**NATIONAL GEOSPATIAL-INTELLIGENCE AGENCY (NGA)
STANDARDIZATION DOCUMENT**

Implementation Practice
Web Mercator Map Projection

2014-02-18

Version 1.0.0

OFFICE OF GEOMATICS

Table of Contents

Appendix A

Appendix B

List of Table and Figures

1. Scope This document contains general technical information on the Web Mercator map projection to include its relationship to the World Geodetic System 1984 (WGS 84) and the risks and impacts associated with its usage as a positioning reference tool for the National System for Geospatial-Intelligence (NSG).

2. Authority

The National Geospatial-Intelligence Agency (NGA) serves as the Functional Manager for Geospatial-Intelligence (GEOINT) to the NSG. As such, it is responsible for defining the WGS 84 and providing guidance regarding its application and use. This includes guidance regarding the ellipsoid, datum, grids, projections, coordinate reference systems, coordinate conversions and coordinate transformations.

The Joint Chiefs of Staff established Position (Point and Area) Reference Procedures policy for unilateral and joint operations of the US Armed Forces, CJCSI 3900.01C, requiring NGA to establish and maintain the WGS 84 and to assist the combatant commanders in determining an appropriate alternative positional reference system should they be unable to use WGS 84. This ensures positional accuracy is retained and is interoperable as data and products are shared and used by the NSG members and partners.

As the NSG Functional Manager for GEOINT, NGA has purposefully not promulgated or promoted the Web Mercator map projection or any of its related variant namings for inclusion in the Department of Defense (DoD) Information Technology Standards and Profile Registry (DISR) or the Intelligence Community Enterprise Standards Baseline (ESB).

3. References

3.1 DoD Documents, Specifications, and Standards

3.1.1 *Department of Defense Directive 5105.60 (DoDD 5105.60), "Subject: National Geospatial-Intelligence Agency (NGA)", July 29, 2009*

3.1.2 *MIL-STD-2401, Department of Defense Standard Practice "Department of Defense World Geodetic System (WGS)", 11 January 1994*

3.1.3 *CJCSI 3900.01C, "Position (Point and Area) Reference Procedures", 30 June 2007*

3.1.4 *CJCSI 6130.01E, "2013 CJCS Master Positioning, Navigation, and Timing Plan (MPNTP)", 1 May 2013*

3.1.5 *NGA.STND.0036_1.0_WGS84, "Department of Defense World Geodetic System 1984, Its Definition and Relationships With Local Geodetic Systems", Fourth Edition [formerly NIMA Technical Report TR8350.2]*

3.2 Non-DoD Publications

3.2.1 *USGS Professional Paper:1395, "Map Projections: A Working Manual", Snyder, John P. 1987*

3.2.2 *OGP Publication 373-7-2 – Geomatics Guidance Note number 7, part 2 – June 2013*

4. Background

4.1 Cartographic theory and the science of map projections extend back nearly 500 years and are well documented in textbooks, publications, and numerous on-line sources. Map projections are mathematical functions that convert coordinates between the 3-dimensional curved surface of the earth and flat 2-dimensional map surfaces. Map projections attempt to maintain certain properties of the curved earth surface onto the flat map surface and accept the resulting distortions and errors as negligible when compared to the map and mission requirements. These desired properties include conformity, equal area, equidistant, and azimuthal or true direction. Internet web services and GIS applications must apply some mathematically derived map projection to visually display 3 dimensional geographic data on a flat 2-dimensional computer screen.

4.2 Web Mercator is a map projection commonly implemented in commercial and open source data sets and internet based visualization applications. Web Mercator mathematics are formulated on a spherical assumption of the earth which conflict with the defined WGS 84 ellipsoid reference system. Therefore, careful considerations must be implemented when Web Mercator is encountered in commercial and open sources data and tool assets sought for use and incorporation into NSG resources and processes.

4.3 The WGS 84 and its predecessor systems back to WGS 60 are based on a global ellipsoid model of the earth that includes the ellipsoid mathematics of map projection theory. The Department of Defense has mandated the WGS 84 as the official positional reference system for use by the DoD. The Global Positioning System (GPS) implements WGS 84 and all GPS obtained position information are relayed by radio signal in WGS 84 coordinates. Therefore, all GEOINT data must implement WGS 84 ellipsoid based formulas to ensure data accuracy and system interoperability between GEOINT data, the GPS, and NSG systems.

4.4 The use of an ellipsoid model of the earth has been the established practice in mapping, charting, and geodesy for over 150 years. The ellipsoid model is a closer representation of the true shape of the earth and provides for accurate position, azimuth, and distance information, as opposed to a spherical model of the earth.

4.5 As the WGS 84 system is based upon an ellipsoid model, GEOINT data referenced to the WGS 84 must be handled by formulas appropriate for its datum and ellipsoid. The formulas used for the Web Mercator map projection are not appropriate for use with WGS 84-referenced data because they involve a spherical, not an ellipsoid, model of the Earth. Map

projection and coordinate differences of over 40 km can occur between spherical based Web Mercator Easting and Northing (plane) coordinates as compared to WGS 84 ellipsoid based Mercator Easting and Northing (plane) coordinates.

5. Web Mercator Overview

5.1 Numerous commercial and open source data sets and internet based visualization applications are using a map projection most commonly known as "Web Mercator". Other terms for Web Mercator include, but are not limited to, "WGS 84 Web Mercator", "WGS 1984 Web Mercator (Auxiliary Sphere)" and "EPSG:3857". The simple conjunction of the words "WGS 84" to "Web Mercator" does not make data compliant to WGS 84 standards and is inherently erroneous, misleading, and potentially a risk for safety of life and safety of navigation.

5.2 The Web Mercator map projection has several defining mathematical formulas and parameters that make data referenced to Web Mercator incompatible with WGS 84 ellipsoid referenced GEOINT. These incompatibilities include:

5.2.1 Spherical equations. The use of simple spherical mathematics to convert ellipsoid based latitude and longitude data to spherical based plane coordinates.

5.2.2 Non-conformal. The use of a point scale factor that varies as a function of the azimuth and creates angular distortions. This means that Web Mercator is not a conformal projection and that true rhumb lines are not straight lines on a Web Mercator projection.

5.3 The mathematical differences described in 5.2.1 and 5.2.2 can cause errors in spherical based Web Mercator plane coordinates of over 40 km compared to WGS 84 ellipsoid based Mercator plane coordinates.

5.4 Visually, Web Mercator plane coordinates appear to portray a correctly rendered 'Mercator-style' map, and numerous layers of Web Mercator data will align on a map or computer screen. The shifts become apparent only when spherical based Web Mercator plane coordinates are compared to WGS 84 ellipsoid based Mercator plane coordinates, e.g. control points, GPS data, and WGS 84 ellipsoid registered maps, data, and GEOINT. Figure A2 in Appendix A provide a visual graphic of this displacement.

6. Implementation Practice

6.1 The WGS 84, including the datum, coordinate system, grid, transformation, gravity model, and map projection information, is NGA's best geodetic model of the Earth and is the official Department of Defense positional reference system. In accordance with References listed in 3.1, WGS 84 ellipsoid based data are required by the NSG to ensure interoperability in relating information from one product to another, to support increasingly stringent accuracy requirements, and to support military, intelligence, and humanitarian activities worldwide.

6.2 NGA does not endorse nor does NGA support the spherical based Web Mercator map projection (and variant namings such as WGS 84 Web Mercator) for the acquisition, visualization, exploitation, and exchange of any GEOINT data for the NSG.

6.3 Any data in the spherical based Web Mercator map projection must be converted to WGS 84 ellipsoid latitude and longitude or to WGS 84 ellipsoid based Mercator plane coordinates before they are used, visualized, exploited, and exchanged for the NSG. This requires knowledge of the original datum information for the data in the Web Mercator map projection.

6.4 Procedures for the conversion from spherical based Web Mercator plane coordinates to WGS 84 ellipsoid based Mercator plane coordinates are specified in the Appendix of this document.

7. Summary

Modern maps, navigation systems, and geodetic applications require a single accessible, global, 3-dimensional reference frame. WGS 84 supports the most stringent accuracy requirements for geodetic positioning, navigation, mapping, and targeting for the NSG. It is essential for global operations, safety of navigation, and interoperability that NSG systems implement and operate on WGS 84 and avoid the use of simple spherical mathematics and spherical models of the earth.

Appendix A

A1. Ellipsoid Mercator Projection

A1.1 The Mercator projection is an ubiquitous map projection that has existed for over 400 years and has several advantages for nautical charting with respect to rhumb lines. It is also a conformal projection that preserves angles and a linear point scale that is equal in all directions around any point. Mercator projection formulas exist for both spherical and ellipsoid mathematics. The formulas used to derive the conformal ellipsoid Mercator coordinates (x = Easting, y = Northing) from ellipsoid Latitude (ϕ) and Longitude (λ) are:

$$x = Easting = a\lambda \tag{1}$$

$$y = Northing = a \ \ln\left[\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\left(\frac{1 - e\sin\phi}{1 + e\sin\phi}\right)^{\frac{e}{2}}\right] \tag{2.a}$$

$$= a \ \tanh^{-1}(\sin\phi) - ae\tanh^{-1}(e\sin\phi) \tag{2.b}$$

$$h = k = \frac{\sqrt{1 - e^2\sin^2\phi)}}{\cos\phi} \tag{3}$$

where:

$\lambda = ellipsoid \ longitude \ in \ radians$

$\phi = ellipsoid \ latitude \ in \ radians$

$a = ellipsoid \ semi\text{-}major \ axis \ [WGS\,84 = 6378137m]$

$e = ellipsoid \ eccentricity \ [WGS\,84 = 8.1819190842622 \times 10^{-02}]$

$h = \ meridian \ scale \ factor$

$k = \ parallel \ scale \ factor$

Note: The above formulas can be found in Ref(3.2.1) and Ref(3.2.2). The two formulas, (2.a) and (2.b), are equivalent.

A2. Web Mercator

A2.1 Numerous web and visualization applications, commodity data, and open source data use a much simplified spherical variant of the Mercator projection for their map images and web applications. Rather than implement the more accurate and conformal ellipsoid mathematics from (1), (2), and (3), software designers implemented the simpler spherical mathematics (see (4) and (5) below) to convert from ellipsoid data to spherical based Mercator

plane coordinates.   Additional simplifications unfortunately produced a non-conformal projection, i.e. the point scale factor varies as a function of the azimuth and creates angular distortions (see (6) below).   The resultant map projection visually appears to be spherical Mercator, but is not conformal projection. The term "Web Mercator" may have been created to bring some order out of the confusion caused by this non-conformal, spherical implementation of the conformal spherical Mercator projection. The formulas used to derive the Web Mercator coordinates (x = Easting, y = Northing) from ellipsoid Latitude (ϕ) and Longitude (λ) are:

$$x = Easting = a\lambda \tag{4}$$

$$y = Northing = a \ \ln\left[\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\right] \tag{5.a}$$

$$= a \ \tanh^{-1}(\sin\phi) \tag{5.b}$$

$$q_\alpha = \frac{a}{a' \cos\phi}$$

$$a' = \frac{\rho\nu}{\nu \cos^2\alpha + \rho \sin^2\alpha}$$

$$\rho = \frac{a(1 - e^2)}{(1 - e^2 \sin^2\phi)^{\frac{3}{2}}} \tag{6}$$

$$\nu = \frac{a}{(1 - e^2 \sin^2\phi)^{\frac{1}{2}}}$$

where:

$\lambda = longitude\ in\ radians$

$\phi = latitude\ in\ radians$

$a = ellipsoid\ semi\text{-}major\ axis\ [WGS\ 84 = 6378137m]$

$e = ellipsoid\ eccentricity\ [WGS\ 84 = 8.1819190842622 \times 10^{-02}]$

$\alpha = \ azimuth, measured\ clockwise\ from\ North$

$q_\alpha = \ the\ scale\ factor\ at\ a\ given\ azimuth$

Note: The above formulas can be found in Ref(3.2.2). The two formulas, (5.a) and (5.b), are equivalent.

A3. Analysis

A3.1 Semi-major and Semi-minor Axes As introduced in Section 4, an ellipsoid model is a closer representation of the earth versus a simple spherical model, as the distance from the center of the earth to the pole (the semi-minor axis) is approximately 20 km less than the distance from the center out to the equator (the semi-major axis). The spherical Web Mercator projection assumes the earth is a perfect sphere and forces the distance from the center to the pole to be the same as the distance from the center to the equator. WGS 84, GPS, DOD and NSG systems are designed to use and incorporate GEOINT data based on the ellipsoid model of the earth and the associated ellipsoid mathematics.

A3.2 Easting Coordinates The Easting coordinate equations are identical for both the ellipsoid based Mercator projection (1) and the spherical based Web Mercator projection (4). The Easting coordinates will be identical in the two projections and do not present any problems.

A3.3 Northing Coordinates Examination of the Northing coordinate equations, (2.a) and (5.a) or (2.b) and (5.b), show additional mathematical terms for the ellipsoid based Mercator projection compared to the spherical based Web Mercator projection. The Northing coordinates will not be identical in the two projections. The additional mathematical terms in (2.a) and (2.b) are required to properly reduce the Northing coordinate of an ellipsoid model compared to the spherical model in the Mercator projection making the ellipsoid Mercator Northing coordinates 'less than' the Web Mercator Northing coordinate.

A3.4 Projection Differences Table A1 lists spherical Web Mercator and ellipsoid Mercator Northing coordinates for selected points at increasing latitudes (similar negative coordinates exist for the Southern latitudes). Note that the relative difference between each set of points is approximately the same for either Projection, (e.g. the distance between Point 1.0 and Point 1.1 for each projection differ by 0.2 meter, as noted in the light grey bars). This implies that relative, or internal, sizes and distances in one projection are similar in the other projection. The absolute difference between the ellipsoid projection and spherical projection is noted in the far right column (dark grey bars). This demonstrates that the location of objects in one projection is far from where the other projection places them.
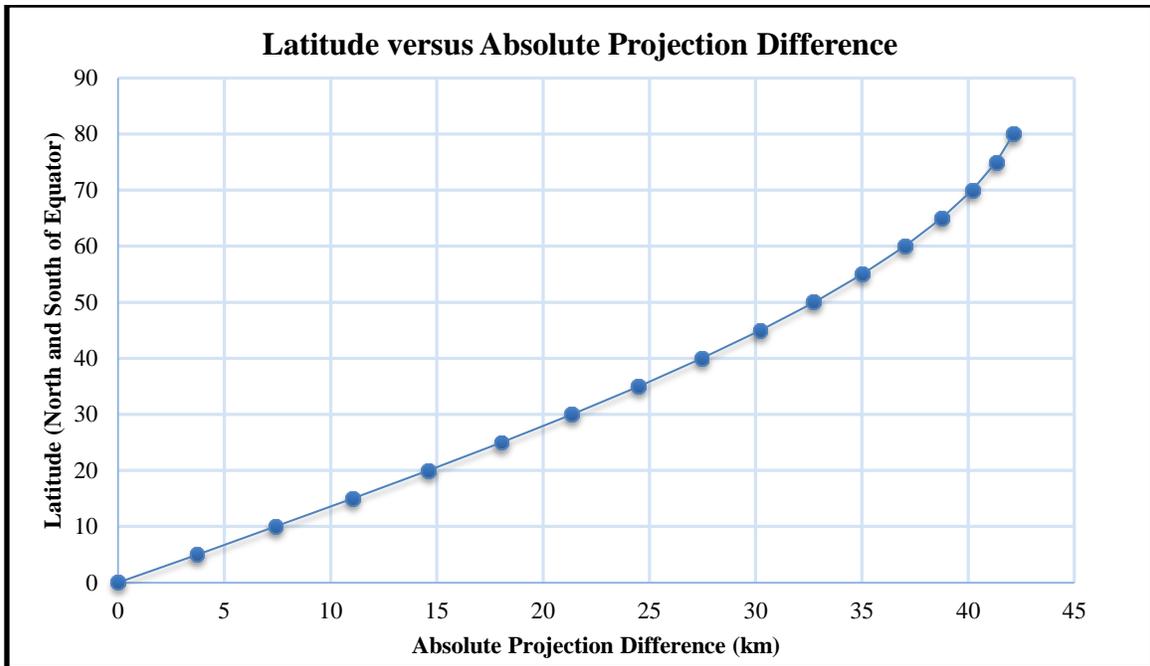
A3.5 Graph of Projection Differences Figure A1 graphs the absolute difference at several latitudes between the spherical Web Mercator Northing coordinates and the WGS 84 ellipsoid based Mercator Northing coordinates. Note that this reduction is not a linear term, but rather a function of the hyperbolic tangent. For convenience, the numbers in Figure A1 are in kilometers.

A3.6 Visual Example Figure A2 provides a visual example of the Northing coordinate shifts when spherical Web Mercator plane coordinates are projected with WGS 84 ellipsoid Mercator plane coordinates. The country outlines of the United Kingdom and Ireland are projected in WGS 84 ellipsoid Mercator (blue) and spherical Web Mercator (red). The WGS 84

graticule is displayed in black. The two insets depict both the Northing shift and how the shift increases at increasing latitudes (33 km at 50°and 36.7 km at 58°).

	WGS 84 Latitude (Deg Min Sec hem)				Spherical Web Mercator Northing (m)	WGS 84 Ellipsoid Mercator Northing (m)	Absolute Projection Difference (m)
Point 1.0	10	00	0.000	N	1,118,889.97	1,111,475.10	7,414.87
Point 1.1	10	00	1.000	N	1,118,921.37	1,111,506.30	7,415.08
					31.40	31.20	
Point 2.0	20	00	0.000	N	2,273,030.93	2,258,423.65	14,607.28
Point 2.1	20	00	1.000	N	2,273,063.83	2,258,456.36	14,607.47
					32.90	32.71	
Point 3.0	30	00	0.000	N	3,503,549.84	3,482,189.09	21,360.76
Point 3.1	30	00	1.000	N	3,503,585.55	3,482,224.61	21,360.94
					35.71	35.52	
Point 4.0	40	00	0.000	N	4,865,942.28	4,838,471.40	27,470.88
Point 4.1	40	00	1.000	N	4,865,982.65	4,838,511.61	27,471.04
					40.37	40.21	
Point 5.0	50	00	0.000	N	6,446,275.84	6,413,524.59	32,751.25
Point 5.1	50	00	1.000	N	6,446,323.95	6,413,572.57	32,751.38
					48.11	47.98	
Point 6.0	60	00	0.000	N	8,399,737.89	8,362,698.55	37,039.34
Point 6.1	60	00	1.000	N	8,399,799.73	8,362,760.29	37,039.45
					61.84	61.74	

Table A1 – Northing Coordinate Projection Differences (in meters)

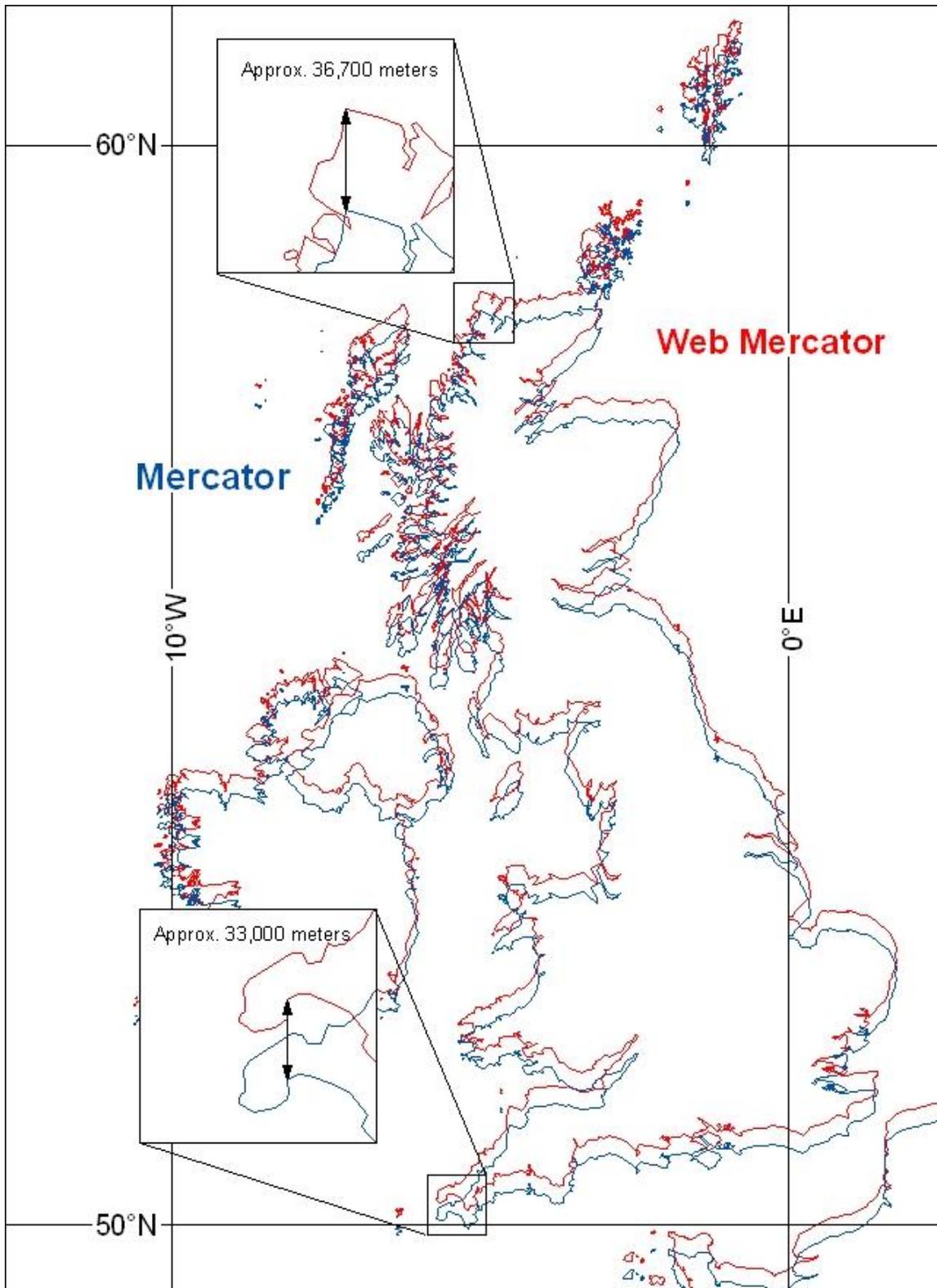**Figure A1 – Latitude versus Absolute Projection Difference (in kilometers)**

Figure A2 – Visual Example of Overlaying Ellipsoid Mercator (blue) and Web Mercator (red), graticule is WGS 84

A4. Mitigation, Solutions, and Transformations

A4.1 Visualization only misconception It might be argued that Web Mercator is only a visualization tool to depict GEOINT data, and that more appropriate tools will be employed for determination of positions, distances, and directions. Visually, Web Mercator Northings and Eastings coordinates appear correct and numerous layers of Web Mercator Northing and Easting data will align on a map or computer screen. Only when compared to WGS 84 ellipsoid Mercator plane coordinates, i.e. control points, GPS data, or WGS 84 ellipsoid registered maps and data, does the shift become apparent. GEOINT data in Web Mercator coordinate data displayed on a screen that 'appears' to be correct can be downloaded and exploited for any analytic purpose. At this point, Web Mercator coordinate data may no longer be 'visualized' and injected into NSG systems that are expecting and designed for WGS 84 ellipsoid based data.

A4.2 Formulas, not the data The underlying fault is not the data, but rather the fault is the use of improper and incorrect equations. Web Mercator is a spherical map projection. GEOINT data that are referred to the WGS 84 datum and ellipsoid should be handled by formulas that are correct for the WGS 84 datum and ellipsoid. Web Mercator applies spherical, not ellipsoid, coordinate conversions. If the original datum information is known before the transformation to Web Mercator, formulas exist to properly transform Web Mercator plane coordinates to ellipsoid Mercator plane coordinates or to WGS 84 ellipsoid latitude and longitude.

A4.3 Obtain GEOINT in pre-visualization form Web Mercator is a map projection derived by conversion of original latitude and longitude coordinate data into easting and northing planar coordinates. The original data was most likely collected and controlled with GPS data, which is WGS 84 ellipsoid data. All attempts should be made to acquire the original underlying commodity or open source data, i.e. WGS 84 latitude and longitude, in its pre-visualization form.

A4.4 Commercial software If the Web Mercator coordinate data is available in a commercial file format, e.g. ESRI ArcGIS shapefile, then that software may be used to transform it to WGS 84 ellipsoid latitude and longitude

A4.5 Math formulas for transforming from Web Mercator

A4.5.1 The formulas for transforming Web Mercator coordinates (x_{wM}, y_{wM}) to WGS 84 ellipsoid Mercator coordinates (x_M, y_M) are defined below.

A4.5.2 In the general case there may be one or more of the following considerations:

(i)   Change of scale (so that a ratio other than 1:1 will describe the representation of the Equator),
(ii)   Reduction for printing,

(iii) Change of units (perhaps associated with the reduction for printing),

(iv) Offsets (so the Equator- prime-meridian-intersection will have non-zero coordinates),

but these are handled in the same way in both systems. In other words, the transformation process is:

(i) Read-in Web Mercator coordinates,

(ii) Undo change of scale, change of units, reduction for printing, offsets,

(iii) Apply equations 9 and 10 below,

(iv) Reapply change of scale, change of units, reduction for printing, offsets.

A4.5.3 The parameters for WGS 84 are:

$$Semimajor\ axis = \ a_{WGS} = 6378137\ meters \tag{7}$$

$$Eccentricity = \ e_{WGS} = \ 8.1819190842622 \times 10^{-02} \tag{8}$$

A4.5.4 Web Mercator uses a sphere for its model of the Earth and therefore one must first determine the radius of the sphere, a_{wM}, used when obtaining the Web Mercator data. To convert the data to WGS 84, one should use the following equations:

$$x_{WGS} = \frac{a_{WGS}}{a_{wM}} \times x_{wM} \tag{9}$$

$$y_{WGS} = \frac{a_{WGS}}{a_{wM}} \times y_{wM} - a_{WGS} \times e_{WGS} \times \tanh^{-1}\left(e_{WGS} \times \tanh\left(\frac{y_{wM}}{a_{wM}}\right)\right) \tag{10}$$

Appendix B
Acronyms

DISR   Department of Defense Information Technology Standards and Profile Registry

DoD   Department of Defense

ESB   Enterprise Standards Baseline

GEOINT   Geospatial-Intelligence

GPS   Global Positioning System

NGA   National Geospatial-Intelligence Agency

NSG   National System for Geospatial-Intelligence

WGS 60   World Geodetic System 1960

WGS 84   World Geodetic System 1984