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On the class of (K-N) QUASI-n-NORMAL OPERATORS on HILBERT SPACE

SIVAKUMAR N
Associate professor,
PG and Research Department of Mathematics,
Hindusthan College of Arts and Science,
Tamil Nadu, India
cbecentumsivakumar@rediffmail.com

BAVITHRA V
Mphil scholar,
PG and Research Department of Mathematics,
Hindusthan College of Arts and Science
Tamil Nadu India
bavithra.srgm@gmail.com

Abstract - In this work we introduce another class of normal operator which is (K-N) quasi n normal operator and given some basic properties. The relation between these operators with another types of normal operators are discussed. Here the results are given by using the conditions of (K-N) quasi normal operators.

Keywords- Hilbert Space, normal operator, quasi normal, n-quasi normal, (k-N) quasi normal, quasi n normal, class(Q) operator.

I. INTRODUCTION

Let H be complex Hilbert Space and $L(H)$ be the algebra of all bounded linear operators. The quasi normal operator was introduced by A. Brown [2] in 1953 and given some properties. The Generalization of the quasi normal operators were introduced by the researchers D. Senthil kumar called K-quasi normal operators. In 2011 O. Ahmed [4], extended it to n-power quasi normal operator. Also in 2015 D. M. Salim and M.K. Ahmed [5], introduced the another class of operator called (K-N) quasi normal operator. In this article we introduce the generalization of the above operator called the (K-N) quasi n normal operator and study some basic properties.

II. DEFINITIONS

A) Normal Operators:

An Operator T on H is said to be normal if $TT^* = T^*T$ and is said to be n-power normal if $T^*T^n = T^nT^*$ where n$\in N$. The class of n-power normal operators is denoted by $[nN]$.

B)Quasi Normal operators:

T$\in B(H)$ is called Quasinormal if $T(T^*T) = (T^*T)T$ and is called n-power Quasinormal if $T^n(T^*T) = (T^*T)T^n$ for some positive integer n.

C) Quasi n normal Operators:

An Operator T on H is called Quasi n normal if $T(T^*T^n) = (T^*T^n)T$ for some positive integer n.

D) K-Quasi-Normal Operators:

An Operator T on H is said to be K-Quasi-normal if it commutes with $(T^*T)^k$ that is $T\ (T^*T)^k = (T^*T)^k T$. If K=1 then K-Quasi-normal becomes Quasi-normal.

E) (K-N) Quasi-Normal Operators:

An Operator T on H is said to be (K-N) Quasi-normal Operator if satisfy the condition $T^{\boldsymbol{k}}(T^*T) = N(T^*T)T^k$, where K is positive integer and N is bounded Operator from a complex Hilbert Space. If N=I, K=1 then T is Quasi-normal.

F) (K-N) Quasi n Normal Operators:

An Operator T on H is said to be (K-N) Quasi n normal Operator if satisfy the condition $T^{\boldsymbol{k}}(T^*T^n) = N(T^*T^n)T^k$ where K is positive integer and N is bounded Operator from a complex Hilbert Space. If N=I, K=1 then T is Quasi n normal.

III. MAIN RESULT

Theorem

A power of (K-N) quasi n normal operator is again a (K-N) quasi n normal operator.

*Proof***:**

Let T be a (K-N) quasi n normal operator. We prove the assertion by using mathematical induction. Therefore T is (K-N) quasi n normal the result is true for $m = 1$

That is,
$$T^{\boldsymbol{k}}(T^*T^n) = N(T^*T^n)T^k \qquad (1)$$

Now we assume that the result is true for $m = n$

that is
$$(T^k(T^*T^n))^n = (N(T^*T^n)T^k)^n \qquad (2)$$

Let us prove the result for $m = n + 1$

That is
$$(T^k(T^*T^n))^{n+1} = (N(T^*T^n)T^k)^{n+1}$$

$$(T^k(T^*T^n))^{n+1} = (T^k(T^*T^n))^n\ T^k(T^*T^n)$$

$$= (N(T^*T^n)T^k)^n N(T^*T^n)T^k \text{[by (1) and (2)]}$$

$$(T^{\boldsymbol{k}}(T^*T^n))^{n+1} = (N(T^*T^n)T^k)^{n+1}$$

Thus the result is true for $m = n + 1$. Therefore T^n is also (K-N) quasi n normal operator for each n.

IV. PROPERTIES OF (K-N) QUASI n NORMAL OPERATORS

The following theorem give some Properties of (K-N) Quasi n normal Operators.

Theorem 4.1

Let $T \in B(H)$ is an Operator if C is commutes with U and V, and $C^2T^k = NC^2T^k$ then T is (K-N) quasi-normal. Where, $B^2 = TT^*, C^2 = T^*T, U = ReT = \frac{T+T^*}{2}, V = ImT = \frac{T-T^*}{2i}$

*Proof***:**

Since $CU = UC, BV = VB$ so, $C^2U = UC^2, B^2V = VB^2$by [1]

Thus $C^2U^k = U^kC^2, B^2V^k = V^kB^2$ then

$$C^2T^k + C^2(T^k)^* = T^kC^2 + (T^k)^*C^2$$

$$C^2T^k - C^2(T^k)^* = T^kC^2 - (T^k)^*C^2$$

This gives $T^kC^2 = C^2T^k$, $T^k(T^*T) = (T^*T)T^k$

And by using the condition $B^2T^k = NB^2T^k$ so we get:

$T^k(T^*T) = N(T^*T)T^k$ then , T is (K-N) quasi-normal.

Theorem 4.2

Let $T \in B(H)$ is an Operator if C is commutes with U and V, and $C^nT^k = NC^nT^k$ then T is (K-N) quasi n normal.

Where, $B^n = T^nT^*, C^n = T^*T^n, U = ReT = \frac{T^n+T^*}{2}, V = ImT = \frac{T^n-T^*}{2i}$

Proof:

Since $CU = UC, BV = VB$ so, $C^2U = UC^2, B^2V = VB^2$

Thus $C^nU^k = U^kC^n, B^nV^k = V^kB^n$ then

$$C^nT^k + C^n(T^k)^* = T^kC^n + (T^k)^*C^n$$

$$C^nT^k - C^n(T^k)^* = T^kC^n - (T^k)^*C^n$$

This gives $T^kC^n = C^nT^k$ and $T^k(T^*T^n) = (T^*T^n)T^k$

And by using the condition $B^nT^k = NC^nT^k$ so we get:

$T^k(T^*T^n) = N(T^*T^n)T^k$ then T is (K-N) quasi n normal.

Theorem 4.3

If $T \in B(H)$ is an Operator such that $C^nU^k = \frac{1}{N}U^KC^n, C^nV^k = \frac{1}{N}V^kC^n$ then T is (K-N) quasi n normal.

Proof:

Since $C^nU^k = \frac{1}{N}U^kC^n, C^nV^k = \frac{1}{N}V^kC^n$ then we have

$C^n(U+iV)^k = \frac{1}{N}(U+iV)^k C^n$ and we have

$C^n T^k = \frac{1}{N} T^k C^n$ therefore,

$$(T^*T^n)T^k = \frac{1}{N} T^k (T^*T^n)$$

$$T^k(T^*T^n) = N(T^*T^n)T^k$$

Then T is (K-N) quasi n normal.

Theorem 4.4

Let $T_{1,}T_2$ be two (K-N) quasi n normal from H to H, such that $T_1{}^k T_2{}^* = T_2{}^k T_1{}^* = T_1{}^* T_2{}^n = T_2{}^* T_1{}^n = 0$ then $T_1 + T_2$ is (K-N) quasi n normal.

Proof:

$$(T_1+T_2)^k[(T_1+T_2)^*(T_1+T_2)^n] = (T_1+T_2)^k[(T_1{}^* + T_2{}^*)(T_1{}^n + T_2{}^n)$$

$$= (T_1+T_2)^k(T_1{}^*T_1{}^n + T_1{}^*T_2{}^n + T_2{}^*T_1{}^n + T_2{}^*T_2{}^n$$

$$= (T_1+T_2)^k(T_1{}^*T_1{}^n + T_2{}^*T_2{}^n)$$

$$= (T_1{}^k + T_2{}^k)(\, T_1{}^*T_1{}^n + T_2{}^*T_2{}^n)$$

$$= (T_1{}^kT_1{}^*T_1{}^n + T_1{}^kT_2{}^*T_2{}^n + T_2{}^kT_1{}^*T_1{}^n + T_2{}^kT_2{}^*T_2{}^n)$$

$$= (T_1{}^kT_1{}^*T_1{}^n + T_2{}^kT_2{}^*T_2{}^n)$$

$$= N\big[(T_1{}^nT_1{}^*)T_1{}^k\big] + N[(T_2{}^nT_2{}^*)T_2{}^k]$$

$$(T_1+T_2)^k[(T_1+T_2)^*(T_1+T_2)^n] = N[(T_1+T_2)^*(T_1+T_2)^n](T_1+T_2)^k$$

Hence $T_1 + T_2$ is (K-N) Quasi n normal.

Theorem 4.5

Let T_1 be (K-N) quasi n normal Operator and T_2 (K-power) quasi n normal Operator. Then their product T_1T_2 is (K-N) quasi n normal Operator if the following conditions are satisfied

- $(T_1T_2)^n = (T_2T_1)^n$
- $(T_1T_{2)} = (T_2T_1)$
- $T_1T_2{}^* = T_2{}^*T_1$

Proof:

$$(T_1T_2)^k(T_1T_2)^*(T_1T_2)^n = (T_1{}^kT_2{}^k)(T_2{}^*T_1{}^*)(T_2{}^nT_1{}^n)$$

$$= (T_1{}^kT_2{}^k)(T_1{}^*T_2{}^*)\ (T_2{}^nT_1{}^n)$$

$$= T_1{}^k(T_2{}^kT_1{}^*)\ (T_2{}^*T_2{}^n)T_1{}^n$$

$$= T_1{}^k(T_1{}^*T_2{}^k)\ (T_2{}^nT_2{}^*)\ T_1{}^n$$

$$= T_1{}^kT_1{}^*(T_2{}^kT_2{}^n)\ T_2{}^*\ T_1{}^n$$

$$= (T_1{}^kT_1{}^*T_2{}^{\ n})\ (T_2{}^kT_2{}^*\ T_1{}^n)$$

$$= N(T_1{}^*T_2{}^n)T_1{}^k)(T_2{}^*\ T_1{}^n)T_2{}^k$$

$$= N({T_1}^* {T_2}^n({T_1}^k{T_2}^*) {T_1}^n{T_2}^k)$$

$$= N({T_1}^* {T_2}^n{T_2}^*{T_1}^k{T_1}^n{T_2}^k)$$

$$= N[{T_1}^*({T_2}^n{T_2}^*) ({T_1}^k{T_1}^n){T_2}^k]$$

$$= N({T_1}^*{T_2}^*) ({T_2}^n{T_1}^n) ({T_1}^k{T_2}^k)$$

$$= N[(T_2T_1)^*(T_2T_1)^n(T_1T_2)^k]$$

$$= N[(T_1T_2)^*(T_1T_2)^n(T_1T_2)^k$$

$$(T_1T_2)^k(T_1T_2)^*(T_1T_2)^n = N[(T_1T_2)^*(T_1T_2)^n(T_1T_2)^k$$

Hence, the product T_1T_2 is (K-N) Quasi n normal.

V. CONCLUSION

In this paper we conclude that the class of (K-N) quasi n normal operators are satisfied the conditions of (K-N) quasi normal operators. We further extend it to any other operator by giving some other conditions.

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