cs488 – Spring 2007

Lambertian Surfaces

‘Lambertian’ is a description of scattering and re-emission of light from the body of a surface. In Figure 1 the circle is a source of light, and the arrow emanating from it a ray of light. Some of the ray is reflected at the horizontal interface of the surface, into the eye, as shown here; the remainder passes through the surface interface into the body of the object. There it is scattered by irregularly-shaped pigmented particles: ‘scattered’ means absorbed by the particle and re-emitted in a new direction; ‘pigmented’ means that the amount of light re-emitted by a particle varies with wavelength. After enough absorption and re-emission to completely randomize the direction of the light path, many more times than shown in the figure, light is re-emitted through the surface interface, interacting no further with the surface.

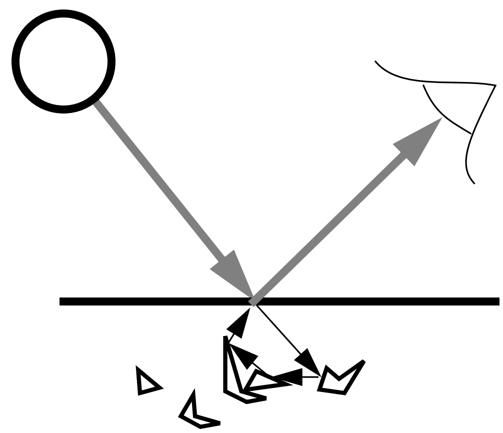

Figure 1.

A suface is called ‘Lambertian’ when the body reflection obeys Lambert’s cosine law, first written down in 1760. This law says that the amount of light emitted from a surface in different directions is proportional to the cosine of the angle between the direction, $\hat{s}$, and the surface normal, $\hat{n}$. That is $E(\hat{s}) = C(\hat{s} \cdot \hat{n})$, where C is a constant. Lambert appears to have observed this relationship empirically*. Now, we will discuss the model that underwrites the cosine law.

When discussing light arriving at a surface from a source of illumination, we noticed that the energy arriving on the surface at x must be measured in the form $E(x)dA$, where $E(x)$ is the energy density at x, measured in units of energy per unit area, for example, and dA is a differential area. Similarly, the light energy emitted from a point source in a particular direction, ω, is $E(\omega)d\omega$, where $d\omega$ is a differential solid angle. The conversion from solid angle to area depends on the distance from the source, r, as the inverse square, and on the cosine of the angle between the direction of the incoming light and the surface normal, as was discussed in class, and in the course notes. When we examine the re-emitted light we

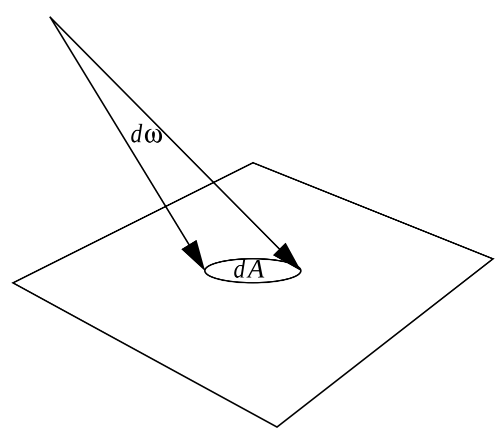

Figure 2.

* C J Scriba, Biography in Dictionary of Scientific Biography (New York 1970-1990).

need to measure it in the same units, energy per unit area, so the energy re-emitted at x is written in the form $E(x)dA$. The next paragraph shows how to calculate the angular distribution of the re-emitted energy.

The key assumption is that light trapped inside the body of the surface is absorbed and re-emitted from pigment particles enough times to be completely randomized. That is, if we put a small area inside the body the amount of light flowing through it in a direction perpendicular to it is independent of the position and orientation of the small area. Figure 3 shows a bold line which is the cross-section of a surface, with a small area dA shown as a break in the line. You can think of this as a hole through which light inside the surface leaves the surface. The circle is the cross-section of a sphere centred on the small area. In the lower half of the sphere, which lies within the surface the direction of light is randomized. Therefore, for any small area on the sphere the light passing through the area perpendicular to the sphere is proportional to the area. Examine the cylinder passing through the hole perpendicular to the surface. Its area on the sphere is exactly the same as the area of the hole, so the amount of light passing through it is proportional to the area of the hole, dA. However, the cross section of a cylinder passing obliquely, also fitting through the hole, as shown, is smaller. Therefore, less light passes through the hole in that direction, because the amount of light is proportional to the cross-section of the cylinder. By how much is it less?

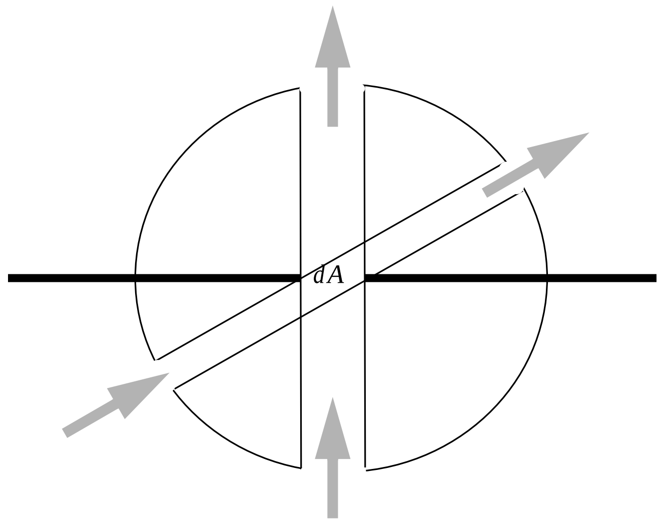

Figure 3.

Suppose the area to be a rectangle perpendicular to the plane of Figure 3. Then the rectangular cylinder at an oblique angle, compared to the perpendicular one, is the same dimension perpendicular to the plane of the figure, but shortened in the plane of the figure. See Figure 4, which is a blown up area near dA in Figure 3. In Figure 4 a line perpendicular to the oblique ray was drawn across the width of the ray, and the angle θ between the ray perpendicular to the surface, which is parallel to the surface normal $\hat{n}$, is put on the diagram. Clearly the width of the oblique ray is $\cos\theta$ times the width of the hole. Thus, $\cos\theta$ is the factor by which the intensity of light emitted obliquely is lessened. This is Lambert's cosine law. Other factors come into play when we consider light entering a detector, such as the eye, when it looks at a Lambertian surface.

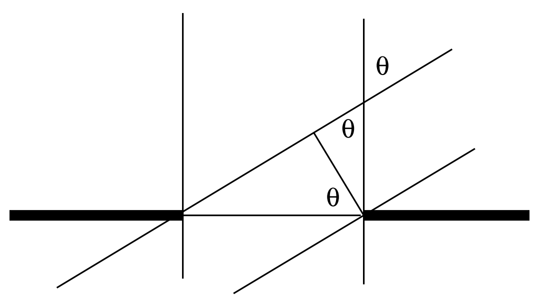

Figure 4.

The detector looks at the surface with a geometry identical to that shown in Figure 4, except that the width of the beam is now proportional to the solid angle centred on the eye, $d\omega$. The hole is now the amount of surface with the solid angle, which is $d\omega/\cos\theta$. Thus, if the light re-emitted perpendicularly at x is $E(x)$ then the light emitted at an angle θ to the normal is $E(x)\cos\theta$. But, the amount of light entering the eye through a given solid angle $d\omega$ is $\frac{d\omega E(x)\cos\theta}{\cos\theta} = E(x)d\omega$, so that the amount of light captured by a detector of fixed aperture (solid angle) is independent of the angle from which the surface is viewed.

Many natural surfaces are close to Lambertian at directions close to the perpendicular, but most depart from Lambert's law when seen at low angles[*]. Makers of paint and other colour coatings try hard to make their products obey Lambert's law[†]. For this reason objects illuminated using Lambert's law alone tend to look artificial.

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* Exercise for the reader. What would a full moon look like if its surface were Lambertian? What does the full moon look like? How, if at all, does the surface of the moon depart from Lambert's law? (Hint. Remember that the surface of the moon is illuminated by the sun, and the moon is close to a sphere. Draw a diagram of the sun/earth/moon system when the moon is full. Assume a Copernican cosmology.)

† Exercise for the reader. Why would a paint manufacturer try to give wall paint a Lambertian surface? How is a painted wall illuminated in most houses? What does it look like? Why? (Hint. Consider first the appearance of an infinite Lambertian plane illuminated from infinity.)