SOCIAL CHOICE AND EQUITY THEORIES: SEEKING THE COMMON GOOD AS A COMMON GROUND

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The communication of basic mathematical and physical notions is generally accepted, as one of the very first concepts that will be exchange between civilisation, since math and physics appears universal. However, such concepts only reflect a small fraction of a much larger body of knowledge to might be share between civilizations. Notoriously, no obvious scheme for exchanging knowledge as been proposed for social sciences, which unlike physical science, are not mathematically formalised. However, some branches of social science have been mathematically formalised. Social choice theory, which deals with the seemingly simple but fundamentally complex problem of finding the optimal "common good" from individual choices, offers a direct path to discuss to notion of democracy. Equity theory, which treats the complex problem of sharing goods or chores, is a very powerful tool to explain the notion of distributive justice. Since democracy and justice provide many societal advantages, such notions are probably universal. Due to the importance of those tools for the survival of civilisation, they are probably among the most useful information to share with another civilisation.

INTRODUCTION

Within the infinite numbers of possible cultures, we might wonder what cultural traits we could possibly share with any extra-terrestrial civilization. Nevertheless, it could be argue that every civilization has to face the same challenge: How to distribute resources between different tasks and/or individuals. Limitation of resources and conflicting interests are likely to be a universal problem. In many cases, no perfect solution exists and cultural tradition plays a role in the choice of the allocation procedure. Being simultaneously universal and culturally oriented renders the allocation problem especially suitable as a topic of discussion between civilizations.

Two theories attempt to solve the problem of a fair resource attribution. Equity theory, which treats of fair sharing methods of goods or chores, is a very powerful tool to explain the notion of distributive justice. Social choice theory, which analyzes the fundamental problem of finding the optimal "common good" from conflicting individual choices, offers a direct path to democracy. Moreover, these two theories are described within a mathematical framework, which ease their translation into an interstellar message. They also generate various paradoxes and impossibilities, both practical and theoretical, which are by themselves an interesting topic of discussion.

EQUITY

First, let us examine the fundamental problem of dividing goods between two persons. The simplest solution, it is to split goods in two equal parts. The basis of this procedure is the principle of proportionality, which extends its roots from the Aristotle's book on ethics. Due to the broad influence of Aristotle on the occidental culture, this

principle is at the base of the system of justice in use in many countries. Still, others visions of justice exist on Earth. For example, this problem presented in the Babylonian Talmud: *Two hold a garment; one claims it all, the other claims half. What is an equitable division of the garnment?* According to the proportionality, the share should be 2/3 and 1/3, but the solution of the Talmud is 3/4 and 1/4. The difference comes from the fact that the Talmud only considers the *contested* part[1].

This is a rather ideal case: claims are restricted and goods are divisible. In real life, claims tend to be unrestricted and often goods are not divisible (ex: car, house, etc). In addition, we do not have always access to an impartial judge. A good sharing rule must not only be neutral for each actor (*equitable*), but must also be *efficient* which means that all actors shall be happy as possible at the outcome of the process. In addition, it shall *not create any envy* between participants at the end of the process. This last property is stronger than proportionality and superseded it, when there are more than two players.

For the construction of an interstellar message, any mathematical notation could be used[2-5]. Here, for sake of simplicity, we will use as much as possible the standard mathematical notation.

In this formalism, the three qualities of a fair share between two actors (a and b) can be described as:

- Equitability: $\sum[U_a(S_a)]= \sum [U_b(S_b)]$
- Efficiency: $\sum [U_a(S_a)]= \sum [U_b(S_b)]$ is maximized
- Envy-freeness: $\sum [U_a(S_a)]\geq \sum [U_a(S_b)]$ and $\sum [U_b(S_b)]\geq \sum [U_b(S_a)]$,

where $\sum$ is the symbol for summation, S_a is the share of the player a, and U_a is the utility function of the player a. The utility can be described as the subjective value given to a good. This is a psycho-physiological perception, that changes from people to people and trough the time for the same people.

Alternation

One very simple rule for non-divisible goods is the alternation: each actor chose an item alternatively. Every child learns this simple algorithm in kindergarten where it is the usual procedure for constituting a sport team.

Strict alternation is not a very good allocation procedure. First, envy-freeness cannot be guaranteed and is not very efficient since often a different allocation, which pleases more the actors, exists. Worst, even equitable sharing is not granted. It is possible to solve partially the problems of envy and inequity. Nevertheless, this method is fundamentally inefficient since it based on an object-by-object comparison instead of a global comparison.

Alternation has the great advantage to be easily extendable to an arbitrary number of choosers. This is why the professional sport teams use it for the allocation of new players. It can also be adapted to handle unequal chooser. Such adaptation is used to build the Gemini telescope observing schedule[6]. In that peculiar case, telescope time must not only be distributed proportionally to each member share but also by scientific value and technical feasibility within each member allocated time. Even if is it is largely used, alternation covers only a small fraction of all possible allocation algorithm.

Divide and Choose

One of the most antique is the divide-and-choose procedure: *One actor (the divider) divides the good(s) in two half, and then the second actor chooses (the chooser) his preferred half*. Since he don't know what part will be chosen the divider is strongly incited to divided the two halves as equally as possible from his perception and the chooser is always guaranteed that he can get at least half of the total value of goods from his standpoint.

Divide and choose				
	U_a	U_b	S_1	S_2
$Object_1$	6	14	X	
$Object_2$	6	14	X	
$Object_3$	17	2	X	
$Object_4$	17	1	X	
$Object_5$	4	4	X	
$Object_6$	6	2		X
$Object_7$	2	21		X
$Object_8$	8	14		X
$Object_9$	17	14		X
$Object_{10}$	17	14		X
$\sum U_a$	100		**50**	50
$\sum U_b$		100	35	**65**

Table 1: Divide and choose method

Here we face a key problem common to almost all procedure: the strategic manipulation by the divider. If the divider knows the value of each (or some) item from the chooser point of view, he could split the lot in such a way that each half worth 80%-20% of the total from is standpoint, and 49-51% for the divider. This trick would lure the divider in choosing the second half leaving the divider with almost all it perceived value.

However, there is some risk in adopting this strategy. For example, it could backfire badly if the chooser also knows the preference of the divider. He could then decide to sacrifice a small fraction of its wealth by choosing the second half! This would send a strong statement to the divider and warn him to never try to fool someone again.
The opposite situation can also occur if the divider and the chooser have both very different needs and if each of them shares this information. In such circumstances, each actor may ends up with 80% of its perceived value of the lot, if they collaborate. Unfortunately, this win-win solution is difficult to achieve with this simple procedure since it needs some exceptional circumstances.

Adjusted Winner

To address this problem, the American mathematician Steven Brams has developed the *adjusted winner procedure*. This method is simple. Each contestant as first to allocate a value to each item disputed; the total value being fixed. Then, the most desired items are temporally given to each contestant. Once all items are distributed, the total value all items received by each contestant is calculated. Then to equilibrate gain an adjustment is made. This is done by an appropriate splitting of the most contested good when possible, or sharing the monetary value of this good after having sold it. This procedure achieved the remarkable feat of being simultaneously equitable, envy-free and efficient.

Like with any other allocation procedure, adjusted winner is sensitive to strategic manipulation. However, since adjusted winner is an efficient procedure and therefore the outcome is expected to be a good as possible, there is a strong initiative to not cheat and play fair.

The following table is given as an example:

	Adjusted winner							
	Utility		Step 1		Step 2		Step 3	
	U_a	U_b	S_a	S_b	S_a	S_b	S_a	S_b
$Object_1$	6	14		X		X		X
$Object_2$	6	14		X		X		X
$Object_3$	17	2	X		X		X	
$Object_4$	17	1	X		X		X	
$Object_5$	4	4	-			X		X
$Object_6$	6	2	X		X		X	
$Object_7$	2	21		X		X		X
$Object_8$	8	14		X		X		X
$Object_9$	17	14	X				13.1	3.1
$Object_{10}$	17	14	X				X	
$\sum U_a$	100		74		74		70.1	
$\sum U_b$		100		63		67		70.1

Table 2: Adjusted winner procedure result

Unfortunately, it is impossible to extend this method to more than two actors. For larger number of actors, only two qualities of a fair deal can be satisfied at the same time. For example, the proportional division is equitable and envy-free but not efficient. Procedures that provide envy-freeness and efficiency[7] or efficiency and equitability[8] have been developed. Maybe, some extra-terrestrial civilisations apply one of those last two procedures in their transaction. This cultural difference is an obvious source of conflict if the complex nature of a fair deal is not understood correctly.

DEMOCRACY

Since the application of fair attribution rule is difficult when the number of participant is large others allocation mechanism have been developed. Those methods are based on the aggregation of individual preference. Even if these procedures are much cruder at the level of individual (therefore more frustrating), they are much easier to apply than fair distribution procedure at the collective level. This trade-off done between efficiency and applicability must be universal and therefore the frustration about politics and democracy it fuel.

At the most fundamental level, democratic processes are procedure of information collection. In this vision, democracy is only an extension of the motto: two heads are better than one. Therefore, democracy generally produces better results in average than despotic governance because decisions have a better chance to be more appropriate to the largest number of individual.

In direct consequence, natural selection alone will favours the emergence of democracy. Indeed, democracy-like processes are observed in animal species. For example, red deers decide to move when about 62% of the adults stand up. Gorillas move when 65% of adults call. Swans fly when signal intensity given by head movements reaches a given threshold. Hamadryas baboons travel in the direction designated by the majority vote. African buffalos move in the average direction of the adult female gaze. Democratic decision appears to be beneficial mostly because they tend to produce less extreme decision[9]. In is book After Contact[10], the American psychologist Albert Harrison concluded also to the profitability of democracy. He noted that democracy is in rapid progression on Earth. Generally, democratic governments are less war prone than totalitarian states, which allows them to spare resources for others issues that are more important.

Despotism beats democracy only if the group is small and if the difference if information between the despot and the rest of the group is large[9]. The American political scientist Robert Dalh has listed of seven fundamental democratic institutions:

elected official, free and fair elections, inclusive suffrage, right to run for an office, freedom of expression, alternative information and associational autonomy. All are related to the free access and unbiased flow of information[11].

Two French scientists have laid mathematical foundations of the analysis of the democracy: Jean-Antoine Nicolas Caritat, marquis de Condorcet and Jean Charles, chevalier de Borda. The first was a pre-eminent mathematician and author of the 1791 French Constitution; the second was a physicist, hero of the American Revolution, who has conceived various scientific instruments. In addition, both distinguish themselves by putting in place the metric system.

The goal of Condorcet was to axiomize the propositions made the Swiss philosopher Jean-Jacques Rousseau in his book Le Contrat Social[12]. His big hope was that social sciences would follow a similar path of progress like the one that has been observed in physics since the Newton's fundamental work, in such a way that his descendant will surpass us in wisdom as in enlightenment[13]. Condorcet first work was to prove the value of the majoritarian rule by probabilistic approach. He demonstrated that error rate of a majoritarian vote is much lower than individual error rate if the error rate of the individual voters is below 50%. This demonstration, known as the Condorcet Jury, has been since be proven over a wide set of initial conditions. Condorcet then tried to extend his conclusion to the problem of multicandidate elections only to uncover one of the major paradoxes of democracy. In the Condorcet's vision, the true winner of an election is the candidate who beats all others candidates in a sequence of pairwise elections. This definition is so compelling that the ability in selecting the Condorcet's winner as become standard benchmark used to compare elections methods.

Even if this condition appears obvious, Condorcet soon discovered that this definition can leads to some apparent inconsistency. For example, a situation can exists where the candidate A beats the candidate B, B beats C and C beats A. Such cycles are inevitable and consist a fundamental and universal limitation to any democratic process. They can happen even with a fully rational and sincere electorate. Therefore, even the most advanced extraterrestrial civilization face the same challenge as us in implementing an effective democracy[14].

Borda's voting method

Borda recognized the value of the Condorcet approach but considered it much to complex to be practical. Instead, he proposed his own election method already in use at the French Académie des Sciences[15]. In the Borda's method, each voter gives his order of preference between each candidate. Each candidate receives a number of points equal to the number of opponent he beats. Much simpler than the Condorcet approach this method was much easier to implement.

However, a weakness soon became apparent: This method is highly sensitive to strategic manipulation. For example, the voter may choose to rate lower a candidate that is a menace for his favourite but who would be otherwise better evaluated. Borda himself accepted this weakness, claming that his method was designed for an honest man. Much worst, outcome of an election using the Borda voting method can be affected by the introduction of dummy candidates. Let us suppose there is two parties with the equal support in the population (A and B), some supporters (10%) of A split and create

a new party a, which is less popular than either A or B in the rest of the population because is more extremist. The election result with 100 electors using Borda's method would be before the split:

Original situation			
#Voters	Preference order	Points for A	Points for B
50	$U_A>U_B$	50	0
50	$U_B>U_A$	0	50
100	$U_B=U_A$	50	50

Table 3: Borda election: original situation

and after the split:

After the split				
#Voters	Preference order	Points for A	Point for a	Points for B
45	$U_A>U_a>U_B$	90	45	0
5	$U_a>U_A>U_B$	5	10	0
50	$U_B>U_A>U_a$	50	0	100
100	$U_A>U_B>U_a$	145	55	100

Table 3: Borda election: after the split

After the split, A win by a large margin thanks to a marginal party a. Even if B partisans try to manipulate strategically their vote by modifying their preference order: B>a>A, this not change much the result: A=95 a=105 and B=100! Only if in a perfect strategic manipulation is carried, 10% of the B supporter choose to vote B>A>a, will they be able to force a draw!

Borda's vote method was used briefly by the French Academy from 1796 to 1803 when Napoleon got it abolished. It has been also used by the science team involved in the Voyager mission planning.

Plurality

What about plurality? It is one of the worst system since it behaves very badly as soon there is more than two choices. Not only, it is not very efficient way of picking the Condorcet winner but also it strongly encourages the strategic voting. Under plurality, giving his vote to a small party is almost equivalent to not vote at all. Therefore, most voters prefer to vote for the less worst of all parties that still as a chance of winning. The direct consequence of such strategy is the emergence of a two parties system: the *Duverger's Law*[16]. It will be difficult for extraterrestrial to understand why we still use a voting scheme that not only poorly measure voters intention but also make them deform there own opinion.

Approval voting

Nevertheless, a simple modification, allowing the elector to vote for more than on candidate, can largely reduce the strategic voting. This method, the approval voting, is also much more likely to select the Condorcet's winner[17]. Its has been use in the longest living democracy in history, the republic of Venice, from 1268 to 1797. It is used now by the many scientific societies (Mathematical Association of America, American Statistical Association, Institute of Electrical and Electronics Engineers, etc) and the United Nations.

Nothing is perfect

Does a perfect election method exist? Scientist have researched it for a long time and created in the process a large number of voting methods: Baldwin, Black, Bucklin, Bullet (anti-plurality), Copeland, Coombs, Dabagh, Dogson, cumulative voting, Hare (Single transferable vote), Instant runoff, Nanson, Range Voting, Two-turn Runoff, Simpson, etc. Unfortunately for us, the answer is no! The American economist Kenneth Arrow demonstrated it in 1951 and received the Nobel Prize of Economics for

his work[18]. Still some methods are better, as we have learned from those researches. Condorcet is generally considered as the best one but is very difficult to apply in practice. Any method relaying on the ordering of candidates are weakened by our inability at ordering effectively more than 4-5 candidates. Moreover, all methods involving vote transfer behave in such way that voting for a candidate may hurt him! Maybe extraterrestrial more advanced than us will use Condorcet without problem or maybe they will use the approval voting, a less efficient method but good enough for most purpose. Over a large sample of civilisations, it would be interesting to see what the prevailing voting schemes are.

CONCLUSION

The existence of such complex decision processes provides powerful insight about species that uses them. For example, they cannot be put in place unless an individual is able to perceive and intellectualized the reality of other individuals. The ability to evaluate the fairness of a situation is likely to appear by natural evolution alone in social animal. For example, capuchin monkey posses a notion of fairness[19]. Otherwise, only despotic decision process is possible. In consequence, we should not expect to meet a democratic beehive-like civilization.

Equitable sharing procedures and electoral procedures are intellectual tools developed to deal with the conflicting individual interest for the best outcome for the group at large. Therefore, both equity and social choice theories are product of civilizations seeking to better manage interactions between individuals with selfish tendencies.

In such circumstances, altruism emerges as priced quality that is encouraged by extra-terrestrial societies, but which is difficult to achieve at the level of individual due to natural tendencies for selfishness[20]. Therefore, social choice and equity theories are topic of discussion common to any civilization that are struggling, like ours, to build a fairer society, we can hardly find a better topic of discussion with our interstellar interlocutor.

Some problems and solutions born from those are likely to be civilization specific, which make them especially interesting in the perspective of an interstellar message construction. Since communication channel capacity and processing power are the ultimate factor limiting the efficiency of social interaction, we might discover that intelligence, wisdom and altruism are closely tied together.

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