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APPLICATION OF FUZZY LINGUISTIC COGNITIVE MAPS TO PRISONER'S DILEMMA

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Abstract. *Fuzzy linguistic cognitive maps are considered. These maps are also applied to simulation of the well-known prisoner's dilemma. This dilemma has provoked various challenges to game theory and decision making. By virtue of our approach we can use fuzzy rules and reasoning when computer models are constructed. We also aim to show that our approach seems usable in general when complicated phenomena of the real world are simulated with computers.*
Keywords: Fuzzy linguistic modeling, Fuzzy cognitive maps, Prisoner's dilemma

1. Introduction.

Complicated phenomena of the real world are still problematic for computer modeling. This is due to the fact that these phenomena can constitute networks which include several nodes with various causal and teleological interconnections. In addition, their nodes may include non-numerical, imprecise or uncertain entities. In particular, we encounter complicated phenomena when we perform research in human sciences because a human being as such is a very complicated object of study.

In human sciences two principal methodological traditions for resolving the foregoing problems are quantitative and qualitative approaches. The former assumes that we can apply similar methods to both the animate and inanimate world, and these methods usually have their origins in natural sciences. In philosophy, this idea of methodological monism is particularly maintained in positivistic approaches.

The qualitative tradition, in turn, presupposes that studies on human beings should apply additional methods that better take into account features characteristic of people such as their intentional behavior. The so-called Geisteswissenschaften (e.g. hermeneutics and phenomenology) usually provide a philosophical basis for this approach [15].

To date there has been a methodological controversy between the quantitative and qualitative traditions but by virtue of certain novel methods in computational intelligence, such as fuzzy systems, neural networks, evolutionary computing and probabilistic reasoning, we can now adopt both of these approaches and also study complicated phenomena better in a computer environment. Below we apply ideas of computational intelligence, in particular fuzzy systems [15,21,22].

As regards fuzzy systems, most of its models are still merely based on fuzzy set theory. This approach is widely adopted in engineering sciences and their applications. However, in human sciences we should apply more actual fuzzy linguistic models in order to obtain more usable results. A firm basis for these linguistic models is suggested in Zadeh's theories on information granulation and computing with words [21,22].

Below, it is our aim to apply fuzzy linguistic models to complicated phenomena of the real world. By virtue of this approach we can use both quantitative and qualitative methods in our model construction. This methodological basis also allows us to study phenomena and construct models that are very problematic or even unresolvable when traditional methods are used.

In our theory formation and model design we principally apply two methods, viz. concept maps [16] and cognitive maps [1,2]. To date it has been problematic to simulate models of this type with computers because they include linguistic or imprecise variables and relationships. Our resolution is that in a computer environment these constructions are transformed into fuzzy linguistic cognitive maps, which are fuzzy linguistic graphs *ipso facto*, and then we can tune and simulate our models effortlessly.

In particular, we focus on one example of a complicated phenomenon, the well-known prisoner's dilemma, because it, and its variations, arouse various problems in decision making and game theory.

Section 2 presents the idea of a fuzzy cognitive map. Sections 3 and 4 consider an application of fuzzy linguistic cognitive maps to prisoner's dilemma. Section 5 concludes our considerations.

2. **Fuzzy Cognitive Maps.** If we aim to construct a computer model of a complicated phenomenon, we first have to specify the variables of our model and assign the interrelationships between these variables. Second, we simulate our model in order to explain and predict phenomena under our study. For example, given the model in Figure 1 with seven variables [10,11], we can provide such what-if questions in our simulations as

what happens to the amount of garbage if the migration into city has a great increase?

If empirical data on behavior of this model in a given period of time is unavailable, our model construction is only based on human intuition or expertise, otherwise we can also apply classical statistics (e.g. regression and path analysis), neural networks, Bayesian networks or evolutionary computing [5,6,8,9,11,12,14,18,19].

Our approach stems from Axelrod's studies on cognitive maps [1,2]. He used numerical matrices in his models which expressed the interrelationships between the variables (or *nodes*). The cell values of a given matrix denoted causal connections between the nodes such as

increase in Node 1 causes increase in Node 2

or

increase in Node 3 causes decrease in Node 4.

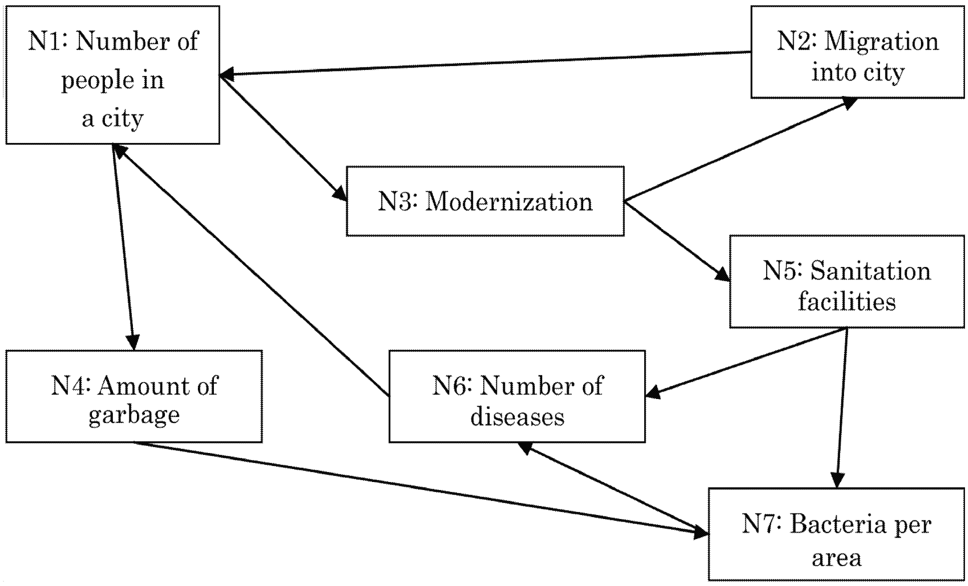

FIGURE 1. Example of a complicated model on public city health issues

For example, if we are only using three values in our matrix, viz. -1, 0 and 1, we can construct a node matrix according to Figure 1 as follows [11,19]:

	Node 1	**Node 2**	**Node 3**	**Node 4**	**Node 5**	**Node 6**	**Node 7**
Node 1	0	0	1	1	0	0	0
Node 2	1	0	0	0	0	0	0
Node 3	0	1	0	0	1	0	0
Node 4	0	0	0	0	0	0	1
Node 5	0	0	0	0	0	-1	1
Node 6	-1	0	0	0	0	0	0
Node 7	0	0	0	0	0	1	0

In this case the cell values -1, 0 and 1 denote *decrease*, *no relationship* and *increase*, respectively. Hence, Node 3 does not affect on Node 1, whereas Node 3 has an increasing effect on Node 2 when the rows denote causes and columns are their respective effects. For example, we can now reason that increase in the number of people in a city increases the amount of garbage because the value of 1 is assigned to our matrix cell (1,4).

More formally, if we first assign initial values to the nodes by also using the values -1, 0 and 1, we can simulate our model by using matrix multiplication. For example, given the initial values to the nodes in an input vector v,

$$v = (v1, v2, v3, v4, v5, v6, v7),$$

matrix multiplication of vector v and our node matrix yield an output vector

$$u = (u1, u2, u3, u4, u5, u6, u7),$$

and the vector components of u will be the new values in our nodes. Hence, for example, in the foregoing model the input v = (0,0,0,0,0,-1,1) yields u = (1,0,0,0,0,1,0).

Since the vector components of the outputs do not necessarily belong to the set {-1,0,1}, we have to use an appropriate transformation function (squashing function) in order to obtain trivalent output component values [10,19].

We can repeat matrix multiplication several times by always using the previous output vector as a new input in the multiplication, i.e., we can use iteration or cycling. By virtue of iteration we can consider future trends related to our variables (c.f. Section 4).

Traditional cognitive maps are based on classical bivalent logic and mathematics. However, we can also use fuzzy cognitive maps when we simulate complicated phenomena in a computer environment [10]. In this context, our node values usually range from 0 to 1 and these values denote the degrees of activation of a node. The degrees of relationship, in turn, can range from -1 to 1 in which case -1, 1 and 0 denote *full negative effect*, *full positive effect* and *no effect*, respectively. In these models a typical transformation function for simulation output vectors is

$$f(x) = 1/(1 + exp(-\alpha \cdot x))$$

in which *exp* is the exponential function and α is a parameter (α >0).

For our public city health model above we can thus establish a node matrix [11,19]

	Node 1	**Node 2**	**Node 3**	**Node 4**	**Node 5**	**Node 6**	**Node 7**
Node 1	0	0	0.6	0.9	0	0	0
Node 2	0.5	0	0	0	0	0	0
Node 3	0	0.6	0	0	0.8	0	0
Node 4	0	0	0	0	0	0	1
Node 5	0	0	0	0	0	-0.8	-0.9
Node 6	-0.3	0	0	0	0	0	0
Node 7	0	0	0	0	0	0.8	0

Given once more the initial node values, v=(0,0,0,0,0,-1,1), matrix multiplication now yields the vector u=(0.3,0,0,0,0,0.8,0). By using the foregoing transformation function (when α=5), we obtain the output vector u=(0.82,0,0,0,0,0,0.98,0). Finally, we provide interpretations to our outputs in a linguistic form by using our intuition.

Several fuzzy cognitive maps in various application areas have been constructed by using the foregoing method, but they have usually included less than ten nodes [19]. In addition, they have been numerical models based on matrix multiplication. However, we still have some problems with these fuzzy cognitive maps.

First, the selection of nodes and specification of the interrelationships between the nodes have often been based on mere intuition or expertise. In order to construct these maps in an automatic manner, evolutionary computing and data sets have been applied to some extent [19]. The second problem is that the prevailing numerical cognitive maps can only use monotonic causal interrelationships between the nodes. Third, time delays arouse problems in numerical maps because certain phenomena occur in a short term whereas others can occur in a long term [4].

If we use appropriate fuzzy linguistic cognitive maps instead, we can resolve at least the second and the third problem above. In linguistic maps we can assign fuzzy linguistic values to the nodes and establish the interrelationships between the nodes with linguistic expressions by using fuzzy linguistic rule sets [4,15,20-22]. Hence, this approach, which is actually applying fuzzy linguistic graphs, allows us to use more versatile values and interrelationships, employ qualitative variables and apply better non-linear modeling. In addition, we can use data sets in order to construct linguistic cognitive maps in an automatic or a semi-automatic manner if we apply neuro-fuzzy systems and fuzzy reasoning methods such as Takagi-Sugeno reasoning.

Below we use fuzzy linguistic models and we apply concept maps [16] and cognitive maps [1,2] as our background theories. In [15] the author has suggested that fuzzy linguistic cognitive maps are powerful when various social and behavioral phenomena related to acts of persons are modeled and this type of research is typical in both quantitative and qualitative modeling.

To date, most complicated models in social and behavioral sciences are constructed according to the researcher's intuition, expertise, interpretation and manual work. By virtue of fuzzy linguistic models we can also consider these problems in a computer environment. Examples of application areas suggested by the author are concept map modelings [16], traditional cognitive maps [1,2], game theory [17], semantic web as well as various other Internet applications [3]. In addition, if data is available, we can construct our computer models by using neural networks, evolutionary computing or multi-regression methods [19]. Below we restrict ourselves to an example in game theory whereas a more thorough examination on fuzzy linguistic cognitive map modeling in human sciences will be presented in the forthcoming paper.

3. **Example of Game Theory, Prisoner's Dilemma.** In formal two-person games, such as those considered in [13,17], problems arise when persons do not behave "rationally". Examples of these paradoxes are the prisoner's dilemma and the game of chicken in game theory as well as certain ethical codes maintained both in Eastern and Western philosophy [17]. We only consider prisoner's dilemma below.

In the traditional prisoner's dilemma we have two "players", viz. I and somebody else, and two strategies, defection and cooperation. Both players aim to obtain maximum personal gain, and their gains depend upon their strategies. Table 1 presents the ranking of gains according to the applied strategies (the first and second ranking in each cell are mine and the other player's rankings, respectively).

Table 1. Ranking of gains in Prisoner's dilemma

	Other player's strategy	
My strategy	Cooperation	Defection
Cooperation	2, 2	4, 1
Defection	1, 4	3, 3

However, the dilemma arises because there are no rational grounds for accepting either of the foregoing strategies. If I defect, I will make my best gain provided that the other player cooperates, but defection is ethically problematic. If I cooperate, I can make my second best gain provided that the other player also cooperates, but if the other person defects, I will make my worst gain. Hence, cooperation is a fairly good strategy and ethically valuable if both players apply it, but will the other player also cooperate?

In mathematical two-person game theory Nash's well-known equilibrium point approach suggests that both players should apply defection for rational reasons because this cell is the equilibrium point in Table 1, but since mutual cooperation makes better gains for them, Nash's approach is also problematic [17].

Hence, at first sight our decision model seems simple but actually it is an example of a complicated phenomenon of the real world which arouses various problems.

We apply the foregoing problem to a concrete example which is originally known as the Buick-selling problem [17], and we use fuzzy models in this context.

Consider that you are selling your car to your friend and both of you attempt to set a fair price for it. In order to do this, you consult a car dealer. The dealer says that his buying price for your car is USD 10 000 but the selling price would be USD 11 000 (thus dealer's profit would be USD 1000). You and your friend should now decide what the fair price for your car is. Is it 10 000, 11 000 or somewhere between these limits?

If we now apply the prisoner's dilemma, we can adopt two strategies. In my case my price can be 11 000 which means that I maximize my gain, but then I "defect" my friend. I can also more or less "cooperate" in which case my price is less than 11 000 but then my gain is worse.

If my friend's price is 10 000, he defects and makes a maximum gain. If his price is more than 10 000, he cooperates and makes a worse gain.

Hence, to be rational, defection makes the maximum gain but then we hurt the other person's feelings, and vice versa. In addition, we do not know which strategy the other person adopts and thus our strategy selection is even more complicated. Hence, we encounter the prisoner's dilemma.

The traditional approach to consider this dilemma is based on bivalent logic in which case we either apply defection or cooperation. In our case we use the fuzzy linguistic approach and thus we can also use degrees of defection or cooperation. For example, a fairly high degree of cooperation (e.g. about 75 %), means simultaneously a fairly low degree of defection (e.g. about 100 % - 75 % = 25 %). In addition, we can use fuzzy numerical or linguistic values and relationships in our models.

In our car sale model my fairly high or about 75 % degree of cooperation (or respectively, fairly low or about 25 % degree of defection) means that I reduce my price from the maximum price to about USD 10 250 because the price can range from 10 000 to 11 000 (i.e., range = 11 000 − 10 000 = 1 000, and my about 75 % degree of cooperation yields the price of about (11 000 − 0.75 · 1000) = 10 250).

My friend, in turn, has to apply about 25 % degree of cooperation (or about 75 % defection) in order to agree with my price above, and as soon as we have agreed upon the price, I can sell my car to him. In the real world, however, the 50 % degree of cooperation seems to be usual, and in our example this policy would mean that our agreed price is 10 500.

From the standpoint of a fuzzy linguistic cognitive map, we can apply the model in Figure 2 when we consider our gains in the car-sale problem. Hence, both persons first set their prices (which are assigned according to their established degrees of cooperation), and then they obtain their gains as feedback according to Table 2 which is a fuzzified rule-based version of Table 1.

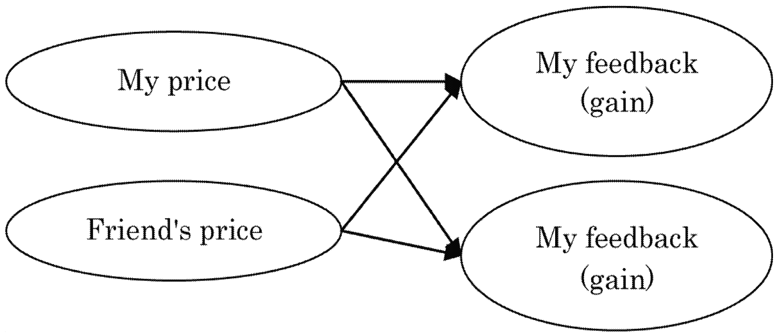

FIGURE 2. Cognitive map model of car-sale problem

In the fuzzy case we thus use fuzzy rules when we specify our gains, i.e., fuzzy rules establish this relationship. Examples of rules based on Table 2 are

if I apply about 100 % degree of cooperation and my friend applies about 100 % degree of cooperation, then we both obtain about 75 % of the maximum gain,

or

if I apply about 100 % degree of cooperation and my friend applies about 0 % degree of cooperation, then my gain is about 0 % and my friend's gain is about 100 % of the maximum gain.

Figure 3 presents a rule base with four tentative fuzzy rules in a zero-order Takagi-Sugeno type reasoning model when training data according to Table 2 and Matlab's^{TM} Fuzzy Logic Toolbox are used. For example, rule 2 in Figure 3 means that if my and my friend's strategies are about 0 % and 100 % degrees of cooperation, respectively, then my degree of gain is 95 % of the maximum. Figure 4 presents the respective decision plane for my degree of gain specification. This fuzzy model yields the degrees of gains in my case but we can use identical model in my friend's case. In this context we have a linear model but naturally non-linear models are also possible.

As soon as we have assigned our degrees of cooperation, we can calculate our respective prices, and finally, we should set the agreed or compromise price which is appropriate to both of us. For example, if my price is 10 700 (30 % degree of cooperation) and my friend's price is 10 200 (20 % degree of cooperation), the compromise price is 10 450 when our compromise is based on the arithmetic mean (which seems to be a widely-used compromise operator). Hence, the agreed price of the car is 10 450.

In the prisoner's dilemma examination, several reasoning models only use once the foregoing strategy tables in their simulations but our problem-setting is much more complicated if we use iteration in this context. Below we consider iteration approach.

TABLE 2. Tentative fuzzy rules for modeling gains in car-sale problem (numbers denote approximate values, my gain is the first number)

	Friend's strategy				
My strategy	**About 100 % cooperation**	**About 75 % cooperation**	**About 50 % cooperation**	**About 25 % cooperation**	**About 0 % Cooperation**
About 100 % cooperation	75%,75%				0%,100%
About 75 % cooperation					
About 50 % cooperation	87.5%,50%		50%,50%		12.5%,62.5%
About 25 % cooperation					
About 0 % cooperation	100%,0%				25%,25%

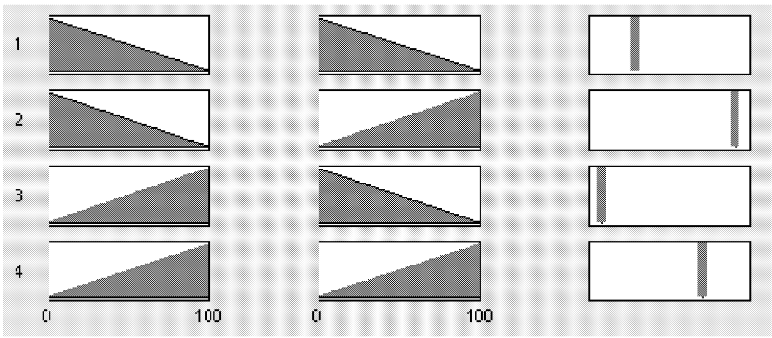

FIGURE 3. Tentative fuzzy rules for my degrees of gains in car-sale model (left input: my strategy, right input: friend's strategy, output: gain ranging from 0 to 100)

4. **Iteration Models and Fuzzy Revenge.** The prisoner's dilemma has opened new frontiers in decision making if we use iteration (or cycling) which means that new decisions are made according to previous ones by using Table 2 [7,17]. An application example is a dealer who conducts business with another person by selling some products to him in a given period of time and in each transaction these persons apply the strategies of Table 2. Hence, in this model our future decisions are made according to the previous ones. For example, if I was cooperating last time but the other person defected me, then next time I would possibly like to take revenge on him by adopting defection.

Consider now that I conduct business with a loyal customer. We thus have several business transactions and in each transaction we must set a price which is satisfactory to both of us, in other words, each time we first set our prices on the basis of our degrees of cooperation and then we assign our compromise price according to the given prices. For example, if each transaction ranges from USD 10 000 to 11 000 and my and my customer's degrees of cooperation are 30 % and 60 %, respectively, my price is

$$11\ 000 - (0.30 \cdot 1\ 000) = 10\ 700$$

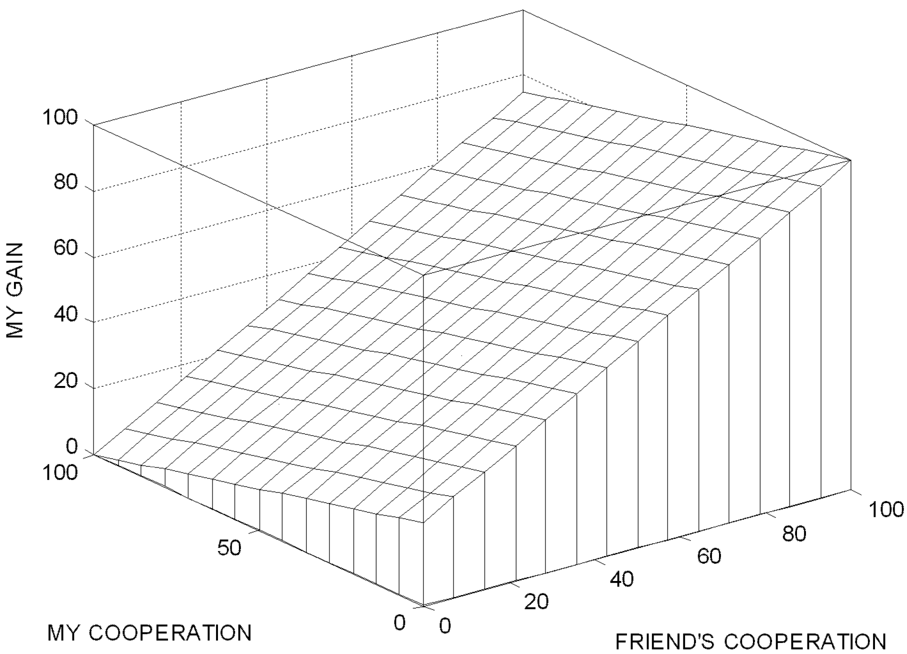

FIGURE 4. Fuzzy model for specifying my degrees of gains in car-sale example

and my customer's price is

10 000 + (0.60 · 1 000) = 10 600.

In order to conduct business, we now have to find an appropriate compromise price, such as the mean of our prices, and thus 10 650 is our agreed price.

Since in iteration models the previous degrees of gains and cooperation of the "players" can have an affect on their future strategies, our model also has to take these aspects into account. Thus, we can use such cognitive maps as in Figure 5 in which my next price depends upon both my previous price and previous degree of gain. The similar conditions concern my customer's pricing.

Consider now that we both adopt similar "fuzzy revenge" strategy by using the metarule

if my previous degree of gain was low or fairly low, then next time I will slightly decrease my degree of cooperation, otherwise I will use my previous degree of cooperation.

In practice, for example, if my previous gain was low or fairly low, I increase my price with USD 100 for the next round, otherwise I set my previous price. My customer, in turn, decreases his price with USD 100 if the previous degree of gain was low or fairly low, otherwise he keeps his price stable.

Our decision network model thus includes five submodels, one for each node in the cognitive map in Figure 5. Figure 6 presents the tentative fuzzy Takagi-Sugeno submodels, and these models were constructed according to the given training data sets. In this model

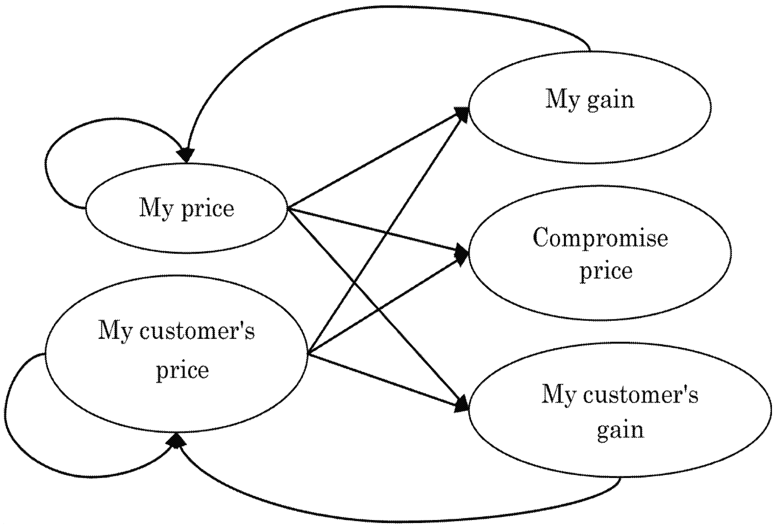

FIGURE 5. Cognitive map model for iteration

the prices were set by using the foregoing metarule, the degrees of gains based on Table 2 and compromise prices were arithmetic means of the set prices.

Figure 7 presents the tentative fuzzy rules for the submodels which yield my and my customer's prices according to previous gains and prices (c.f. Figure 6). In this context Takagi-Sugeno's zero-order grid technique models in Matlab were used in order to preserve better the conceivability of these models. In addition, a simple Matlab subroutine (Matlab's m-file) was used to supervise the simulation and collect the simulation results according to the node values.

In practice the supervision subroutine generated an $(m+1)\times n$ output matrix when m was the number of iterations and n was the number of nodes. The first row of the matrix included the initial values of the nodes, and the values in the second row were obtained according to the values in the first row and the foregoing fuzzy reasoning operations or mathematical functions. In the third row, in turn, we obtained the values according to the second row and our reasoning operations, etc.:

Node 1 initial value	Node 2 initial value	Node 3 initial value	Node 4 initial value	Node 5 initial value
Value yielded by the previous row and reasoning operation	Value yielded by the previous row and reasoning operation	Value yielded by the previous row and reasoning operation	Value yielded by the previous row and reasoning operation	Value yielded by the previous row and reasoning operation
Value yielded by the previous row and reasoning operation	Value yielded by the previous row and reasoning operation	Value yielded by the previous row and reasoning operation	Value yielded by the previous row and reasoning operation	Value yielded by the previous row and reasoning operation
...	...	...	...	...

Figures 8 and 9 present examples of simulation results when my and my customer's initial degrees of cooperation were 20% and 70%, respectively, i.e., our respective initial

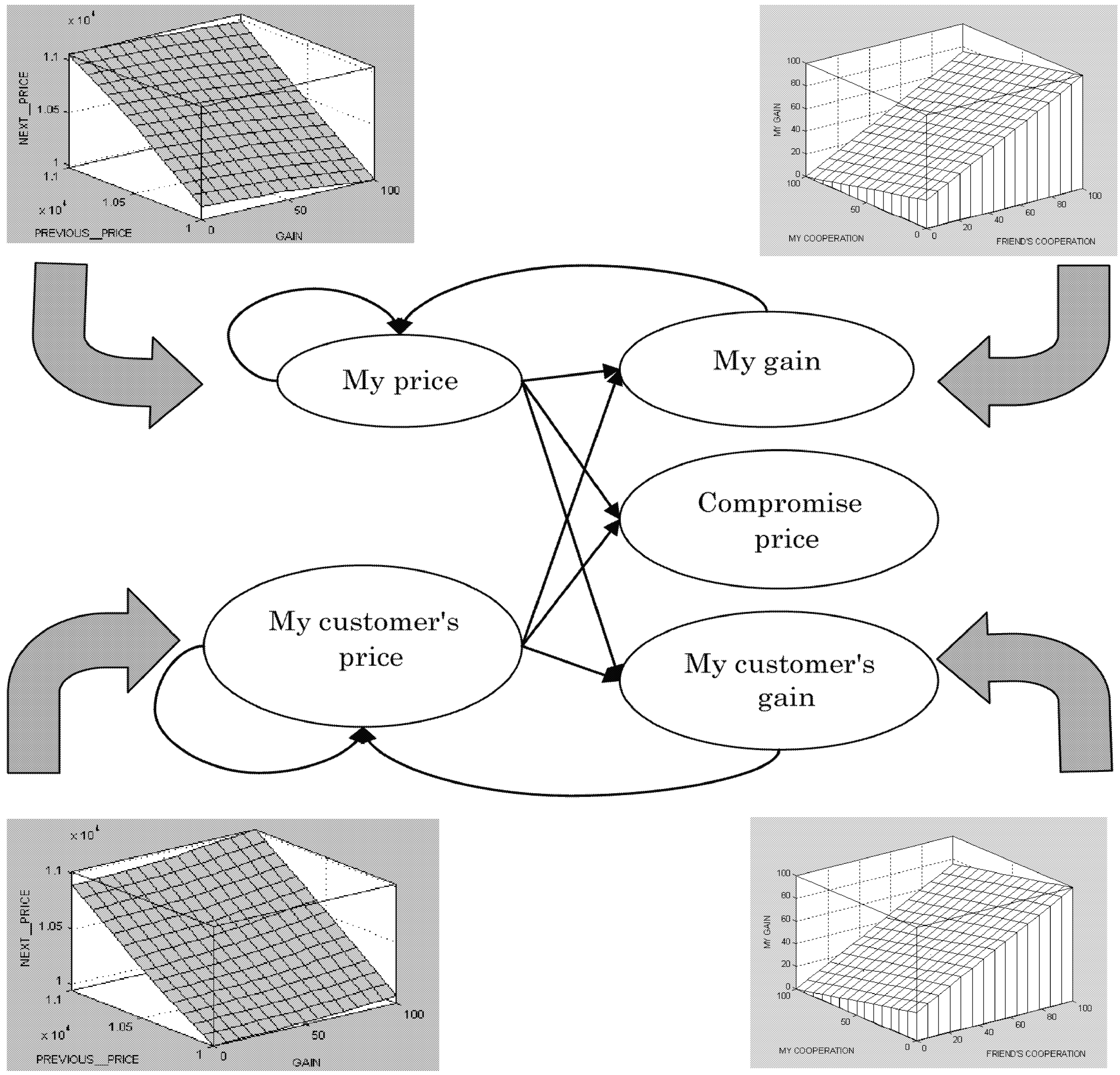

FIGURE 6. Fuzzy submodels for iteration model of loyal customer

prices were USD 10 800 and 10 700. Since both of us applied the fuzzy revenge metarule mentioned above, we seem finally to end up with full defection (i.e., degrees of gain are finally about 25% and the dealer sets maximum prices and the customer minimum prices) and similar results were obtained when alternative initial values were used. Hence, our revenge rule seems to lead to defection in general. Compromise prices were the arithmetic means of the set prices.

Various conventional decision strategies are available for iteration models but no general optimal theoretical solution has been suggested yet if the best strategy means the best total gain. On empirical grounds one of the best strategies seems to be that a person cooperates at the first stage and then he uses the same strategy which the other person used at the previous stage. Our fuzzy-revenge metarule above was based on this idea to

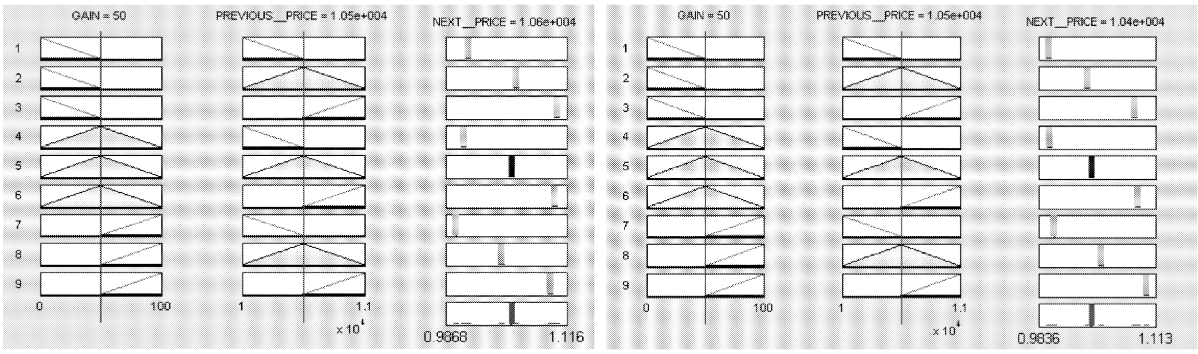

FIGURE 7. Fuzzy rules for setting prices for me (left) and for my customer (right) according to previous degrees of gains and prices

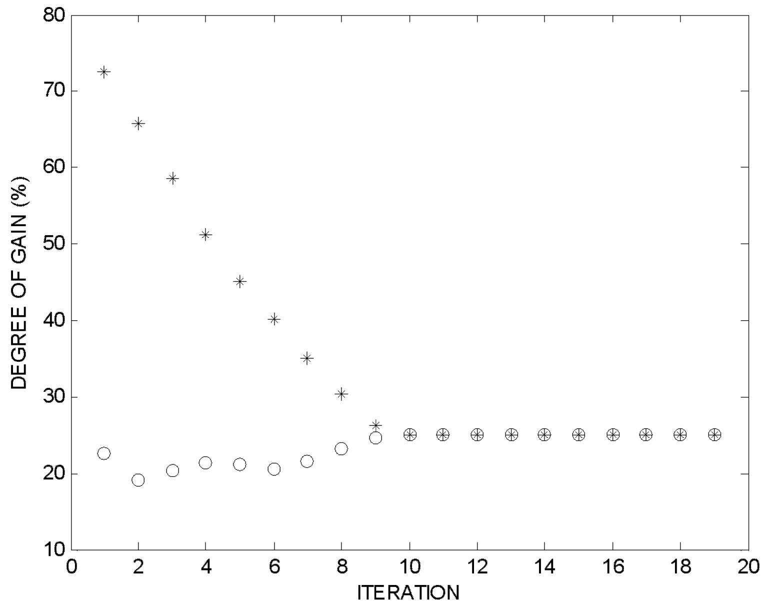

FIGURE 8. An example of simulation results in degrees of gains when the initial values for prices were USD 10 800 and 10 700 (* = dealer, o = customer)

some extent. Another good strategy seems to be that both persons constantly cooperate [17].

The construction of the foregoing fuzzy linguistic cognitive map is only based on intuition, and we still lack methods for utilizing data sets in automatic map construction. As will be presented in the author's forthcoming paper, we can resolve this problem at least partially by using appropriate supervision algorithms, semi-automatic methods based on regression analysis and analysis of variance as well as neuro-fuzzy tools such as Matlab's Anfis.

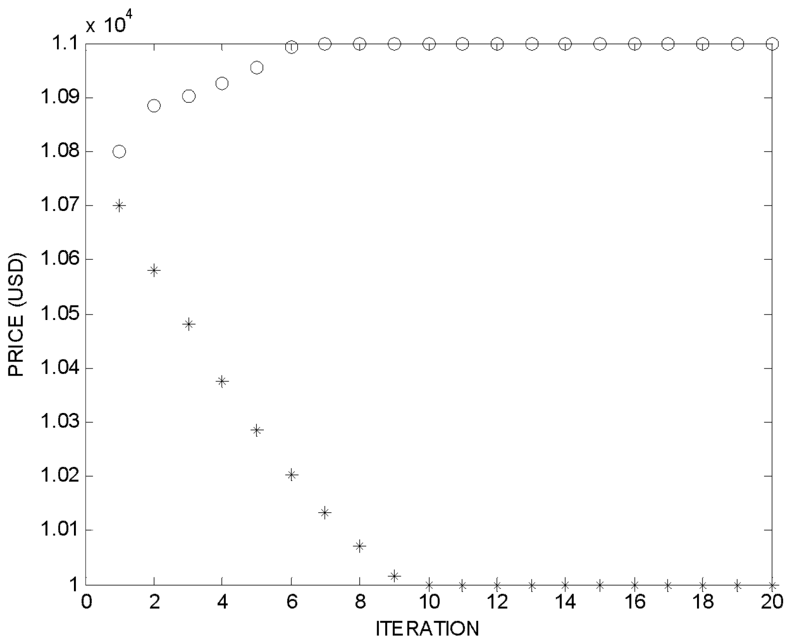

FIGURE 9. Example of simulation results in setting prices when the initial values were USD 10 800 and 10 700 (o = dealer, * = customer)

5. **Conclusions.** Our aim was to consider computer modeling of complicated phenomena in the real world. We suggested that if concept maps, cognitive maps and fuzzy systems are used, we can simulate better the behavior of these phenomena in a computer environment.

Various cognitive map modeling approaches were considered at a general level. We stated that both traditional and fuzzy numerical maps are problematic because they are linear models which only use numerical values and simple monotonic causal relationships. They also are unable to take time delays into account. On the other hand, by virtue of neural networks and evolutionary computing, we can also construct these maps automatically with given data sets if necessary.

In order to construct more usable and versatile cognitive maps, basic features of fuzzy linguistic cognitive maps were considered. These maps can use approximate linguistic values, various interrelationships and non-linear modeling because fuzzy variables, fuzzy rule sets and fuzzy reasoning can be applied. However, we still lack automatic methods based on data sets when we construct these maps. Neuro-fuzzy systems, regression analysis and analysis of variance seem to resolve this problem at least partially but further studies are still required in this area.

As a concrete example, the well-known prisoner's dilemma was examined because this dilemma has aroused various problems in decision theory due to its complicacy. Our fuzzy linguistic model seemed simple, usable and user-friendly in this context because it allows human-like application of approximate and linguistic entities. Hence fuzzy linguistic cognitive maps of this type can enhance computer model construction in both quantitative and qualitative research and they also allow us to construct models in several such conditions in which this type of work was impossible before.

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