NEGATIVE REFRACTION BY PHOTONIC CRYSTAL

R. Srivastava and S. Srivastava

Department of Physics
Udai Pratap Autonomous College
Varanasi 221002, India

S. P. Ojha

C. C. S. University
Meerut 200005, India

Abstract—Negative refraction has been the subject of considerable interest and it may provide the possibility of a variety of novel applications. It arises in metamaterials that have simultaneous negative permittivity ε and permeability μ, where in-homogeneities are much smaller than the wavelength of the incoming radiation. Recently it has been shown that photonic crystals (PCs) may also exhibit negative refraction although they have a periodically modulated positive permittivity ε and permeability μ. We have theoretically studied the negative refraction in one-dimensional (1D) photonic crystals (PCs) consisting dielectric ZnSe with air. By using transfer matrix method and block theorem we have studied the photonic band structure and group velocity and with the help of group velocity we have obtained the frequency bands of negative refraction. We found that negative refraction may occur near the low frequency edge of the second and fourth bandgaps. We have also studied the effective index of refraction and the transverse position shift through PC that can characterize negative refraction more explicitly.

1. INTRODUCTION

The optical properties of materials that are transparent to electromagnetic (EM) waves can be characterized by an index of refraction and is given by the Maxwell's relation $n = \sqrt{\varepsilon.\mu}$ where ε is the relative dielectric permittivity and μ is the relative permeability of the medium. Generally, ε and μ are both positive in ordinary materials.

While ε could be negative in some materials, no natural materials with negative μ are known.

For certain structures, which are called metamaterials, the effective permittivity and permeability, possess negative values. It means in such materials the index of refraction is less than zero, and therefore, phase and group velocity of an electromagnetic (EM) wave can propagate in opposite directions. This phenomenon is called the negative index of refraction and was theoretically proposed by Veselago [1]. Furthermore, light incident from a conventional right-handed material to metamaterials, will bend to the same side as the incident beam, and for Snell's law to hold, the refraction angle should be negative.

A periodic array of artificial structures called split ring resonators (SRRs) suggested by Pendry et al. [2] which exhibit negative effective μ for frequencies close to the magnetic resonance frequency. Smith et al. [3, 4] reported the experimental demonstration of meta-materials by stacking SRR and thin wire structures as arrays of 1D and 2D structured composite meta-materials (CMM). Experimental observation of negative refraction in meta-materials is verified by Shelby et al. [5]. Recently negative refraction has been the subject of considerable interest which may provide the possibility of a variety of novel applications of very interesting new phenomena like the super-lens effect [6–10]. A negative refractive index arises in meta-materials that have simultaneous negative permittivity ε and permeability μ, where in homogeneities are much smaller than the wavelength of the incoming radiation and it has been demonstrated at microwave wavelengths [11–13]. On the other hand, it has been shown that this fantastic phenomenon may also occur in ordinary materials, which are called photonic crystals (PCs) at optical wavelengths [14–17].

Photonic crystals are artificial structures, which have periodic dielectric structures with high index contrast, the resultant photonic dispersion exhibits a band nature analogous to the electronic band structure in a solid, and the propagation of electromagnetic waves are forbidden in the photonic band gap (PBG). There also shows an extraordinary strong nonlinear dispersion at wavelengths close to the bandgap, and photonic crystals can refract abnormally light at these wavelengths under certain conditions as if they had a negative refractive index, which is referred to as negative refraction effect [18–27]. In photonic crystals, light travels as Bloch waves, which travel through crystals with a definite propagation direction despite the presence of scattering. In the long wavelength regime, the average direction of energy propagation can be shown to be the same as the direction of group velocity [28].

The realisation of negative refraction and its experimental verification has been done by Valanju et al. [29]. They show that the transmission of energy is possible when the wave comes in a range of frequencies, which combine to form energy packets. A wave with just one frequency component could be bent in the wrong way, but this is irrelevant because real light never has just one frequency. Cubukcu et al. has been first to demonstrate negative refraction phenomenon in a two-dimensional (2D) photonic crystals in the microwave regime [30]. Further experimental studies proved that carefully designed photonic crystals are candidates for obtaining negative refraction at microwave (18) and infrared [31] frequency regimes. The super-prism effect is another exciting property arising from the photonic crystals [19, 32]. Extensive numerical [21, 33, 34] and experimental studies [35–37] provided a better understanding of negative refraction, focusing and sub-wavelength imaging in photonic crystal structures. Masaya Notomi showed that refraction-like behaviour could be expected to occur in photonic crystals that exhibit negative refraction for certain lattice parameters.

Boedecker and Henkel [38] mentioned that the simple one-dimensional Kronig-Penney model provided an exactly soluble example of a photonic crystal with negative refraction. In this paper, we studied the negative refractive behaviors in 1D photonic crystal by the Bloch theory and transfer matrix method. With the help of the group velocity and the transmittance, we get frequency bands of negative refraction. When negative refraction occurs in photonic crystals, the transmission wave will shift adversely from the incidence point in x axis at the end face.

2. THEORY

To study the propagation of electromagnetic waves in one-dimension photonic crystal we choose a particular x-axis through the material along the direction normal to the layers. The Maxwell's equation for light propagating along the x-axis may be written as

$$\frac{d^2 E(x)}{dx^2} + \left(\left(\frac{\omega}{c} n(x) \right)^2 - \beta^2 \right) = 0 \tag{1}$$

With

$$\beta = \frac{\omega}{c} n(x). \sin(\theta_i) \quad \text{and} \quad i = 1, 2$$

where β is the propagation wave vector constant, c is the velocity of light and $n(x)$ is the periodic refractive index profile of the structure

and is given by

$$n(x) = \begin{cases} n_1, & 0 < x < a \\ n_2, & a < x < d \end{cases} \tag{2}$$

with $n(x) = n(x+d)$ where d is the periodicity of lattice, a and b are the thickness of the alternate regions which have refractive indices n_1 and n_2. The geometry of the structure is shown in the Figure 1.
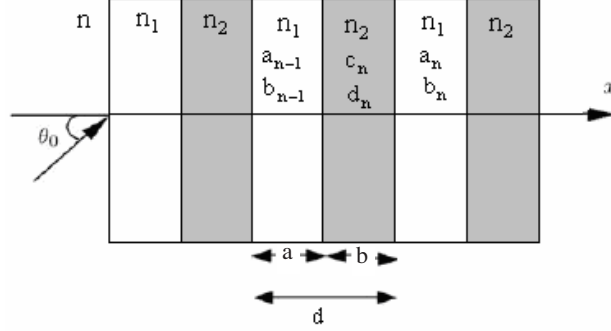

Figure 1. Periodic refractive index profile of the structure having refractive indices n_1 and n_2 respectively.

The solutions of Equation (1), in any region, are the combinations of left- and right-traveling waves. These solutions and their derivatives must be continuous at the interfaces of the dielectric medium.

The general solution of Equation (1) can be written as

$$E_1(x) = a_n e^{-1ik_1(x-d)} + b_n e^{1ik_1(x-d)} \tag{3}$$

$$E_2(x) = c_n e^{-1ik_2(x-nd+a)} + d_n e^{1ik_2(x-nd+a)} \tag{4}$$

where

$$k_i = \left(\left(\frac{\omega}{c} n_i \right)^2 - \beta^2 \right)^{\frac{1}{2}} = \frac{n_i.\omega}{c} \cos(\theta_i),$$

$$\theta_i = \sin^{-1} \left(\frac{n_0. \sin \theta_0}{n_i} \right) \quad \text{and} \quad i = 1, 2$$

a_n, b_n, c_n and d_n are related through the continuity condition at the interface $x = (n-1)d$ and $x = (n-1)d + b$. The continuity condition leads to the matrix equation which relates the coefficient in the first layer of the nth cell is given as

$$\begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix} = T_n \begin{bmatrix} a_n \\ b_n \end{bmatrix} \tag{5}$$

T_n is the called transfer matrix and is given by

$$T_n = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \tag{6}$$

Now using Bloch theorem

$$E(x + d) = e^{ikd} E(x) \tag{7}$$

where k is known as the Bloch wave vector, and for determination of K as a function of eigen value the equation can be written as

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \cdot \begin{bmatrix} a_n \\ b_n \end{bmatrix} = e^{i.K.d} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \tag{8}$$

By solving Equations (6) and (8) and after a few simple straight forward steps we obtain the expression for the dispersion relation which is given as

$$K(\omega) = \frac{1}{d} \cos^{-1} \left(\cos(k_1.a).\cos(k_2.b) - \frac{1}{2} \left(\frac{k_2}{k_1} + \frac{k_1}{k_2} \right).\sin(k_1.a).\sin(k_2.b) \right) \tag{9}$$

The reflection and transmission can be related easily between the plane wave amplifications.

$$\begin{pmatrix} t \\ 0 \end{pmatrix} = M \begin{pmatrix} 1 \\ r \end{pmatrix} \tag{10}$$

And $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$ with $M_{11} = m_{11}U_{N-1} - U_{N-2}$, $M_{21} = m_{21}U_{N-1}$, $M_{12} = m_{12}U_{N-1}$, $M_{22} = m_{22}U_{N-1} - U_{N-2}$ and $U_N = \frac{\sin[(N+1)).K(\omega).d]}{\sin[K(\omega).d]}$.

And

$$t = M_{11} - \frac{M_{12}.M_{21}}{M_{22}} \tag{11}$$

In the photonic crystal group velocity (V_g) has a proper meaning and it governs the energy flow of a light beam and the Bloch wave vector in photonic crystal is given by $k = \beta.\hat{x} + K.\hat{z}$. So the group velocity (V_g) in photonic crystal is expressed as [39]

$$V_g = V_{gx}.\hat{x} + V_{gz}.\hat{z}$$

where V_{gx} and V_{gz} are the components of the group velocity in the x and z axis respectively.

$$V_{gx} = \frac{\partial \omega}{\partial \beta} = -\frac{\partial K / \partial \beta}{\partial K / \partial \omega} \quad \text{and} \quad V_{gz} = \frac{\partial \omega}{\partial K} = \frac{1}{\partial K / \partial \omega}$$

And resultant group velocity is

$$V_g = \sqrt{(V_{gx})^2 + (V_{gz})^2} \tag{12}$$

The x component of the group velocity V_{gx} may cause the transverse position shift S_x along x-axis after a beam passes through PC, and is defined as [39]

$$S_x = V_{gx}.\tau \tag{13}$$

where τ is the group delay time through the structure.

3. RESULTS AND DISCUSSION

For the numerical calculation, we have taken air as dielectric with ZnSe. The refractive index of air and ZnSe is $n_1 = 1$ & $n_2 = 2.3$ and the thickness $a = 0.75\%$ of d & $b = 0.25\%$ of d respectively, where d is the total stack thickness. The total number of layer $N = 10$.

Figure 2 shows the photonic band structure verses normalized frequency for oblique incidence $\theta = 45°$. It is clear from Figure 2 that the bandwidth of odd number bandgap is wide but the bandwidth of even number bandgap is very narrow. It is noticeable here that the lattice constant d is arbitrary; thus, the result obtained here is valid for arbitrary wavelengths and the existence of bandgap is possible as long as $d \cong \lambda$.

Figure 3 shows the x component of group velocity V_{gx} verses normalized frequency. On Comparing Figures 2 and 3, it is clear that at the bandgap edge, there is the strong group velocity dispersion. With the increase of frequency, V_{gx} decreases from positive value to negative value, falls of sharply to the negative minimum at the edge of band and then jumps to the positive maximum, afterwards, V_{gx} decreases again, and cycles in this manner. $V_{gx} < 0$ indicates the energy flow may tend to the negative direction of x axis, so negative refraction phenomenon might occur at some frequency in the bandgap. At normal incidence V_{gx} always equals to zero hence the oblique incidence of wave is the necessary condition for negative refraction. From Figure 4, we observed that the transmittance is zero in the first and third bandgap, i.e., at these frequencies the waves are completely reflected and do
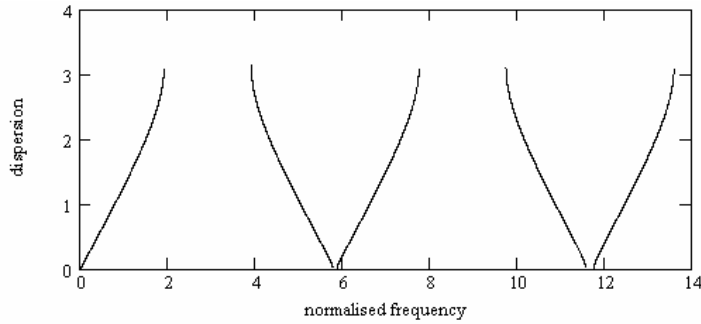
Figure 2. Dispersion Vs normalized frequency for $a = 0.75$ and $b = 0.25$ and refractive index of air/ZnSe is $n_1 = 1$ and $n_2 = 2.3$ at angle $= 45°$.
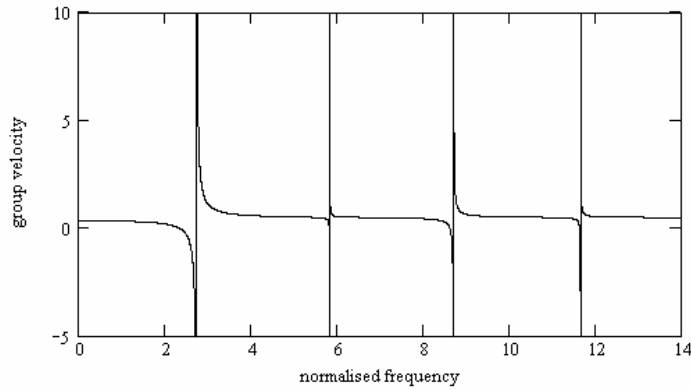

Figure 3. Group velocity Vs normalized frequency for $a = 0.75$ and $b = 0.25$ and refractive index of air/ZnSe is $n_1 = 1$ and $n_2 = 2.3$ at angle $= 45°$.

not pass through the crystal but in the second and fourth bandgaps, the transmittance is not zero, which means part of wave can pass. Relating with Figure 3, we observed that there exists the negative group velocity in the low frequency edge of the second bandgap, at the same time; the transmittance is not zero, so the transmission wave will bend to the negative direction of x axis, which is the negative refraction phenomenon. According to the parameters in Figure 3, the normalized frequency band for negative refraction lies between 5.722 and 5.738 in the second band gap and 11.528 and 11.542 in the fourth, but the transmission energy is less than 5%. Although in the first and third
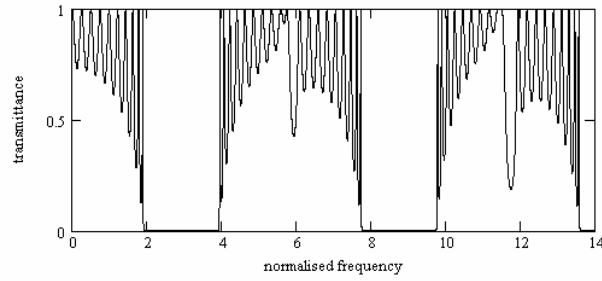
Figure 4. Transmittance Vs normalized frequency for $a = 0.75$ and $b = 0.25$ and refractive index of air/ZnSe is $n_1 = 1$ and $n_2 = 2.3$ at angle $= 45°$.
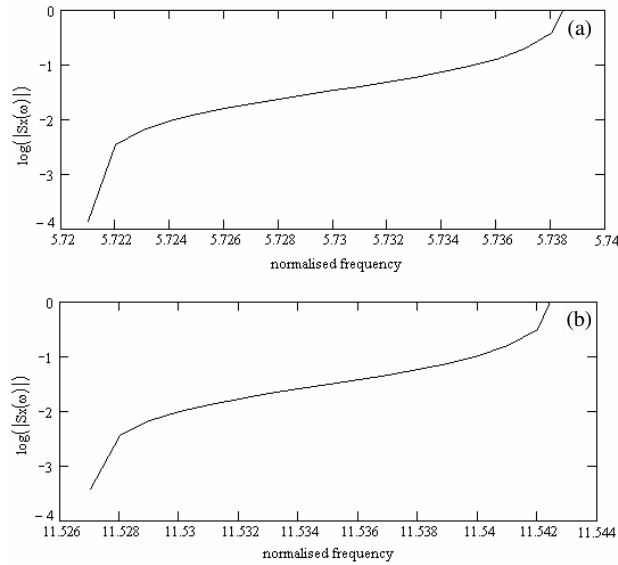

Figure 5. Logarithm of absolute value of transverse position shift (a) second bandgap and (b) the fourth bandgap vs normalized frequency.

bandgaps, some group velocity is negative so there is no transmission of energy and the negative refraction can not happen. Therefore, we can conclude that negative refraction may occur in the frequency near the low frequency edge of the second and fourth bandgap because of the strong group velocity dispersion. Figure 4 shows the transmittance vs normalized frequency and Figure 5 shows the logarithm of absolute value of position shift with 10 as the base, in the frequency band of

negative refraction. Because the magnitude of V_{gx} in most part of the bandgap is too small, so the shifts at these frequencies are not obvious. Only in an extremely narrow frequency band, the beam shows great shift. When the shift reaches the negative minimum, it will jump to the positive maximum suddenly, that means, the negative refraction switches to the normal refraction

4. CONCLUSIONS

In conclusion, we demonstrated theoretically that negative refraction may occur near the low frequency edge of the second and fourth bandgaps in 1D PCs for an oblique incidence of wave. These unique properties of refracting Bloch photons have the potential to perfect the design of the integrated photonic systems.

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