Reactance Chart*

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Summary—**The time-honored chart of reactance, frequency, inductance, and capacitance is extended in range, with added scales of susceptance, wavelength, and time constant. Simple geometric patterns are given which enable the chart to be used for the direct solution of various problems such as the bandwidth of a resonant circuit, some properties of a transmission line (wave impedance and delay), and half sections of the usual kinds of constant-k filters (low-pass, high-pass, band-pass).**

I. Introduction

SINCE 1928, the reactance chart has been one of the outstanding aids to radio engineers. It is here presented as Figs. 6 and 7 (see pages 1394 and 1395) with the following improvements:

1. A wavelength scale is added at the top of the chart giving a direct conversion between frequency and wavelengths.
2. A time-constant scale is added at the bottom, giving the time constant CR or L/R, or the time delay in a transmission line or filter.
3. A conductance or susceptance scale is added at the left side, the reciprocal of resistance or reactance.
4. The frequency scale of Fig. 7 covers from 0.1 cycle to 100,000 Mc (3 mm wavelength) in two ranges. The scales on the left side are common to both ranges. The other scales have upper and lower sets of units. All the upper units are used together for the low-frequency range, the lower units for the high-frequency range.

The following instructions describe the use of the chart in many problems, including the complete design of constant-k filters.

II. General Instructions

A point on the chart is the intersection of four lines, one each for frequency f, reactance X, inductance L, and capacitance C. Any two of these quantities determines the point and thereby the other two quantities.

Fig. 6 is a single square of the chart, enlarged for accuracy of reading. If accuracy is desired, the problem is first computed on Fig. 6, all except the decimal point. Then Fig. 7 is employed to locate the decimal point and to give a rough check on the number.

In locating a number on the inductance or capacitance scales of Fig. 6, it is important that the decimal point be shifted by an even number of places.

If accuracy is not required, only Fig. 7 is used. This is good for one or two significant figures and the decimal point is located in the numbers and units on the scales.

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III. Abbreviations

c = cycle per second
s = second
m = meter
Ω = ohm
℧ = mho
h = henry
f = farad
M = mega (10^6)
μ = micro(10^{-6}).

IV. Reactance and Resonance

Disregarding the sign of the reactance, the basic formulas for this chart are as follows:

$$X = \omega L = 2\pi f L \qquad \text{ohms (1)}$$

$$X = \frac{1}{\omega C} = \frac{1}{2\pi f C} \qquad \text{ohms (2)}$$

in which

X = reactance of L or C (ohms)
f = frequency (cycles per second)
$\omega = 2\pi f$ = radian frequency
L = inductance (henries)
C = capacitance (farads).

If L and C form a resonant circuit as in Fig. 1, these two formulas merge into the resonance formulas

$$X_0 = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}} \qquad \text{ohms (3)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \text{radians per second. (4)}$$

In which the subscript "0" denotes resonance.

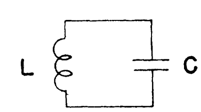


Fig. 1—Resonant circuit.

Fig. 2 shows the pattern representing the above four formulas for reactance and resonance.

The susceptance corresponding to reactance X is $B = 1/X$. This is given on the reciprocal scale on the left side.

The wavelength corresponding to frequency f is

$$\lambda = \frac{c}{f} = \frac{2\pi c}{\omega} \qquad \text{meters (5)}$$

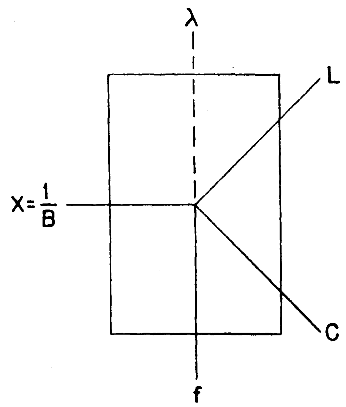


Fig. 2—Solution of reactance and resonance.

in which

$c = 3 \times 10^8$ meters per second
$= 300$ meters per microsecond
$=$ speed of light.

V. Resistance and One Kind of Reactance

A combination CR or L/R behaves as a low-pass or high-pass filter. Fig. 3 shows the typical combinations of this kind. Each presents a voltage attenuation ratio of $1/\sqrt{2}$ (3 db) at a nominal cutoff frequency f_c such that

$$R = \omega_c L \quad \text{or} \quad \frac{1}{\omega_c C} \qquad \text{ohms. (6)}$$

Fig. 4 shows the solution of this problem on the chart.

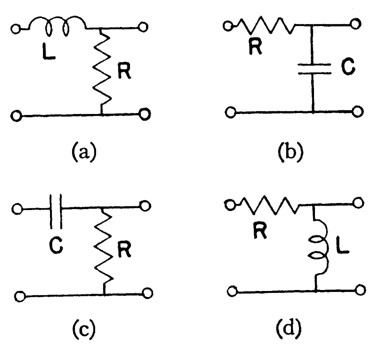


Fig. 3—Resistance and one kind of reactance: (a) and (b) low-pass; (c) and (d) high-pass.

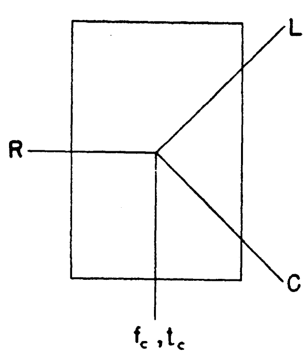


Fig. 4—Solution of low-pass and high-pass filters.

Each of the networks of Fig. 3 has a charging or discharging time constant

$$t_c = \frac{1}{\omega_c} = \frac{L}{R} \quad \text{or} \quad CR \qquad \text{seconds. (7)}$$

An extra scale below the chart gives this time constant. It is noted that the subdivisions of this scale are provided by diagonal lines meeting to form a "*V*" at the bottom edge of the chart where each subdivision should be. The time constant is the radian period corresponding to the frequency.

VI. Resistance and Resonance

A sharply resonant circuit with resistance as shown in Fig. 5(a) behaves as a band-pass filter. Relative to the peak, it presents a voltage attenuation ratio of

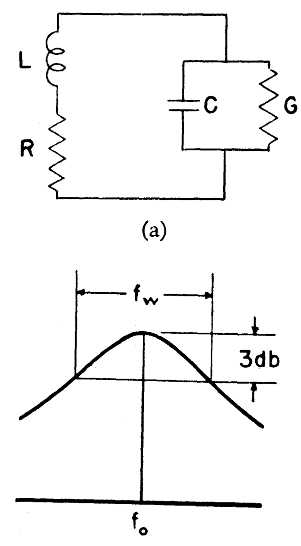


Fig. 5—Resistance in a resonant circuit.

$1/\sqrt{2}$ (3 db) at a nominal bandwidth f_w indicated in Fig. 5(b). The series R and shunt G contribute parts of the bandwidth as follows:

$$f_w = f_u + f_v \ll f_0 \tag{8}$$

$$\omega_u = \frac{1}{t_u} = \frac{R}{L}; \qquad \omega_v = \frac{1}{t_v} = \frac{G}{C} \tag{9}$$

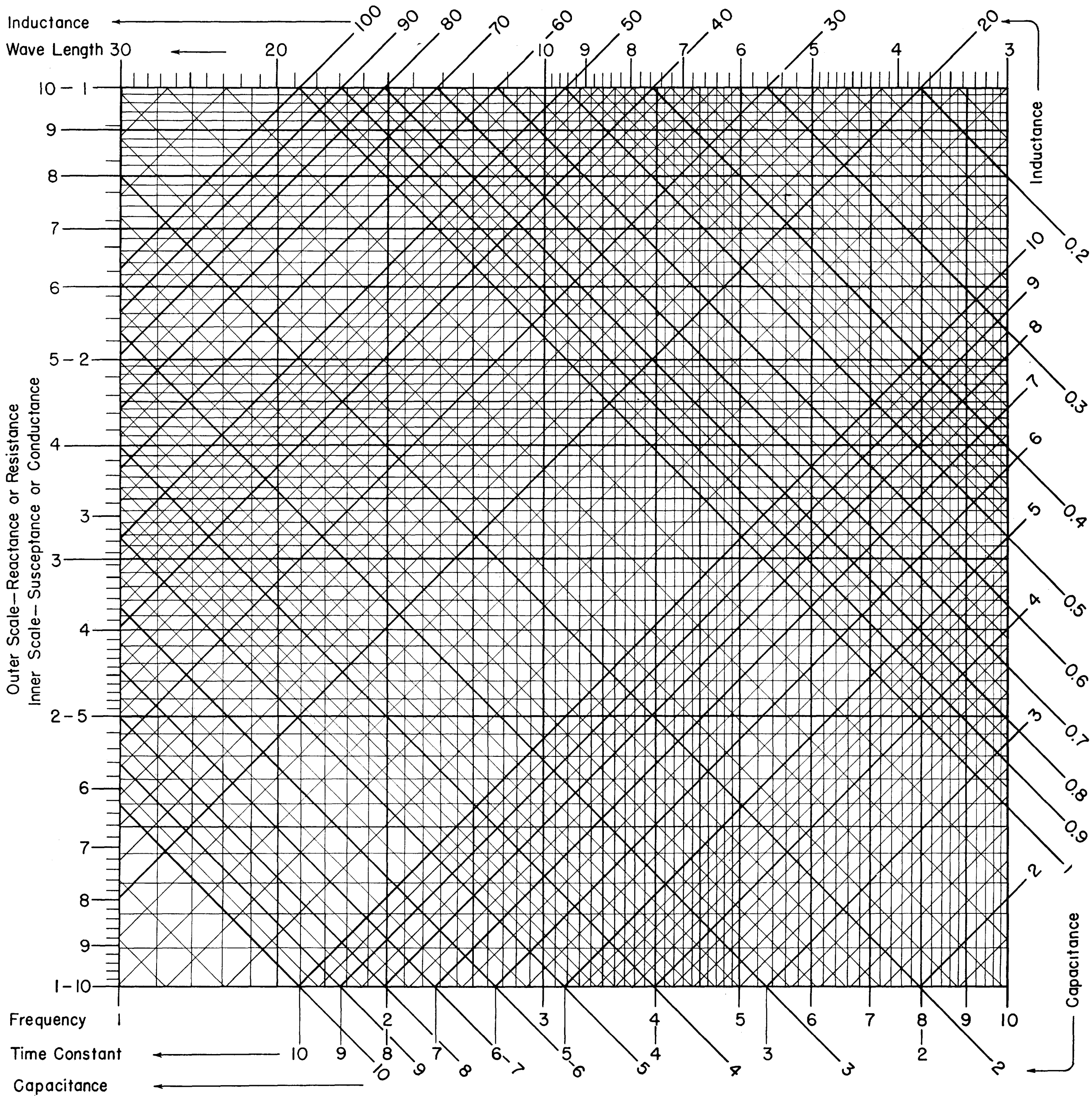
$$\omega_w = \frac{1}{t_w} = \frac{R}{L} + \frac{G}{C} \tag{10}$$

in which

$t_u =$ time constant of L/R
$t_v =$ time constant of C/G
$t_w =$ half time constant of damping of free oscillations in the resonant circuit.

Fig. 8 shows the pattern of all these relations and others on the chart, involving five points of intersection. Since the bandwidth does not involve the frequency of

Note. On the scales of inductance and capacitance, both decimal points must be shifted by even numbers of places, or both by odd numbers of places, to interchange the scales on this chart with the actual values in henries (Mh, h, µh, µµh) and farads (f, µf, µµf, µµµf). This rule must be followed if both inductance and capacitance appear in the same computation.



$$X \text{ or } R = 2\pi f L = \frac{1}{2\pi f C} = \sqrt{\frac{L}{C}} = \frac{L}{t_c} = \frac{t_c}{C}$$

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{X}{2\pi L} = \frac{1}{2\pi C X} = \frac{1}{2\pi t_c} = \frac{300}{\lambda} \times 10^6$$

$$L = \frac{X}{2\pi f} = \frac{1}{4\pi^2 f^2 C} = CR^2 = R t_c = \frac{t_c^2}{C}$$

$$t_c = \sqrt{LC} = \frac{L}{R} = CR = \frac{1}{2\pi f} = \frac{\lambda}{2\pi \times 300} \times 10^{-6}$$

$$C = \frac{1}{2\pi f X} = \frac{1}{4\pi^2 f^2 L} = \frac{L}{R^2} = \frac{t_c}{R} = \frac{t_c^2}{L}$$

$$\lambda = 2\pi\sqrt{LC} \times 300 \times 10^6 = \frac{300}{f} \times 10^6 = 2\pi t_c \times 300 \times 10^6$$

Fig. 6—Enlargement of one square.

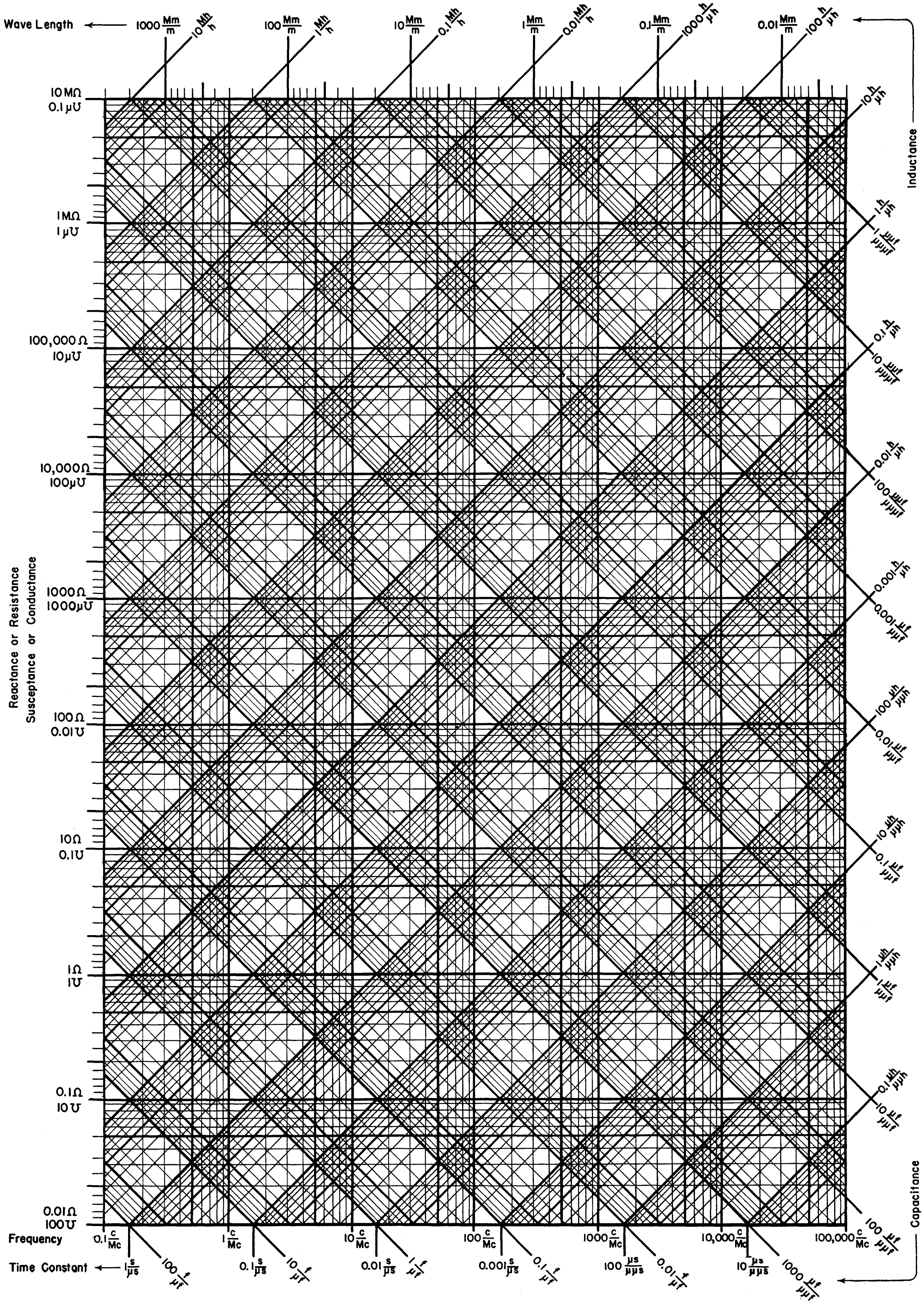


Fig. 7—Reactance chart.

resonance, the bandwidth may be determined separately.

The intersection of R and L determines f_u while that of G and C determines f_v. The sum of these is the nominal bandwidth f_w, which also determines the half time constant of damping, t_w on the time-constant scale.

The total effect of R and G may be regarded as concentrated in an equivalent series resistance R' or shunt resistance R'', such that

$$R' = \omega_w L \quad \text{or} \quad R'' = \frac{1}{\omega_w C}\cdot \tag{11}$$

These formulas are closely associated with the ratio of reactance to resistance

$$Q = \frac{f_0}{f_w} = \frac{X_0}{R'} = \frac{R''}{X_0} \gg 1. \tag{12}$$

The equivalent series or shunt resistance, R' or R'', is determined by the intersection of f_w with L or C, respectively. This pattern of Fig. 8 gives a simple conversion between equivalent series and shunt resistance if either is given, together with L and C.

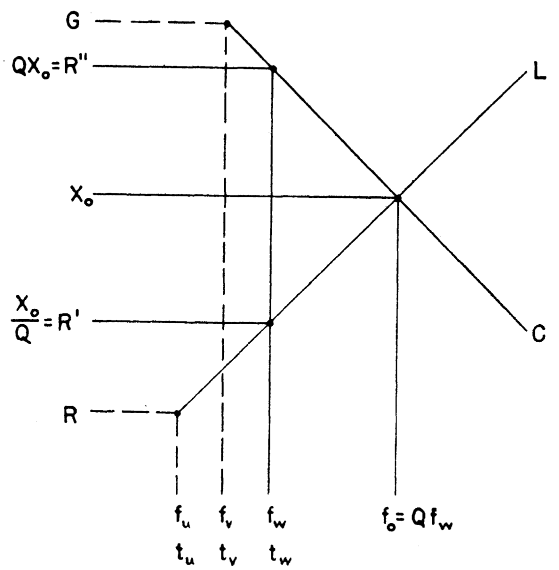


Fig. 8—Bandwidth of a resonant circuit.

The remaining point in Fig. 8 is the resonance point like Fig. 2. If either the series R or the shunt G is absent, this pattern simplifies by the disappearance of the dotted lines, because either f_u or f_v disappears and the other merges into f_w.

It may happen that the pattern of Fig. 8 for a given problem is divided between the two frequency ranges on the chart. This is true of circuits resonant in the neighborhood of 1 Mc. If many problems are to be encountered in such a region, a third scale may be added in red ink (or may be imagined) which is the mean of the two ranges marked on Fig. 7.

VII. Low-Pass and High-Pass Filters

Low-pass and high-pass filters of the constant-k type are shown in Fig. 9. The half section is the logical (but not usual) basis for filter design formulas, and is used in this treatment.

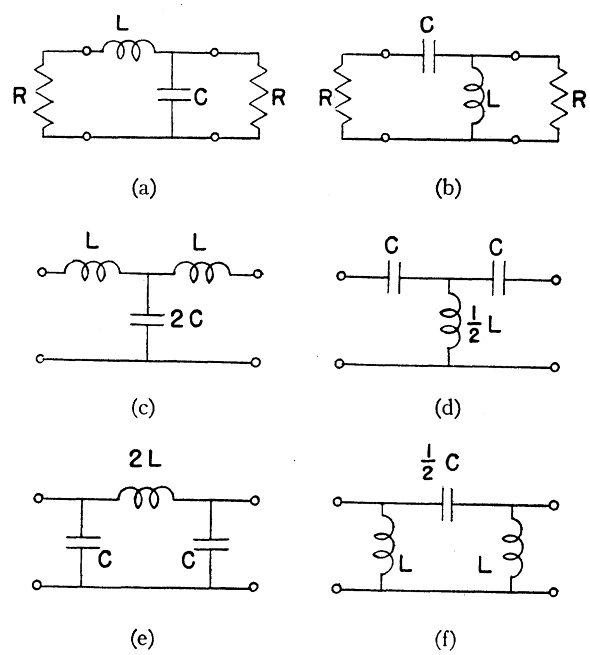


Fig. 9—Low-pass and high-pass filters to be solved by Fig. 4: (a) and (b) half sections; (c) and (d) mid-series sections; (e) and (f) mid-shunt sections.

On this basis, the low-pass and high-pass formulas are the same as (6) above, R being the nominal or "mid-band" image impedance. Therefore, these filters are designed directly by the pattern of Fig. 4.

In a low-pass filter, the time constant t_c is the delay per half section, based on the "mid-band" phase slope at frequencies much less than the cutoff frequency.

Either of these half-section filters with R on both sides is simply a critically damped resonant circuit whose frequency of resonance is f_c. Critical damping can be obtained by resistance on only one side of the half section if the value is changed to $2R$ in series or $R/2$ in shunt.

VIII. Band-Pass Filters

Band-pass filters of the constant-k type are shown in Fig. 10. If the cutoff frequencies are f_1 and f_2, the design formulas may be expressed as follows

$$\omega_2 - \omega_1 = \frac{R}{L_1} = \frac{1}{RC_2} \tag{13}$$

$$\omega_1\omega_2 = \frac{1}{C_1L_1} = \frac{1}{C_2L_2}\cdot \tag{14}$$

These two expressions have in common L_1 and C_2. In (13) they are involved with R and the bandwidth $f_2 - f_1$, and in (14) with C_1, L_2, and the mean frequency $\sqrt{f_1 f_2}$.

These relations are respectively analogous to (11) and (4) above, from which follows the pattern of Fig. 11. The three points of intersection are determined by R, f_2, and f_1 in the usual practical problem, but the design is determined by any set of quantities sufficient to locate the three points. The pattern is shown for cases in which the bandwidth is less or greater than the mean frequency.

A band-rejection filter is computed by the same procedure, by merely interchanging the series and parallel arms in the half section of Fig. 10(a).

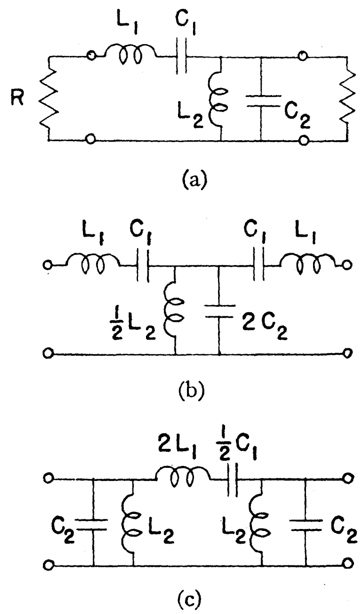


Fig. 10—Band-pass filters. (a) Half section. (b) Mid-series section. (c) Mid-shunt section.

IX. Transmission Lines

The transmission line of Fig. 12 has two characteristics of interest here, its wave impedance and its time of delay.

The wave impedance is

$$R = \sqrt{\frac{L}{C}} \tag{15}$$

in which L and C are uniformly distributed inductance and capacitance of any length of the line. This equation is solved by the pattern of Fig. 4.

The time of delay is

$$t_e = \sqrt{LC} \tag{16}$$

in which L and C are the total values for a certain length of line. This formula also is solved in Fig. 4.

Both of these formulas apply also to any number of sections of low-pass filter, L and C being the total series inductance and shunt capacitance.

Acknowledgment

The writer expresses his special appreciation of the skill and ingenuity of E. A. Owen in preparing the charts.

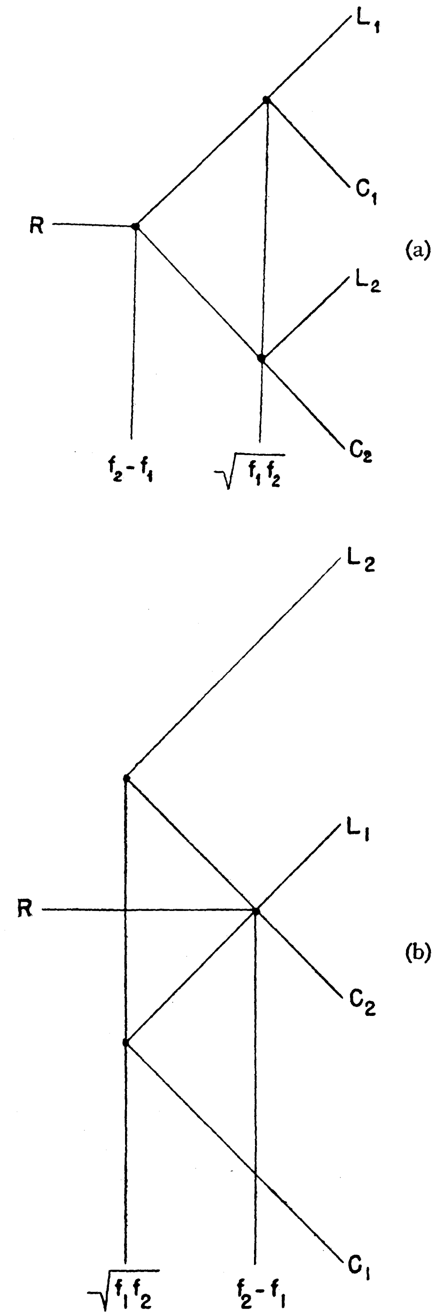


Fig. 11—Solution of band-pass filters.

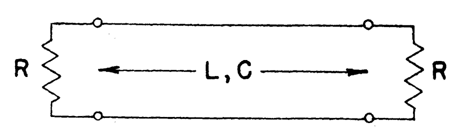


Fig. 12—Transmission line, to be solved by Fig. 4.

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