
6. 23,120,000,000,000 (twenty three trillion one hundred twenty billion)
7. a. \$3
b. \$400
8. forty six quadrillion eight hundred thirty trillion fifty billion
9. 70,000,000,000,839,000
10. True

THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #1.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE #2.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE YOU SHOULD CONSULT A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.
