14.01 Midterm # 1, October 3rd, 1996, 7:30-9:30 pm
(With Answer Key)

Question I: True or False (35 points).

a) Suppose income rises by one percent and consumption of a good rises by 0.8 percent. Then the good is normal but not superior.

- **True**. Since consumption rose when income did, the good is normal, and since consumption rose by less than 1 percent, the good is not superior.

b) If the demand for a good is very elastic, then a shift in the supply curve will lead to a very small change in both the equilibrium price and quantity.

- **False**. If the demand curve is very elastic, a change in the supply curve will cause a small change in the price but a large change in the equilibrium quantity.

c) A firm already owns capital worth $500,000 and uses all of it in producing its output. It also uses labor and has a wage bill of $500,000. If the firm sells its output for a million dollars, then while it has zero accounting profits, the firm has half a million dollars in economic profits.

- **False**. The firm has zero economic profits and half a million dollars in accounting profits, not the other way around; the opportunity cost of the firm's capital is half a million dollars.

d) Generally speaking, price ceilings reduce consumers' surplus.

- **False**. Whether consumer surplus goes up or down depends on the elasticities of the demand and supply curves (one part of the area under the demand curve and above the price line increases, while another part decreases).

e) An agricultural price support will cause a production shortage for the good in question.

- **False**. A price support gives suppliers an incentive to overproduce (relative to what consumers wish to purchase) and hence will cause a surplus, not a shortage.

f) Two goods are complements when a change in the first good's price leads to an increase in consumption of both goods.

- **Oops**. This question should have read “if a *fall* in the first good’s price. . .". As such, anyone who gave a reasonable explanation of their answer got credit. For the record, the question would have been true for a fall in the first good's price.

g) Suppose a person has a backward-bending labor supply curve and that the person's wage is very, very high. Then if the wage rises a little bit, the income effect will dominate the substitution effect, so that the person will reduce labor supply.

- **True**. By definition, a person with a backward-bending labor supply curve will reduce labor supply when her wage is very high (i.e. high enough to put her in the backward-bending portion of the curve) and it rises a little bit. The reason is that the income effect of her raise makes leisure sufficiently more attractive than the price (substitution) effect that makes leisure more expensive.

Question II (40 Points)

Suppose Pat cares about consumption and leisure and has the utility function $u(c, l) = \alpha \ln c + (1-\alpha) \ln l$, where Pat's consumption is measured (in dollars) by c and leisure is $l = 24 - L$, where L is the number of hours of labor Pat supplies to the labor market. Suppose also that Pat can work at the hourly wage w and that Pat's labor income is subject to the constant tax rate τ, so that Pat's after-tax earnings for each hour of labor Pat supplies are $w(1-\tau)$. Pat also has nonlabor income, which we will assume is untaxed, equal to y.

a) Write out Pat's budget constraint. What is the implicit after-tax price of leisure Pat must pay?

- Noting that $L = 24 - l$, we can rewrite this budget constraint as:

  $$c + (1-\tau)\, w\, l = 24\,(1-\tau)\, w + y.$$

  That is, Pat spends c dollars on market goods (consumption) and $(1-\tau)\, w\, l$ dollars on leisure. Hence the after-tax price of an hour of leisure is $(1-\tau)\, w$. In part (a), you got 6 points for giving the correct budget constraint, and 2 points for the correct implicit after-tax price of leisure. In terms of partial credit, if you made a mistake on the budget constraint, you generally got 3 points out of 6. The most common mistake, by far, was writing: $c + (1-\tau)\, w\, l = (1-\tau)\, wL + y$.

b) Derive Pat's labor supply function (keeping in mind that Pat will never have negative hours of labor) as a function of the tax rate, the wage, and the preference parameter α. Suppose that for Pat $\alpha = 1/3$. What is the value of y below which Pat will supply positive hours of labor to the labor market?

- Pat's labor supply function may be derived by observing that with Cobb-Douglas preferences, expenditure on a good is always a constant share of income (if you didn't remember this fact, either the usual MRS/price relationship or a LaGrangean optimization program would get you that answer), with the share equal to the good's Cobb-Douglas preference parameter. Hence we have:

  $$(1 - \tau)\, w\, l = (1 - \alpha)\,[24\,(1 - \tau)\, w + y],$$

  since $(1 - \alpha)$ is the preference parameter for leisure and the price of leisure was seen to be $(1 - \tau)\, w$. Dividing both sides of this equation by $(1 - \tau)\, w$, we get Pat's leisure demand function; using the fact that $L = 24 - l$, we get Pat's labor supply function:

  $$L(\tau) = 24\,\alpha - \frac{(1 - \alpha)\, y}{(1 - \tau)\, w} \qquad \text{. . . (1)}$$

  Actually, this equation will hold only if the righthand side is positive, since Pat cannot work negative hours. Thus Pat's true labor supply function is the equation above if the RHS is non-negative and zero otherwise.

  If Pat has $\alpha = 1/3$, then Pat's labor supply will be equal to: $8 - (2/3)\, y / (1 - \tau)\, w$; hence the maximum nonlabor income Pat can have if labor supply is to be positive is: $y^* = 12\,(1 - \tau)\, w$. That is, if Pat's nonlabor income is not less than 12 times Pat's after-tax wage, Pat will not choose to work for labor income.

  In part (b), you got 4 points for correctly setting up the optimization condition, i.e. that the MRS between consumption and leisure equals its price ratio. If you had the wrong price ratio, or made other mistakes, you generally received 2 points out of 4 here. You got 3 points for taking your results from MRS=price ratio, and substituting that into the budget constraint they found in part (a). You received 3 points for deriving the correct labor supply function. You got 2 points for noting that the labor supply function only holds if the expression L(t) is greater than or equal to 0. If the expression is negative, then $L(\tau)$ is still 0. You received 3 points for solving for the maximum nonlabor income y. If your budget constraint from part (a) was incorrect, you could still receive up to 7 points in part (b) [4 points for MRS=price ratio, and 3 for plugging into your budget constraint].

c) Suppose that Pat chooses $L > 0$. Remembering that the tax rate is τ and that Pat works $L(\tau)$ hours at wage w, write out the government's revenues $R(\tau)$ from Pat's labor income (remember that the notation $L(\tau)$ and $R(\tau)$ means that L and R are considered functions of τ). For any value of α, find the value τ^* that maximizes the government's revenue from Pat's labor income?

- Pat works $L(\tau)$ hours, where $L(\tau)$ is given by equation (1) above, earns w dollars per hour on the labor market, and pays a tax of τ cents per dollar earned. Hence the government receives revenues equal to: $R(\tau) = w\,\tau\,L(\tau)$ or:

$$R(\tau) = 24\,\alpha\,w\,\tau - \frac{(1-\alpha)\,y\,\tau}{(1-\tau)} \qquad \ldots (2)$$

Setting $R'(\tau) = 0$, we have

$$24\,\alpha\,w - \frac{(1-\alpha)\,y}{(1-\tau)^2} = 0 \qquad \ldots (3)$$

It is easy to see that the LHS of this equation is decreasing in τ, so the revenue function is globally concave; this fact means that any solution to the first order condition is a (unique) maximum. Solving this equation for τ, we have

$$\tau^* = 1 - sqrt\left[\frac{(1-\alpha)\,y}{24\,\alpha\,w}\right] \qquad \ldots (4)$$

In part (c), you got 4 points for noting that $R(\tau) = Lw\tau$. You got an additional point for setting its derivative equal to 0. If you incorrectly specified $R(\tau)$ [many people wrote $R(\tau) = Lw\tau$], you received 2 out of 5 points for setting up the problem. You got 5 points for correctly solving for the revenue-maximizing τ.

d) Some politicians claim that an income tax cut will actually {\it increase} government tax collections. Suppose for Pat that $\alpha = 1/3$, $y = 3$, and $w = 10$; under these conditions, Pat is typical of many American workers. What value would the tax rate have to be for a tax cut to increase government revenue from Pat's labor income? (You may use the approximation *sqrt* (1/40) = 1/6.) Suppose that Pat's tax rate were 29% and that Pat was a typical American worker. Evaluate the policitians' claim that tax cuts will increase government tax collections.

- Plugging in the parameter values to equation (4), we find that $\tau^* = 5/6 = 83.3$ %. Given Pat's much lower tax rate, it is not plausible that government tax revenues will increase when the tax rate is cut.

In part (d), you got 3 points for finding $\tau^* = 5/6$. You got 2 points for noting that tax cuts increase revenue only when τ is *greater* than 5/6. You got 2 points for pointing out that the politicians' claim is false. If you didn't get $\tau^* = 5/6$., you generally still received 2 points partial credit if you provided a good explanation for your answer.

Question III (25 Points)

Consider a firm that makes sprockets from two inputs, labor and capital, according to the production function $Q = F(K, L) = K^{\beta} L^{1-\beta}$, where $\beta < 1$.

a) Suppose the firm has not yet made a decision as to the size of its factory; that is, it has not yet chosen how much capital to purchase, so it is operating in the Long Run. What can you say about the firm's long run returns to scale?

- This production function is clearly of the Cobb-Douglas form; Cobb-Douglas production functions always exhibit long run constant returns to scale. To see this fact, simply observe that

$$F(\lambda K, \lambda L) = (\lambda K)^{\beta} (\lambda L)^{1-\beta} = \lambda K^{\beta} L^{1-\beta} = \lambda F(K, L) \qquad \text{. . . (5)}$$

  which is the definition of CRS production. Part (a) was worth 9 points. To get full credit, you needed not just to recognize that the production function was CRS because the sum of the exponents was one, but also to show how the exponents are related to returns to scale (either in words or preferably by showing what happens to output when both inputs are increased by the same proportion).

b) Suppose that in the short run the firm is committed to a capital input of $\bar{K}$, which minimizes costs at the current output and input prices. If output or input prices should change the firm may want to adjust its output level, even though it cannot change its capital input. What can you say about the firm's returns to scale in the short run?

- Now we have capital stock fixed in the short run at $K = \bar{K}$. Thus when we consider returns to scale, only labor is allowed to be increased proportionately, so that we have

$$F(\bar{K}, \lambda L) = \bar{K}^{\beta} (\lambda L)^{1-\beta} = \lambda^{1-\beta} \bar{K}^{\beta} L^{1-\beta} = \lambda^{1-\beta} F(\bar{K}, L) \qquad \text{. . . (5)}$$

  and since \$\b < 1\$, the production function exhibits *decreasing* returns in the short run. Part (b) was also worth 9 points. Again, people who said that it was decreasing returns because the exponent on L was less than one did not get full credit *unless* they explained the relationship between the exponent and the extent of returns to scale.

c) Suppose the wage rate doubles but the prices of output and capital remain unchanged. In response, the firm has reduced labor input to $\tilde{L}$ so as to maximize short run profits. Will the new input combination $\bar{K}, \tilde{L}$ maximize long-run profits? Explain your answer.

- Part (c) was worth 7 points. People generally got at least half credit if they recognized that the firm's short-run response (with capital fixed) would differ from its long-run response. Actually demonstrating this mathematically is difficult; people were generally better off if they just noticed that capital and labor could substitute for one another in the long-run production function, so a firm would want to do so if the price of one input changed. The extent of credit varied with the quality and clarity with which these ideas were expressed, and graphs of the isoquants and isocost lines of the short-run and long-run average cost curves tended to help. People who put that the firm's short run response might not be optimal in the long-run because other things might change, like the output price or the price of capital, lost credit; the question stated explicitly that these did not change, and that you were concerned only with the firm's response to a doubling of the wage.