

**Physics 23 Fall 1993**  
**Lab 2 - Adiabatic Processes**

Theory

This laboratory is a study of the adiabatic expansion of three gases: helium, air, and carbon dioxide. The experiments are carried out at a pressure of approximately one atmosphere and at room temperature ( $\sim 295^\circ \text{K}$ ). Under these conditions the gases are close to ideal in behavior, and so the theoretical values of the various quantities to be measured will be those of an ideal gas.

An adiabatic process is one that takes place so rapidly that no heat transfer takes place between the surroundings and the system, or within the system. Two types of adiabatic processes are examined. In one, a container is filled to a slight overpressure with a gas, and then a stopper is popped so as to make the gas decompress quickly. In the other, a ball is bounced on a cushion of gas, alternately compressing and expanding it without allowing heat transfer to take place.

An ideal gas is a collection of point particles that are non-interacting (except for momentary collisions). The translation kinetic energy of the particles is related to the temperature by the equation

$$\frac{1}{2} mv^2 = \frac{3}{2} kT$$

where  $k$  is Boltzmann's constant,  $1.38 \times 10^{-16}$  erg/deg. If the gas is made up of diatomic or triatomic molecules, there will also be a rotational part to the kinetic energy. The amount of any gas is given by the number of moles  $n$ , one mole being  $6.023 \times 10^{23}$  particles. The pressure inside a container of gas is determined by the number of moles, the temperature, and the volume of the container. They are related by the equation of state of an ideal gas

$$pV = nRT$$

The specific heat of a gas is the heat energy added to the gas per degree of temperature rise. The value of the specific heat depends on the condition under which the heat is added. If the heat is added at constant volume the heat energy goes entirely into increasing the internal kinetic energy of the particles. If the heat is added at constant pressure, the gas can expand while the heat is added and the gas does work on the expanding walls of the container. Consequently, the specific heat at constant pressure ( $C_p$ ) is greater than the specific heat at constant volume ( $C_v$ ). As derived in section 20.5 of Ohanian, the two are related by

$$C_p = C_v + R$$

By applying the first law of thermodynamics and using the adiabatic condition of zero heat transfer, it can be shown (see the appendix at the end of this writeup or section 20.6 of Ohanian)

that the pressure and volume of an ideal gas obey the equation (called the adiabatic law)

$$pV^\gamma = \text{constant}$$

during an adiabatic process. The exponent  $\gamma$  is the ration of the specific heats,  $C_p / C_v$ .

The experimentally measured values of the two specific heats and the ratio  $\gamma$ , can be explained from a simple theoretical model and calculation. The principle of equipartition of energy is invoked to assign  $(1/2)RT$  per mole per degree of freedom to the internal energy. Since

$$C_v = \left( \frac{dQ}{dT} \right)_{\text{constant volume}} = \frac{dU}{dT}$$

(because no work is done at constant volume), we would expect to have  $C_v = (1/2)R$  per mol per degree of freedom.

The number of degrees of freedom and the value of  $C_v$  depend on the complexity of the internal energy of the gas. A monatomic gas can only move with three translational degrees of freedom, so  $C_v = (3/2)R$ . A diatomic gas (dumbbells) can move with the same three translational modes and can rotate in two dimensions as well; therefore, in this case  $C_v = (5/2)R$ . A triatomic gas can rotate in three dimensions, so  $C_v = 3R$ . Therefore, the values of  $C_p$  for the three cases are  $(5/2)R$ ,  $(7/2)R$ , and  $4R$ ; and the values of  $\gamma$  from this simple theory are 1.67, 1.4, and 1.33 respectively. Actual experimental values of  $\gamma$  for helium, air, and carbon dioxide are:

He	1.67
Air	1.40
CO <sub>2</sub>	1.30

### The Method of Clement and Desormes

A simple and straightforward method of measuring  $\gamma$  was developed in the 19th century by Clement and Desormes. A container of gas is pressurized slightly above atmospheric pressure and allowed to come to equilibrium temperature with the surroundings. Then a stopper is popped momentarily to decompress the gas to one atmosphere. The gas cools during this expansion. The temperature immediately after decompression can be calculated by combining the ideal gas law with the adiabatic law. Let  $p_1$  be the pressure before decompression,  $p_0$  the atmospheric pressure,  $V_c$  the volume of the container,  $V$  the volume of the expanded gas,  $T_0$  the room temperature, and  $T$  the temperature after decompression. We have

$$p_1 V_c = p_0 V, \quad p_1 V_c = nRT_0, \quad p_0 V = nRT$$

which can be combined to give

$$p_1^{-1} T_0 = p_0^{-1} T$$

or

$$\frac{T_0}{T} = \left(\frac{p_0}{p_1}\right)^{\gamma-1}$$

After decompression, the gas warms up until the final temperature is again  $T_0$  and the pressure is  $p_2$ , with  $p_2 > p_0$  but  $p_2 < p_1$ . From the ideal gas equation ( $n$  and  $V$  are constant), we have  $p_0 T_0 = p_2 T$ , so that

$$\frac{T}{T_0} = \frac{p}{p_0}$$

and therefore

$$\frac{p_2}{p_0} = \left(\frac{p_1}{p_0}\right)^{\gamma-1}$$

In the experiment the pressures  $p_1$  and  $p_2$  are only slightly greater than  $p_0$ :  $p_1 = p_0 + \Delta p_1$ ,  $p_2 = p_0 + \Delta p_2$ , so the above equation can be written

$$\left(1 + \frac{\Delta p_2}{p_0}\right) = \left(1 + \frac{\Delta p_1}{p_0}\right)^{\gamma-1}$$

Since the term on the right in the bracket on each side is much less than 1, we can apply the binomial expansion to get

$$1 + \left(\frac{\Delta p_2}{p_0}\right) = 1 + (\gamma - 1)\left(\frac{\Delta p_1}{p_0}\right)$$

with the final result

$$\Delta p_2 = \frac{\gamma - 1}{\gamma} \Delta p_1 \quad (1)$$

Despite the algebraic gymnastics needed to arrive at this equation, the expression is simple and it can be applied easily in the experiment. An oil manometer is used to read  $p_1$  (the pressure before the cork is popped) and  $p_2$  (the pressure the gas comes to when back at room temperature) directly.

Another more sophisticated method of measuring  $\gamma$  was developed by Rüchardt. A container of gas has a long glass tube emerging from the top. A ball bearing, exactly fitting the precision glass tube, is dropped into it. The ball bounces on the gas below, compressing it adiabatically on the downswing and letting it expand adiabatically on the upswing. The period of the oscillations of the bouncing ball is directly related to the  $\gamma$  of the gas and the geometry of the system.

Let  $V$  be the enclosed volume,  $p_0$  the atmospheric pressure,  $m$  the mass of the ball, and  $A$  the cross-sectional area of the tube. The pressure in the container will be  $p = p_0 + mg/A$ . If the ball is stationary,  $mg = A(p - p_0)$ . If the ball is pressed down a distance  $dx$ , compressing the gas adiabatically, the volume change of the gas will be  $dV = Adx$ . The pressure rise will be

$$dp = \left( \frac{dp}{dV} dV \right) \quad .$$

Since  $pV^\gamma = C$  (a constant) for an adiabatic process,

$$\frac{dp}{dV} = \frac{d(C/V^\gamma)}{dV} = \left( -\frac{\gamma}{V} \right) (pV^\gamma) \quad ;$$

therefore,  $dp = -(\gamma p/V)Adx$ , and the force on the ball will be

$$dF = A dp = - \left( \frac{\gamma p}{V} \right) A^2 dx \quad .$$

This is the equation obeyed by a mass on a spring with a force constant

$$k = \frac{\gamma p A^2}{V} \quad .$$

The period of oscillation is given by  $T = 2\pi (m/k)^{1/2}$  so that

$$k = \frac{4\pi^2 m}{T^2} = \frac{\gamma p A^2}{V} \quad .$$

Thus, we can calculate the  $\gamma$  of the gas from the period of oscillation of the ball

$$= \frac{4\pi^2 mV}{\gamma p A^2 T^2} \quad . \quad (2)$$

The derivation of this equation makes use of one of the most universal principles in mechanics: when an object experiences a restoring force proportional to the displacement, the resulting motion

is simple harmonic. This point is explained in more detail in Chapter 14 of the Ohanian.

## References

The following are the sections in the texts which are pertinent to this lab. They should be read before coming to lab.

1. Ohanian's Physics Chapter 19
2. Ohanian's Physics Chapter 20 sections 20.1 - 20.3, 20.5 - 20.6
3. Ohanian's Physics Chapter 21 section 21.1
4. Tipler's Modern Physics Chapter 2

## Experimental Purpose

The purpose of this lab is to experimentally measure the ratio of specific heats for helium, air and carbon dioxide using two methods: (1) the method of Clement and Desormes and (2) the method of Ruchardt. In the first method, the ratio of specific heats will be computed from the expression

$$= \frac{p_1}{p_1 - p_2}$$

In the second method the ratio of specific heats will be computed from the expression

$$= \frac{4 \cdot mV}{pA^2 T^2}$$

In both cases, the experimentally measured values will be compared to the theoretically predicted values.

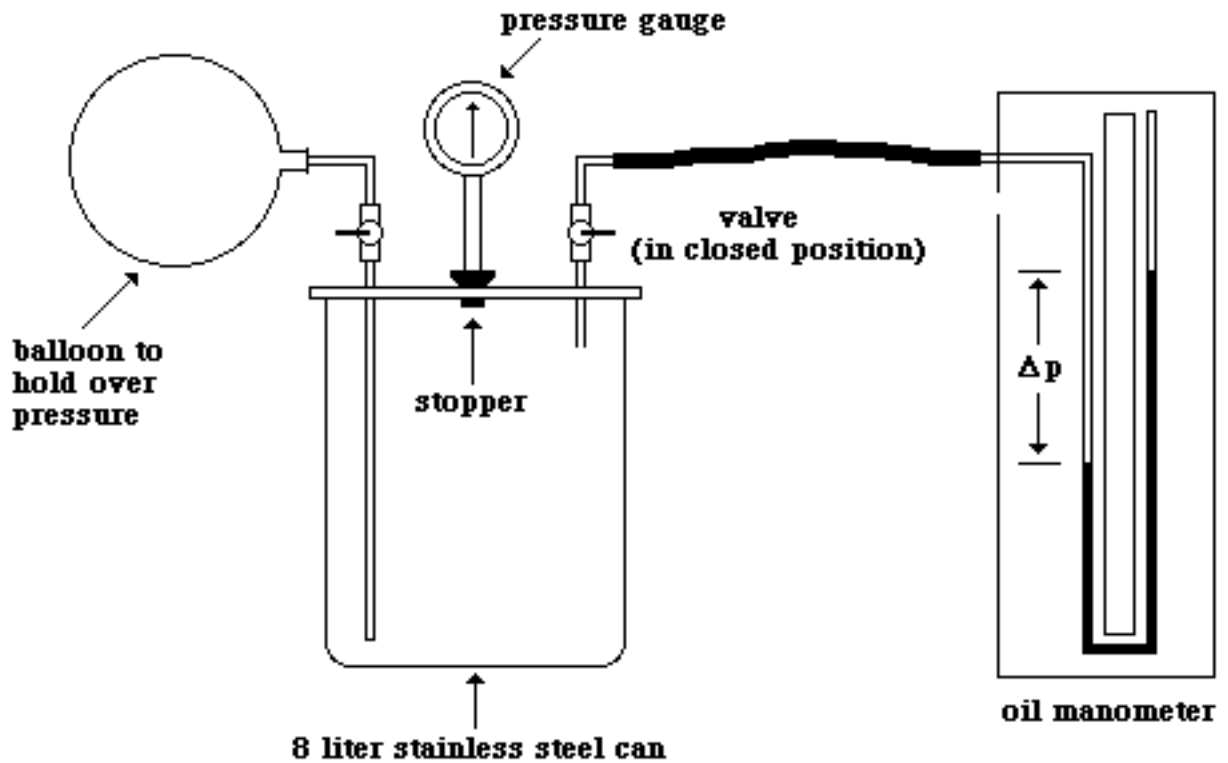
## Procedure

The main piece of apparatus is an 8 liter stainless steel can with a brass lid with a hole in it soldered on. The can has two ports with valves for evacuating it or filling it with gas, a dial pressure gauge, and a rubber stopper. There is also an oil manometer for measuring pressures, a vacuum pump, gas cylinders, balloons, and an attachment for measuring by Ruchardt's method. A schematic diagram of the apparatus is shown on the next page. Before beginning a few comments on the use of the equipment is necessary.

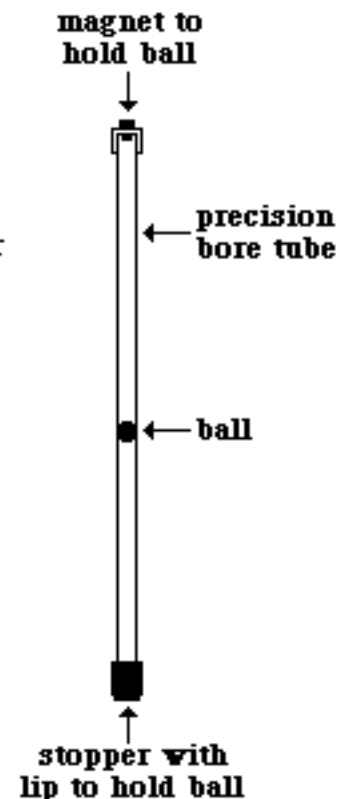
- a. The valves on the can ports are open when the black valve stem is pointing straight out

from the valve. They are closed when they are at right angles to the straight out position.

- b. When evacuating the cans, the cans often make banging noises as the sides of the cans adjust to the pressure differences. This is normal, but may be alarming if you are not expecting it.
- c. The volume of the cans are nominally 8 liters. The actual volume of the can is stamped somewhere on the top of the brass lid.
- d. Only use the oil manometers for can pressures near one atmosphere. Failure to do so may cause the oil to either be sucked into the can or be blown out the other end of the manometer.
- e. Please use extreme care when handling the precision glass tubing used with Ruchardt's method. Each of the tubes cost \$125.00 and must be imported from Germany. Also, please take special care of the magnet used to hold the ball at the top of the tube. It is easily misplaced and it also is difficult to replace. Except when releasing the ball, please keep it in



The apparatus used to measure the ratio of the specific heats of the gases. With a slight overpressure, the stopper is popped and replaced. The ratio ( $\gamma$ ) can be found by knowing the original overpressure and the final pressure. The tube at the right replaces the stopper in Ruchardt's method; the frequency of the bounces of the ball is used to find  $\gamma$ .



the top of the glass tube.

- f. Any dust entering the precision glass tubing will cause deterioration of the performance of the apparatus. Should the ball begin to stick in the tube rather than oscillating, cleaning it with ethyl alcohol will help the situation. If your tube needs to be cleaned, see your TA.

#### Part I - The Method of Clement and Desormes

1. Attach a balloon to one of the ports and a vacuum pump to the other. Make sure there is a rubber stopper in the hole in the top of the can. Open both valves on the can, turn on the vacuum pump and evacuate the can. When the can is completely evacuated, close the valve attached to the vacuum pump and then shut off and detach the vacuum pump from the can. Attach the compressed air line to the port to which the vacuum pump was connected. Open the valve on the can connected to the air line and then slowly turn on the compressed air and admit enough air to the can to expand the balloon to approximately one foot in diameter. Shut off the compressed air, close the balloon valve and disconnect the air line. At this point, the balloon valve should be closed and the other valve should be open allowing the pressure in the can to be one atmosphere. Take the can your lab station and wait approximately five to ten minutes for the air in the can to come into thermal equilibrium with its surroundings.

Connect the manometer to the free port, keeping the balloon valve closed. Gently open the manometer port and check to see if the oil levels in the arms of the manometer change. If the levels do change, then the air inside the can has not had enough time to come into thermal equilibrium with its surroundings. (Why do the levels change if the gas in the can is not at thermal equilibrium?) In that case, remove the stopper and allow more time for thermal equilibrium to occur. Replace the stopper and repeat this process until the levels do not change.

Open the balloon valve gently until the difference in the oil levels in the arms of the manometer is approximately 400 mm. Close the balloon valve. You are now ready to take a measurement, but wait five minutes for the can to come into thermal equilibrium.

To make a measurement, loosen the stopper without letting out the gas. When it is ready to pop, read the manometer pressure ( $p_1$ ). Since pressures are relative, you only need to record the difference in the two arms in millimeters. Now pop the cork and replace it as quickly as possible. The pressure will drop and start to climb back. The manometer cannot respond fast enough to go to zero. After five or ten minutes the pressure will have come back to the final value  $p_2$ . Substitute  $p_1$  and  $p_2$  into equation (1) and compute a value for  $\gamma$ . Repeat the measurement with air two more times, using the balloon to pressurize the can. Compute an average value for  $\gamma$  for air.

2. Repeat step 1 using either helium or carbon dioxide instead of air. The only differences in the procedures are as follows.



- a. When admitting the gas to the evacuated can, you must use a specific port, the choice of which depends on the gas that you are using. One of the ports is marked BOTTOM and the tube goes all the way to the bottom of the can. Use this port to admit CO<sub>2</sub>; use the other, which just goes in the top, to admit helium (why?). In either case, the balloon will need to be shifted to the other port before evacuating the can.
  - b. To admit the gas to the can after it has been evacuated, use the following procedure. Attach the supply tube from the gas tank to the appropriate port of the can. Make sure that the main valve on the top of the gas tank is full open (full counterclockwise). Slowly screw in the flow screw on the regulator until the proper rate of flow is established. When the balloon has been inflated to approximately one foot in diameter, close the balloon valve and unscrew the flow screw until it is loose and gas is no longer coming out the supply tube. The pressure on the flow pressure gauge should read zero. Disconnect the supply tube from the can.
  - c. When connecting the manometer to the can and checking to see if the gas is in thermal equilibrium with the surroundings, do not remove the stopper if the gas is not in thermal equilibrium. Instead, disconnect the manometer from the port, wait until you think equilibrium has been achieved, and then momentarily open the valve on the manometer port to bring the pressure to one atmosphere. Reconnect the manometer, open the manometer valve and recheck the pressure in the can. Repeat this process until the oil levels in the two arms of the manometer do not change. This method will limit the introduction of air into the can.
3. Repeat step 2 for the remaining gas.

## Part II - Ruchardt's Method

4. Before beginning the measurement procedures, a few comments on the system parameters are needed:
  - a. The mass of the ball is 16.6 grams.
  - b. The volume  $V$  in equation (2) is the combined volume of the can and the glass tube. The glass tube has a cross sectional area of  $2.01 \text{ cm}^2$ . A meterstick is provided to measure the length of the glass tube. When measuring its length, be sure to include the part inside the rubber stopper at the bottom of the tube and the part in the teflon sleeve at the top of the tube. The volume of the tube can be computed from these two pieces of information.
  - c. The pressure  $p$  in equation (2) is not atmospheric pressure, although you need the atmospheric pressure to compute  $p$ .  $p$  is computed using the expression

$$p = p_0 + \frac{mg}{A}$$

where  $p_0$  is the atmospheric pressure,  $m$  is the mass of the ball,  $g$  is the acceleration of gravity and  $A$  is the cross sectional area of the glass tube. The atmospheric pressure can be measured using the barometer mounted on the side of one of the chalkboards. Your TA will show you how to use it. Note that the units of the atmospheric pressure measured in this way will be mm of Hg. The atmospheric pressure will need to be converted to units of dynes/cm<sup>2</sup> before it can be used in the above equation for  $p$ .

Make the necessary measurements and compute  $V$  and  $p$ .

5. Evacuate the can and fill it with air in the same way that you did in the first paragraph of step 1. Take the can your lab station and wait approximately five to ten minutes for the air in the can to come into thermal equilibrium with its surroundings.

The precision glass tube used for this part is mounted in a ring stand assembly. Do not remove the tube from this assembly. Before moving it, make sure that all of the clamps holding the tube are tight. Carefully place the precision glass tube assembly in the stopper hole in the top of the can. Make whatever adjustments are necessary to make the tube fit firmly in the hole and to make the tube vertical. When done making the adjustments, check to see that all of the clamps holding the tube are still tight, that the magnet is in the top of the tube and that the ball is resting at the bottom of the tube. If the ball is stuck somewhere in the middle of the tube, ask your TA to clean the tube.

At this point the balloon valve should be closed and the other valve should be open. Close the open valve. Open the balloon valve. The overpressure in the balloon will force the ball up to the top of the tube where the magnet will hold it. When the ball is at the top of the tube, close the balloon valve and then open the other valve momentarily to drop the can pressure to one atmosphere. You are now ready to make a measurement.

Lift the magnet off the top of the tube and the ball will drop. Time the bounces for about five periods. Compute the period of oscillation. Substitute the measured period and the other needed parameters into equation (2) and compute a value of  $\gamma$  for air. Repeat two or three times, first opening the balloon valve to drive the ball to the top, placing the magnet back, relieving the pressure, and then dropping the ball. Compute an average value for  $\gamma$  for air. Remove the precision glass tube assembly from the can.

6. Evacuate the can and fill it with with CO<sub>2</sub> (or He) using the same method as you used in step 2. Be sure to use the proper port (BOTTOM for CO<sub>2</sub>; the other for He) when filling the can. Flush out the precision glass tube with the gas to be used by running the gas through it for approximately thirty seconds. Flush from the bottom if you are using CO<sub>2</sub> and flush from the top

if you are using He. Place the glass tubing back into the stopper hole and repeat the measurements made in step 5 using this new gas.

7. Repeat step 6 for the remaining gas.

### Lab Report

You should use the lab notebook format described by your TA in his/her comments at the beginning of the lab session. Your lab report should include the answers to all of the questions asked in the introduction or procedure, all raw and derived data, and an estimate of the magnitude and sources of error in any data recorded. When answering any question or when giving any comparison or explanation, always refer to specific data to support your statements. For this lab, also include the following:

1. computations of  $\gamma$  for each of the gases for each of the two methods; and
2. a comparison of the experimentally measured values of  $\gamma$  with the theoretically predicted values given in the theory section of this writeup.

## Appendix

By using the first law of thermodynamics, the ideal gas law, and some definitions of specific heats, it can be shown that, in an adiabatic process, an ideal gas obeys the relation

$$pV = \text{constant}$$

An adiabatic process is one in which no heat is transferred to or from the system - the system in this case being one mole of an ideal gas. The constant will be defined.

The first law states that  $dU = Q - W$ . The specific heat at constant volume,  $C_v = (dQ/dT)_v$ , is the heat added divided by the temperature change. At constant volume,  $dV = 0$  and therefore

$$W = 0, \quad dU = dQ$$

and

$$C_v = \left( \frac{dU}{dT} \right)_v .$$

Since, for an ideal gas,  $U = (1/2)fRT$ , where  $f$  is the number of degrees of freedom per molecule,

$$C_v = \frac{1}{2} f R$$

and

$$dU = C_v dT$$

This last equation, although it was derived by considering a constant-volume process, is valid for any process involving an ideal gas. This is so because  $U$  depends on temperature only, and  $dU$  is the same for any  $dT$ , regardless of what caused the change.

For an adiabatic process,  $Q = 0$ , so

$$dU = -W = -p dV = C_v dT .$$

From the ideal gas law,

$$pV = RT \quad (n=1), \quad p dV + V dp = R dT$$

and we have that

$$R = C_p - C_v .$$

Combining the above equations yields

$$p dV = V dp = \frac{-p dV(C_p - C_v)}{C_v} .$$

Defining the ratio  $\gamma = C_p/C_v$ , and collecting terms, we have

$$p dV + V dp = 0$$

Dividing this equation by  $pV$  gives

$$\left(\frac{dV}{V}\right) + \frac{dp}{p} = 0$$

This can be integrated, with the result

$$\ln V = -\ln p + C$$

And finally, if  $\ln pV = C$ , then

$$pV = \text{constant}$$

as advertised.