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THESIS

**A LITTORAL COMBAT MODEL
FOR LAND-SEA MISSILE ENGAGEMENTS**

by

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September 2007

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A LITTORAL COMBAT MODEL FOR LAND-SEA MISSILE ENGAGEMENTS

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ABSTRACT

This thesis develops a Littoral Combat Model of interactions between Naval Ships at sea and Anti-Ship Cruise Missile Batteries on land. The Littoral Combat Model seeks to answer the question: Is a modern naval force capable of effectively operating in the dangerous littoral environment? The model is derived from a combination of Hughes' Salvo Model and Lanchester's Equations. Cases are developed using either direct fire or area fire weaponry by the sea based force. Land forces deliver aimed fire with missiles, which the ships defend against. A number of embellishments are utilized to provide an in-depth analysis of the interaction. Application of the model with two representative scenarios shows (1) that attacking effectively first remains an important advantage and (2) that accurate direct fire weapons used by the sea based force against the batteries ashore will often overcome Admiral Nelson's warning that "A ship's a fool to fight a fort." However, naval area fire (e.g., naval gunnery) is a key weakness in these inherently complicated littoral engagements, unless used in large volume and backed by sufficient Defensive Power in the sea based force.

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EXECUTIVE SUMMARY

Admiral Horatio Nelson declared that "A ship's a fool to fight a fort" over two hundred years ago. This quote has remained a mantra of naval warfare. Forces in fortified positions ashore had significant advantages over those at sea, such as a greater ability to endure and repair damage. In today's navies, which posses guided munitions, superior scouting capabilities, and substantial defensive weaponry, the question arises: Is a ship still a fool to fight a fort?

Before developing a model that can help answer this question, it is important to understand two important force-on-force attrition models that have already been developed: Hughes' Salvo Model and Lanchester's Differential Equations. The equations of the Salvo Model measure combat interactions between two forces at sea. In this model, pulses of Striking Power from one force are degraded by the Defensive and Staying Power of the other force, and the resulting attrition measured. The Salvo Model helps quantify the results of warfare at sea, but it is not well suited to measuring the attrition of forces on land. Lanchester's Equations, dealing with both direct and indirect fire situations, are used for land combat. The overall depletion of a force is found by the product of a set rate of attrition that an individual combatant in the opposing force can inflict and the number of opposing forces implementing this rate.

Combining attributes of both the Salvo Model and Lanchester's Equations produces a Littoral Combat Model which measures the results of combat interactions between forces at sea and on land. The assumptions inherent in the Littoral Combat Model follow directly from the assumptions adopted for both the Salvo Model and Lanchester's Equations. Important ones to note are:

(1)   There is a Uniform distribution of fire for both forces.

(2)   Losses are measured in units put out of action.

(3)   Each engagement consists of pulses of Offensive Power exchanged between the two forces.

The Littoral Combat Model considers combat interactions as combinations of circumstances and situations. Circumstances are defined by the force's scouting efficiency. If the force at sea has superior scouting, then a Sea Force Initial Attack will occur. If the force on land has superior scouting, then a Land Force Initial Attack will occur. If neither is the case, meaning both forces have equal levels of scouting, then a Simultaneous Attack will occur. Situations are dependent upon the naval force's method of attrition. In this work, three distinct situations are studied: direct fire munitions (e.g., Tomahawk Land Attack Missiles), indirect fire missiles (e.g., The Navalized Army Tactical Missile System), and indirect fire gunnery munitions (e.g., 5"/62 Mk 45 Mod 4 Gun System). All three situations require their own set of unique equations.

The Littoral Combat Model uses many of the terminology, symbology, and embellishment concepts from the Salvo Model. Two unique ideas were added, however: pulses of Naval Gunnery and the Munitions' Lethality, Targeted Area Ratio (MTR). As the Littoral Combat Model relies on pulses of Offensive Power (e.g., a salvo of missiles), a series of individual gunfire rounds are combined together to form the 'pulse.' MTR is the ratio of the total lethal area of a single indirect fire munition to the total area targeted; it is used so as to reduce the number of parameters in the Littoral Combat Model.

The Fractional Exchange Ratio (FER) is utilized as the model's Measure of Effectiveness. FER is the ratio of the fraction of sea based forces destroyed to the fraction of land based forces destroyed. It is presented as:

$$FER = \frac{A_{\text{Fraction Destroyed}}}{B_{\text{Fraction Destroyed}}} = \frac{\Delta A / A}{\Delta B / B}$$

The FER indicates the eventual winner of the engagement, assuming no other forces are added and that all the initial parameter values stay constant. A FER of one indicates parity between the two forces, a FER greater than one results in an eventual victory for the land based forces, and a FER less than one implies an eventual victory for the sea based force. The farther the FER value from one, the larger the percentage of units from the victorious force survive the engagement.

The Littoral Combat Model's parameters are as follows:

A : Sea based force.

B : Land based force.

ΔA : Change in size of the sea based force.

ΔB : Change in size of the land based force.

α_1 : Number of well-aimed sea based direct fire missiles per ship.

α_2 : Number of well-aimed sea based area fire missiles per ship.

α_3 : Number of well-aimed individual gunnery rounds per ship.

β : Number of well-aimed land based missiles per land element.

a_1 : Staying Power per ship of the sea based force.

a_3 : Defensive Power per ship of the sea based force.

σ_i : Targeting Capability of force i, $0 \le \sigma \le 1$.

ρ_i : Deception Capability of force i, $0 \le \rho \le 1$.

δ_i : Defensive Readiness of force i, $0 \le \delta \le 1$.

MTR: Munitions' Lethality, Targeted Area Ratio.

> Where: $\text{MTR} = area_{Lethality} / area_{Total}$

> $area_{Lethality}$: The area covered by individual munitions in a salvo.

> $area_{Total}$: The uncertain location of a target.

The three cases of the Littoral Combat Model are:

(1)   Land force ASCM's versus sea force direct fire munitions:

$$\Delta B = \sigma_A \alpha_1 A \qquad\qquad \Delta A = \frac{\sigma_B \rho_A \beta B - \delta_A a_3 A}{a_1}$$

(2)   Land force ASCM's versus sea force area fire missiles:

$$\Delta B = \alpha_1 (MTR_{msl}) AB \qquad\qquad \Delta A = \frac{\sigma_B \rho_A \beta B - \delta_A a_3 A}{a_1}$$

(3)   Land force ASCM's versus sea force area fire gunnery:

$$\Delta B = \alpha_2 (MTR_{gun}) AB \qquad\qquad \Delta A = \frac{\sigma_B \rho_A \beta B - \delta_A a_3 A}{a_1}$$

Two distinct scenarios were developed to better understand the Littoral Combat Model. These scenarios permit a variety of analyses, allowing both a further understanding of the model's mathematical nature and an opportunity to extract broad themes about force-on-force combat between units on land and those at sea. For each scenario, all the parameters relating to the sea based force are varied and the different FER levels calculated and graphed. The exact parameter values necessary to attain certain FER levels are found and tabled. The trends and patterns evident in these graphs and tables form the basis of the analysis.

Seven major themes emerge from the analysis:

(1)   Area fire weaponry requires more Staying Power than direct fire weaponry to thoroughly attrite the land based force.

(2)   Good scouting by the sea based force reduces the size of the force necessary to overcome the land based force.

(3)   Increasing the quality of targeting information brings dramatic benefits for the sea based force.

(4)   Defensive Power can prolong the battle, but Striking Power wins the fight quickly.

(5)   A certain level of "unfairness" exists when a land based force armed with direct fire weapons engages a sea based force armed with indirect fire weapons.

(6)   There exists a Total Victory value in some of the parameters.

(7)   The energies of the sea based force should be invested in attacking effectively first.

This last theme deserves particular comment. Attacking effectively first is dependent on good scouting, Figure 1 demonstrates this importance (this graph is best

viewed in color). Here the effect Sea Force Size has on FER is shown. On the Y axis are FER values from 0 to 3 and on the X axis are Sea Force Size values, ranging from 0 to 10. The blue lines represent Sea Force Initial Attacks, the red lines represent Land Force Initial Attacks, and the green lines represent Simultaneous Attacks. Solid lines are direct fire missiles, dashed lines are area fire missiles, and dotted lines are naval gunnery. Note the very exponential lines seen in the graph, having steep declines and a clear flattening out, this is a common trend of the Littoral Combat Model. The nature of the Littoral Combat Model equations causes such non-linear behavior.

Figure 1.   FER Levels with Increasing Sea Force Size

Moving from attack circumstances with good scouting by the sea based force, resulting in a Sea Force Initial Attack, to those with poor scouting, resulting in a Land Force Initial Attack, the size of the required force increases. In situations where the initiative rests with the land force, so that the sea based force must absorb its attack, sufficient forces at sea are necessary to offset the attrition that will occur, hence the red lines that appear on the far right. In situations where the sea based force has the initiative, and attacks the land based force first, only forces used to thoroughly attrite the land based force in one salvo are necessary, hence the blue lines appear on the far left. The size of

the sea based force need not be overly large if an initial attack can be guaranteed. Conversely, a Land Force Initial Attack requires a sufficiently large sea based force to ensure low FER values. In both scenarios, a certain minimum number of sea based ships are necessary to attain a FER of one, a situation of parity. The model shows that for these fairly representative scenarios, very small numbers of ships (1-3) are in great peril, even under favorable attack circumstances and situations.

With the Littoral Combat Model, analysts have a unique and interesting method of better understanding the way that force-on-force attrition will occur in the littoral environment. The simplicity of the model is necessary in overcoming the very convoluted nature of combat between forces on land and those at sea. The Littoral Combat Model makes it readily apparent that a ship may no longer be "a fool to fight a fort." The increased accuracy of weapons and the presence of defensive weapons onboard ships has caused the playing field between the force on land and the force at sea to be evened. In order to take full advantage of these facts, however, improved scouting and anti-scouting methods are required; they ensure the sea based force attacks effectively first. Without this first attack, the chances of a complete victory by the sea based force, when not having very superior forces, are slim.

I. INTRODUCTION

A. PAST, PRESENT & FUTURE OF LAND-SEA COMBAT INTERACTION

The concept of a Land-Sea Engagement is central in naval operations; it has been in the past and will continue to be so in the future. Often, this engagement is an 'uphill' fight for the naval force due to the inherent advantages (more stable platforms, quicker renewability of damaged assets, etc.) of the land based force.

Examples from the past and present demonstrate this point. The Royal Navy of the Napoleonic Wars dealt with shore-based mobile gun batteries as they tried to blockade the numerous small towns along the French coastline.[1] Almost all of the Union Navy's major engagements during the American Civil War took place against Confederate shore fortifications. Teddy Roosevelt and the Rough Riders charged up Kettle Hill because the defenses surrounding Santiago prevented the U.S. Navy from getting to the Spanish fleet inside the harbor. The Anzac's suffered their terrible losses on the beaches of Gallipoli because the Royal Navy's battleships were unable to overcome Turkish shore defenses in the Dardanelles.[2] Finally, the Exocet Anti-Ship Cruise Missile (ASCM) attack on the HMS GLAMORGAN seriously hampered the ability of Admiral Woodward's destroyers and frigates to supply Naval Gunfire Support in the final days of the Falklands Campaign, due to concerns of a repeated incident.[3] Such events are not confined to naval history; the ASCM attack on the INS HANIT off of the Lebanese coastline on July 16, 2006 points to the modern version of the same dilemma: the requirement for sea based forces to operate close to shore and the vulnerability they have in such situations.

The concept of Area-Denial, the application of maximum force to deny a limited amount of area to the enemy, has become the common tactic for smaller nations unable to effectively engage opposing forces on an equal basis. Admiral Mike Mullen, the Chief of Naval Operations, believes defeating this Area-Denial tactic is an important task for the

U.S. Navy: "To be effective in the multitude of missions that await us, the Navy must be capable of assuring access--at a time and place of our choosing--throughout the maritime domain."[4]

In an effort to assure this access, naval forces must overcome many different threats within the diverse regions of the oceans. The presence of land based ASCM batteries is certainly one of the more obvious threats. Today, the ASCM is the most potent weapon system used by forces ashore against naval forces. The inability of a navy to operate near the shore against this threat could doom efforts to influence events ashore.

B.   TYPES OF LAND-SEA COMBAT INTERACTIONS

There are similarities and variations in any land-sea engagement in the littoral environment. The method of the engagement varies from a high-tech series of direct fire missile salvos used against mobile shore batteries equipped with supersonic ASCM's, like the truck-mounted SS-N-22 Sunburn launch system, to traditional sea based gun systems trying to destroy fixed ASCM batteries. Similarities do exist, however, since in both cases each side is attempting to emit Offensive Power in order to knock out the opponent. In addition, the sea based forces in both cases have a defensive capability and a level of Staying Power that degrades the land based force's Offensive Power.[5] These two elements offer the sea based force the ability to prolong the fight.

The nature of the environment also complicates the situation. A Coastal ASCM battery, easily hidden in the usual clutter of human activity near the ocean, and thus difficult for the forces at sea to detect, generally has an advantage. Assuming that the ASCM battery is mobile, a frequent configuration of today's coastal defense systems, it also possesses an ability to attack at a time and place of its choice, further complicating the sea based force's operations. The land based force has an easier "detect to engage process" than the naval force: the flatness of the sea contrasts sharply with the jaggedness and clutter of the land. But a well equipped and trained naval force, operating in a favorable environment, can make this process more difficult. It is the interaction between these advantages and disadvantages which serves to complicate the land-sea engagement.

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Mobility and range are also elements in the engagement; bringing advantages and disadvantages,for either side, depending on the circumstances. Mobility and range have been cited as the best tactics for the sea based force to employ when faced with a strong land based force.[5] Yet they are not always options; given a mobile land based force, superior land based weapons systems, or the necessity of operating in a certain area, mobility and range may be nullified. Truck mounted ASCM batteries can negate a sea based force's mobile advantage. The chokepoints of the world's oceans serve to funnel naval operations into specific areas, no matter the desires of the commander. The inability of the sea based force to always avoid the threat means that, in some cases, a direct confrontation is necessary.

C.   ANALYSIS OF LAND-SEA COMBAT INTERACTIONS

If the sea based force does have to directly confront the land based force, what factors give it an edge? Elements of the conflict benefiting the sea based force quickly come to mind: The ability to accurately target the land based force, Defensive Power for protection from the land based force's Offensive Power, and launching enough Offensive Power to 'swamp' the land based force. Unfortunately, resources, time and tactical constraints may limit some, or all, of these options. Thus, it is necessary to examine the different elements of the engagement in detail and determine their relative importance. In determining these relative levels, questions are answered which help better understand the relationship between possible outcomes and specific force make-ups.

To enable this analysis of the land-sea engagement, it is necessary to construct a model useful for the study of interactions between forces in the littoral region. This model should not sacrifice too much detail or make too many assumptions so as not to degrade its usefulness. The foundation of such a model comes from two highly respected analytic representations: Hughes' Salvo Model and Lanchester's Differential Equations.

Utilizing these respected models permits construction of a Littoral Combat Model, providing a broad analytical perspective for the engagement process between land based forces, armed with ASCM's, and sea based forces operating in littoral waters. This work develops two different scenarios that showpiece likely situations a naval force

might face. One is a Down-Scale type of conflict, where a small sea based force operates near a limited number of potent missile batteries. The other, an Up-Scale type of conflict, has the sea based force confront a larger, more powerful threat. The Littoral Combat Model evaluates these different scenarios, finding what parameters most benefit the sea based force. The analysis of the two scenarios looks for results, specifically beneficial parameters and trends, which could assist a naval force against Anti-Access or Area-Denial threats from the shore. With the knowledge gained, a better understanding of doctrine and equipment requirements can be achieved. This understanding, if applied, can help to improve the efficiency and lethality of a naval force when it operates in the dangerous littoral environment. Land based power, especially ASCM's from shore batteries, is a threat that has shown itself to be potent in the past and will, no doubt, continue to be potent in the future.

D.   THESIS BREAKDOWN

This thesis seeks to both introduce the Littoral Combat Model and relate some impressions gained from an analysis of its outputs. Chapter II presents the Salvo Model and Lanchester's Differential Equations, giving an overview of each model and comparing their similarities and differences. Chapter III introduces the Littoral Combat Model, looking at its different assumptions, parameters, and configurations. To fully demonstrate the Littoral Combat Model, actual problems are solved using parameter numbers that reflect a mini-scenario developed especially for its illustration. Chapter IV introduces the two scenarios that formed the basis of the analysis; the parameter values for each scenario and the resulting baseline outputs are given. Chapter V looks at the results of the analysis, describing seven themes from the data output, supported by pertinent graphs and tables. Chapter VI contains the conclusions, broad concepts derived from the themes, and a series of future work ideas revolving around the Littoral Combat Model. Appendix A presents a series of graphs and tables that were derived from an analysis of these scenarios with the Littoral Combat Model. The graphs presented in this work are best viewed in color.

II.   OVERVIEW OF CURRENT FORCE-ON-FORCE MODELS

A.   INTRODUCTION

The following is a brief description of the two models that are the basis for the Littoral Combat Model: the Pulsed Salvo Model and Lanchester's Differential Equations. This chapter explains each separately and then describes the differences and similarities.

B.   A PULSED SALVO MODEL

The Salvo Model provides a method for comparing the military worth of warship capabilities using simple equations that investigate the interaction between combat characteristics.[5] The model's output is the fraction of forces remaining after two forces exchange a pulse of Offensive Power, usually in terms of ASCM's, taking into account the Defensive Power available to both sides and the ability of either side to take these 'hits' and continue to operate.[6]

The idea of pulses of Offensive Power is a key concept of the model. Realizing that the presence of missile combat significantly changed the nature of naval warfare, the model was an attempt to define this new type of conflict and quantify its attributes. It first must be understood that previous naval battles centered on the gun and its ability to emit Offensive Power over some set timeframe, resulting in a cumulative degradation of the enemy. In missile combat, however, a force may defeat its opposition in only one or two pulses of Offensive Power. In effect, there was a transition from the near continuous style of duels between big guns to a set of pulses between missile-armed combatants.

The following assumptions are made for the Salvo Model:

(1)   Offensive (Striking) Power is the number of accurate and functioning ASCM's that are launched in each pulse.

(2)   The targeting spread of ASCM's is uniform across the opposing force. That is, no one ship is singled out as a target.

(3)   The Defensive Power of the force is able to eliminate all accurate and functioning ASCM's until it is saturated. This saturation of the defenses results in ASCM 'leakers' getting through and hitting the force at sea.

(4)   Staying Power defines a ship's ability to withstand a pulse of Striking Power and still function, it equates to staying in the fight vice staying afloat.

(5)   Hits on the force diminish its fighting strength linearly.

(6)   There is no range advantage inherent in the model, both sides are equally likely to target the opposing side and achieve hits.

The Salvo Model is presented here in its simplest form:

$$\Delta A = \frac{\beta B - a_3 A}{a_1} \tag{1}$$

$$\Delta B = \frac{\alpha A - b_3 B}{b_1} \tag{2}$$

Where:

A : One force in the engagement.

B : The other force in the engagement.

ΔA : Change in A force levels.

ΔB : Change in B force levels.

α : Offensive Power per ship of A forces.

β : Offensive Power per ship of B forces.

a_1 & a_3 : Staying Power & Defensive Power per ship of A forces.

b_1 & b_3 : Staying Power & Defensive Power per ship of B forces.

Embellishments added to the model cause increased complications, but are useful case-specific tools in further determining the relative worth of force attributes in an engagement. The embellishments make Offensive Power and Defensive Power a function of a number of other factors, such as force readiness, actual targeting capability, and human performance. The embellishments' effect on Offensive Power and Defensive Power are as follows:

$$\alpha' = \sigma_A \tau_A \rho_B \alpha \tag{3}$$

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$$a'_3 = \delta_A \tau_A a_3 \tag{4}$$

$$\beta' = \sigma_B \tau_B \rho_A \beta \tag{5}$$

$$b'_3 = \delta_B \tau_B b_3 \tag{6}$$

Where:

σ_i : Targeting Capability of force i, $0 \leq \sigma \leq 1$.

ρ_i : Deception Capability of force i, $0 \leq \rho \leq 1$.

δ_i : Defensive Readiness of force i, $0 \leq \delta \leq 1$.

τ_i : Training Multiplier of force i, $0 \leq \tau \leq 1$.

When using this deterministic model to study the relation between different force compositions, a number of results become apparent. First among these is the importance of Offensive Power in the engagement. Any amount of Defensive Power and Staying Power can be overcome, with large enough pulses of missiles. However, this works both ways and may result in a Pyrrhic Victory. To describe this phenomenon, Hughes coined the term Tactical Instability, where one force has the offensive potential to 'swamp' the opposing side, but lacks the Defensive Power or Staying Power to keep itself from being destroyed. The Salvo Model points out the importance of scouting in the engagement process. The force that can detect and strike the other first, before its own force is discovered and engaged, gains big, and sometimes decisive, advantages, even if it is a markedly inferior force in any or all of its attributes.[5]

The Salvo Model is useful in studies of naval combat, as it brings forth certain elements of ship design and naval operations which are not available in other, more land-centric, models. The realization of the System-of-Systems nature of naval combatants, and the effect this has on the outcome of combat, was one of the greatest innovations of this combat model.

C.   THE LANCHESTER DIFFERENTIAL EQUATIONS

Lanchester's Laws are the basis for many differential equations that have been developed to model force attrition, both in combat and non-combat situations.[7]   Two particular equations provide the basis for Lanchester's Laws, these are the aimed-fire combat model (or Square Law) and the area-fire combat model (or Linear Law). Whereas the Salvo Model saw the degradation of a force occurring over a series of one or more pulses of Offensive Power, the Lanchester equations assume a flow of continuous fire.

For the Square Law equation, the degradation is the product of the numbers of shooters and their individual firing rates. The aimed-fire (direct-fire) model is as follows; note the simplicity of the arrangement:

$$dA/dt = -\beta B \tag{7}$$

$$dB/dt = -\alpha A \tag{8}$$

Where:

β : Rate at which one B kills A forces.

α : Rate at which one A kills B forces.

This continuous form can be approximated in an interval (Δt of time) by the difference equation variant shown below. Note that the direct-fire model assumes a constant rate of attrition over the time of the engagement.

$$\Delta A = \beta B \Delta t \tag{9}$$

$$\Delta B = \alpha A \Delta t \tag{10}$$

Where:

ΔA : Change in A force levels.

ΔB : Change in B force levels.

Δt : Time interval.

The Linear Law equation is nearly as simple, it depends both on the numbers and attrition rate of the attacking force, as well as on the numbers of the attacked force. The formulation is shown below:

$$dA/dt = -\phi AB \tag{11}$$

$$dB/dt = -\varepsilon BA \tag{12}$$

Where:

ϕ : Attrition Coefficient of the B forces.

ε : Attrition Coefficient of the A forces.

Again, this continuous form can be approximated in a Δt interval of time by the difference equation variant. Note that, as with the direct-fire model, the area-fire model assumes a constant rate of attrition over the time of the engagement.

$$\Delta A = \phi AB\Delta t \tag{13}$$

$$\Delta B = \varepsilon BA\Delta t \tag{14}$$

To better understand the Attrition Coefficient, target density and rate of fire must be appreciated. The area fire concept (e.g., the "artillery duel") is a commonly used variant of Lanchester's Linear Law, and helps illustrate the Attrition Coefficient. Specifically, by breaking down the size of the attacked force and the individual lethal area of the munitions of that force, divided by the total targeted area, a useful application of area fire is derived. Let B number of guns fire at a rate β into a targeted area, $area_A$, containing a force A, with each individual B munition covering an area, $lethal_B$. Likewise, have a force of A guns, fire at a rate α into a targeted area, $area_B$, containing the B force, with each individual A munition covering an area $lethal_A$. Each side will then degrade each other, within each time step, as the following approximation of the continuous form dictates:

$$\Delta A = \beta(lethal_B / area_A)AB\Delta t \tag{15}$$

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$$\Delta B = \alpha(lethal_A / area_B)BA\Delta t \qquad (16)$$

Where:

β : Firing Rate of B forces over time.

α : Firing Rate of A forces over time.

$lethal_i$: Lethal area of force i.

$area_i$: Total targeted area containing force i.

These two models, direct fire and area fire, can be altered into many different forms, the Ambush (Deitchman) model and the Logarithmic model are examples.[8]

D.   COMPARING THE TWO MODELS

Two important differences between the Salvo Model and Lanchester's Equations stand out: the presence of degradation factors and the nature of the engagement. Within the Salvo Model, Defensive Power and Staying Power thwart Offensive Power and its ability to further reduce the attacked force. Lanchester's basic equations lack these defensive components. As to their nature, to say that one model is perfectly suited to land warfare and one model is unique to sea warfare would be an exaggeration. Certainly some land forces could, when modeled, be given a Defensive Power component; likewise many naval forces lack any amount of Staying Power worth mentioning. But generally speaking, the breakdown is understood to occur.

The Salvo Model is, to a first approximation, sound for analyzing the effect of land based Offensive Power on a sea based force whilst Lanchester's direct and area fire formulas can be well suited to model aggregate attrition of a land based force. These concepts are assumed throughout the rest of this work.

III. THE LITTORAL COMBAT MODEL

A. INTRODUCTION

The Littoral Combat Model is now introduced with numerical examples for its application. The model deals with three distinct situations, stemming from the type of weapon systems used by the sea based force when attacking land based forces: (1) land attack missiles with direct fire accuracy, (2) land attack missiles with area fire accuracy, and (3) naval gunnery. The method of attrition of the sea based force remains constant across all three different situations: ASCM's fired from the land based force.

First, the assumptions that provide the foundation for the model are introduced, followed by the Measure of Effectiveness (MOE) of the model, the Fractional Exchange Ratio (FER). Next, the equations relating to the three different situations are introduced, in the process re-iterating the concept of area fire and introducing a variable that assists us in modeling this specific type of combat. Also presented are a series of embellishments which permits better representation of additional factors in littoral combat.

After these preliminaries, the chapter presents the formal set of Littoral Combat Model equations for each situation. Each is accompanied by a concise mini-scenario, so that the reader may see qualitative issues represented by quantitative variables which, when applied to the equations, produce a descriptive result of the combat interaction. The last sections describe some of the model's peculiarities and limitations.

B. MODEL ASSUMPTIONS

Listed here are the important assumptions made with this model. It should be noted that all assumptions from the Salvo Model and Lanchester's Equations are adopted in the Littoral Combat Model. Additional assumptions are made which tie these two together and deal with certain circumstances unique to littoral combat. Further explanations about these assumptions are in the following sections.

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(1)   There is a uniform distribution of fire for both land and sea based forces.

(2)   Each of the forces (land & sea) are homogenous in nature, i.e., all units within the force utilize the same weapons systems and have the same operational characteristics.

(3)   Each engagement consists of a pulse of Offensive Power between the two forces.

(4)   Sea based missilery has two types of fire:

    (a)   Direct fire, assuming precision munitions.

    (b)   Area fire, assuming indirect munitions.

(5)   Sea based gunnery has area fire characteristics.

(6)   All land based missile fire has direct fire characteristics, i.e., guided, precision missiles.

(7)   The sea based force has a certain area of uncertainty with regards to land targets in area fire situations.

    (a)   The size of the area of uncertainty defines the targeting capability of the sea based force in an area fire scenario (e.g., a large $area_{Total}$ indicates poor targeting compared to a small $area_{Total}$).

    (b)   A Munitions' Lethality, Targeted Area Ratio (MTR) is used to describe this dynamic relationship between area of uncertainty ($area_{Total}$) and lethal area ($area_{Lethal}$). This corresponds to the probability of hitting a specific target in an area fire situation.

(8)   There is no land based defensive capability.

(9)   Sea based Defensive Power includes hard kills (e.g., Area Air Defense, CIWS) and soft kills (e.g., chaff, jamming, other Electronic Warfare).

(10)   Targeting Capability can degrade the given Offensive Power in all engagements for either force.

    (a)   Targeting Capability is a function of a unit's "Detect to Engage" process, and the ability of its opponent to counter this process.

(11)   Deception Capability affects the opponent's Offensive Power in all engagements.

    (a)   Deception Capability is a unit's capability to reduce the Striking Power of the opposing force after it has been launched, thus degrading the numbers actually engaged by the defensive systems.

(12)   Defensive Readiness affect's Defensive Power in all engagements.

        (a)   Defensive Readiness is a function of a unit's capability to accurately utilize all of the defensive 'shots' allotted.

(13)   Losses for both sides are measured in units put out of action.

(14)   Sea based forces have an inherent Staying Power that defines the number of hits a unit can receive before being put out of action.

(15)   Land based systems consist of "delicate" ASCM missiles on either mobile or non-mobile launching platforms, these are put out of action by a single hit.

(16)   In situations where naval gunnery is engaging land based missile forces, the gunnery salvo is composed of the number of rounds the gun system emits during a defined unit of time. This is its 'pulse' of Offensive Power. The unit of time represents the salvo decision time plus the flight time of the incoming missiles as if it, and the gun, commenced their pulses simultaneously.

(17)   One force may launch an initial, unanswered salvo upon the other in a single pulse.

        (a)   In the case of an initial attack by either force, this unanswered salvo will be modeled and the results measured. Then the opposing force will fire a counter-salvo with its surviving units.

(18)   There are no range limitations for either side; both sides are always assumed to be within range of the other's weaponry.

(19)   Both sides are assumed to have sufficient weapon reloads to continue the engagement to completion. This lets us measure not just the effects of the single salvo with the MOE, but the entire engagement.

C.   CHANGES IN FORCE LEVELS & A MEASURE OF EFFECTIVENESS: THE FRACTIONAL EXCHANGE RATIO

The basic force parameters of the model are as follows:

A : Sea based force.

B : Land based force.

ΔA : Change in size of the sea based force.

ΔB : Change in size of the land based force.

The model seeks to find values for ΔA and ΔB that are then applied to A and B so that the reduced size of the forces after a pulse can be computed:

$$A_{t+1} = A_t - \Delta A_t \tag{17}$$

$$B_{t+1} = B_t - \Delta B_t \tag{18}$$

The use of a Fractional Exchange Ratio (FER) to measure the results of salvo warfare upon two opposing forces has become common.[5],[9] The FER takes the output resulting from the salvos and determines the relative loss to either side. It is found as follows:

$$FER = \frac{A_{\text{Fraction Destroyed}}}{B_{\text{Fraction Destroyed}}} = \frac{\Delta A / A}{\Delta B / B} \tag{19}$$

The end-state goal for the sea based force is Sea Control. This dictates two post-salvo objectives:

(1)   The presence of remaining sea based forces to maintain Sea Control.

(2)   The destruction of all land based forces emitting Offensive Power against sea based forces.

These two objectives can be formulated and observed through the FER.

The proportion of sea based forces destroyed:

$$A_{\text{Fraction Destroyed}} = \Delta A / A \tag{20}$$

Observation 1: Note that objective #1 requires a result such that $\Delta A < A$, with $\Delta A = 0$ being preferred. This preference implies we should have: $\Delta A / A = 0$

The proportion of land based forces destroyed:

$$B_{\text{Fraction Destroyed}} = \Delta B / B \tag{21}$$

Observation 2: Note that objective #2 requires a result such that $\Delta B = B$, implying that: $\Delta B / B = 1$.

Because of these two observations, a FER value as follows is desired:

$$FER = \frac{A_{\text{Fraction Destroyed}}}{B_{\text{Fraction Destroyed}}} = \frac{\Delta A / A}{\Delta B / B} = \frac{0}{1} = 0 \qquad (22)$$

Obviously FER = 0 is a 'best case' scenario for the sea based force, and will not always be present. The desired FER must take the form $0 \leq FER < 1$, with a value of zero being preferred.

D.   SEA BASED FORCE ATTRITION

Sea based force attrition is common across all three of the situations defined by the Littoral Combat Model. It is assumed that the land based forces utilize ASCM's, no matter what type of weaponry the sea based force is using. The Salvo Model supplies this formula.

The central idea of the Salvo Model:

$$\text{Change in Own Force Level} = \frac{\left(Power_{\text{Offensive}} - Power_{\text{Defensive}} \right)}{Power_{\text{Staying}}} \qquad (23)$$

Apply the appropriate parameters:

β : Offensive Power of the land based force.

a_1 : Staying Power of the sea based force.

a_3 : Defensive Power of the sea based force.

Then construct the basic force level change formula, as seen in the Salvo Model.

$$\Delta A = \frac{\beta B - a_3 A}{a_1} \qquad (24)$$

Note the robustness of sea based force attrition offered by this simple model. The method of attrition need not always be land based ASCM's, as discussed in this model. It could take the form of land based aircraft armed with direct attack rockets and bombs. In such a case, perhaps the land based force is described as the airfield and β is the number

of aircraft that can be sortied. The method of attrition could also take on the form of an 'asymmetric' attack, with β being the number of UAV's sent to crash into the ships at sea. The Littoral Combat Model possesses the ability to analyze an assortment of different conditions, offering the user the ability to investigate a great array of possible combat scenarios.

E.   LAND BASED FORCE ATTRITION WITH DIRECT FIRE MISSILES

Lanchester's Aimed Fire model, applied as a discrete pulse, provides the equations for the attrition of the land based force with direct fire missiles. This model's defining parameter, rate of attrition, can be equated to the Salvo Model's Offensive Power. Due to assumptions eight and fifteen above, this Offensive Power is not degraded by a Defensive Power or Staying Power.

The basic idea of the Aimed Fire model is presented:

$$\text{Change in Force Level} = \text{Rate of Attrition}_{\text{Opponent}} * \text{Force Level}_{\text{Opponent}} \qquad (25)$$

The Littoral Combat Model's parameters are added:

α_1 : Number of well-aimed direct fire sea based missiles per each ship per salvo (Striking Power).

Giving the basic force level attrition formula for a direct fire missile with one kill of a land based force per well-aimed shot:

$$\Delta B = \alpha_1 A \qquad (26)$$

F.   AREA FIRE AND MTR

As the next two situations deal with area fire it is worth further understanding this weapon type. In area fire situations, some total area is being subjugated to fire from a munition incapable of covering the whole area in one round. In order to increase attrition upon the enemy with area fire weaponry, the targeted area could be decreased, indicating a higher degree of targeting information, or the area of the munitions' lethality can be

increased, covering more of the targeted area. In many combat situations, it is preferable to decrease the size of the entire targeted area as this results in less collateral destruction.

Taking into account the dynamics of area fire situations, and in an attempt to decrease the number of parameters inserted into a model already crowded with inputs (helping to improve its simplicity and computing efficiency), a new parameter is developed that combines both the total targeted area and the lethal area of the weapons systems. The Munitions' Lethality, Targeted Area Ratio (MTR) is used throughout this work in lieu of inputting both a lethal area of the munition and the total targeted area in every instance. It is presented here as:

$$MTR = Area_{Lethality} \big/ Area_{Total} \qquad (27)$$

Where:

$Area_{Lethality}$: The lethal area of one round of munition.

$Area_{Total}$: The total targeted area.

MTR is essentially the probability that an individual point target is hit by a random shot, having a certain lethal area, when fired into a designated total area. Understanding MTR as a probability is probably the easiest way of dealing with it as a parameter.

As the MTR increases to one, it describes the closing together of the total targeted area and the area that can be made lethal by a single munition. A MTR of one may be thought of as a perfectly aimed shell which must strike its target. For most current combat situations, very low MTR's are likely the norm; such low-value MTR's are the ones utilized in this analysis. Note, however, that future weapons systems have the potential for much greater accuracy and lethality, with a corresponding increase in MTR.

Comparing the effectiveness of area fire versus direct fire weapon types is a component of this study. A caveat must be applied however: The nature and type of the different weapons systems may make it hard to place them in either category. 'How precise is a precision weapon?' is an eternal question of combat analysis.

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G. LAND BASED FORCE ATTRITION WITH AREA FIRE MISSILES

Lanchester's Area Fire Model is used for land based force attrition using an area fire missile. The basic idea:

$$\text{Change in Force Level} = Power_{\text{Offensive}} \frac{Area_{\text{Lethal}}}{Area_{\text{Total}}} \text{Force Level}_{\text{Own}} * \text{Force Level}_{\text{Opponent}} \quad (28)$$

The appropriate parameters:

α_2 : Number of well-aimed area fire sea based missiles per each ship per salvo (Striking Power).

MTR_{msl} : Munitions' Lethality, Targeted Area Ratio for area fire missiles.

The basic force level change formula for an area fire missile salvo:

$$\Delta B = \alpha_2 MTR_{msl} AB \quad (29)$$

H. LAND BASED FORCE ATTRITION WITH NAVAL GUNNERY

Many naval forces are armed with gunnery systems in addition to land-attack missiles. These gunnery systems have a long and effective history of use in Coastal Combat and the coming of the missile age has done little to demean the many beneficial aspects a sea based gunnery system can bring to an engagement.

It is assumed that naval gunnery acts in an area fire mode. Though many systems are being developed that permit naval gunnery to be much more precise than in previous times, the current and near-future uses will continue to have area fire characteristics. These concepts are the same as seen in Formula 28.

Offensive Power takes on a very different meaning with naval gunnery. Whereas before it was the number of missiles in a salvo, now it is the number of effective rounds fired by the sea based gun system over a certain period of time. This period of time is defined by the situation. If the gun begins firing as soon as the opposing missile salvo launches, then it is the amount of time the gun can continue firing until a missile, if it overcomes the defenses, hits the ship. Note also that many gun systems serve dual

purposes of land attack and air defense, so the amount of time spent engaging the target on land may be very short, as the gun will need to switch to a defensive mode. Since many gunnery systems are defined by their rate of fire, applying a time limit for the salvo opportunity to this rate produces a solid number that can be used within the model.

The following parameters are used:

α_3: Firing Rate of the sea based gunnery system; number of individual gunnery rounds fired per each ship per salvo before the damage from an enemy missile salvo must be taken into account (Striking Power).

MTR_{gun}: Munitions' Lethality, Targeted Area Ratio for the gunfire system.

The basic force level change formula for naval gunnery:

$$\Delta B = \alpha_3 MTR_{gun} AB \tag{30}$$

I.   EMBELLISHMENTS

As in the Salvo Model, certain embellishments are applied to enable further analysis of these engagements. These come at the cost of increased complexity for the model. Three of the basic embellishing factors defined by the salvo model are utilized:

σ_i: Targeting Capability of force i, $0 \leq \sigma \leq 1$.

ρ_i: Deception Capability of force i, $0 \leq \rho \leq 1$.

δ_i: Defensive Readiness of force i, $0 \leq \delta \leq 1$.

The training multiplier could certainly be used, but in the interest of keeping the model as simple as possible and with the knowledge that such a factor is very hard to define, it has been left out of the model.

These factors are then applied to the basic parameters as follows:

$$\alpha_1' = \sigma_A \alpha_1 \tag{31}$$

$$\beta' = \sigma_B \rho_A \beta \tag{32}$$

$$a_3' = \delta_A a_3 \tag{33}$$

J. FINAL FORMULA AND EXAMPLE OF THE SHIP LAUNCHED LAND ATTACK MISSILE [DIRECT FIRE] VS. COASTAL ANTI-SHIP MISSILE BATTERY MODEL

The two formulas for the first case, including embellishments, are:

$$\Delta B = \sigma_A \alpha A \qquad (34)$$

$$\Delta A = \frac{\sigma_B \rho_A \beta B - \delta_A a_3 A}{a_1} \qquad (35)$$

To present an example of the direct fire land attack missile engaged against a coastal ASCM battery the following scenario is used. Suppose there is a sea based force composed of four ships, each ship having two direct fire missiles in its salvo. These ships are capable of shooting down two incoming missiles and they suffer a mission kill after only one missile hit. The sea based force's Targeting and Deception Capabilities are at half of their full value, perhaps due to the cluttered shipping environment in which they operate. Finally, their Defensive Readiness is also at half of its full value, possibly due to large amounts of friendly forces in the region, cluttering the tactical picture and delaying active response. The land based force consists of six launching sites, each with four missiles in its salvo. Consistent with the cluttered shipping environment, their Targeting Capability is at half of its full value. The values for parameters of both the sea based force and the land based force are shown in Table 1.

Table 1.   First Situation's Mini-Scenario Parameters

Sea Based Force Parameters				Land Based Force Parameters	
Sea Based Force Size	A = 4	Targeting Capability of the Sea Based Force	σ_A = .5	Land Based Force Size	B = 6
Number of Well-Aimed Direct Fire Missiles	α_1 = 2	Deception Capability of the Sea Based Force	ρ_A = .5	Number of Well-Aimed ASCMs	β = 4
Staying Power	a_1 = 1	Defensive Readiness of the Sea Based Force	δ_A = .5	Targeting Capability of the Land Based Force	σ_B = .5
Defensive Power	a_3 = 2				

Assuming both forces fire a salvo at the same time, neither one gaining a surprise advantage, the attrition amount after this first pulse would be:

$$\Delta B = \sigma_A \alpha_1 A = .5 * 2 * 4 = 4 \tag{36}$$

$$\Delta A = \frac{\sigma_B \rho_A \beta B - \delta_A a_3 A}{a_1} = \frac{.5*.5*4*6 - .5*2*4}{1} = 2 \tag{37}$$

Giving us a FER of:

$$FER = \frac{A_{\text{Fraction Destroyed}}}{B_{\text{Fraction Destroyed}}} = \frac{\Delta A/A}{\Delta B/B} = \frac{2/4}{4/6} = \frac{.5}{.667} = .75 \tag{38}$$

The sea based force, given sufficient inventory over the course of the engagement, will be able to destroy all of the land based force (FER < 1). Note that the sea based force was able to completely negate its opponent, even though it started off with a salvo size of only 8 missiles as opposed to the land based force's salvo size of 24 missiles. This brief example shows the value of Defensive Power for a sea based force.

K.   FINAL FORMULA AND EXAMPLE OF THE SHIP LAUNCHED LAND ATTACK MISSILE [AREA FIRE] VS COASTAL ANTI-SHIP MISSILE BATTERY MODEL

The two formulas for the second case, including embellishments, are:

$$\Delta B = \alpha_2 MTR_{msl} AB \tag{39}$$

$$\Delta A = \frac{\sigma_B \rho_A \beta B - \delta_A a_3 A}{a_1} \tag{40}$$

The scenario is slightly modified for this type of engagement, in order to demonstrate the workings of the model. Let the sea based force again consist of four ships, this time each ship having a salvo size of ten area fire missiles. Each munition, a rocket systems armed with a load of cluster bombs, has a lethal area of 100 square yards. The enemy is known, from targeting information, to be within an area of 10,000 square yards. The Deception Capability and Defensive Readiness values remain at half of their full value. The Staying Power and Defensive Power take their previous values of one and two respectively. The land based force keeps all of its previous values: six launchers with four missiles each and a targeting parameter of half its full value. Table 2 summarizes each of the parameters for this mini-scenario.

Note that the important factor of Targeting Capability of the sea based force and the Deception Capability of the land based force for the scenario have not been discussed. Both of these parameters are inherent in the $area_{Total}$ parameter.

Table 2. Second Situation's Mini-Scenario Parameters

Sea Based Force Parameters				Land Based Force Parameters	
Sea Based Force Size	A = 4	Deception Capability of the Sea Based Force	ρ_A = .5	Land Based Force Size	B = 6
Number of Well-Aimed Direct Fire Missiles	α_2 = 10	Defensive Readiness of the Sea Based Force	δ_A = .5	Number of Well-Aimed ASCMs	β = 4
Staying Power	a_1 = 1	Area of Munitions Lethality	$area_{Lethality}$ = 100	Targeting Capability of the Land Based Force	σ_B = .5
Defensive Power	a_3 = 2	Total Targeted Area	$area_{Total}$ = 10000		

First, find the Munitions' Lethality, Targeted Area Ratio:

$$MTR_{msl} = \frac{Area_{Lethality}}{Area_{Total}} = \frac{100}{10000} = .01 \tag{41}$$

In the first salvo, the following attrition occurs:

$$\Delta B = \alpha_2 MTR_{msl} AB = 10*.01*4*6 = 2.4 \tag{42}$$

$$\Delta A = \frac{\sigma_B \rho_A \beta B - \delta_A a_3 A}{a_1} = \frac{.5*.5*4*6 - .5*2*4}{1} = 2 \tag{43}$$

Giving a FER as follows:

$$FER = \frac{A_{\text{Fraction Destroyed}}}{B_{\text{Fraction Destroyed}}} = \frac{\Delta A/A}{\Delta B/B} = \frac{2/4}{2.4/6} = \frac{.5}{.4} = 1.25 \tag{44}$$

The land based force will win, destroying all of the naval forces and having survivors (FER > 1). Assuming sufficient inventory and steady rates of fire for each launcher, the remaining elements of the coastal ASCM battery will continue to be a threat for any other naval force.

L. FINAL FORMULA AND EXAMPLE OF THE NAVAL GUNNERY VS COASTAL ANTI-SHIP MISSILE BATTERY MODEL

The two formulas for the third case, including embellishments, are:

$$\Delta B = \alpha_3 MTR_{gun} AB \tag{45}$$

$$\Delta A = \frac{\sigma_B \rho_A \beta B - \delta_A a_3 A}{a_1} \tag{46}$$

The scenario is again modified to demonstrate an engagement where the sea based force solely relies on its gunnery systems. Let the sea based force consist of four ships, each ship firing 60 rounds from their gunnery systems in a pulse before suffering attrition. Each round has a lethal area of 60 square yards and the enemy is targeted within an area of 60,000 square yards. The Deception Capability and Defensive Readiness values remain at half of their full value. The Staying Power and Defensive Power parameters keep their previous values of one and two respectively. The land based force keeps all of its previous values: six launchers with four missiles each and a targeting parameter of half its full value. Table 3 summarizes each of the parameters for this mini-scenario.

Table 3.   Third Situation's Mini-Scenario Parameters

Sea Based Force Parameters				Land Based Force Parameters	
Sea Based Force Size	$A = 4$	Deception Capability of the Sea Based Force	$\rho_A = .5$	Land Based Force Size	$B = 6$
Number of Well Aimed Direct Fire Missiles	$\alpha_3 = 60$	Defensive Readiness of the Sea Based Force	$\delta_A = .5$	Number of Well-Aimed ASCMs	$\beta = 4$
Staying Power	$a_1 = 1$	Area of Munitions Lethality	$area_{Lethality} = 60$	Targeting Capability of the Land Based Force	$\sigma_B = .5$
Defensive Power	$a_3 = 2$	Total Targeted Area	$area_{Total} = 60000$		

Note, as with the area fire missile, that Targeting Capability of the sea based force and the Deception Capability of the land based force have been left out. Also, in this scenario the rate of fire (the number of rounds sent down-range) has been greatly increased, as is generally the case when comparing a missile system to a gunnery system.

In keeping with reality, the MTR of the munitions has been decreased, with a smaller lethal area and a larger targeted area.

The Munitions' Lethality, Targeted Area Ratio:

$$MTR_{gun} = \frac{Area_{Lethality}}{Area_{Total}} = \frac{60}{60000} = .001 \qquad (47)$$

In the first salvo, the following attrition rates occur:

$$\Delta B = \alpha_3 MTR_{gun} AB = 60*.001*4*6 = 1.44 \qquad (48)$$

$$\Delta A = \frac{\sigma_B \rho_A \beta B - \delta_A a_3 A}{a_1} = \frac{.5*.5*4*6 - .5*2*4}{1} = 2 \qquad (49)$$

Giving a FER as follows:

$$FER = \frac{A_{\text{Fraction Destroyed}}}{B_{\text{Fraction Destroyed}}} = \frac{\Delta A/A}{\Delta B/B} = \frac{2/4}{1.44/6} = \frac{.5}{.24} = 2.08 \qquad (50)$$

Here again the land based force is the victor, destroying all of the naval forces (FER > 1).

As both of the last two mini-scenarios demonstrate, the potency of the direct fire weapon (the shore-based ASCM battery) against an area fire weapon (the area fire missiles or naval gunnery systems) appears to be quite high. This will be an area of further study.

M.   SIMULTANEOUS ATTACK VERSUS SURPRISE ATTACK

The three examples presented above assumed that both sides have released their Offensive Pulse at approximately the same time, a Simultaneous Attack. This type of equal timing is, however, rarely the case in an actual engagement. Through superior scouting capability, one side will, most likely, gain enough information about the enemy to enact a first, unanswered attack. The gathering of information continues to prove itself one of the more important factors playing a role in salvo warfare.[5], [9], [10]

This analysis uses different circumstances of attack to describe the effects of scouting (or lack thereof): Sea Force Initial Attack, Land Force Initial Attack, and Simultaneous Attack.

N.   LIMITATIONS AND A NOD TO REALITY

The results presented by the Littoral Combat Model are mere insights, not predictions. The simplicity of the model gives it a great ability to produce a clear insight into problems, but prevents it from making quantitative predictions about actual engagements. The model ignores losses to specific ships. The assumed uniform distribution of fire negates any ability to quantify the loss of one or another ships in the sea based force. Modifying the model in future studies to enable this analysis is possible, as was done with the Salvo Model.[11] This type of detail may be important in some tactical situations. For instance, during an Amphibious Assault, the loss of a transport ship full of Marines and their equipment may be more detrimental to the force's overall mission objective than the loss of an escort ship. Another limitation deals with the assumptions relating to the gunnery system's engagement with a shore-based ASCM battery. The two types of weapons systems represent distinctly different situations, a quick pulse of direct missile fire versus a continuous flow of area gunfire over time. These assumptions serve to simplify the model while not, hopefully, degrading its analytical value. By using a firing rate of rounds per minute of a gun system, and then applying the flight time of the attacking missile system, a fairly accurate pulse value is, hopefully, provided.

Modern gunfire weapons systems, using the latest precision fire technology, are capable of a much higher level of accuracy than previously attained. As gunfire systems are fielded with this increasing gunnery and improved lethality, the MTR values may rapidly increase. Though this work did not place gunnery systems in a direct fire or near direct fire situation, it is certainly possible for this to be the case.

O.   PREVENTING DIVISION BY ZERO

Readers will note the possibility of division by zero during the Land Force Initial Attack. If the initial surprise attack manages to wipe out the sea based force, so that the change in the sea based force is greater than the sea based force's original value, then there will be no attrition of the land based force during the sea based force's re-attacking opportunity, as shown:

$$\text{If } \Delta A \geq A \text{ and } \Delta A / A = k, \text{ then } k \geq 1 \tag{51}$$

$$\text{If } \Delta A / A = k \text{ and } k \geq 1, \text{ then } \Delta B = 0 \text{ so } \Delta B / B = 0 \tag{52}$$

$$\text{Thus } FER = \frac{A_{\text{Fraction Destroyed}}}{B_{\text{Fraction Destroyed}}} = \frac{\Delta A/A}{\Delta B/B} = \frac{k}{0} \tag{53}$$

When FER goes to infinity, a computing tool cannot deal with this situation. To ensure that a denominator of zero in the FER equation will never occur, the sea based force, A, is forced to artificially retain a negligible force level of .01, even if it should be annihilated.

$$\text{RULE: If } \Delta A > A, \text{ then let } A = 0.01, \text{ which gives } \Delta B \neq 0, \text{ and thus } \Delta B / B > 0 \tag{54}$$

Sample problems during model verification found that a value of .01 adequately described the situation occurring: FER values show that the land based force has inflicted a catastrophic defeat on the sea based force.

P.   REASONS FOR A NON-LINEAR RELATIONSHIP BETWEEN PARAMETERS

In the initial runs of the model very exponential curves were discovered when viewing the output. In investigating the reason for these extreme curves, a breakdown of the model for the FER equation with all different attack situations in the Simultaneous Attack circumstances was conducted; Equations 55 through 57 show the results of this breakdown. Note the interactions that exist between the different parameters in the final equation (on the far right-hand-side of each figure). The presence of different quotients

and products hints at the resulting very non-linear curves. Large value parameters mixing with small value parameters produce situations where even a slight change in values brings dramatically changed results.

In the following examination of the analysis results, the very non-linear curves common to the Littoral Combat Model are prevalent. These curves play a role in better understanding the very dynamic relationship between parameters of the Littoral Combat Model. These dynamic relationships lead to some of the themes of land-sea combat interactions.

$$FER = \frac{\Delta A}{A} \bigg/ \frac{\Delta B}{B} = \frac{\dfrac{\beta' B - a'_3 A}{a_1}}{A} \bigg/ \frac{\alpha'_1 A}{B} = \frac{B}{a_1 A}\left[\frac{\beta' B}{\alpha'_1 A} - \frac{a'_3}{\alpha'_1}\right] \tag{55}$$

$$FER = \frac{\Delta A}{A} \bigg/ \frac{\Delta B}{B} = \frac{\dfrac{\beta' B - a'_3 A}{a_1}}{A} \bigg/ \frac{\alpha'_3 AB}{B} = \frac{1}{a_1 A}\left[\frac{\beta' B}{\alpha'_3 A} - \frac{a'_3}{\alpha'_3}\right] \tag{56}$$

$$FER = \frac{\Delta A}{A} \bigg/ \frac{\Delta B}{B} = \frac{\dfrac{\beta' B - a'_3 A}{a_1}}{A} \bigg/ \frac{\alpha'_2 AB}{B} = \frac{1}{a_1 A}\left[\frac{\beta' B}{\alpha'_2 A} - \frac{a'_3}{\alpha'_2}\right] \tag{57}$$

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IV. SCENARIO PRESENTATION

A. INTRODUCTION

This short chapter describes the two scenarios used to analyze the Littoral Combat Model, defined as Down-Scale and Up-Scale. Neither scenario is likely to be identical to a real-life situation, but they are broadly similar to a number of modern settings in the realm of littoral warfare. The baseline scenario numbers for each of the parameters are presented first, followed by the resulting FER's.

B. DOWN-SCALE SCENARIO

The Down-Scale scenario features a force of Littoral Combat Ships (LCS's) operating near a coastline controlled by a well-funded and organized insurgent force. The LCS force must operate within sight of the shoreline in order to conduct its mission, e.g. blockade, surveillance, or amphibious assault preparation. The insurgent forces have an Area-Denial capability in the form of a small number of truck mounted ASCM's. The analysis presumes future land attack capabilities for LCS are present, consisting of both land attack gunnery systems and missiles.

The land based force consists of three launch units, each armed with two sub-sonic ASCM's ready to fire. Because the LCS force is close to the shore, the land force possesses a high Targeting Capability ($\sigma_B = .8$).

The sea based force consists of four LCS platforms with land attack capabilities. The ships each contain two Non-Line Of Sight (NLOS) Launch Systems totaling 30 NLOS missiles. The reservation of rounds for other land attack missions permits a single salvo of four direct fire missiles or ten indirect fire missiles for each ship (there being a need for more direct fire missiles as a defense against fast attack craft). Each ship possesses a single Mk 110 57mm gun mount with a firing rate of 220 rounds per minute. Assuming the enemy missile has a time of flight of around 25 seconds, and that each gun possesses dud rounds or other firing errors, the gunfire pulse is 80 rounds. The LCS

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defensive ability is relatively good, but it is degraded by the proximal combat environment, causing short reaction times. Thus, Defensive Power is assessed as negating two incoming missiles for each LCS. A single incoming ASCM is capable of causing a mission kill of a LCS platform, so Staying Power has a value of one.

Due to the high number of friendly aircraft in the vicinity, causing constant IFF problems, and the inherent difficulties sensors and communications face in the littoral environment, the Defensive Readiness level is low ($\delta_A = .2$). Also lowered is Deception Capability, again due to the proximity of the launch sites and the ship's short reaction time, preventing full deployment of appropriate deception measures ($\rho_A = .8$). Finally, the LCS squadron's own Targeting Capability is not perfect, due to deception and counter-targeting measures by the insurgents ($\sigma_A = .1$).

The following two equations present the MTR values assumed for both the area fire missile and area fire gunnery of the sea based force:

$$MTR_{msl} = \frac{Area_{Lethality}}{Area_{Total}} = \frac{5000\,ft^2}{500,000\,ft^2} = .01 \tag{56}$$

$$MTR_{gun} = \frac{Area_{Lethality}}{Area_{Total}} = \frac{2000ft^2}{2,000,000\,ft^2} = .001 \tag{57}$$

Note the greatly increased $Area_{Total}$ of the sea based gunnery system compared to that of the area fire missile. An assumption is made that utilization of the NLOS missile in its area fire mode appreciably 'narrows' the area in which the target may be located. This assumption is based on the firing rates of the two platforms. The gunnery system has a high firing rate and a larger number of rounds in the magazine, thus permitting its use in situations with a wider targeted area. The total targeted area for the gunnery system is 222,222 square yards and the total targeted area for the area fire missile system is 55,556 square yards; for purposes of comparison, the standard U.S. Football field is 6400 square yards. In either case, the scenario does not assume blind firing into a large area but rather firing into the generally known area of the enemy. Table 4 contains the values of the different parameters utilized for the baseline in the Down-

Scale scenario. The Fractional Exchange Ratios of this baseline Down-Scale Scenario for all of the different attack types (Sea Force Initial, Land Force Initial, and Simultaneous) and all the different situations (direct fire missile, area fire missile, and naval gunnery) are displayed in Table 5.

Table 4.   Down-Scale Scenario Baseline Parameters

Sea Based Force Parameters						Land Based Force Parameters	
Sea Based Force Size	A = 4	Staying Power	a_1 = 1	Targeting Capability of the Sea Based Force	σ_A = .1	Land Based Force Size	B = 3
Direct Fire Missiles	α_1 = 4	Defensive Power	a_3 = 2	Deception Capability of the Sea Based Force	ρ_A = .8	Number of Well-Aimed ASCMs	β = 2
Area Fire Missiles	α_2 = 10	Munitions Targeting Ratio, Msl	$MTR_{Missile}$ = .01	Defensive Readiness of the Sea Based Force	δ_A = .2	Targeting Capability of the Land Based Force	σ_B = .8
Gunfire Rounds	α_3 = 80	Munition Targeting Ratio, Gun	MTR_{Gun} = .001				

Table 5.   Down-Scale Scenario Baseline FER's

Engagement Situation	Sea Force Initial Attack	Land Force Initial Attack	Simultaneous Attack
LAM (direct) versus ASCM	0.09	2.39	1.05
LAM (area) versus ASCM	0.44	3.18	1.4
Naval Gunnery versus ASCM	0.79	3.98	1.75

C.   UP-SCALE SCENARIO

The Up-Scale scenario revolves around a contested transit by a group of VLS-equipped surface warships through a narrow oceanic strait. The land forces possess a large supply of powerful ASCM's, and are attempting to close the strait. The surface warships seek to overcome this attempt at Area-Denial.

The land force possesses eight ASCM launch units, each launch unit containing four super-sonic ASCM's. The narrowness of the straits and the possession of adequate surveillance capabilities enhances the land based force's Targeting Capability ($\sigma_B = .8$).

The sea force making the straits transit is composed of six guided missile destroyers (DDG's). Each ship has a salvo size of four land attack missiles in both the direct attack and indirect attack situations, the small number is caused by a need for a reserve of land attack missiles for future use and the presence of many other types of missiles in the VLS magazines (e.g., Standard Missiles for Air Defense). The ships are armed with the 5"/62 gun system, with a firing rate of 20 rounds per minute. Due,

however, to the flight time of the land force's supersonic missiles and the different types of ready ammunition kept available, only 11 rounds make up the gunfire pulse. The DDG's are capable of sustaining one hit before a mission kill. Due to the narrowness of the straits and the speed/maneuverability of the incoming ASCM's, the DDG's are only able to shoot down, assuming they have perfect alertness, two of the incoming missiles. The land based force has been able to severely reduce the effectiveness of the sea based force's Targeting Capability ($\sigma_A = .1$). The speed of the ASCM's and the large size of the possible threat area has provided the sea based force with mediocre Deception Capability ($\rho_A = .6$) and Defensive Readiness ($\delta_A = .6$).

The following two equations present the MTR values assumed for both the area fire missile and area fire gunnery of the sea based force:

$$MTR_{msl} = \frac{Area_{Lethality}}{Area_{Total}} = \frac{5000\,ft^2}{500,000\,ft^2} = .01 \tag{58}$$

$$MTR_{gun} = \frac{Area_{Lethality}}{Area_{Total}} = \frac{4000ft^2}{1,142,857\,ft^2} = .0035 \tag{59}$$

Again, note the disparity between the $Area_{Total}$ of the two weapons systems, caused by the different uses of direct and area fire munitions. The total targeted area for the gunnery system is 126,984 square yards and the total targeted area for the area fire missile system is 55,556 square yards. As in the Down-Scale Scenario, this is a situation where the rounds are not being sent indiscriminately into a large area, but instead are used against an enemy whose general position is known but against which the weapons are not able to provide direct fire. Table 6 shows the parameters utilized for the baseline in this scenario. The Fractional Exchange Ratios of this baseline Up-Scale Scenario for all of the different attack types (Sea Force Initial, Land Force Initial, and Simultaneous) and all the different situations (direct fire missile, area fire missile, and gunnery) are presented in Table 7.

Table 6.   Up-Scale Scenario Baseline Parameters

Sea Based Force Parameters						Land Based Force Parameters	
Sea Based Force Size	A = 6	Staying Power	$a_1 = 1$	Targeting Capability of the Sea Based Force	$\sigma_A = .1$	Land Based Force Size	B = 8
Direct Fire Missiles	$\alpha_1 = 4$	Defensive Power	$a_3 = 2$	Deception Capability of the Sea Based Force	$\rho_A = .6$	Number of Well-Aimed ASCMs	$\beta = 4$
Area Fire Missiles	$\alpha_2 = 4$	Munitions Targeting Ratio, Msl	$MTR_{Missile} = .01$	Defensive Readiness of the Sea Based Force	$\delta_A = .6$	Targeting Capability of the Land Based Force	$\sigma_B = .6$
Gunfire Rounds	$\alpha_3 = 11$	Munition Targeting Ratio, Gun	$MTR_{Gun} = .0035$				

Table 7.   Up-Scale Scenario Baseline FER's

Engagement Situation	Sea Force Initial Attack	Land Force Initial Attack	Simultaneous Attack
LAM (direct) versus ASCM	0.48	8.57	2.4
LAM (area) versus ASCM	1.08	10.71	3
Naval Gunnery versus ASCM	1.2	11.13	3.12

D.   VARYING FER'S

Note from Table 5 and Table 7 the wide range of FER values. These values change dramatically from situations of LAM (direct) versus ASCM with the land force being surprised to those of LAM (area) versus ASCM with the sea force being surprised. These dramatic differences are not surprising, given the nature of both the Littoral Combat Model and littoral combat itself. Changes in situation and circumstance play a major role in determining the outcome of the engagement, more so than individual parameter inputs. The nine different combinations of situation and circumstance displayed in the two tables represent nine very diverse engagement types, the differences displayed amongst the FER's demonstrate this diversity.

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V. ANALYSIS OF RESULTS

A. INTRODUCTION

Seven major themes arose from an analysis of the 11 sea force parameters and are presented in this chapter, accompanied by pertinent graphs and tables that reinforce the concepts. In the analysis, each of the individual sea force parameters were varied over a range of numbers while other parameters remained constant and the FER's noted. Also, EXCEL's Premium Solver was used to find specific parameter values necessary to attain FER levels of 1, .5, and .1. A deeper examination of the interaction between MTR and Striking Power and its effect on FER values for the area fire situations was also conducted. All of the collected data generated during the course of the analysis is presented in Appendix A. The graphs presented are best viewed in color.

B. THE FIRST THEME

Area fire weaponry requires more Staying Power than direct fire weaponry to thoroughly attrite the land based force.

The Staying Power graphs, Figure 2 for the Down-Scale Scenario and Figure 3 for the Up-Scale Scenario, contain the same trend. First though, an explanation of the graphs is necessary. On the Y axis are FER values from 0 to 3 and on the X axis are the Staying Power values, ranging from 1 to 10 hits. The blue lines represent Sea Force Initial Attacks, the red lines represent Land Force Initial Attacks, and the green lines represent Simultaneous Attacks. Solid lines are direct fire missiles, dashed lines are area fire missiles, and dotted lines are naval gunnery. Note the very exponential lines seen in the graph, with steep declines and a clear flattening out. This is a common trend of the Littoral Combat Model and is found in many of the graphs. The nature of the LCM equations causes such non-linear behavior (see Chapter III, Section P).

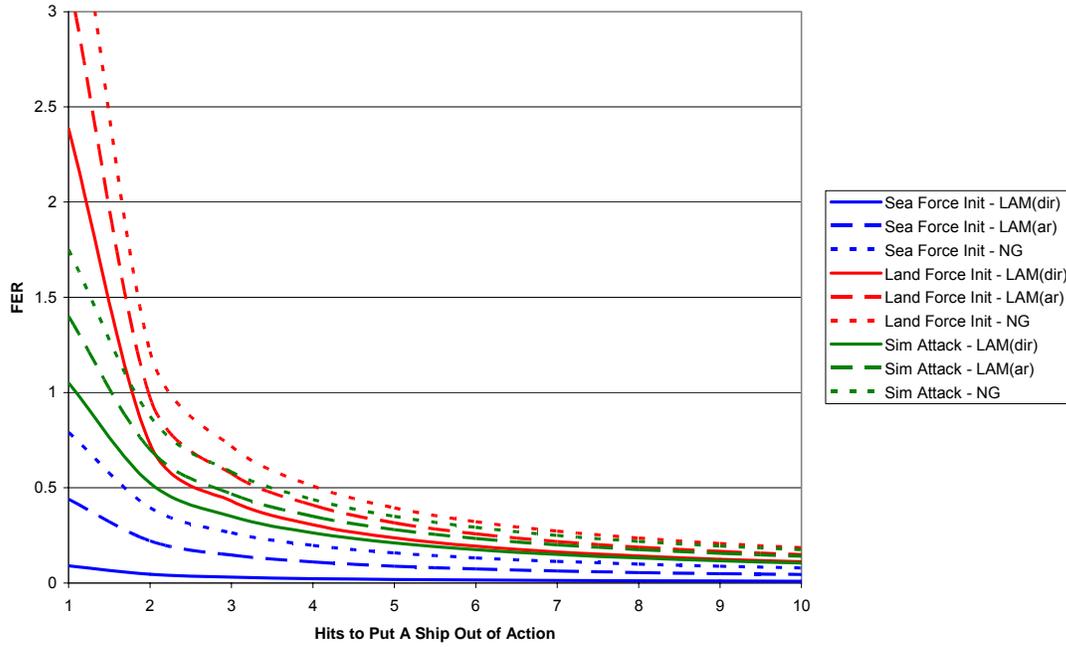Figure 2.   Down-Scale Scenario FER Levels with Increasing Staying Power Values.

Figure 3.   Up-Scale Scenario FER Levels with Increasing Staying Power Values.
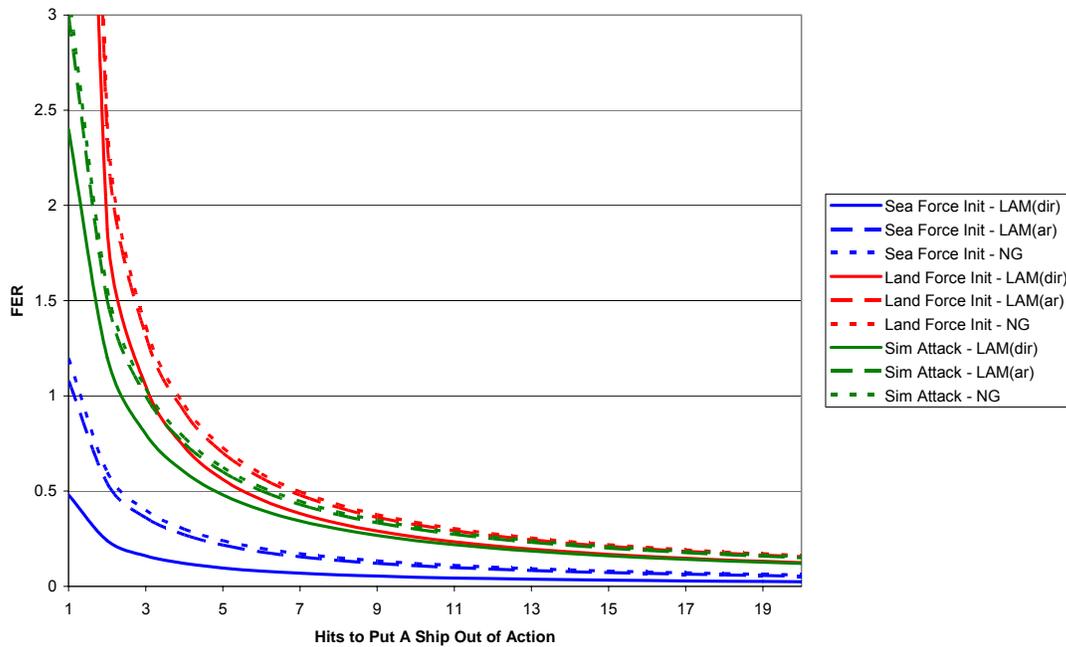
Note in Figures 2 and 3 the change from direct fire weapons to area fire weapons, across all three different attack circumstances, the amount of Staying Power necessary to win the fight increases. This result is intuitive; area fire weaponry requires a cumulative effort to deplete the opposing force, as random fall of shot requires a large number of rounds to guarantee a hit. The force, therefore, needs to be able to stay in the fight long enough to deliver this decisive number of salvos (whether gun or missile); Staying Power enables this cumulative effect.

C.   THE SECOND THEME

Good scouting by the sea based force reduces the size of the force necessary to overcome the land based force.

Note the Sea Force Size graphs, Figure 4 for the Down-Scale Scenario and Figure 5 for the Up-Scale Scenario. Moving from attack circumstances with good scouting by the sea based force, resulting in a Sea Force Initial Attack, to those with poor scouting, resulting in a Land Force Initial Attack, the size of the required force increases. In situations where the initiative rests with the land force, so that the sea based force must absorb its attack, sufficient forces at sea are necessary to offset the attrition that will occur, hence the red lines that appear on the far right. In situations where the sea based force has the initiative, and attacks the land based force first, only forces used to thoroughly attrite the land based force in one salvo are necessary, hence the blue lines appear on the far left. The size of the sea based force need not be overly large if an initial attack can be guaranteed. Conversely, a Land Force Initial Attack requires a sufficiently large sea based force to ensure low FER values. In both scenarios, a certain minimum number of sea based ships are necessary to attain a FER of one, a situation of parity. The model shows that for these fairly representative scenarios, very small numbers of ships (1-3) are in great peril, even under favorable attack circumstances and situations.

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Figure 4.   Down-Scale Scenario FER Levels with Increasing Sea Force Size Values.

Figure 5.   Up-Scale Scenario FER Levels with Increasing Sea Force Size Values.
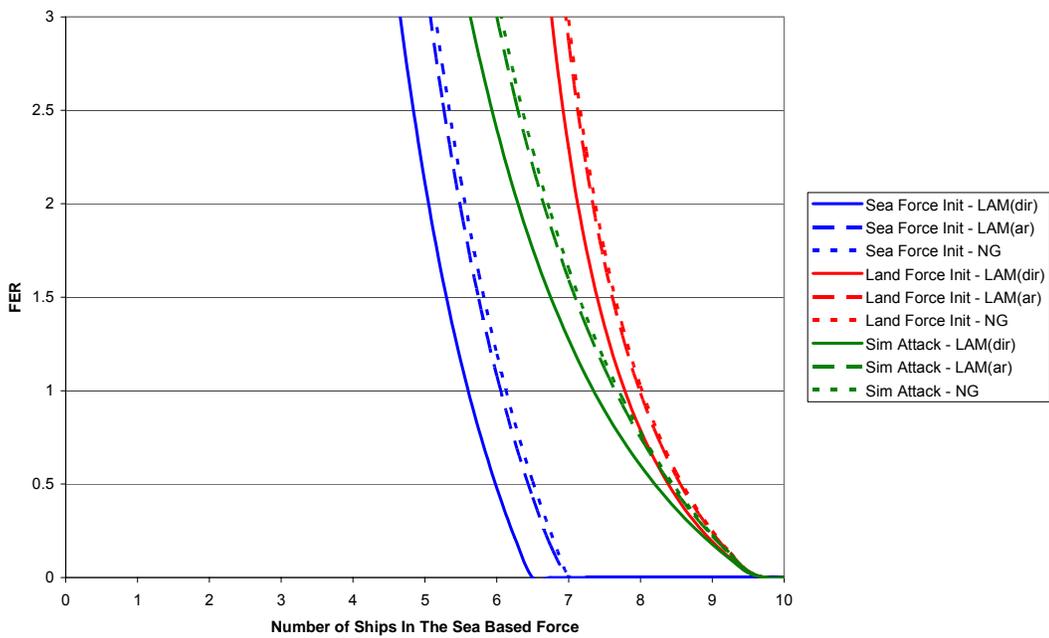
D. THE THIRD THEME

Increasing the quality of targeting information brings dramatic benefits to the outcome of the engagement for the sea based force.

Targeting information is an important element of land-sea combat interactions. Even though a force may be able to get off a first strike, achieving a surprise attack, its targeting information may be poor and its munitions, consequently, fail to hit targets. This situation can be exacerbated by effective counter-targeting efforts by the defender. In Figure 6 for the Down-Scale Scenario and Figure 7 for the Up-Scale Scenario the results of Targeting Capability, σ_A, for the sea based force, as it is varied from none (zero) to perfect (one), are presented. Note that Targeting Capability for the sea based force is only used with LAM(dir) munitions, as the MTR parameter contains the measurement of this capability for area fire situations.

Figure 6.   Down-Scale Scenario FER Levels with Increasing Sea Force Targeting Capability Values.

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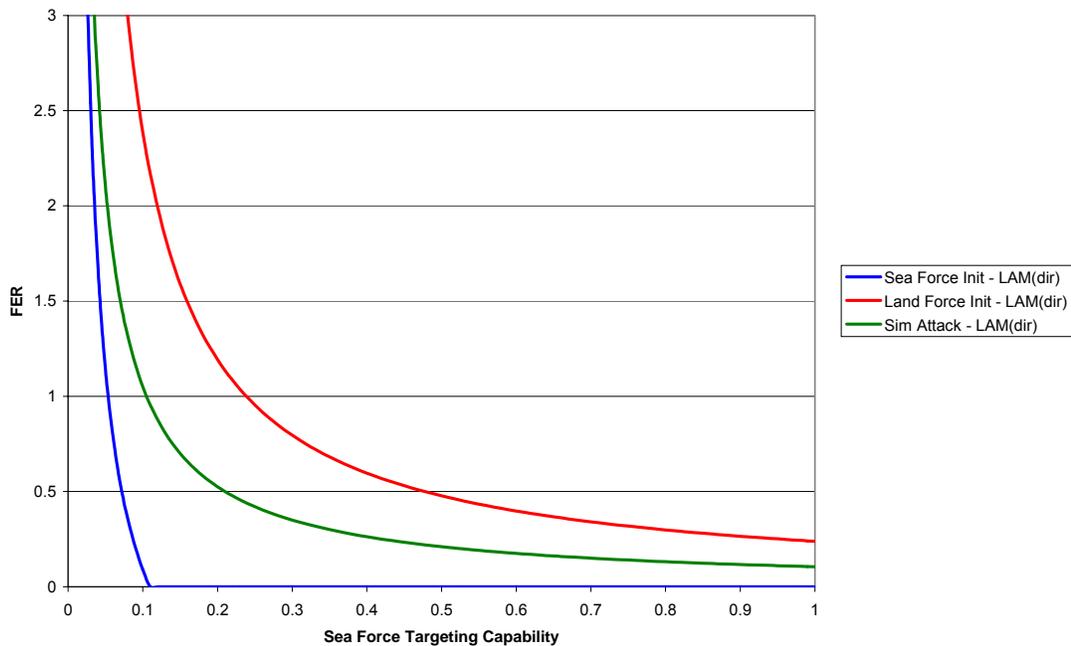
Figure 7.   Up-Scale Scenario FER Levels with Increasing Sea Force Targeting
Capability Values
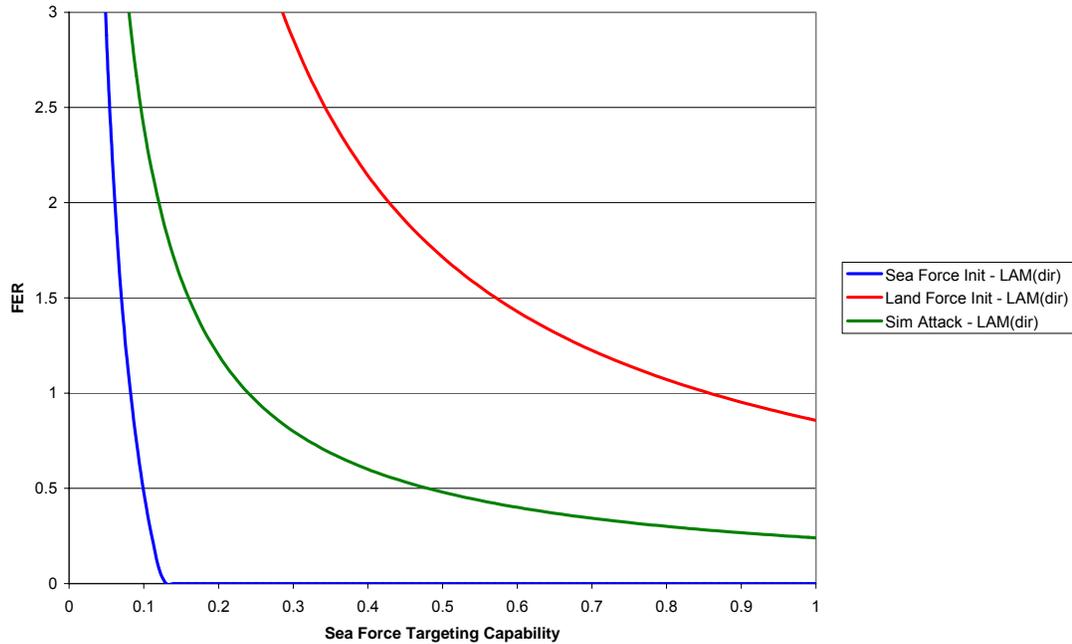
At low levels, for example $\sigma_A = .1$, even a minor increase in Targeting Capability brings about dramatic decreases in FER. This is true for all of the attack circumstances. The benefits of these improvements tend to flatten out as the line moves to higher levels of Targeting Capability, due to the exponential nature of the FER curves. At lower levels of Targeting Capability, all possible efforts should be made to improve this capability in order to ensure these beneficial results. At higher levels, such improvements may not be worth the cost, in terms of resources and time. Of course, every different scenario, with its own unique set of parameter values shaping the results, will have a particular 'knee of the curve.'

E.   THE FOURTH THEME

Defensive Power can prolong the battle, but Striking Power wins the fight quickly.

This theme compares the relationship between two important parameters, Striking Power and Defensive Power. Either, if they are large enough, can produce a FER below one. Yet when the results of the analysis are viewed, Striking Power achieves this low FER more readily than Defensive Power. For the Down-Scale Scenario Figure 8 shows

the direct fire Striking Power and Figure 9 the Defensive Power. For the Up-Scale Scenario Figure 10 shows direct fire Striking Power and Figure 11 the Defensive Power. It is obvious in both scenarios that Defensive Power can, if large enough, defeat the pulse of a land based force, no matter the circumstances of the attack. In many cases, however, the amount of Defensive Power necessary for this to occur is impractically large, especially when the land based force conducts an initial surprise attack. Note the dramatic effect even minor amounts of Striking Power can have on the FER, especially at lower levels. Keep in mind, however, that in some cases, such as a Land Force Initial Attack, large amounts of Striking Power are necessary to achieve the desired low levels of FER.

A related, theoretical, concept should be noted. A force with strong Defensive Power may be able to successfully survive its opponent's attack. If, however, it lacks adequate Striking Power, it will not be able to thoroughly attrite the other force. From this perspective, large amounts of Defensive Power prevent defeat, but do not gain victory.

Figure 8.   Down-Scale Scenario FER Levels with Increasing LAM(dir) Values

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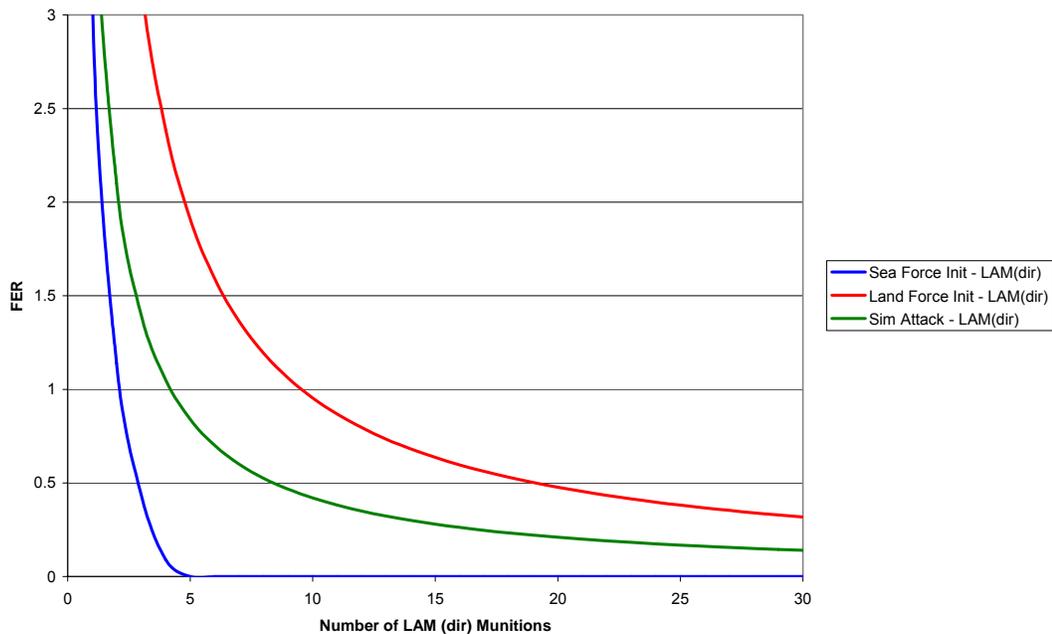
Figure 9.   Down-Scale Scenario FER Levels with Increasing Defensive Power Values
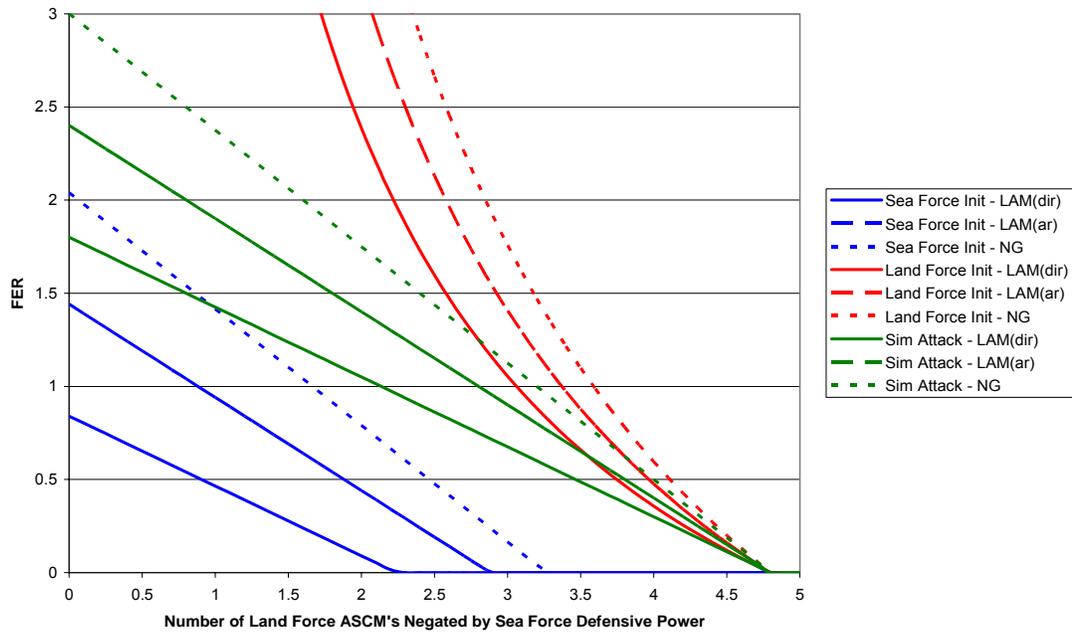
Figure 10.   Up-Scale Scenario FER Levels with Increasing LAM(dir) Values
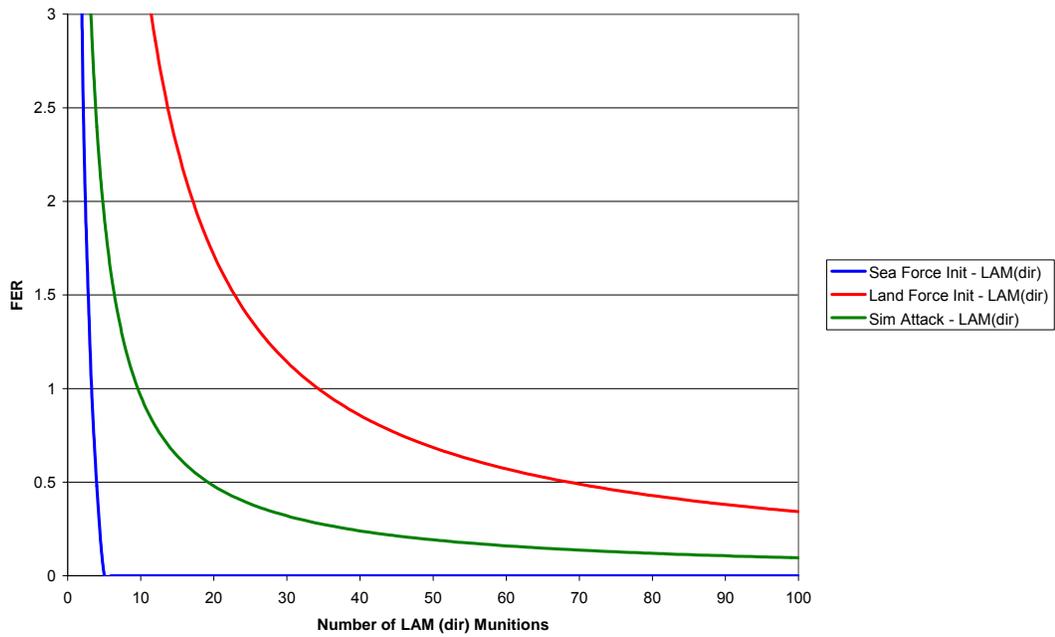
Figure 11.   Up-Scale Scenario FER Levels with Increasing Defensive Power Values
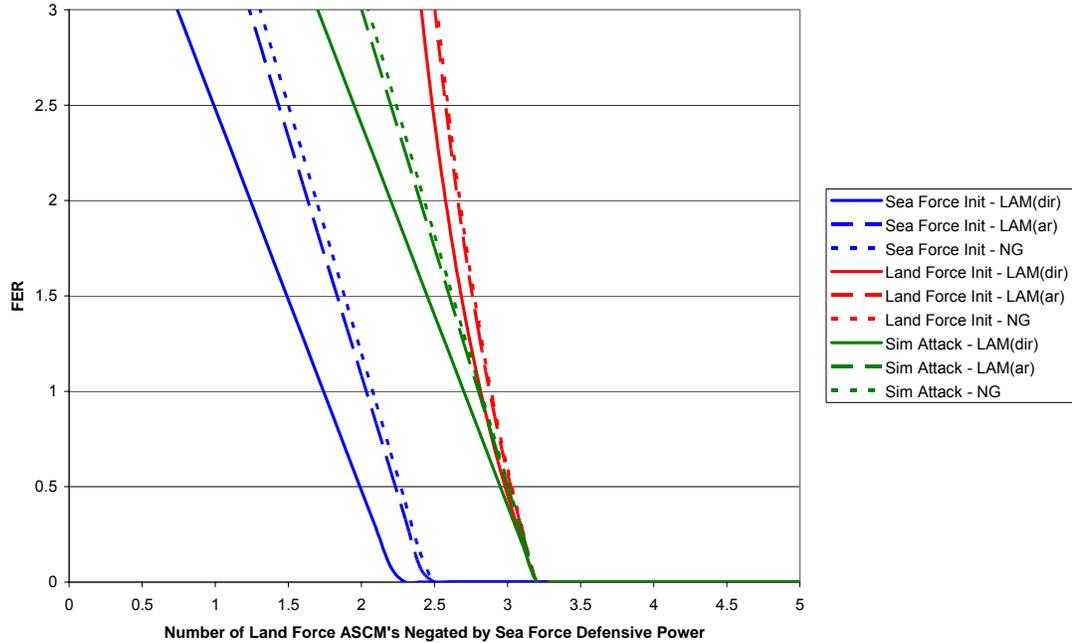
F.   THE FIFTH THEME

A certain level of "unfairness" exists when a land based force with direct fire weapons engages a sea based force with indirect fire weapons.

When the sea based force must rely on area fire munitions to attrite a land based force firing aimed weapons, the sea based force is at a marked disadvantage. Table 8 for the Down-Scale Scenario and Table 9 for the Up-Scale Scenario demonstrate this trend. Tables 8 and 9 represent different Gunnery Striking Power values necessary to obtain certain FER levels. At the top of each column are the desired FER levels (1, .5 & .1), grouped into their specific attack circumstances. Under each of these FER levels are the Gunnery Striking Power values necessary, given the situation. For example, in a Sea Force Initial Attack 71.43 individual naval gunnery rounds per ship per pulse are required to obtain a FER of 1.

In order to achieve low FER levels, the sea based force must expend plentiful amounts of ammunition as its area fire weaponry attempts to attrite the land based forces. During this expenditure, which can take a long period of time, the land based forces will

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have the ability to fire more missiles at the ships at sea, requiring a higher Staying Power in these ships. In situations where the land based force uses direct (aimed) fire ASCM's, only shipborne area fire weapons with high rates of gunfire, very accurate lethal munitions or large amounts of Defensive Power can work to attain low FER levels.[1]

Table 8.   Down-Scale Scenario Gunnery Striking Power Values Needed to Attain Specific Levels of FER

Alpha Three, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
Navy Guns vs. ASCM	71.4285865	95.89045	132.0755	318.1819	636.3643	3181.832969	140	280.0003	1400.007

Table 9.   Up-Scale Scenario Gunnery Striking Power Values Needed to Attain Specific Certain Levels of FER

Alpha Three, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
Navy Guns vs. ASCM	11.74168	14.16765	16.97313	122.449	244.8984	1224.492	34.28574	68.57146	342.8599

G.   THE SIXTH THEME

There exists a Total Victory value with some of the parameters.

The exponential curves of the Littoral Combat Model do not conduct an infinite flattening out. Rather, there are exact parameter values that lead to a FER of zero. A FER of zero represents Total Victory, it can clearly be seen in Figures 4, 5, 9 & 11.

Total Victory values are present in parameters that can, on their own, win the engagement. For example, Figures 4 and 5 show the number of sea based forces necessary to attain winning FER levels. These graphs demonstrate that if the naval force can bring enough ships to the scene, they will either overwhelm the force on land with Striking Power or provide a complete umbrella of Defensive Power protection, perhaps even both. The word 'overwhelming' needs to be caveated by one of the key assumptions

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[1] Future combat systems have the potential for greatly increased MTR values with their improved accuracy and lethality. The MTR values presented in this work will continue to increase into the future, creating even more of an advantage for the sea based force when attacking forces on land with area fire munitions.

of the model, a uniform distribution of fire. A real situation may see a land based force "gang-up" on a smaller portion of the overall force at sea, or vice versa. In this case, the concept of Total Victory by a certain parameter value may be debunked.

In some cases, the Total Victory value can be unrealistically high. For example, a Land Force Initial Attack in the Up-Scale Scenario using naval gunnery requires 1224.5 individual naval gunnery rounds to obtain a FER of .1, as show in Table 9. In the same circumstances and situation a much higher number of individual naval gunnery rounds, 128,398,686, is necessary to obtain a FER of zero (to the precision required by Excel's Premium Solver).

It is also worth observing that there are cases where there may not be any sea based force left to shoot back, such as when there is an overwhelming Land Force Initial Attack. Even a Total Victory amount of Defensive or Striking Power cannot save a force which has failed to adequately scout out the enemy's presence.

H.   THE SEVENTH THEME

The energies of the sea based force should be invested in "attacking effectively first."[5]

The final theme represents, in many ways, a compilation of the ideas discussed in the previous six themes. In order to win the land-sea combat interaction efficiently, the sea based force should concentrate on scouting and attacking effectively first. Note the relative position of the Sea Force Initial Attack lines compared to the Land Force Initial & Simultaneous Attack lines in all of the preceding graphs, and in all of the graphs found in Appendix A. In every case, a Sea Force Initial Attack results in a lower FER when at the same parameter value as the other two attack circumstances.

This point is illustrated with Figures 12 through 14, 3D graphs of the effect of MTR_{msl} and area fire missile Striking Power on FER in the Up-Scale Scenario. The Y axis represents MTR_{msl} while the X axis represents area fire missile Striking Power. The Z axis shows the resulting FER for each of the different combinations of these two sea force parameters. Figure 12 shows a Sea Force Initial Attack. There is a high spike in the

far back corner of the graph, representing the only time that the FER rose above a near-zero level. This is caused by a very small number of inaccurate munitions being fired. In Figure 13, the Simultaneous Attack, and Figure 14, the Land Force Initial Attack, there is no solitary spike but instead, a steadily decreasing sloping surface, representing a continuous drop in FER as the combination of MTR and Striking Power improves. The disparity between these three graphs, a spike in one and a steady slope in the others, demonstrates the quantitative advantage of "attacking effectively first." The sea based force should devote the resources necessary to conduct this first effective attack with precision munitions, namely the gathering of scouting and targeting information.

A peculiarity of area fire weapons is also visible in these graphs: the ability to reduce Striking Power by increased targeting and/or lethality. In every attack circumstance or situation, a low FER level is attained when the sea based force has either (1) a high rate of fire with fairly inaccurate or non-lethal weaponry causing a low MTR (located on the far left of each graph) or (2) a low rate of fire with relatively accurate or lethal weapons causing a high MTR (located on the far right of each graph).

Figure 12.   Up-Scale Scenario FER Levels with Improving MTR(msl) & Greater LAM(ar) Values for a Sea Force Initial Attack
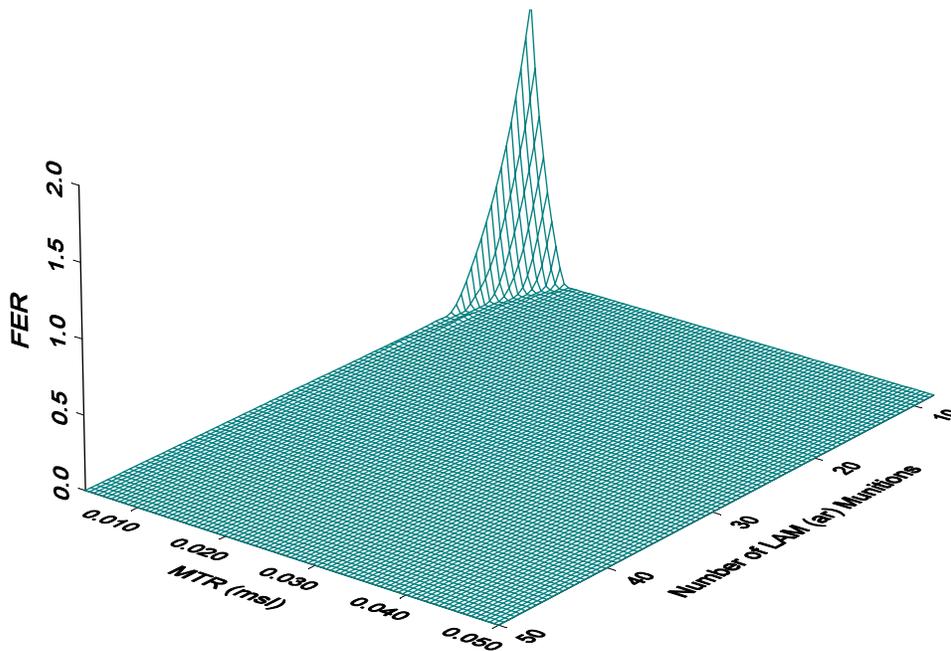
Figure 13. Up-Scale Scenario FER Levels with Improving MTR(msl) & Greater LAM(ar) Values for a Simultaneous Attack
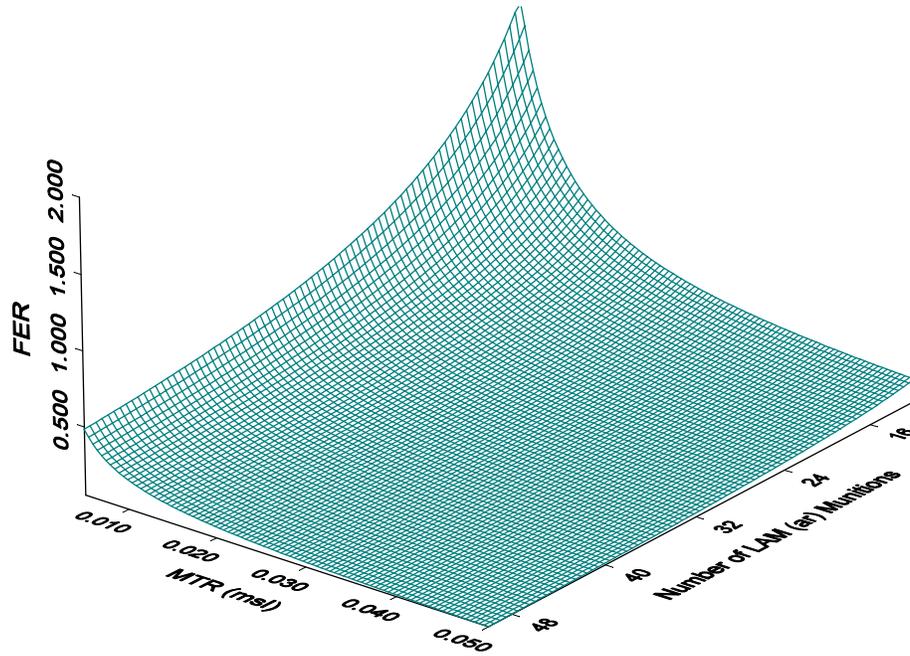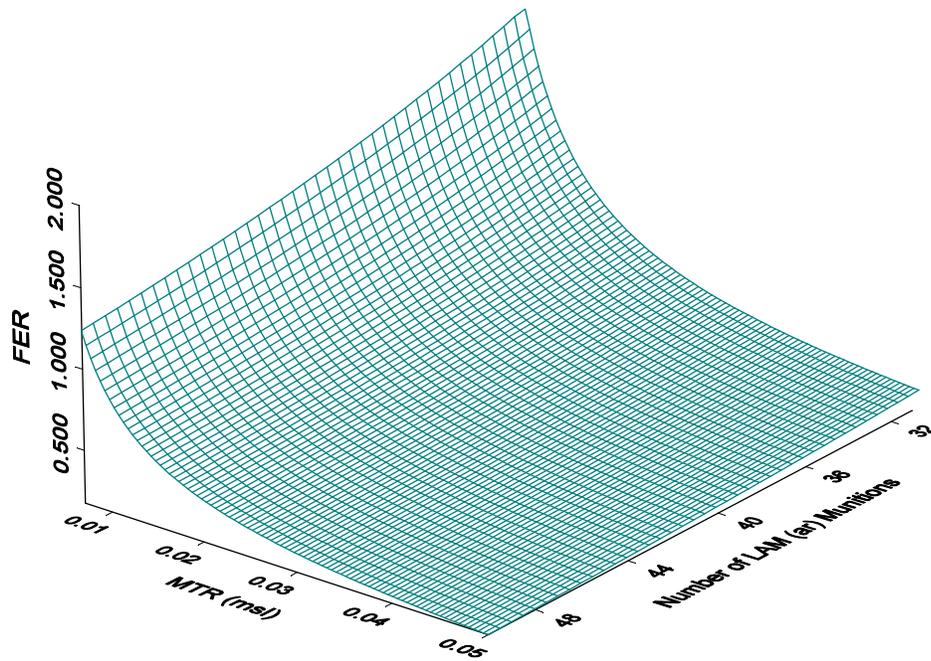

Figure 14. Up-Scale Scenario FER Levels with Improving MTR(msl) & Greater LAM(ar) Values for a Land Force Initial Attack

The advantages of an effective first attack are also shown by a comparison of Striking Power for all three attack circumstances and all three attack situations. Table 10 for the Down-Scale Scenario and Table 11 for the Up-Scale Scenario represent the necessary Striking Power values of all the different circumstance and situation combinations. A comparison of equivalent FER levels for the Sea Force Initial Attack with the other attack circumstances, across the three different attack situations, shows that Striking Power has to increase significantly in a Land Force Initial Attack and even for a Simultaneous Attack. For example, in the Up-Scale Scenario for a Land Force Initial Attack 122.45 individual naval gunnery rounds are required to achieve a FER of 1, but in a Sea Force Initial Attack the same FER is achieved with only 11.74 individual naval gunnery rounds, a considerable decrease. This type of drastic change is common to all of the different attack circumstances and situations. Attacking first can appreciably reduce the amount of firepower expended in attriting the land based force.

Table 10.   Down-Scale Scenario Striking Power Values Needed to Attain Specific Levels of FER.

Striking Power, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	2.14285746	2.876713	3.962265	9.545457	19.09094	95.45488608	4.200002	8.4	42.00042
LAM(ar) vs. ASCM	7.14286058	9.589045	13.20755	31.81819	63.63643	318.1832969	14	28.00003	140.0007
Navy Guns vs. ASCM	71.4285865	95.89045	132.0755	318.1819	636.3643	3181.832969	140	280.0003	1400.007

Table 11.   Up-Scale Scenario Striking Power Values Needed to Attain Specific Levels of FER.

Striking Power, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	3.287671	3.966943	4.752477	34.28573	68.5715	342.8585	9.600002	19.20003	96.0008
LAM(ar) vs. ASCM	4.109589	4.958679	5.940594	42.85715	85.71436	428.5718	12.00001	24.00002	120.0004
Navy Guns vs. ASCM	11.74168	14.16765	16.97313	122.449	244.8984	1224.492	34.28574	68.57146	342.8599

VI. CONCLUSION

A. CONCLUSIONS

The Littoral Combat Model provides new ways to represent the combat interactions between sea based and land based forces when they engage each other in the littorals. Different attack circumstances, based on which side manages to get off the first offensive pulse, and different attack situations, based on the method used by the sea based force to attrite the forces on land, define the engagement, and are taken into account by the Littoral Combat Model. The Littoral Combat Model rests on the premise that pulses of Striking Power (i.e., salvos) play a dominant role in these land-sea combat interactions.

In studying the results of the Littoral Combat Model, it becomes apparent that a ship is not always "a fool to fight a fort." Whereas the ships in previous eras of naval warfare had to deal with time consuming gun battles against entrenched shore batteries, the modern accurate missile systems and efficient targeting ability of today's combatants make the ship a much more competitive weapon system than heretofore. It is now possible for a naval surface task force with direct fire weapons and good targeting to take on potent anti-ship cruise missile shore batteries and, given favorable circumstances, come away the victor. No longer does the land based force possess the advantage of superior Staying Power and the sea based force the disadvantage of having to cause cumulative attrition with powerful but inaccurate gunfire.

Nevertheless, it is also apparent that a fight between forces on land and those at sea is difficult for the sea based force. The right circumstance and situation must be present to assure success. A fleet, in the wrong scenario, cannot successfully challenge shore batteries of ASCM's.

"Attacking effectively first" is the goal of the naval commander when attacking forces on land. Scouting out the enemy to assure an initial surprise attack, targeting accurately all possible land based weapons sites, and attacking with the largest amount of

Striking Power available are three concepts the Littoral Combat Model demonstrates are necessary for the commander at sea to employ.

Like any model, the Littoral Combat Model should only be used as an adjunct and assist. It is a simplification of a complicated scenario and is only as good as the data given. Nor can its simplicity deal with all of the complexities of tactical situations. In the end, the output provided should only be used as input for the commander's decision making process; the results of this model are suggestions to problems in naval warfare, not answers.

Littoral battles between forces on sea and land are a major factor in naval operations now and into the future. With the Littoral Combat Model a method of modeling this unique, complex situation is provided. With this new set of equations and relationships it is possible to dig deeper into the mechanisms of a type of naval warfare that has been given too little analytical thought.

B.   AREAS FOR FURTHER STUDY

There are many topics for further research. The following is a short list of some of them.

(1)   A deeper understanding of the nature of the parameter inputs to the Littoral Combat Model, looking at two-way, three-way, etc interactions that might exist between these parameters and the real-world technologies and doctrine they represent.

(2)   Development of a heterogeneous model that goes past the uniformity of platform, weapon system, and unit targeting inherent in the present Littoral Combat Model. Given the complex and multi-faceted nature of littoral naval combat, especially those that concern amphibious warfare, a model that can deal with destruction of amphibious ships not involved fundamentally in the firing process would be an important step.

(a)   In developing this heterogeneous model, the study should look at the effect of forces both on sea and land. The infrastructure and organization of the missile batteries on land are key components of the battle, not just the launchers themselves.

(3)   Looking at certain types of tactics and doctrine to be used for littoral naval combat could be accomplished with the Littoral Combat Model. Perhaps the necessity of dispersion or concentration tactics by the sea based force

changes as the situation changes. Looking at what factors determine the 'right amount' of Striking Power or Defensive Power to utilize is also important.

    (a)   This study would be especially relevant if it looked at actual naval forces in actual geographic areas, giving a broader understanding of real force requirements for specific missions.

(4)   Development of a stochastic Littoral Combat Model would help understand how the stochastic nature of the real world plays a role in these land-sea combat interactions and could give more insight into the problem.

(5)   Expanding the application of the Littoral Combat Model to situations where the land based force is using gunnery systems (e.g., the Russian Bereg 130mm Self-Propelled Coastal Defense Artillery System) to engage the force at sea vice ASCM's. This would change the nature of the degradation of the sea based forces.

(6)   A study of sea based gunnery systems expected to provide precision fire in the near future could be made with the Littoral Combat Model. In treating these precision gunfire systems as direct fire weapons, rather than the area fire weaponry of their predecessors, while retaining the same characteristics of multiple rounds in a single pulse of offensive power, a better understanding of the value of these new systems in future engagements could be gained.

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APPENDIX A: COLLECTED DATA FROM THE LITTORAL COMBAT MODEL

A.   DESCRIPTION

The following pages are the complete sets of data produced by the Littoral Combat Model for the two scenarios described previously. In the first section, the scenarios' baseline parameters and FER values are repeated for the reader's benefit. In the following 12 sections, tables and graphs outline the changes that occur when a parameter value is varied across a range of numbers.

The tables in these sections give necessary parameter values to attain desired FER levels of 1, .5, and .1 for all the different combinations of circumstance and situation of attack. Note that within some of these tables, especially the embellishment parameters, which take a value between zero and one, the full FER value is not attained. In these instances there is recorded the best FER possible with the maximum value allowed. For example, in Table 29, Up-Scale Scenario Sea Force Targeting Capability, the values for FER of .5 and .1 under Land Force Initial Attack have ".857 @ 1," indicating that the best possible FER obtained was .857 when the Sea Force Targeting Capability was one ($\sigma_A = 1$).

The graphs show the change in FER on the Y axis caused by the changing parameter value on the X axis. The blue lines represent Sea Force Initial Attacks, the red lines represent Land Force Initial Attacks, and the green lines represent Simultaneous Attacks. Solid lines are direct fire missiles, dashed lines are area fire missiles, and dotted lines are naval gunnery. The graphs presented are best viewed in color.

The last section shows the FER produced by changing the Striking Power values of the two area fire situations along with the MTR of that particular situation. A graph exists for each attack circumstance in each of the two scenarios. The Y axis represents the MTR values while the X axis represents the Striking Power values. The Z axis shows the resulting FER for each of the different combinations of these two sea force parameters.

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The Appendix is presented to give the reader the full set of data collected and as a means of further supporting the conclusions presented in the thesis.

B. BASELINE SCENARIO PARAMETER AND FER VALUES

Table 12.   Down-Scale Scenario Baseline Parameters

Sea Based Force Parameters						Land Based Force Parameters	
Sea Based Force Size	A = 4	Staying Power	a_1 = 1	Targeting Capability of the Sea Based Force	σ_A = .1	Land Based Force Size	B = 3
Direct Fire Missiles	α_1 = 4	Defensive Power	a_3 = 2	Deception Capability of the Sea Based Force	ρ_A = .8	Number of Well-Aimed ASCMs	β = 2
Area Fire Missiles	α_2 = 10	Munitions Targeting Ratio, Msl	$MTR_{Missile}$ = .01	Defensive Readiness of the Sea Based Force	δ_A = .2	Targeting Capability of the Land Based Force	σ_B = .8
Gunfire Rounds	α_3 = 80	Munition Targeting Ratio, Gun	MTR_{Gun} = .001				

Table 13.   Down-Scale Scenario Baseline FER's

Engagement Situation	Sea Force Initial Attack	Land Force Initial Attack	Simultaneous Attack
LAM (direct) versus ASCM	0.09	2.39	1.05
LAM (area) versus ASCM	0.44	3.18	1.4
Naval Gunnery versus ASCM	0.79	3.98	1.75

Table 14.   Up-Scale Scenario Baseline Parameters

Sea Based Force Parameters						Land Based Force Parameters	
Sea Based Force Size	A = 6	Staying Power	a_1 = 1	Targeting Capability of the Sea Based Force	σ_A = .1	Land Based Force Size	B = 8
Direct Fire Missiles	α_1 = 4	Defensive Power	a_3 = 2	Deception Capability of the Sea Based Force	ρ_A = .6	Number of Well-Aimed ASCMs	β = 4
Area Fire Missiles	α_2 = 4	Munitions Targeting Ratio, Msl	$MTR_{Missile}$ = .01	Defensive Readiness of the Sea Based Force	δ_A = .6	Targeting Capability of the Land Based Force	σ_B = .6
Gunfire Rounds	α_3 = 11	Munition Targeting Ratio, Gun	MTR_{Gun} = .0035				

Table 15.   Up-Scale Scenario Baseline FER's

Engagement Situation	Sea Force Initial Attack	Land Force Initial Attack	Simultaneous Attack
LAM (direct) versus ASCM	0.48	8.57	2.4
LAM (area) versus ASCM	1.08	10.71	3
Naval Gunnery versus ASCM	1.2	11.13	3.12

C.   SEA FORCE PARAMETER SENSITIVITY ANALYSIS

Table 16.   Down-Scale Scenario Sea Force Values Needed to Attain Specific Levels of FER

Sea Force, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	2.94367791	3.376927	3.99999	4.845484	5.689065	7.774485681	4.072254	5.160883	7.649512
LAM(ar) vs. ASCM	3.41255771	3.91864	4.625108	5.180399	6.068391	8.087255019	4.511529	5.633279	8.000009
Navy Guns vs. ASCM	3.79805387	4.356424	5.131938	5.455753	6.368048	8.306139729	4.86546	6.000001	8.241541

Table 17.   Up-Scale Scenario Sea Force Values Needed to Attain Specific Levels of FER

Sea Force, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	5.60274	5.982656	6.372173	7.783064	8.388232	9.258629	7.349419	8.19938	9.243957
LAM(ar) vs. ASCM	6.053758	6.437381	6.824253	7.988072	8.557786	9.320817	7.649504	8.418746	9.311017
Navy Guns vs. ASCM	6.129297	6.51257	6.897886	8.022408	8.585305	9.330382	7.698514	8.453607	9.321244

Figure 15.   Down-Scale Scenario FER Levels with Increasing Sea Force Values
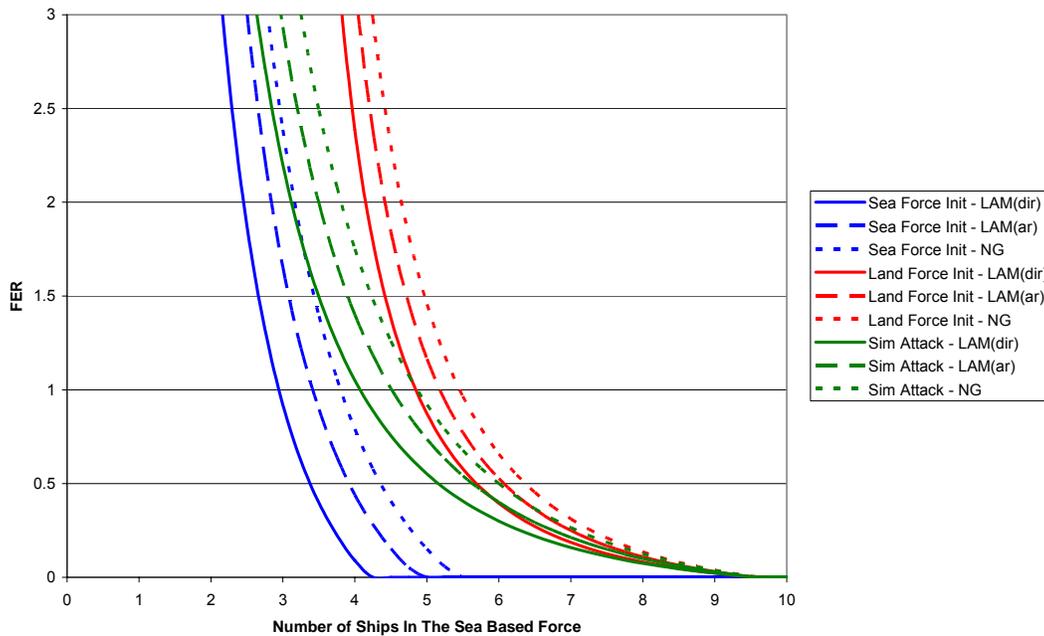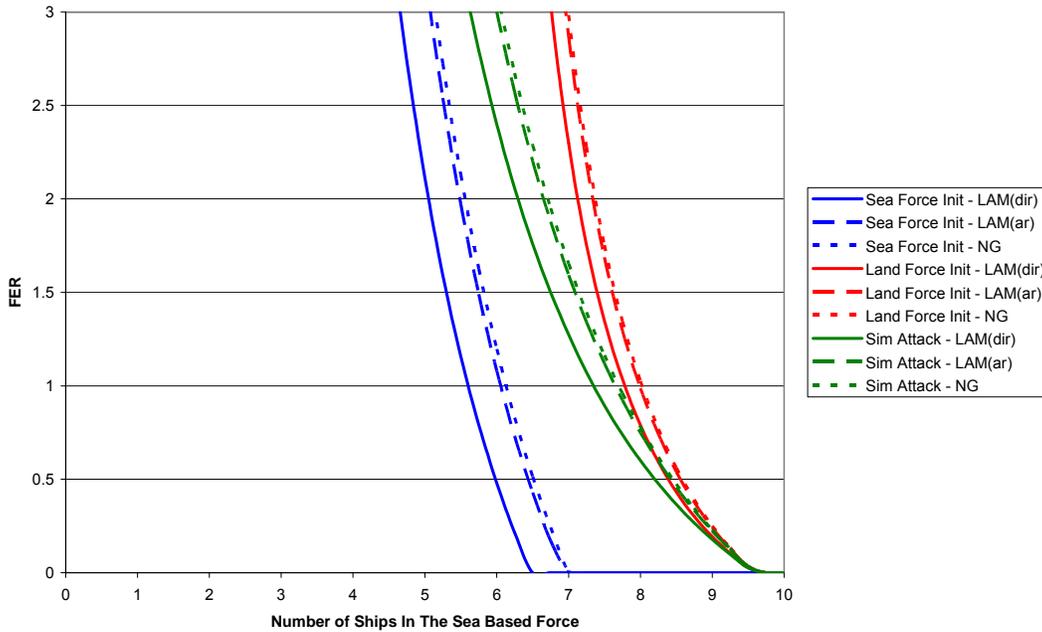

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Figure 16.   Up-Scale Scenario FER Levels with Increasing Sea Force Values

D.   LAND ATTACK MISSILE (DIR) STRIKING POWER SENSITIVITY ANALYSIS

Table 18.   Down-Scale Scenario LAM (dir) Striking Power Values Needed to Attain Specific Levels of FER.

Alpha One, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	2.14285746	2.876713	3.962265	9.545457	19.09094	95.45488608	4.200002	8.4	42.00042

Table 19.   Up-Scale Scenario LAM (dir) Striking Power Values Needed to Attain Specific Levels of FER.

Alpha One, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	3.287671	3.966943	4.752477	34.28573	68.5715	342.8585	9.600002	19.20003	96.0008

Figure 17.   Down-Scale Scenario FER Levels with Increasing LAM (dir) Striking Power
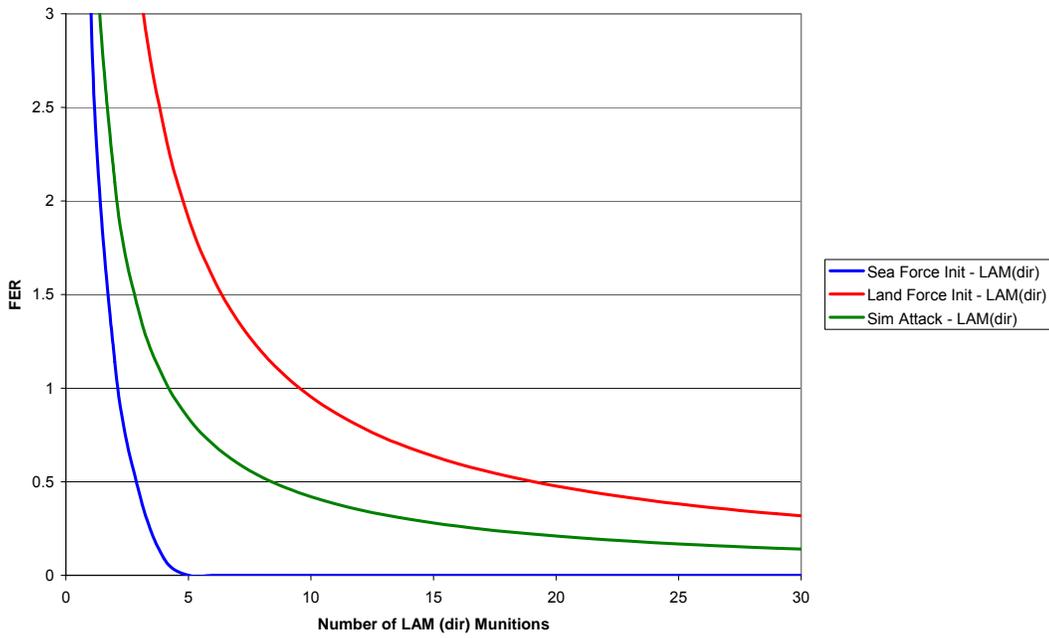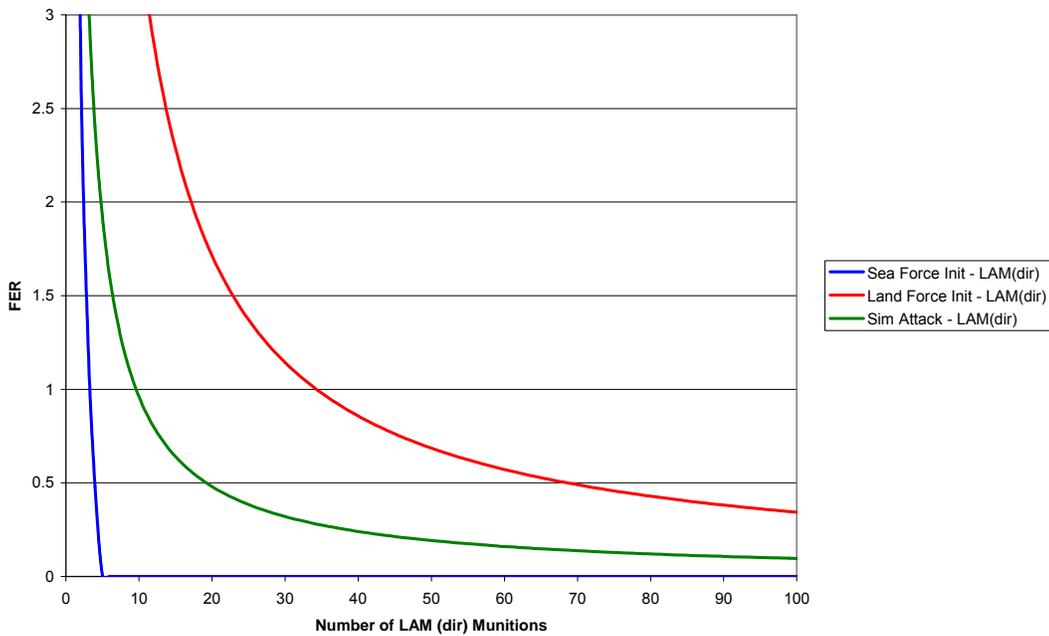Values.

Figure 18.   Up-Scale Scenario FER Levels with Increasing LAM (dir) Striking Power
Values.

E.   LAND ATTACK MISSILE (AREA) STRIKING POWER SENSITIVITY ANALYSIS

Table 20.   Down-Scale Scenario LAM (ar) Striking Power Values Needed to Attain Specific Levels of FER.

Alpha Two, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(ar) vs. ASCM	7.14286058	9.589045	13.20755	31.81819	63.63643	318.1832969	14	28.00003	140.0007

Table 21.   Up-Scale Scenario LAM (ar) Striking Power Values Needed to Attain Specific Levels of FER.

Alpha Two, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(ar) vs. ASCM	4.109589	4.958679	5.940594	42.85715	85.71436	428.5718	12.00001	24.00002	120.0004

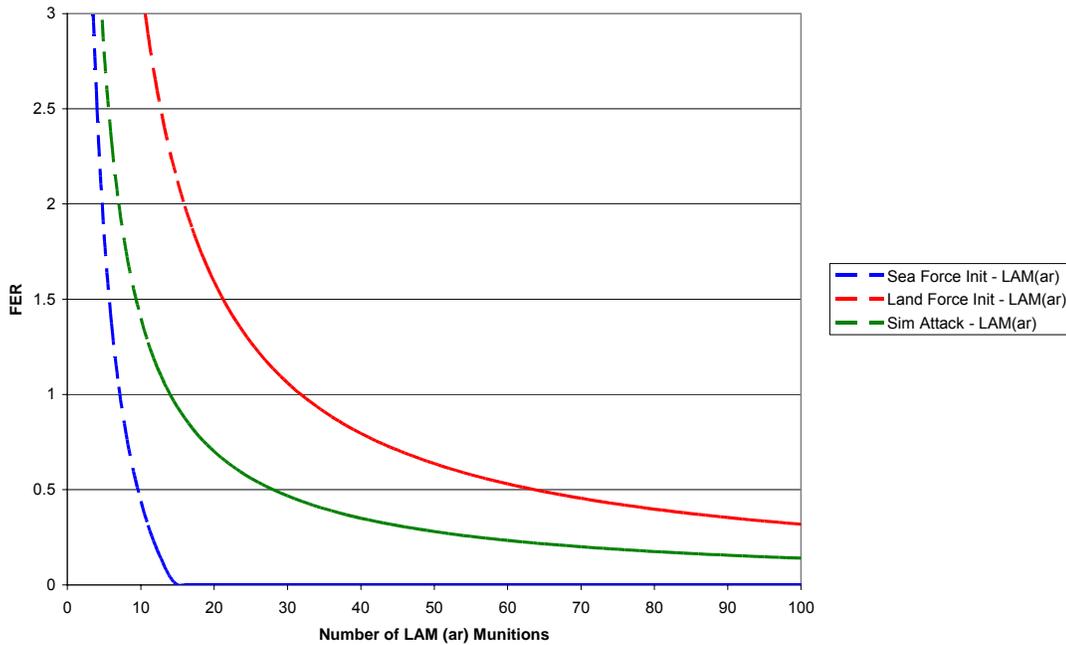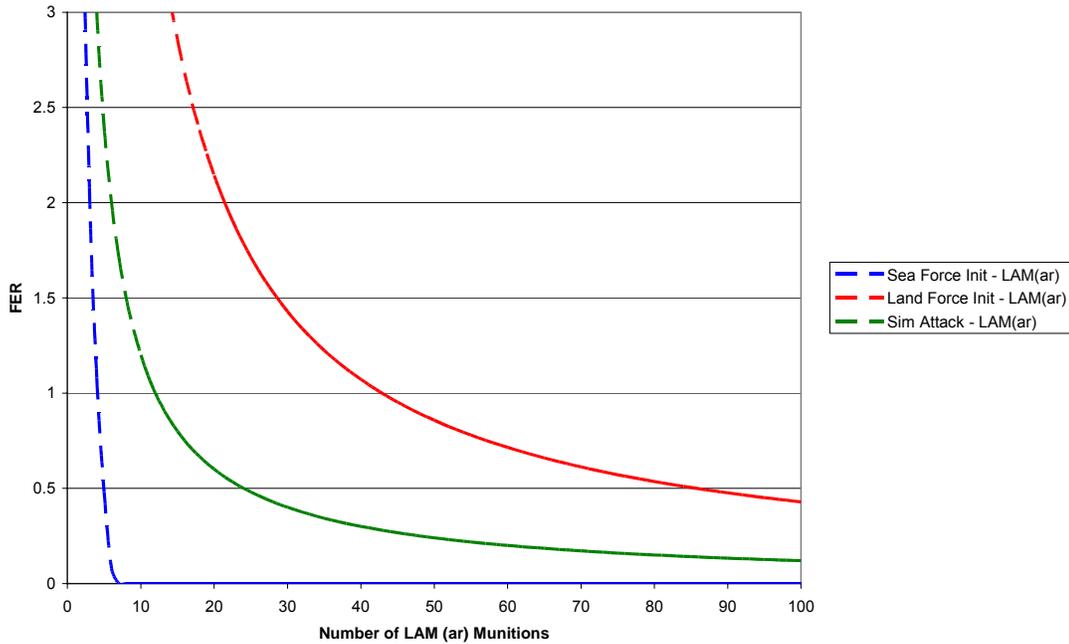Figure 19.   Down-Scale Scenario FER Levels with Increasing LAM (ar) Striking Power Values.

Figure 20.   Up-Scale Scenario FER Levels with Increasing LAM (ar) Striking Power
Values.

F.   NAVAL GUNNERY STRIKING POWER SENSITIVITY ANALYSIS

Table 22.   Down-Scale Scenario Gunnery Striking Power Values Needed to Attain
Specific Levels of FER.

Alpha Three, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
Navy Guns vs. ASCM	71.4285865	95.89045	132.0755	318.1819	636.3643	3181.832969	140	280.0003	1400.007

Table 23.   Up-Scale Scenario Gunnery Striking Power Values Needed to Attain
Specific Levels of FER.

Alpha Three, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
Navy Guns vs. ASCM	11.74168	14.16765	16.97313	122.449	244.8984	1224.492	34.28574	68.57146	342.8599

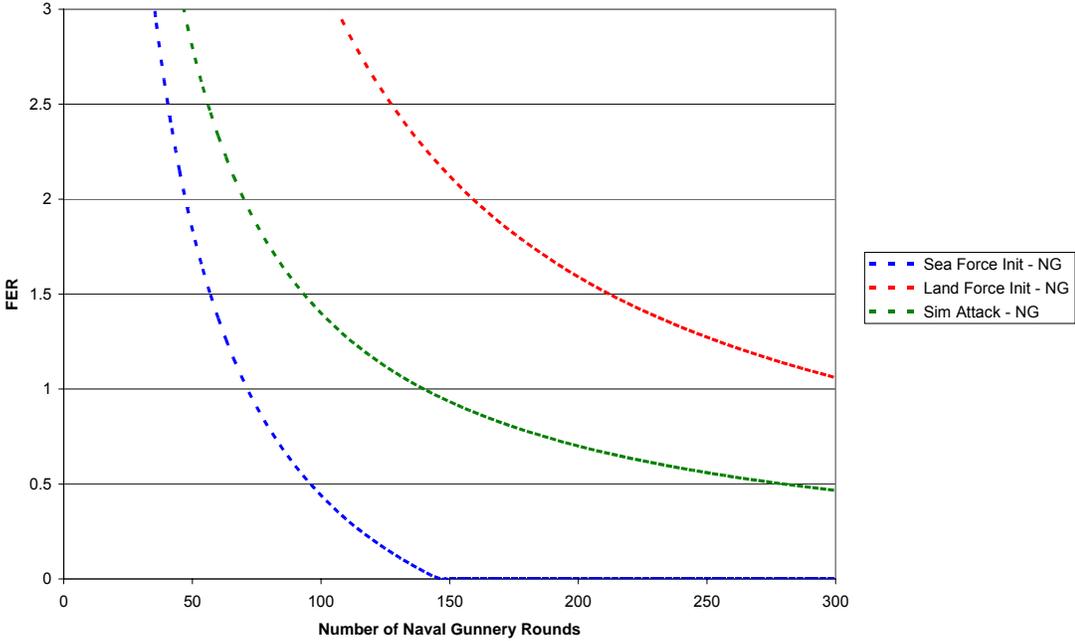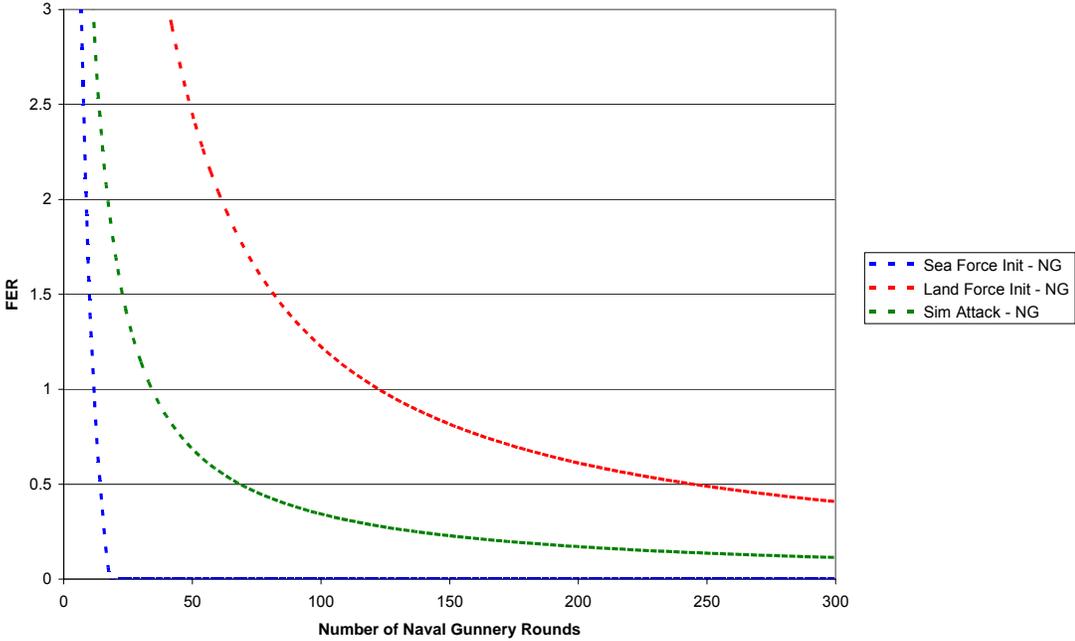Figure 21.   Down-Scale Scenario FER Levels with Increasing Gunnery Striking Power Values.

Figure 22.   Up-Scale Scenario FER Levels with Increasing Gunnery Striking Power Values.

G. STAYING POWER SENSITIVITY ANALYSIS

Table 24.   Down-Scale Scenario Staying Power Values Needed to Attain Specific
Levels of FER.

A One, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	0.09	0.18	0.9	1.61	2.660001	11.06003223	1.05	2.100001	10.50008
LAM(ar) vs. ASCM	0.43999984	0.880001	4.400004	1.960001	3.360006	14.56012729	1.4	2.800004	14
Navy Guns vs. ASCM	0.79000032	1.580001	7.900061	2.31	4.060006	18.06009545	1.750001	3.500002	17.50007

Table 25.   Up-Scale Scenario Staying Power Values Needed to Attain Specific
Levels of FER.

A One, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	0.48	0.96	4.800028	3.120001	5.520002	24.72006	2.4	4.800006	24.00011
LAM(ar) vs. ASCM	1.08	2.16	10.80002	3.720001	6.720007	30.72014	3.000001	6.00001	30.0001
Navy Guns vs. ASCM	1.196884	2.393768	11.96892	3.836885	6.953775	31.88893	3.116885	6.233775	31.16911

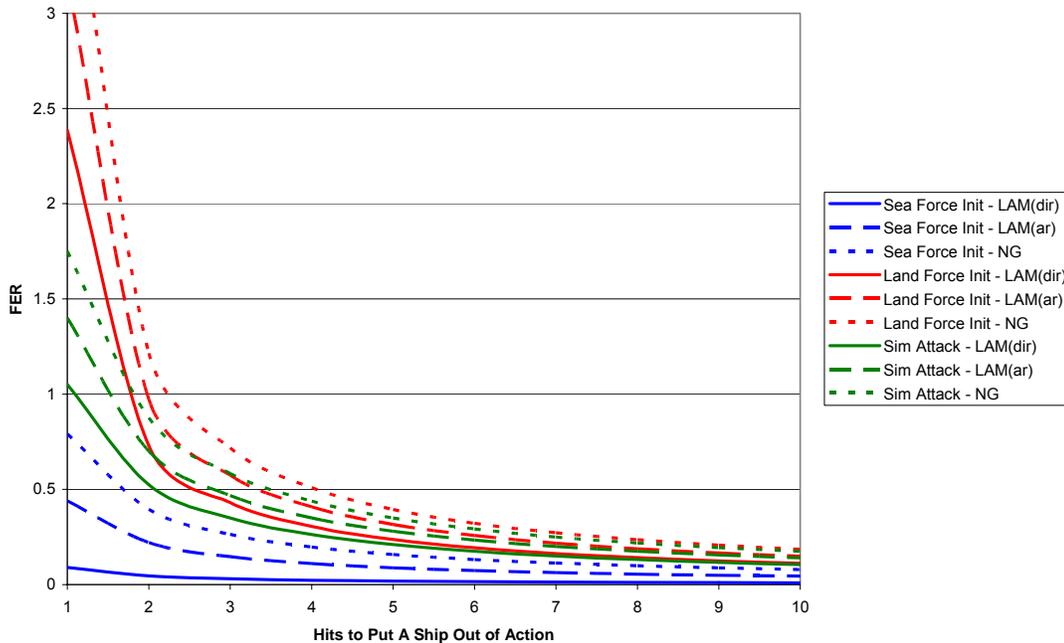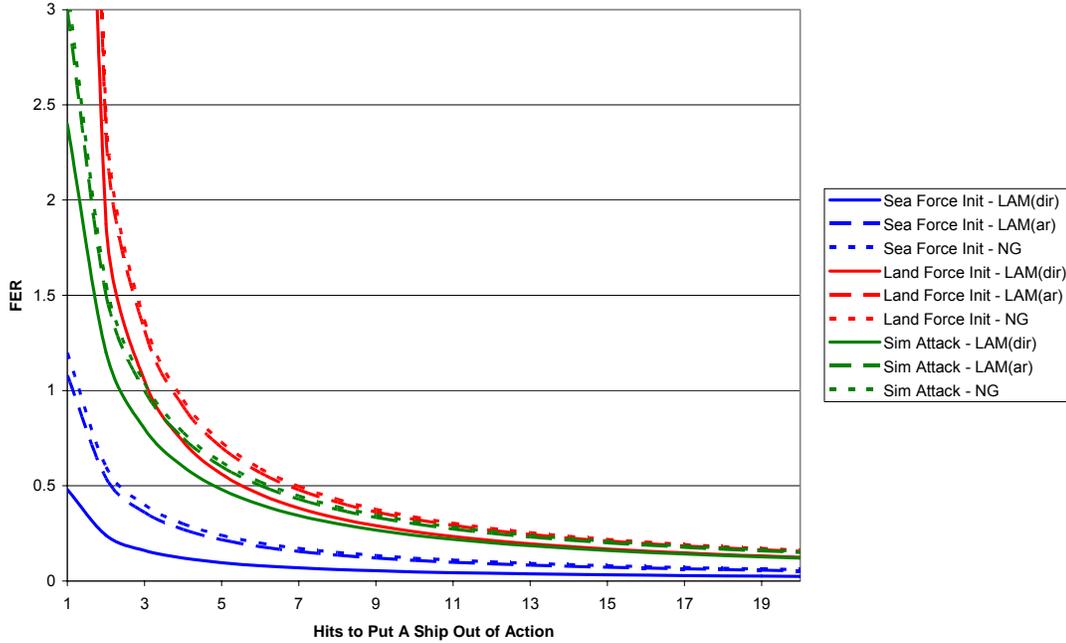Figure 23.   Down-Scale Scenario FER Levels with Increasing Staying Power Values.

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Figure 24. Up-Scale Scenario FER Levels with Increasing Staying Power Values.

H. DEFENSIVE POWER SENSITIVITY ANALYSIS

Table 26. Down-Scale Scenario Defensive Power Values Needed to Attain Specific Levels of FER.

A Three, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	.84 @ 0	0.906667	1.973333	3.06087	3.747369	4.546835826	2.133333	3.466669	4.533333
LAM(ar) vs. ASCM	0.879998	1.879998	2.68	3.371429	3.966667	4.6076927	2.8	3.8	4.6
Navy Guns vs. ASCM	1.6639984	2.464	3.104	3.587879	4.110346	4.644961923	3.2	4	4.64

Table 27. Up-Scale Scenario Defensive Power Values Needed to Attain Specific Levels of FER.

A Three, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	1.74	1.99	2.19	2.815385	2.982609	3.151456	2.700001	2.95	3.15
LAM(ar) vs. ASCM	2.032	2.232	2.392	2.877419	3.021429	3.160938	2.8	3	3.16
Navy Guns vs. ASCM	2.0758	2.2683	2.4223	2.887246	3.027432	3.162369	2.815	3.0075	3.1615

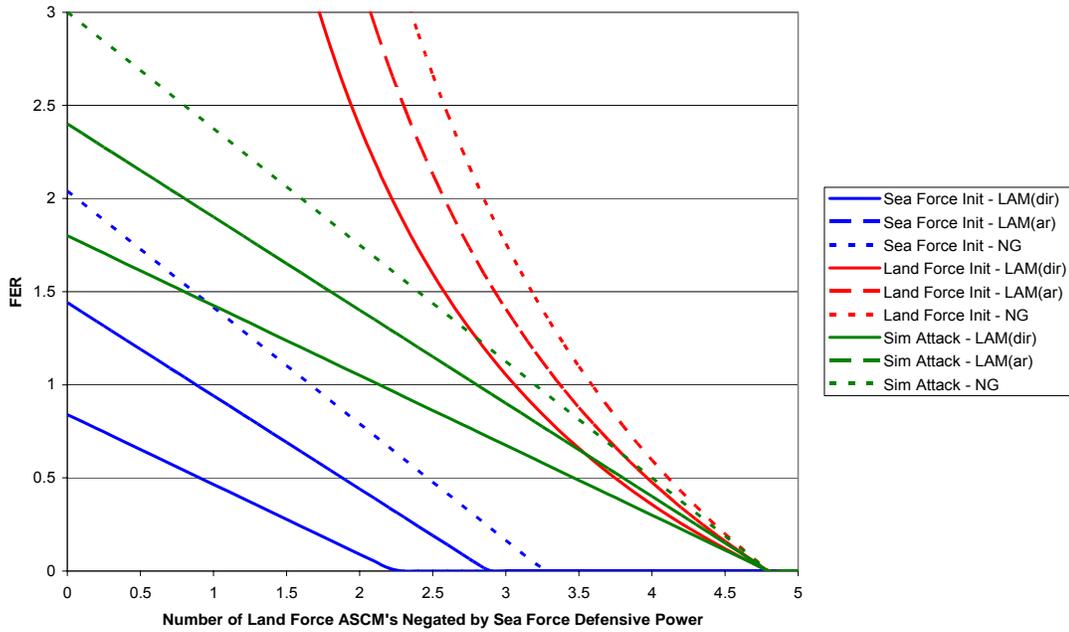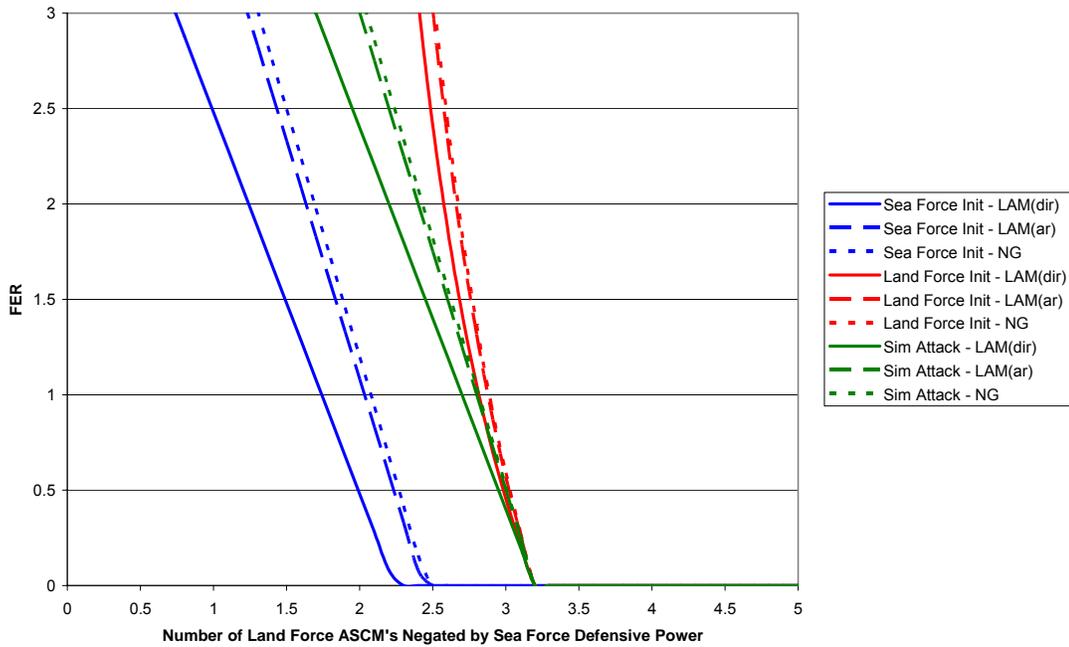Figure 25.   Down-Scale Scenario FER Levels with Increasing Defensive Power Values.

Figure 26.   Up-Scale Scenario FER Levels with Increasing Defensive Power Values.

I. SEA FORCE TARGETING CAPABILITY SENSITIVITY ANALYSIS

Table 28.   Down-Scale Scenario Sea Force Targeting Capability Values Needed to Attain Specific Levels of FER.

Sigma A, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	0.05357142	0.071918	0.099057	0.238636	0.477273	*0.2386 @ 1*	0.105	0.21	*.105 @ 1*

Table 29.   Up-Scale Scenario Sea Force Targeting Capability Values Needed to Attain Specific Levels of FER.

Sigma A, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	0.082192	0.099174	0.118812	0.857143	.857 @ 1	.857 @ 1	0.24	0.480001	.24 @ 1

Figure 27.   Down-Scale Scenario FER Levels with Increasing Sea Force Targeting Capability Values.
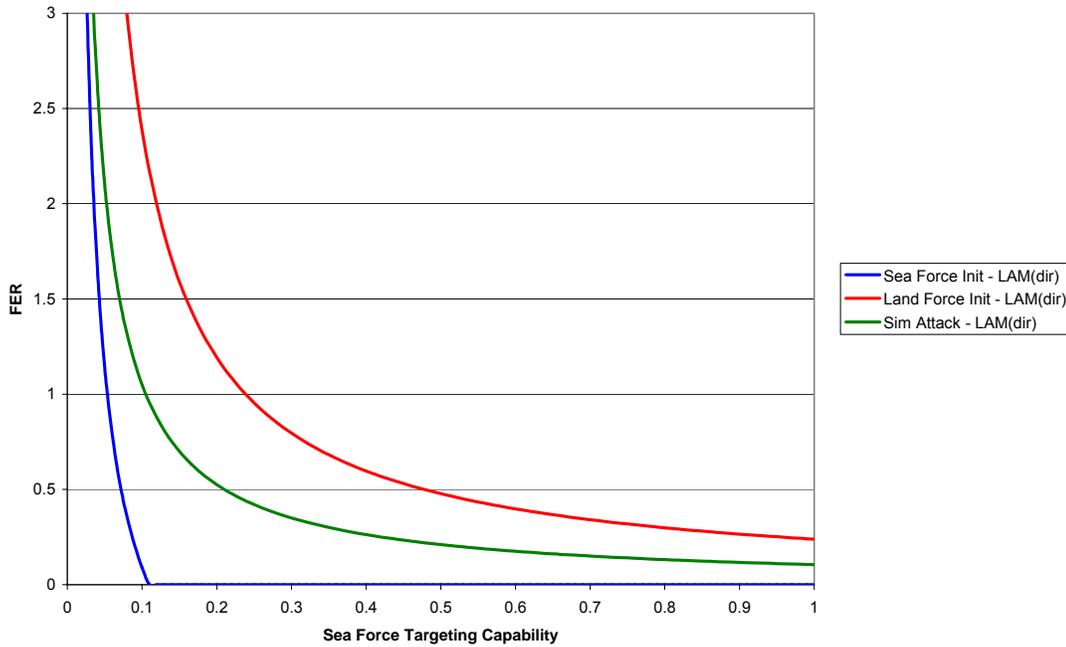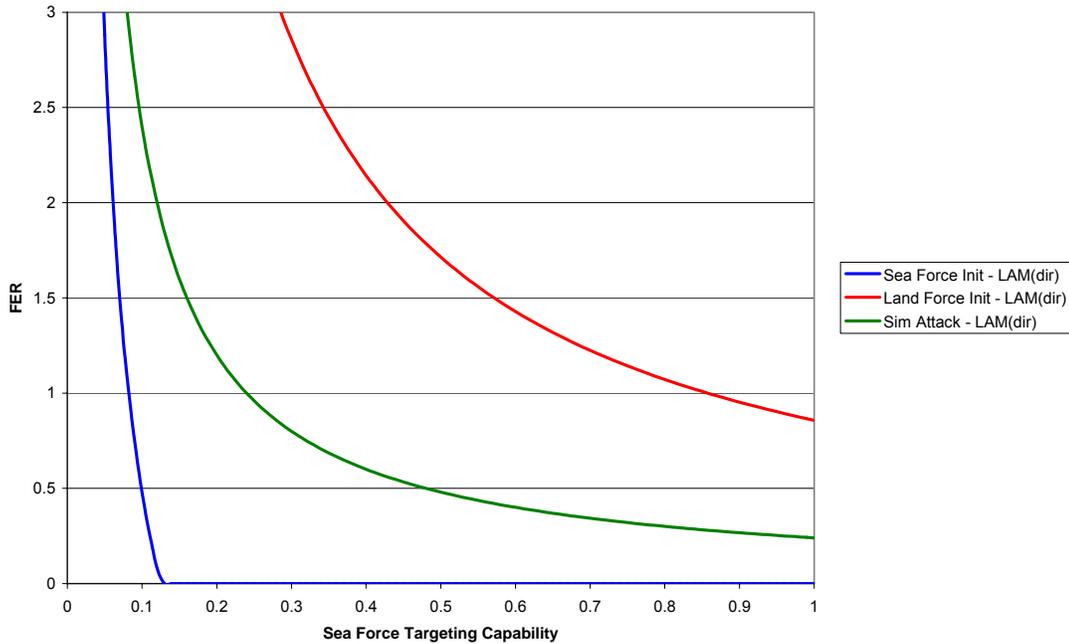
Figure 28.   Up-Scale Scenario FER Levels with Increasing Sea Force Targeting Capability Values.

J.   SEA FORCE DECEPTION CAPABILITY SENSITIVITY ANALYSIS

Table 30.   Down-Scale Scenario Sea Force Deception Capability Values Needed to Attain Specific Levels of FER.

Rho A, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	.3 @ 1	.3 @ 1	0.809525	0.623188	0.508772	0.375527111	0.777778	0.555555	0.377778
LAM(ar) vs. ASCM	.8 @ 1	0.833333	0.611111	0.571429	0.472222	0.365384505	0.666666	0.5	0.366667
Navy Guns vs. ASCM	0.88235333	0.686274	0.529412	0.535353	0.448276	0.359173053	0.6	0.466667	0.36

Table 31.   Up-Scale Scenario Sea Force Deception Capability Values Needed to Attain Specific Levels of FER.

Rho A, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	0.669643	0.602679	0.549107	0.447115	0.415761	0.384102	0.46875	0.421875	0.384375
LAM(ar) vs. ASCM	0.592105	0.542763	0.503289	0.435484	0.408482	0.382324	0.45	0.4125	0.3825
Navy Guns vs. ASCM	0.581518	0.534582	0.497033	0.433641	0.407357	0.382056	0.447187	0.411094	0.382219

Figure 29.   Down-Scale Scenario FER Levels with Decreasing Sea Force Deception
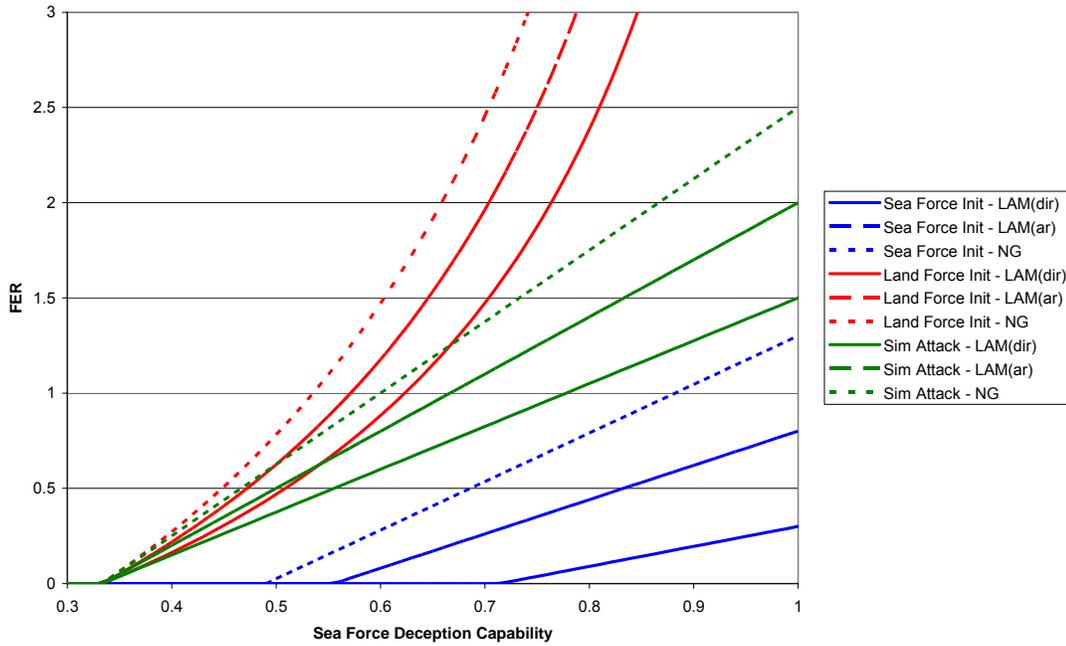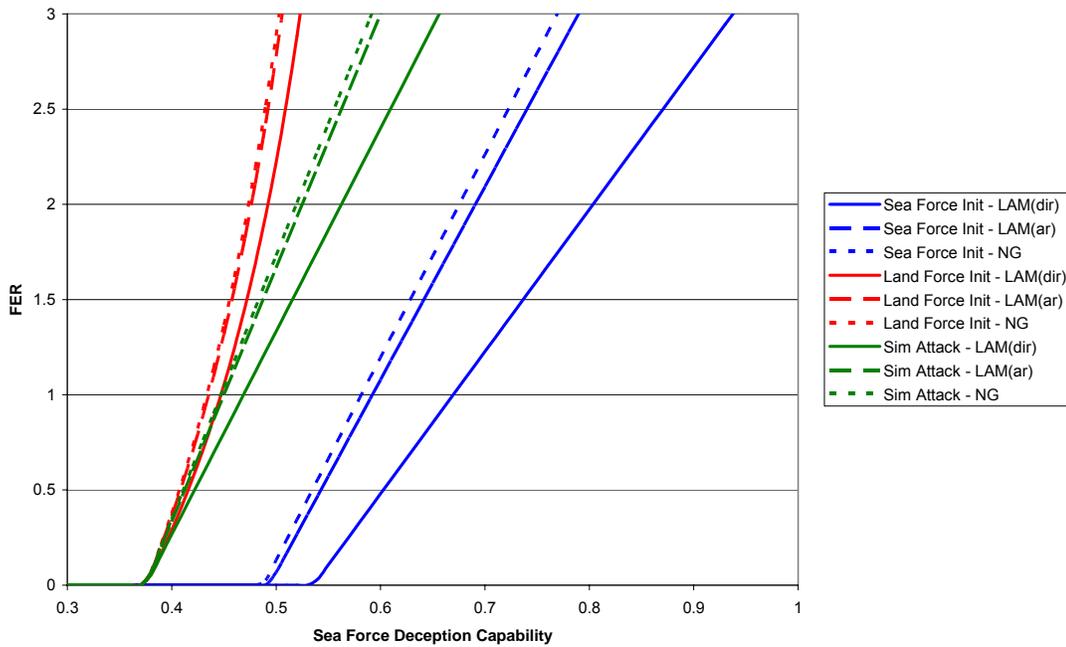Capability Values.

Figure 30.   Up-Scale Scenario FER Levels with Decreasing Sea Force Deception
Capability Values.

K. SEA FORCE DEFENSIVE READINESS SENSITIVITY ANALYSIS

Table 32. Down-Scale Scenario Sea Force Defensive Readiness Values Needed to Attain Specific Levels of FER.

LilDeltaA, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	.84 @ 0	0.090667	0.197333	0.306087	0.374737	0.454683628	0.213333	0.346667	0.453333
LAM(ar) vs. ASCM	0.0879998	0.188	0.268	0.337143	0.396667	0.460769353	0.28	0.38	0.46
Navy Guns vs. ASCM	0.16639984	0.2464	0.3104	0.358788	0.411035	0.464496273	0.32	0.4	0.464

Table 33. Up-Scale Scenario Sea Force Defensive Readiness Values Needed to Attain Specific Levels of FER.

LilDeltaA, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(dir) vs. ASCM	0.522	0.597	0.657	0.844615	0.894783	0.945437	0.81	0.885	0.945
LAM(ar) vs. ASCM	0.6096	0.6696	0.7176	0.863226	0.906429	0.948281	0.84	0.9	0.948
Navy Guns vs. ASCM	0.62274	0.68049	0.72669	0.866174	0.90823	0.948711	0.8445	0.90225	0.94845

Figure 31. Down-Scale Scenario FER Levels with Increasing Sea Force Defensive Readiness Values.
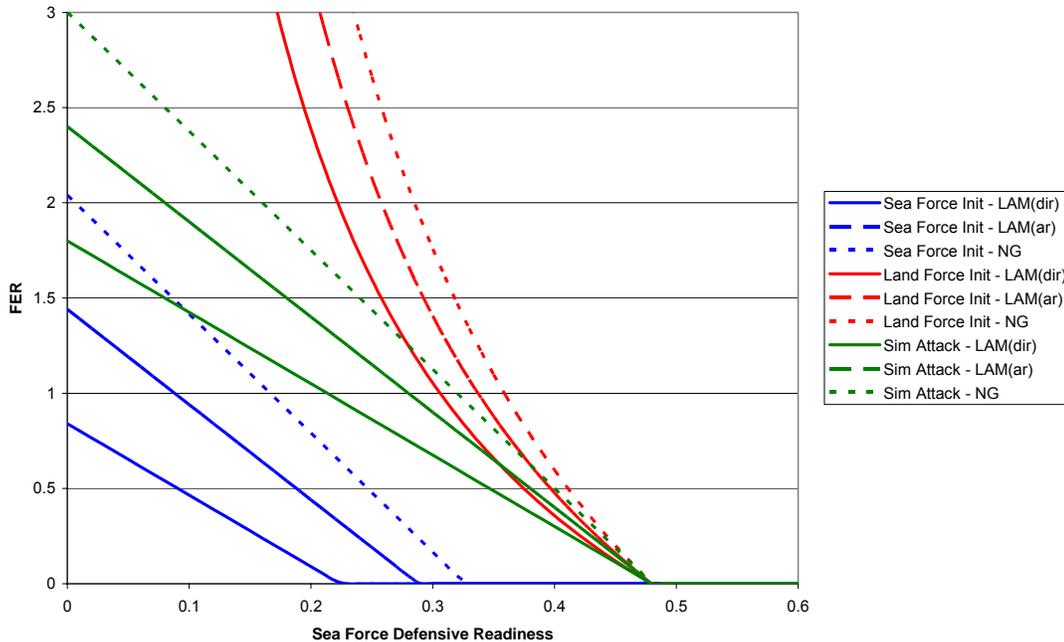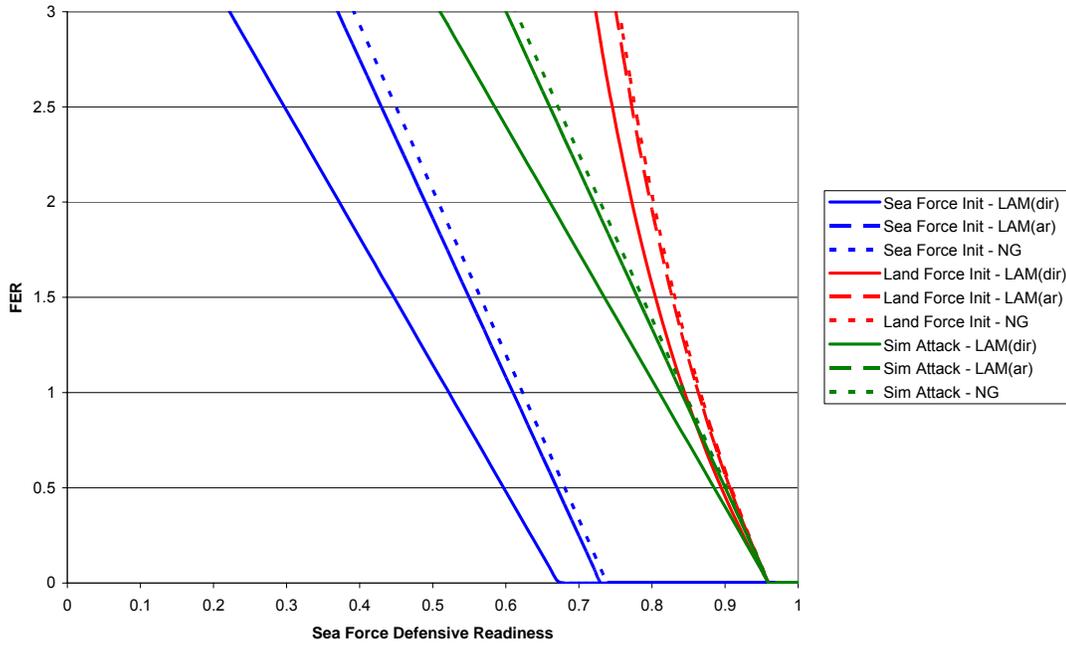
Figure 32.   Up-Scale Scenario FER Levels with Increasing Sea Force Defensive
Readiness Values.

L.   MUNITIONS' LETHALITY, TARGETED AREA RATIO (AREA MISSILE) SENSITIVITY ANALYSIS

Table 34.   Down-Scale Scenario MTR (msl) Values Needed to Attain Specific
Levels of FER.

MTR Msl, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(ar) vs. ASCM	0.00714286	0.009589	0.013208	0.031818	0.063636	0.318182797	0.014	0.028	0.140001

Table 35.   Up-Scale Scenario MTR (msl) Values Needed to Attain Specific Levels of
FER.

MTR Msl, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
LAM(ar) vs. ASCM	0.010274	0.012397	0.014851	0.107143	0.214286	.107 @ 1	0.03	0.06	0.300001

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Figure 33.   Down-Scale Scenario FER Levels with Increasing MTR (msl) Values.
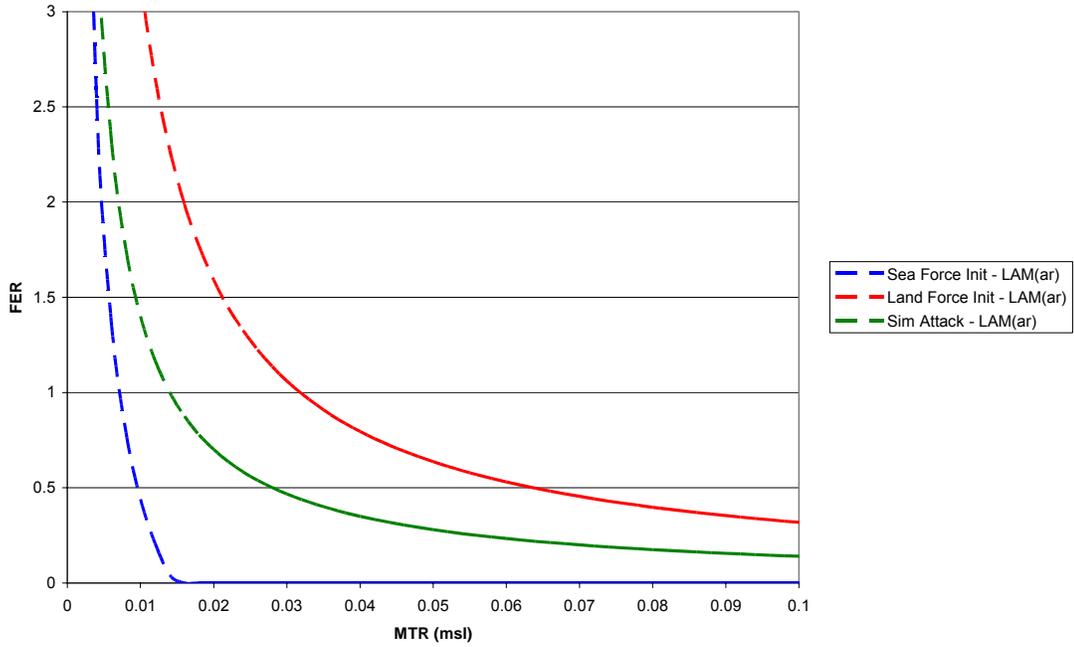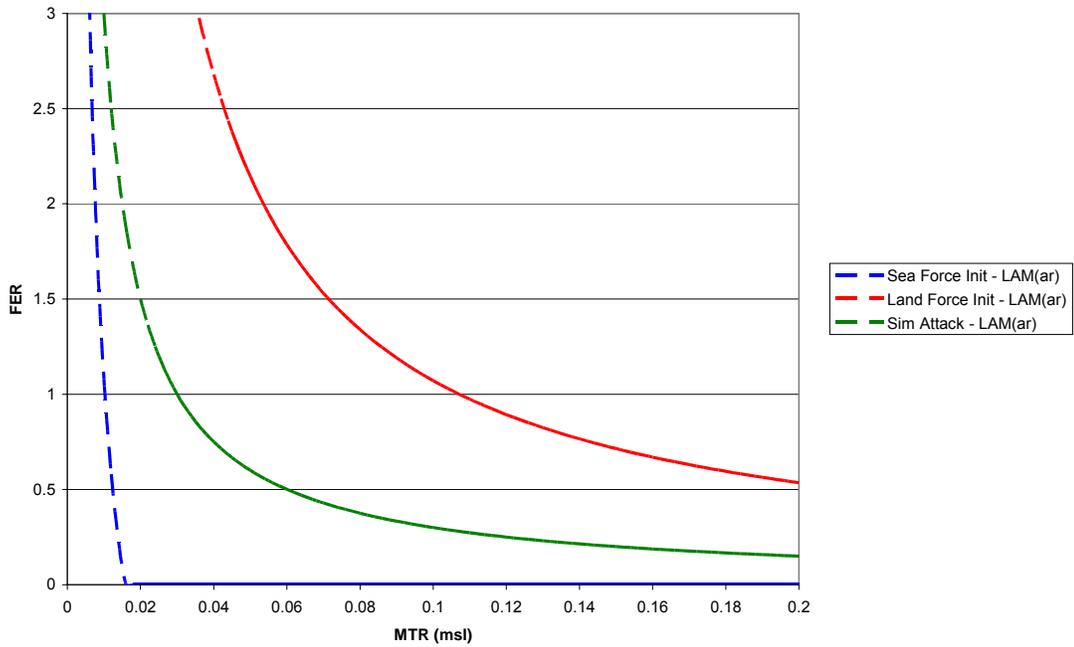

Figure 34.   Up-Scale Scenario FER Levels with Increasing MTR (msl) Values.

M. MUNITIONS' LETHALITY, TARGETED AREA RATIO (NAVAL GUNNERY) SENSITIVITY ANALYSIS

Table 36.   Down-Scale Scenario MTR (gun) Values Needed to Attain Specific Levels of FER.

MTR Gun, Down Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
Navy Guns vs. ASCM	0.00089286	0.001199	0.001651	0.003977	0.007955	0.039772955	0.00175	0.0035	0.0175

Table 37.   Up-Scale Scenario MTR (gun) Values Needed to Attain Specific Levels of FER.

MTR Gun, Up Scale	Attack Circumstances								
	Sea Force Initial Attack			Land Force Initial Attack			Simultaneous Attack		
Desired FER Values	1	0.5	0.1	1	0.5	0.1	1	0.5	0.1
Navy Guns vs. ASCM	0.003736	0.004508	0.005401	0.038961	0.077922	0.389613	0.010909	0.021818	0.109091

Figure 35.   Down-Scale Scenario FER Levels with Increasing MTR (gun) Values.
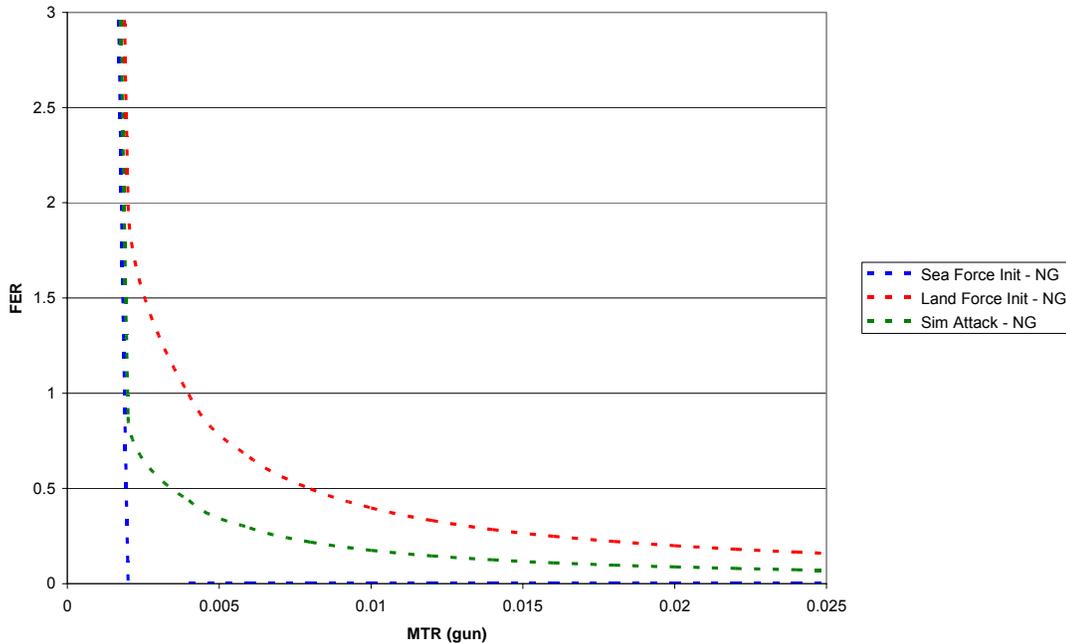
Figure 36.   Up-Scale Scenario FER Levels with Increasing MTR (gun) Values.
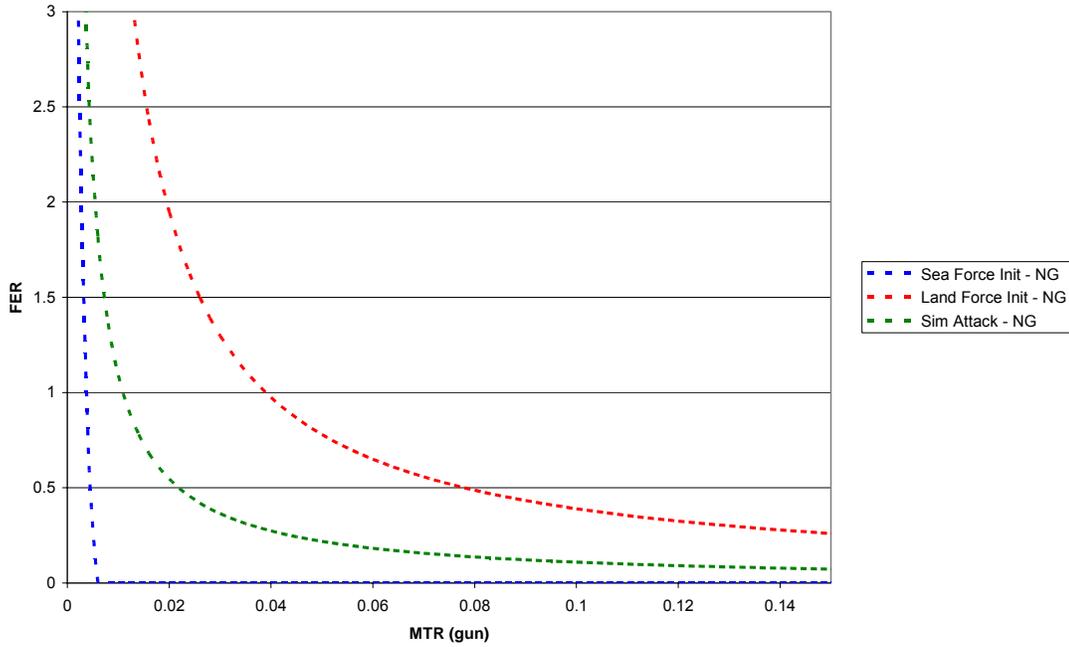
N.   STRIKING POWER & MTR SENSITIVITY ANALYSIS

Figure 37.   Down-Scale Scenario FER Levels with Improving MTR (msl) & Greater
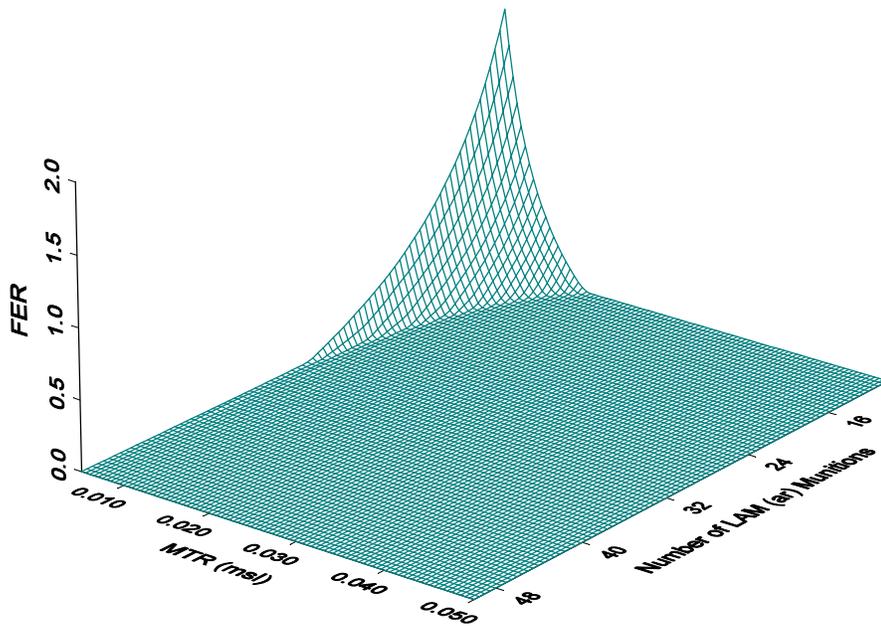LAM (ar) Values for a Sea Force Initial Attack.

Figure 38.   Down-Scale Scenario FER Levels with Improving MTR (msl) & Greater
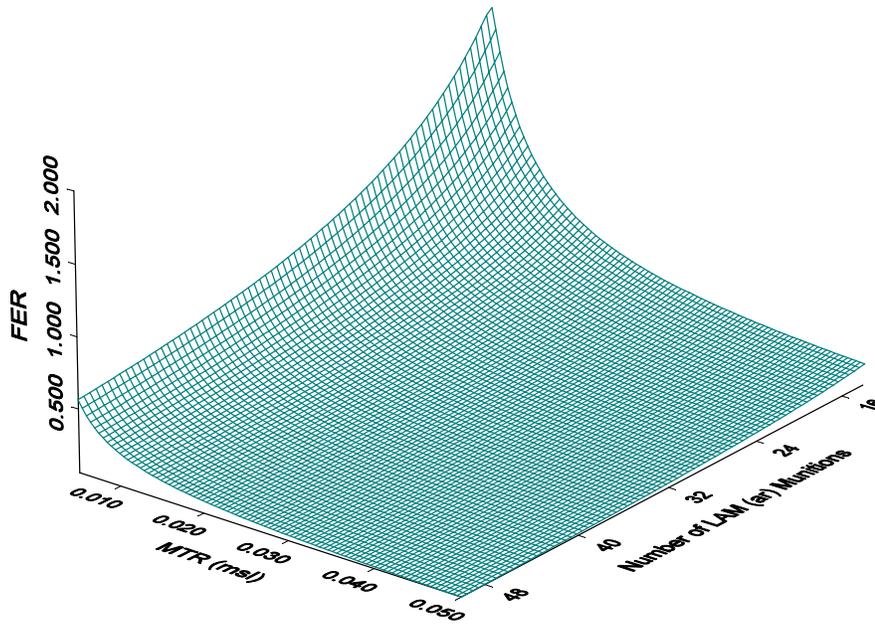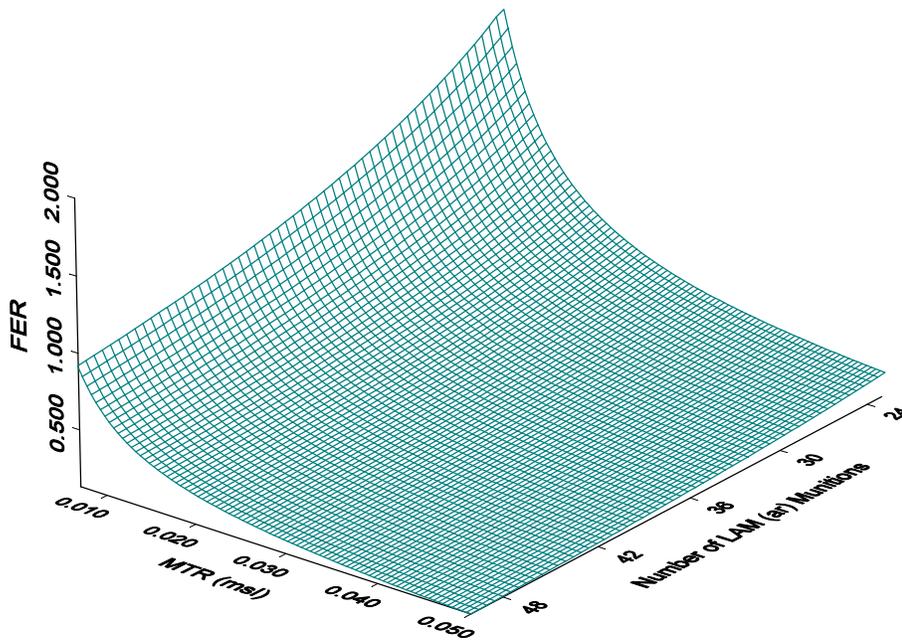LAM (ar) Values for a Simultaneous Attack.

Figure 39.   Down-Scale Scenario FER Levels with Improving MTR (msl) & Greater
LAM (ar) Values for a Land Force Initial Attack.

Figure 40.   Up-Scale Scenario FER Levels with Improving MTR (msl) & Greater LAM (ar) Values for a Sea Force Initial Attack.
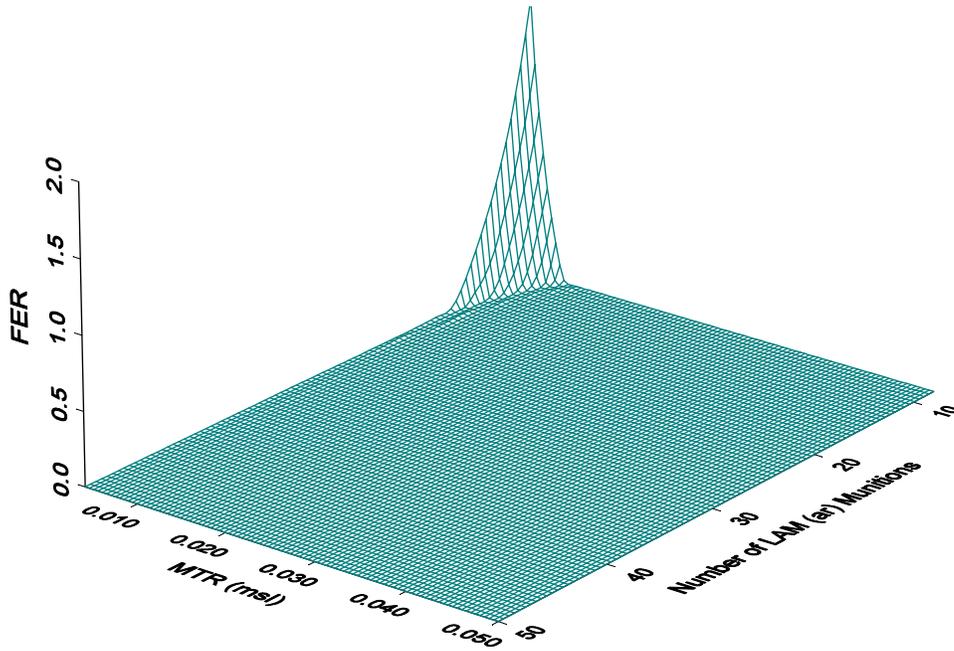

Figure 41.   Up-Scale Scenario FER Levels with Improving MTR (msl) & Greater LAM (ar) Values for a Simultaneous Attack.
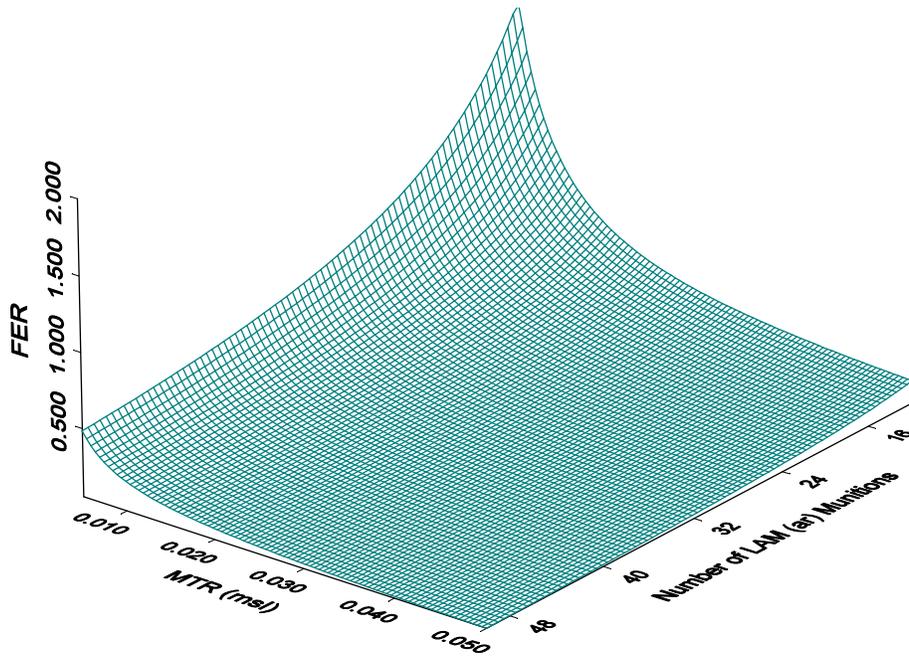
Figure 42.   Up-Scale Scenario FER Levels with Improving MTR (msl) & Greater LAM (ar) Values for a Land Force Initial Attack.
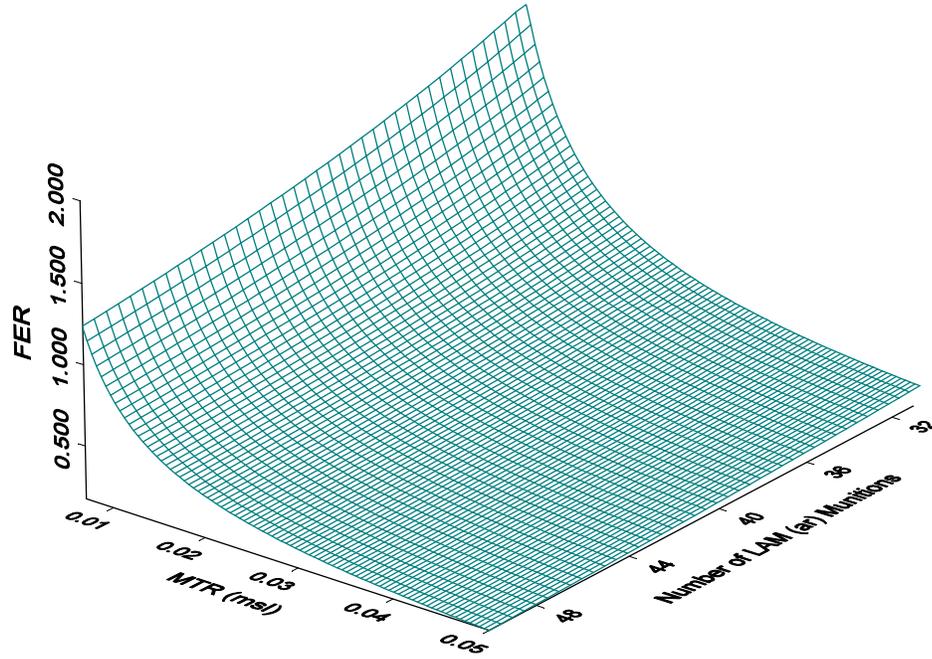
Figure 43.   Down-Scale Scenario FER Levels with Improving MTR (gun) & Greater Naval Gunnery Values for a Sea Force Initial Attack.
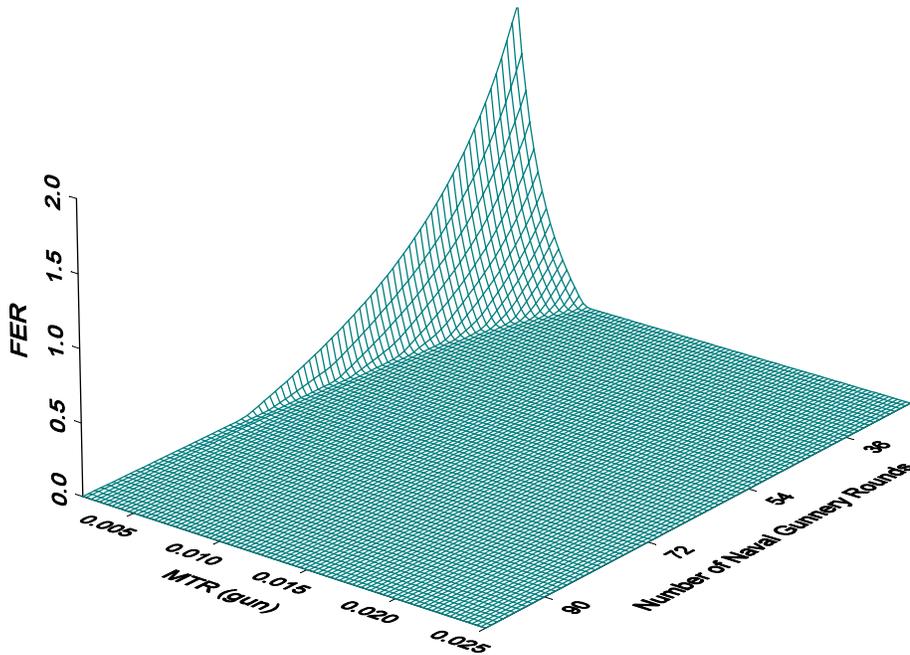
Figure 44.   Down-Scale Scenario FER Levels with Improving MTR (gun) & Greater
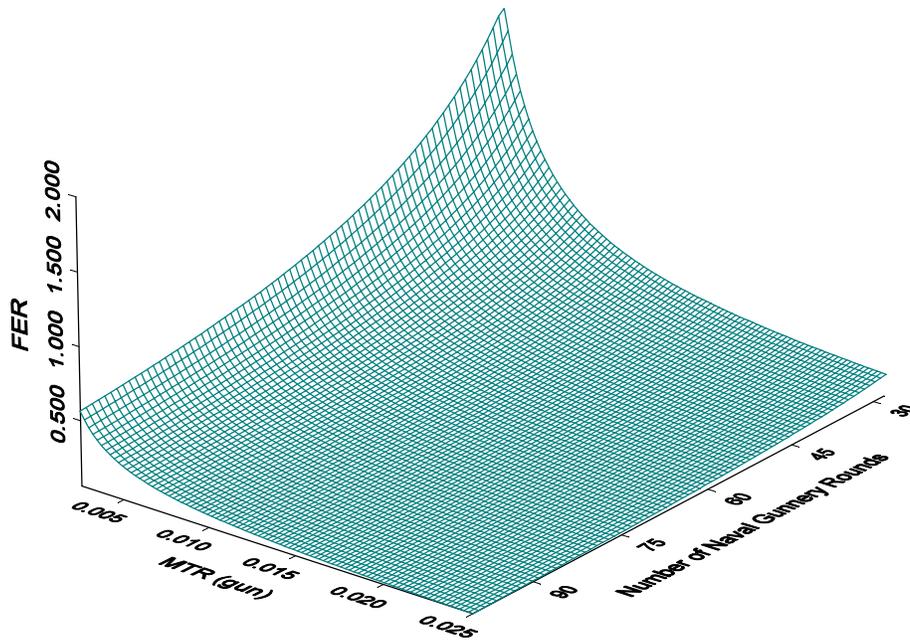Naval Gunnery Values for a Simultaneous Attack.

Figure 45.   Down-Scale Scenario FER Levels with Improving MTR (gun) & Greater
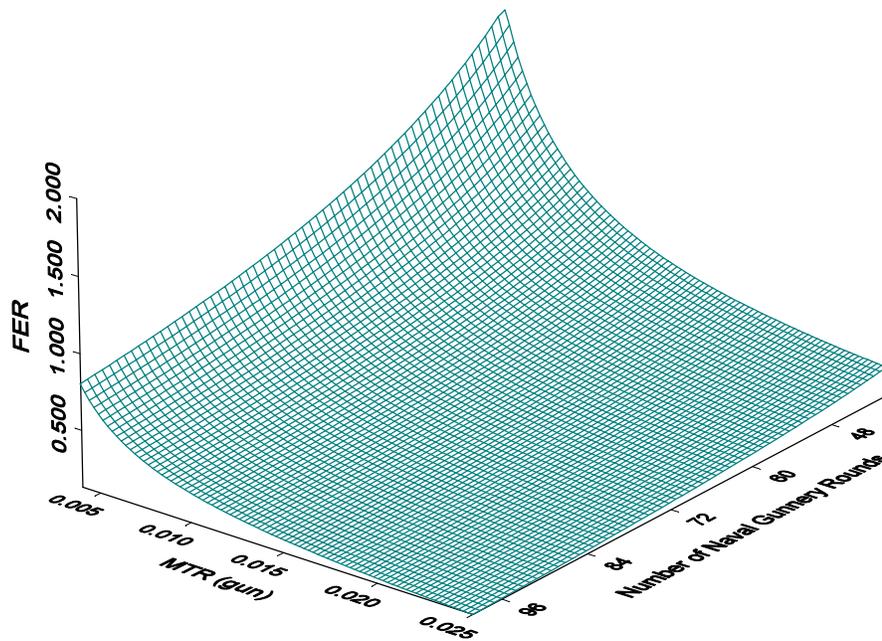Naval Gunnery Values for a Land Force Initial Attack.

Figure 46.   Up-Scale Scenario FER Levels with Improving MTR (gun) & Greater Naval Gunnery Values for a Sea Force Initial Attack.
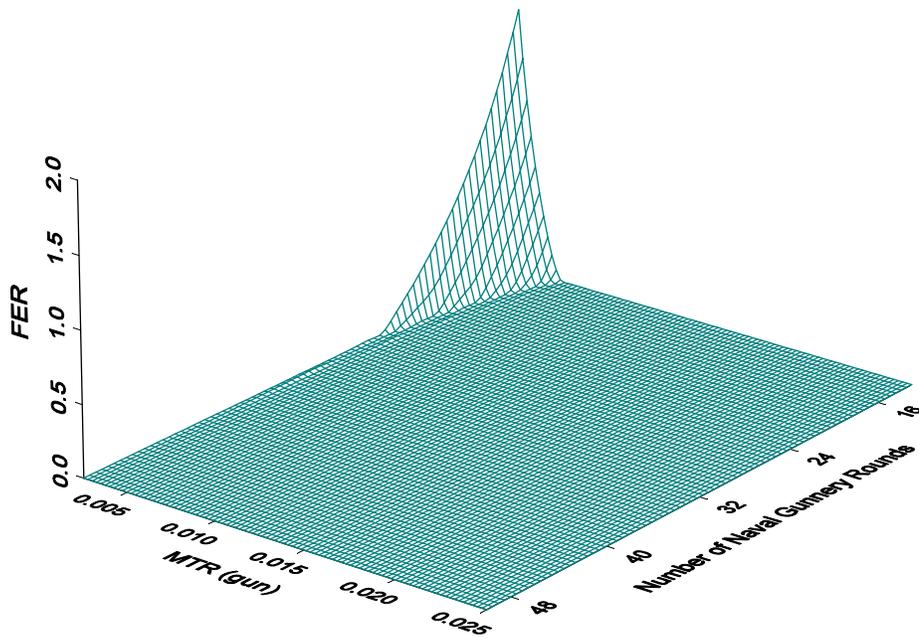
Figure 47.   Up-Scale Scenario FER Levels with Improving MTR (gun) & Greater Naval Gunnery Values for a Simultaneous Attack.
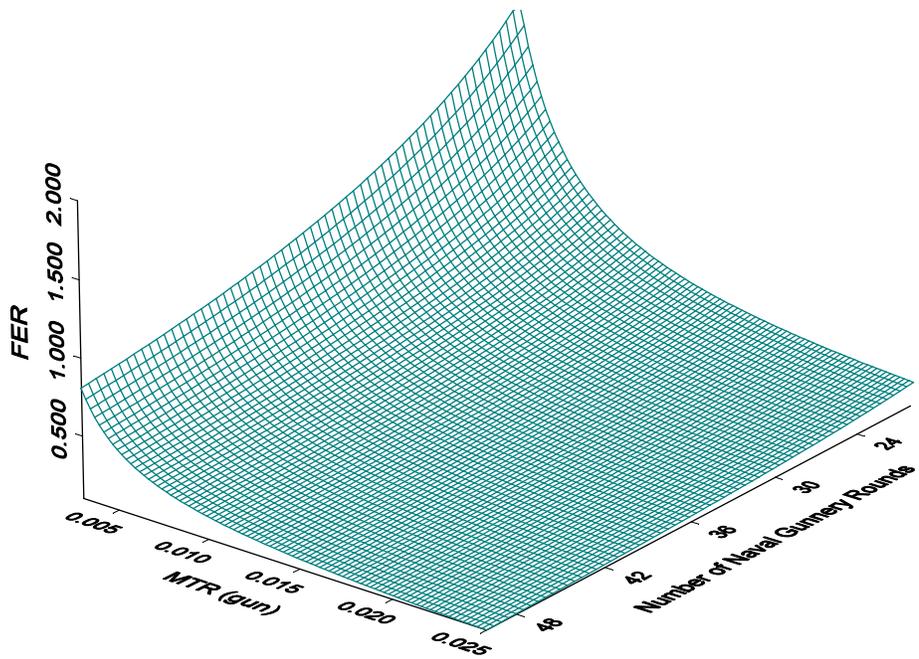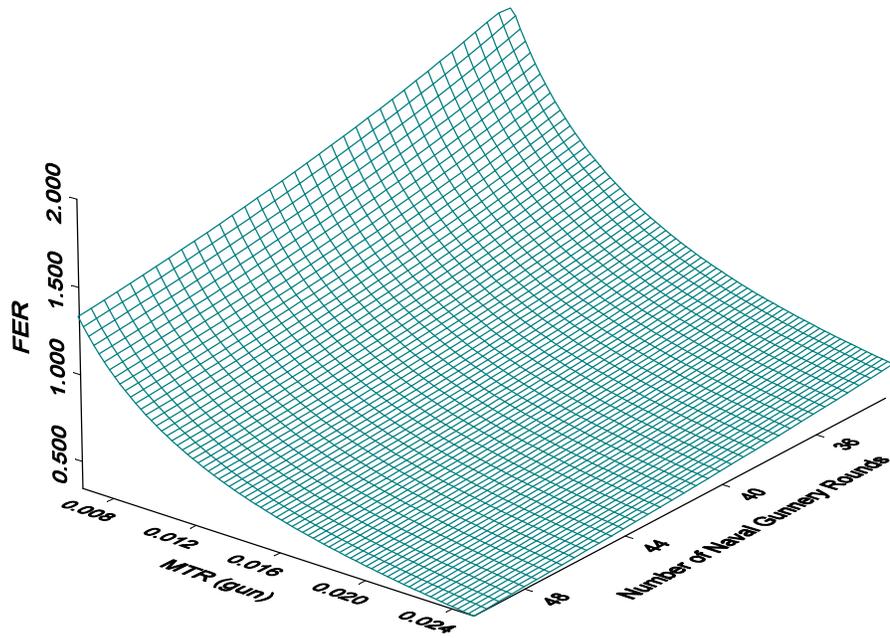
Figure 48.   Up-Scale Scenario FER Levels with Improving MTR (gun) & Greater Naval
Gunnery Values for a Land Force Initial Attack.

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