REGENERATIVE FREQUENCY DIVIDERS

A TUTORIAL PAPER

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1. ABSTRACT

The intent of this tutorial paper is to introduce the concept of regenerative circuits (primarily dividers). This introduction is supported with the mathematical derivations of the most basic performance characteristics. The theory is followed by examples of circuit configurations and finally the various emerging applications where these systems are utilized.

2. HISTORICAL PERSPECTIVE

The concept of regenerative frequency modulation can be traced back to a 1922 patent by J. W. Horton [20]. At the time, however, due to a very involved and complex circuit implementation of the idea, it did not draw too much attention. It wasn't until the early advancements in the telecommunications (i.e. telephony and television) that the concept gained a relatively wider audience. In 1939, R. L. Miller published his paper [20], presenting the theory and applications of the principle of regeneration, that earned him a credit of being recognized as the precursor in this field.

To date, numerous implementations of the concept were reported in the literature, and theory was developed to describe some aspects of operation like steady state stability regions and phase noise characteristics.

3. PRINCIPLES OF OPERATION — THEORY

The basic idea behind frequency regeneration is to create an oscillation at a sub-harmonic of the input signal with the aid of a feedback network. A simplified block diagram representing such a system is presented in Fig. 1.

In general, for this system to work and maintain stability, it is required that at the desired output frequency, the loop gain was greater than unity to sustain the oscillation when the input

signal is present, and that the gain was smaller than one in the absence of the input, to ensure no spurious oscillations.
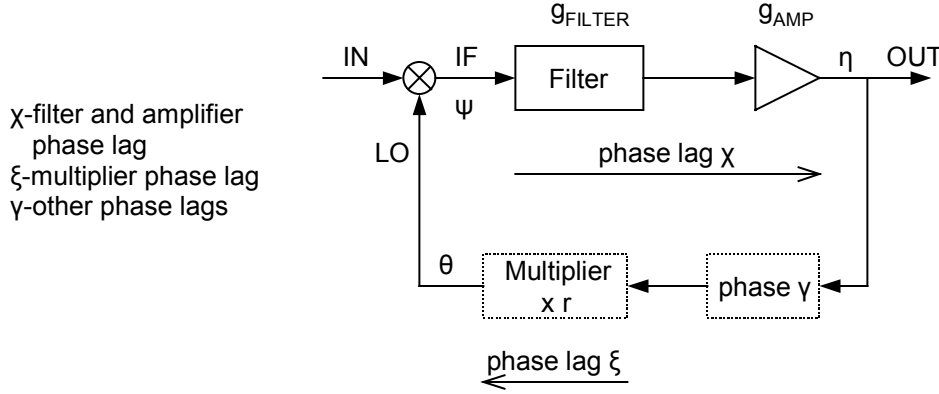

χ-filter and amplifier
   phase lag
ξ-multiplier phase lag
γ-other phase lags

Figure 1. Block diagram of a regenerative circuit [10].

For the starting condition it is assumed that originally there is a harmonic component of the injected signal present in the feedback, to start the oscillation. This assumption is very realistic and the origin of such signal can be attributed to thermal noise or to the transients that take place when the input is initially injected into the circuit [18]. Considering the simplest case of a divide-by-two system, by having a desired output frequency component f_{OUT} available for mixing with the input at f_{IN}, the sidebands at f_{OUT} and at $3f_{OUT}$ are created (since $f_{OUT}= ½ f_{IN}$). By filtering out the $3f_{OUT}$ component, the output builds up and the oscillation is sustained.

Different division ratios can be obtained using higher order modulators, where the general expression for the output frequency is of the following form [20]:

$$f_{n,m} = n \cdot f_{LO} \pm m \cdot f_{IN} \Rightarrow f_{OUT} = \frac{m}{n \pm 1} \cdot f_{IN} \qquad (1)$$

where n and m are mixing mode indexes.

Also, by including a harmonic generator in the feedback path, it is possible to obtain much higher division ratios as described by the following formula [20]:

$$f_{OUT} = \frac{m}{r \cdot n \pm 1} \cdot f_{IN} \tag{2}$$

where r is the multiplication factor of the harmonic generator.

4. PHASE NOISE

The analysis reproduced in this section follows exactly the derivations presented by Rubiola et al. in [10]. Their analysis shows that both the amplitude noise of the output signal, A_{OUT}, and the output phase noise, ψ, are functions of θ, where θ is the phase of the feedback signal at the mixer input. Because of that it is not possible to tune the system for maximum A_{OUT} while minimizing the phase noise, that is, design optimizing both parameters. In fact, it can be shown that for the maximum amplitude of the output signal, the phase noise gain also reaches its peak, and as the amplitude decreases, so does the noise. Decreasing the amplitude, unfortunately, leads towards system instability, potential problems with the startup and decreases the operating bandwidth. A compromise is possible, however. It is due to the fact, that at the point where the amplitude reaches its maximum value, the circuit is operating in saturation and therefore detuning it by a small amount does not affect the amplitude significantly while the phase noise gain diminishes more rapidly. This can lead to improvements on the order of 10 dB in the phase noise level. [10]

The first step in the analysis is to show that there is a correlation between the noise in the amplitude and the noise in the phase of the mixer's output signal. We start by identifying the signals at the terminals of the mixer block:

$$X_{IN}(t) = A_{IN} \cdot \cos(2\pi f_{IN}t) \tag{3}$$

$$X_{LO}(t) = A_{LO} \cdot \cos(2\pi f_{LO}t + \theta) \tag{4}$$

$$X_{IF}(t) = A_{IF} \cdot \cos(2\pi f_{OUT}t + \psi) \tag{5}$$

Equation (5) can be rewritten as follows:

$$x_{IF}(t) = \sum_{n,m} I_{n,m} \cdot \cos(2\pi f_{OUT} t + \beta_{n,m}) \qquad (6)$$

where I and β are the amplitude and phase of a mixing mode n, m, respectively.

From the first part of equation (1) and equations (3) and (4), β is found to be:

$$\beta_{n,m} = n \cdot \theta + \alpha_{n,m} \qquad (7)$$

where $\alpha_{n,m}$ is the phase offset of the mixing mode n, m.

Now, since the elements of the sum in equation (6) are phasors, it is easy to see that for different modes, having different value of n, a small perturbation in θ will result in different angles of β for the component phasors (according to equation (7)), thereby introducing fluctuations in the magnitude of the resulting phasor. This fact and equation (7) prove that both the amplitude noise, and the output phase noise ψ, are functions of θ.

Next, assuming small perturbations in θ due to the presence of different mixing modes, the output phase noise can be linearized as follows:

$$\psi(\theta + d\theta) = \psi(\theta) + G \cdot d\theta \qquad (8)$$

where G is the phase noise gain, and is defined as dψ/dθ.

It is this parameter G, that will describe the phase noise characteristics of the system. The derivation starts with the phase equation of the feedback loop:

$$r \cdot (\psi + \chi + \gamma) + \xi = \theta \qquad (9)$$

and two equations for the output phase:

$$\eta = \frac{\theta - \xi}{r} - \gamma \qquad (10)$$

and

$$\eta = \psi + \chi \qquad (11)$$

(Note that it is appropriate to relate phases of different frequencies, since all signals are coherent).

In the analysis, only the amplifier and the multiplier are considered as the major sources of noise. To identify the contributions of each one of them to the overall phase noise figure, it is convenient to consider them separately. Thus, first by excluding the effect on the noise due to the multiplier, the following two assumptions can be made:

i) γ is constant, therefore $d\gamma = 0$, and

ii)   the multiplier is noiseless, therefore $d\xi = 0$.

Then, by differentiating equations (9) and (10), we get:

$$r \cdot d\psi + r \cdot d\chi = d\theta \qquad\qquad (12)$$

and

$$d\eta = \frac{1}{r} \cdot d\theta \qquad\qquad (13)$$

Substituting (12) into (13), given that $G = d\psi/d\theta$, we obtain:

$$\frac{d\eta}{d\chi} = \frac{1}{1 - r \cdot G} \qquad\qquad (14)$$

By performing similar operations and excluding the effect of the opamp, which allows to assume that $d\gamma = 0$, and $d\chi = 0$, the following expression can be arrived at:

$$\frac{d\eta}{d\xi} = \frac{G}{1 - r \cdot G} \qquad\qquad (15)$$

Having found the contributions of the amplifier and the multiplier, the total noise phase can be formulated as:

$$S_\eta(f) = \left[\frac{1}{1 - r \cdot G}\right]^2 \cdot S_\chi(f) + \left[\frac{G}{1 - r \cdot G}\right]^2 \cdot S_\xi(f) \qquad\qquad (16)$$

where, the S(f) terms represent noise power spectral densities, with S_χ and S_ξ being defined by the component characteristics and S_η being the generated noise.

Equation (16) indicates that the feedback loop attenuates S_χ and S_ξ, since G is negative (G>0 is possible only if the perturbing mode is greater than the main mode, which would not be the case under desired, stable operating point).

This property makes regenerative dividers very attractive for applications where low phase noise frequency division or high spectral purity frequency sources are required.

As a final remark, it should be noted that the $S_\chi(f)$ term in equation (16), is affected by the opamp's ability to attenuate at frequencies corresponding to higher mixing products. Namely, considering the case of a divide-by-two system, the constituent of $S_\chi(f)$ due to the thermal noise would be double, if the bandwidth of the opamp was wide enough not to attenuate signals around $3f_{IN}/2$ thus allowing the noise around this image frequency to mix with the input, generating extra noise at the desired $f_{IN}/2$. This pertains to the systems where there is no image reject filter in the feedback path. [4]

5. STABILITY

A theory on the stability aspect of the regenerative dividers have been presented in a few papers [11], [14], [19]. All of them share the same conclusion, which is the existence of phase bands, and consequently, frequency ranges where the operation is stable. These stable phase bands, however, depend on the topology of the divider or precisely, on the mixer's principle of operation, which is either (i) small signal, where a single transistor performs the mixing, or (ii) large signal, where a Gilbert multiplier is driven like a commutator that merely switches the input signal, as outlined in Section 7.

The analyses presented in the literature limit to the case of a divide-by-two regenerative divider and therefore it will also be the scope of this section.

As the first case, the large signal switching mixer is considered. Derksen, et al. offered a theory in [11], that will be followed in short, below.

First, note, that to adopt the system of Figure 1 to the case of a divider by two, no multiplier is present in the feedback and the stray phase lags, also in the feedback, are ignored, making γ = 0. It is further assumed that the phase and the amplitude of the signals are not mutually related, which is not in contradiction with the conclusions of the previous section where only the noise of both parameters was found to be correlated.

Based on these assumptions, the amplitude and phase equations describing the steady state operation can be found by inspection:

$$A_{LO} = g_{AMP} \cdot g_{FILTER} \cdot A_{IF} \tag{17}$$

and $\theta = \psi + \chi + 2k\pi$ where k is an integer   (18)

Also, subject to the large signal operation, the output of the mixer can be written as:

$$x_{IF}(2\pi f_{OUT}) = sign(x_{IN}) \cdot sign(x_{LO})$$

$$= sign[cos(2\pi(2f_{OUT})t)] \cdot sign[cos(2\pi f_{OUT}t + \theta)] \tag{19}$$

where, x_{IN} and x_{LO} are defined in (3) and (4).

Equation (19), then, yields the output:

$$x_{OUT} = A_{OUT} \cdot cos[2\pi f_{OUT}t + \psi(\theta)] \tag{20}$$

where, by applying Fourier analysis to equation (19),

$$\psi(\theta) = \begin{cases} 0 & \text{for} & \theta = 0 \\ \arctan[(\sqrt{2} - \cos\theta)/\sin\theta] - 90° & \text{for} & 0 < \theta < 45° \\ \arctan[\cos\theta/(\sqrt{2} - \sin\theta)] - 90° & \text{for} & 45° \le \theta < 135° \\ \arctan[-(\sqrt{2} + \cos\theta)/\sin\theta] - 90° & \text{for} & 135° \le \theta < 180° \end{cases}$$

(21)

Stability is then guaranteed, subject to the following condition [19]:

$$|\psi'(\theta)| < 1 \tag{22}$$

Applying (22) to (21), the total loop phase periodic stable interval can be determined:

$$19° < \theta < 71° \qquad \text{and} \qquad 109° < \theta < 161°$$

$$-35° < \psi < -55° \quad \text{and} \quad -125° < \psi < -145°$$
$$55° < \chi < 125° \quad \text{and} \quad 235° < \chi < 305° \quad\quad (23)$$

These results were confirmed experimentally [11]. (This is done by measuring the loop delays for the lower and upper frequency limits of a stable band and converting it into phase delay).

A rather involved analysis was also presented by Immovilli and Mantovani in [19], which is based on the assumption of small signal operation. This theory predicts stability for the following periodic phase bands:

$$0° < \chi < 90° \quad \text{and} \quad 270° < \chi < 360° \quad\quad (24)$$

By comparing (23) to (24), it can be observed that for the case of small signal operation, the boundaries of stability fall exactly in the center of the stable phase bands for the large signal operation case and span intervals that are entirely unstable according to (23). While there is no contradiction in this due to assumptions that exclude each other, prior to [11], a paper by Harrison [14] was published where the author, justifying it, applied Immovilli and Mantovani's "small signal" theory to the "large-signal" case of double balanced diode mixer and confirmed the predictions of that theory with impressive accuracy of experimental results.

6. DESIGN PROCEDURE

The basic approach to the design of regenerative dividers is outlined in [16]. It is suggested to initially optimize the circuit for self-sustained oscillation around the desired output frequency. Next the amount of feedback should be reduced such that the oscillation would be prevented in the absence of the input signal. At this point the circuit is tuned in a traditional way, that is for maximum amplitude of the output. Should the application of the circuit, however, require stringent noise characteristics, the system can be slightly "detuned" according to the motivation given in [10].

Independently of the above, a consideration should be given to the design of the loop components in a manner that the loop phase delay dependence on operating frequency is minimized. This way, the bandwidth of the divider could be maximized for a given topology [11].

7. CIRCUIT TOPOLOGIES AND PERFORMANCE CHARACTERISTICS

The architectures of regenerative circuits can be lumped into two distinct classes. An intuitive implementation would be to incorporate all the necessary building blocks, that is, the mixer, the amplifier and the filter, exactly the way they are configured in the concept block diagram of Figure 1. In fact, this was the approach in the very early stages, which later on evolved around the Gilbert multiplier circuit. The second class of circuits emerged by recognizing the regenerative divider, in terms of principles of operation, as a derivative of an oscillator. Therefore a single transistor realization was employed.

The latter topology is depicted in Figure 2a, and was first presented in the early 80's by Cornish and later by Rauscher [18] and Stubbs et al. [16]. Raucher's circuit achieves regeneration with the aid of nonlinearity in the transconductance of the transistor and utilizes transistor's gain for the required amplification. One of the identified problems with this topology was a severe performance deterioration in the presence of two closely spaced tones [18]. In general, the major disadvantage, however, is a very narrow stable bandwidth, while there are couple of desired characteristics like very high operating frequency and low power consumption. A few variations on this topology were presented to address its shortcomings or further enhance its performance. One way of doing so is to replace the transistor with a Darlington pair [15] (Figure 2b). This configuration offers a trade-off between the bandwidth and the maximum frequency of the input signal. For inputs lower than f_T the achievable bandwidth can be up to 50%. By increasing the frequency of operation to about f_{max}, only a few per cent bandwidth is possible [15].
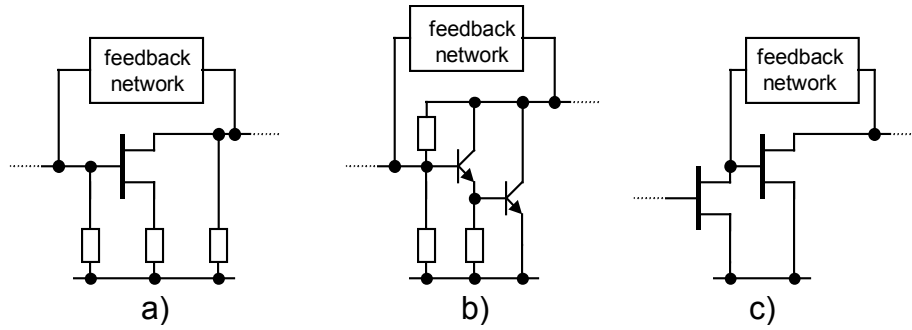
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Figure 2. Small signal operation configurations: a) single transistor, b) Darlington, c) double transistor.

Yet another alternative approach is to use two transistors instead of one [9], as shown in Figure 2c. Here, the first device is responsible for amplification and the cascaded one for regeneration. The enhancement of this configuration is in the form of increased bandwidth, up to around 20 % [9].

The other class of circuits, based on the Gilbert multiplier in a feedback configuration is shown in figure 3. This topology has a definitely much broader bandwidth compared to the single device circuits, although, the power consumption is inferior. The limit on the input frequency approaches f_T for a given technology which is quite competitive for the circuits discussed before with the exception for the Darlington case. The first implementations of the regenerative circuits that incorporated the Gilbert multiplier, also appeared in the 80's [17], [13]. This kind of mixer provides amplification by itself and configuring it in feedback mode suffices to achieve regenerative action. Derksen et al., in [17], showed that by including an additional transimpedance amplifier, the main characteristics of the system, like input sensitivity, maximum frequency and bandwidth were much improved. These results were later confirmed by Ichino et al. [13].
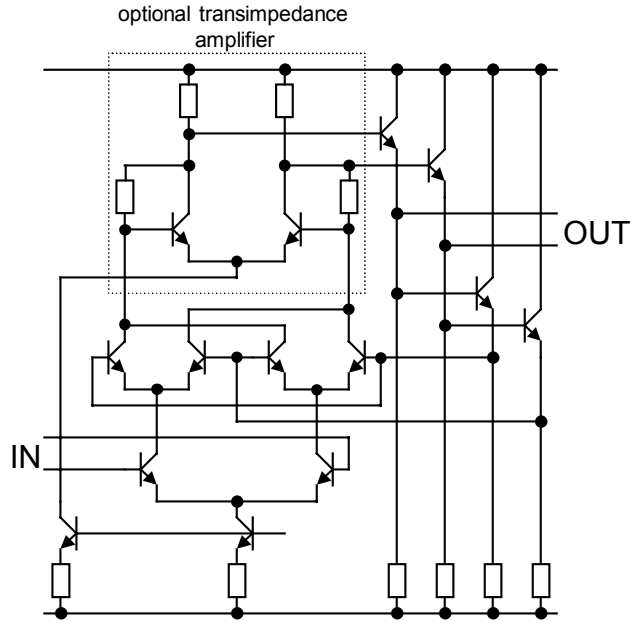
Figure 3. Large signal configuration based on the Gilbert multiplier with an optional transimpedance amplifier [17].

Today, still the main disadvantage of the double balanced mixer topology are very high power requirements unless the operating frequency spec is much relaxed ([2] as an example).

Table 1 below presents a survey of the various designs that have been published, with a breakdown into two major categories, 1-5 and 6-8, as identified above.

#	topology	Technology	f_in [GHz]	BW	power [mW]	function	year	ref.
1	single transistor (MIC)	GaAs FET	16			/2	1984	18
2	single transistor (MMIC)	GaAs FET	10	8%		/2	1986	16
3	Darlington pair (MMIC)	Si bipolar/f_t=10GHz	17.7	18%		/2	1987	15
4	two transistors (MMIC)	GaAs FET	29	20%		/2	1993	9
5	single transistor (MMIC)	0.15um pHEMT-GaAs/f_t=100GHz	94	3%	8	/2	2000	1
6	double-balanced Gilbert cell	Si bipolar	5.3	72%	135	/2	1985	17
7	double-balanced Gilbert cell	Si bipolar/f_t=20GHz	18	71%	690	/8	1989	13
8	double-balanced Gilbert cell	SiGe bipolar/f_t=75GHz	63	73%	470	/2	1999	3

Table 1. Implementations of regenerative circuits and their characteristics.

8. APPLICATIONS

The reason for so much attention towards regenerative circuits in the recent decades is twofold. First of all, they offer a few advantages over conventional flip-flop dividers that currently dominate in the applications. Some of the benefits of these new dividers include higher operating frequency, lower power consumption [3], [13], and improved phase noise performance [10]. Secondly, the regenerative dividers can be utilized with surprising versatility.

Apart from straight division/multiplication, which is a "day-job" for regenerative circuits, a number of interesting applications have been presented in the literature.

One of the well "marketable" properties of regenerative dividers is their impressive phase noise performance. Their noise floors were reported at −165 dB (−175 dB for a "detuned" divider), for both sub-GHz [10], and much higher frequencies [4]. Utilizing this characteristic, Groslambert et al. were able to demonstrate an application where a series of dividers served as a reference signal for phase noise measurement of a crystal oscillator at 10 MHz [12].

More recently, the concept of regenerative division was incorporated into autonomous RF ICs. Wang et al. designed a clock recovery IC for next generation of SONET applications, operating at a rate of 40 and 20 Gbit/s [6].
In another RF application, presented by Maligeorgos and Long, a regenerative multiplier/divider combination employed in a receiver IC, facilitates a generation of phase quadrature signals and allows for a lower frequency VCO, simultaneously achieving an LO with reduced spectral noise [2].

As indicated by [2] and [6], the regenerative circuits are very well suited for the receiver/transmitter wireless building blocks, and in the near future, are expected to become very prominent especially in this sector of electronics.

9. MISCALLENOUS

For the interested reader, a list of references on other related aspects of the regenerative theory is found below:

[14] – Steady state solutions for the phase and amplitude, and starting condition for the regenerative divider based on a diode double balanced mixer.

[7] – ½ divider implementation using LSB-SSB modulation.

[5] – CAD analysis approach to microwave regenerative dividers using Voltera series method.

[8] – Study of the influence of lowpass filters on the divider input sensitivity.

10. CONCLUDING REMARKS

The major characteristics of regenerative circuits like stability regions and phase noise performance that were outlined along with some circuit configurations and real-life applications of the concept should be regarded merely as a short compendium of the knowledge on these topics and as a literature survey. All presented material has been previously published by various authors, and if a thorough analysis is required the reader is suggested to consult the original publications.

It is fair to say, that at this point, most of the theoretical work on the topic has been completed, which is evident with the recent publications focusing primarily on the applications side. As outlined in Section 8, the regenerative circuits offer the characteristics and performance which enable upgrades to new generations in current RF applications. Because of that, it is expected that soon, if it's already not the case, they will be found in a number of production ICs.

11. REFERENCES

[1].   S. Kudszus, W. H. Haydl, M. Neumann, M. Schlechtweg, "94/47-GHz regenerative frequency divider MMIC with low conversion loss*", IEEE Journal of Solid-State Circuits*, Vol. 35, No. 9, Sep. 2000, p.1312-7.

[2]   J. Maligeorgos, J. Long, "A 2V 5.1-5.8GHz image-reject receiver with wide dynamic range", *2000 IEEE International Solid-State Circuits Conference*, February 9, 2000, p.322-323.

[3].   J. Müllrich, W. Klein, R. Khlifi, H.-M. Rein, "SiGe regenerative frequency divider operating up to 63 GHz", *Electronics Letters*, Vol. 35, No. 20, 30 Sep. 1999, p.1730-1.

[4].   E. S. Ferre-Pikal, F. L. Walls, "Microwave regenerative frequency dividers with low phase noise", *IEEE Transactions on Ultrasonic s, Ferroelectrics, and Frequency Control*, Vol. 46, No. 1, Jan. 1999, p.216-9.

[5].   K.-K. M. Cheng, C.-W. Fan, "A novel approach to the analysis of microwave regenerative analog frequency dividers", *IEEE Microwave and Guided Wave Letters*, Vol. 8, No. 7, July 1998, p.266-7.

[6].   Z.-G. Wang, M. Berroth, A. Thiede, M. Rieger-Motzer, P. Hoffman, A. Hülsmann, K. Köhler, B. Raynor, J. Schneider, "40 and 20 Gbit/s monolithic integrated clock recovery using a fully-balanced narrowband regenerative frequency divider with 0.2 μm AlGaAs/GaAs HEMTs", *Electronics Letters*, Vol. 32, No. 22, 24 Oct. 1996, p.2081-2.

[7].   A. A. Ciubotaru, "Technique for achieving wideband ½ regenerative frequency division using LSB-SSB modulation", *Electronics Letters*, Vol. 31, No. 6, 16 March 1995, p.425-6.

[8].   A. A. Ciubotaru, "Influence of lowpass filter on input sensitivity of ½ regenerative frequency divider", *Electronics Letters*, Vol. 30, No. 23, 10 Nov. 1994, p.1906-8.

[9].   A. Suárez, E. Ngoya, P. Savary, M. Camiade, J. C. Sarkissian, R. Quéré, "Broadband design and simulation of frequency dividers in the millimetric band", *23rd European Microwave Conference Proceedings, Madrid, Spain, 6-10 Sep. 1993*, Vol. 1, p.777-80.

[10].   E. Rubiola, M. Olivier, J. Groslambert, "Phase noise in the regenerative frequency dividers", *IEEE Transactions on Instrumentation and Measurement*, Vol. 41, No. 3, June 1992, p.353-60.

[11].   R. H. Derksen, V. Lück, H.-M. Rein, "Stability ranges of regenerative frequency dividers employing double balanced mixers in large-signal operation", *IEEE Transactions on Microwave Theory and Techniques*, Vol. 39, No. 10, Oct. 1991, p.1759-62.

[12].   J. Groslambert, M. Olivier, E. Rubiola, "High spectral purity frequency sources using low noise regenerative frequency dividers", *Proceedings of the 45th Annual Symposium on Frequency Control*, 29-31 May 1991, p.636-9.

[13].   H. Ichino, N. Ishihara, M. Suzuki, S. Konaka, "18-GHz 1/8 dynamic frequency divider using Si bipolar technologies", *IEEE Journal of Solid-State Circuits*, Vol. 24, No. 6, Dec. 1989, p.1723-8.

[14]. R. G. Harrison, "Theory of regenerative frequency dividers using double-balanced mixers", *IEEE 1989 MMT-S International Microwave Symposium Digest, Long Beach, CA, USA, 13-15 June 1989*, Vol. 1, p.459-62.

[15]. I. Kipnis, "20 GHz frequency-divider silicon bipolar MMIC", *Electronics Letters*, Vol. 23, No. 20, 24 September 1987, p.1085-7.

[16]. M. G. Stubbs, S. P. Stapleton, "A single stage monolithic regenerative ½ analog frequency divider", *IEEE Gallium Arsenide Integrated Circuit Symposium. Technical Digest. 28-30 Oct.* 1986, p.199-201.

[17]. R. H. Derksen, H.-M. Rein, K. Wörner, "Monolithic integration of a 5.3 GHz regenerative frequency divider using a standard bipolar technology", *Electronics Letters*, Vol. 21, No. 22, 24 Oct. 1985, p.1037-9.

[18]. C. Rauscher, "Regenerative frequency division with a GaAs FET", *IEEE Transaction on Microwave Theory and Techniques*, Vol. MMT-32, No. 11, Nov. 1984, p.1461-8.

[19] G. Immovilli, G. Mantovani, "Analysis of the Miller frequency divider by two in view of applications to wideband FM signals", *Alta Frequenza*, Vol. XLII, No. 11, November 1973, p.313E(583)-323E(593).

[20]. R. L. Miller, "Fractional-Frequency Generators Utilizinig Regenerative Modulation", *Proceedings of the I.R.E.,* July 1939, p.446-57.