

14.01 Final Exam – Thursday, December 19, 1996

Questions 1–5 each are worth 50 points. Answer all questions. You have 3 hours.

Question #1: True or False (Remember to explain your answers.)

- a. Price subsidies to farmers increase social welfare because they ensure that enough food will be produced to feed the country.
- b. Coase's First Theorem states that, if there is costless information, costless bargaining, and clearly set property rights, then there will be a Pareto efficient allocation of resources.
- c. Since research and development creates positive externalities, the amount of R&D done by profit-maximizing firms will be too low in the absence of subsidies or other incentives to increase R&D.
- d. A monopolist always has an upwardly sloping supply curve.
- e. In a perfectly competitive economy with perfect information, Pareto inefficient competitive equilibria could occur if some agents have zero endowments.
- f. Selling “discount” and “premium” brands of cigarettes is one way for tobacco companies increase their profits.
- g. A perfectly discriminating monopsonist will hire as many workers as a perfectly competitive market for labor services.
- h. In an uncertain environment with risk-averse agents, full insurance is always socially optimal, even when insurance companies cannot monitor policyholders' behavior perfectly.

Question #2: Consumer Theory

Hafthor and Sula, recently betrothed, are deciding how much to spend on their wedding. Hafthor, a graduate student at MIT, makes a subsistence wage, and can contribute nothing to the wedding. Sula's utility function is given by the following expression:

$$U(c, l, m) = \alpha_1 \ln c + \alpha_2 \ln l + (1 - \alpha_1 - \alpha_2) \ln m$$

where c represents consumption, l represents leisure, and m represents the amount of “wedding” units consumed. Assume that the price of consumption is equal to 1. The price per wedding unit is p_m . Sula's wage rate is w , and she has no nonlabor income. Sula has 24 units of time to divide between labor and leisure.

- a. Write down Sula's budget constraint. (Note that Sula has to divide her labor income between consumption and the wedding.)
- b. State the optimality conditions for Sula's utility maximization. (*Hint: This problem is similar to the two-good case where Sula optimizes over consumption and leisure. The only difference is that we have an additional consumption good, the wedding.*)
- c. Suppose $w = 10$, $\alpha_1 = 1/3$, $\alpha_2 = 1/2$, $p_m = 60$. What are Sula's optimal choices of m , c , and l ?
- d. In part (c), you should have computed a value for m^* , the utility-maximizing number of wedding units, that is strictly less than 1. (If you didn't, you can still give intuitive answers for this section and part (f).) Suppose that Sula's parents would like a more lavish wedding, with $m = 1$. They offer to contribute an amount equal to $p_m(1 - m^*)$ to the wedding. Assume that Sula treats this additional income as cash, i.e., her nonlabor income has now risen from 0 to $p_m(1 - m^*)$. Assume the parameters are equal to those values specified in part (c). Given the generous contribution from her parents, is Sula's optimal choice of m now $m = 1$? If your answer is “yes,” did Sula's parents overpay? Could they have contributed a smaller amount, whereby Sula would have still chosen $m = 1$? If your answer is “no,” explain why the contribution of Sula's parents was not sufficient for Sula to increase m to $m = 1$.
- e. Find the minimum amount of nonlabor income y which Sula's parents would have to contribute to Sula in order for her to choose $m = 1$. Again, assume the parameters are equal to those values specified in part (c).
- f. Use your results in parts (d) and (e) to answer this question:

The U.S. Environmental Protection agency has recently found that many states have spent too little on environmental protection. In an attempt to correct the imbalance, Congress plans to appropriate additional funds to any state whose environmental spending falls short of \$100 per capita. According to the appropriation formula, a state with per capita spending equal to $X < 100$ gets a grant equivalent to $(100 - X)$ per capita. Suppose that the additional funds are treated by the state legislatures as unrestricted income. Will the planned appropriations raise state environmental spending to the level of \$100 per capita? Explain.

Question #3: Imperfect Competition

Two firms (named "1" and "2") are the only sellers of lysine. Consumers regard both firms' brands of lysine as identical. Let Q_1 and Q_2 denote the respective lysine outputs of the two firms. The market demand curve for lysine is: $P = 60 - Q$, where P is the market price, and $Q = Q_1 + Q_2$ is the total market quantity. Firm 1 has a constant marginal cost equal to 12, while firm 2 has a constant marginal cost equal to 6. Both firms have zero fixed costs.

- a. If the two firms could collude, what would be the values of Q_1 and Q_2 that maximize their joint profits? What is the market price P in this case?
- b. Compute the Cournot equilibrium values of Q_1 , Q_2 , and P .
- c. Suppose that Firm 2 can commit itself in advance to producing output Q_2 ; in other words, Firm 2 is the Stackelberg leader. What level of output will it choose, and what will Q_1 and P be?
- d. Draw a graph that illustrates how you found the Cournot and Stackelberg equilibria. (It does not have to be exactly to scale, but clearly label your axes and any other lines you draw.) At the Stackelberg equilibrium, is Q_2 the output level that firm 2 would choose if it thought that firm 1's output would stay at Q_1 ?
- e. Making reference to the graph you just drew, what can you say about the Stackelberg equilibrium price compared to the Cournot equilibrium price? Is the Stackelberg price always lower? Sometimes lower? Does it depend on which firm is the Stackelberg leader? Provide an intuitive explanation for your answer.

Question #4: Externalities and Efficient Regulation

Note: You are not required to use mathematical models to answer this question. Whether or not you use mathematical models, your answer will be graded on the quality of your economic analysis, including its precision, clarity, and logical accuracy.

Suppose that the Republic of Znosko-Borovska has a competitive paper-products industry with more than one supplier. In order to produce any positive output of paper products, each of these firms will have to generate some positive amount of toxic waste, which pollutes the Bogoljubow River. Consumers in Znosko-Borovska want to have paper products, but they also value a clean river, so that their welfare is decreasing in the total amount of toxic waste dumped in the river. Paper producers can reduce the amount of toxic waste dumped into the Bogoljubow River, but only at cost. Their cost functions for reducing waste are characterized by strictly positive and increasing *marginal* cost for each unit of waste reduction. Reduction of waste dumping is of no direct benefit to paper producers (e.g., there are no public relations problems from polluting). The production function for paper products is independent of the amount of waste dumped generated. Finally, assume that all profits from the sale of paper products are distributed to individuals who are not affected by (and don't care about) pollution of the Bogoljubow River.

- a. First, suppose that the Premier of Znosko-Borovska has direct political connections with the paper industry. He chooses not to regulate the industry at all. Any company may dump as much toxic waste as it pleases without being penalized at all. From a social welfare standpoint, will there be too much, too little, or an efficient amount of pollution? Be careful to explain what measure of social welfare you are using to answer the question, and explain *why* you think your answer is right given that measure.
- b. After weeks of sustained protest, the old prime minister is replaced by a new reform-minded leader, who wants to regulate the dumping of toxic wastes in the Bogoljubow River. The new prime minister decides to retain you as an outside economic advisor. You have been asked to determine whether regulating with prices (i.e. imposing a per-unit emissions tax on paper firms' pollution) or quantities (imposing a ceiling on each firm's dumping of wastes) is a better approach. You inquire of the Prime Minister (and of officials in the Znosko-Borovskan Interior Ministry) about the costs and benefits of pollution control.
- c. You are told that the marginal emissions reduction cost curves (the "MERC" curve) is the same for all firms. Moreover, the Ministry has a general idea of the relative slopes of the MERC curve and the curve representing the marginal social benefits of reducing emissions (which are simply the marginal social costs of the

emissions themselves). The Ministry does not know the exact formulas for these curves, however. What do you tell the Prime Minister?

- d. Now the Interior Ministry tells you that the MERC curves actually vary across firms. The Ministry thinks it has a pretty good idea of what the distribution of emission reduction costs is, but it does not know what the cost is for each individual firm. Therefore the Prime Minister realizes that she must impose either the same emissions tax rate or the same limit on emissions quantity on every firm. If her goal is also to reduce emissions in the most efficient way, which should she choose: the tax or the quantity limit?
- e. Now suppose that the new Prime Minister asks if you have any alternative ideas for toxic waste reduction besides direct quantity regulation. She would like to work things so that the total level of waste dumping does not exceed a prespecified level L . The Interior Ministry, however, still doesn't know which firms have which MERCs. Can you think of a way to lower the cost of emissions reduction? Explain why you think your idea is more efficient.

Question #5: A General Equilibrium Problem

Consider the following island exchange economy. There is one good, call it bananas, and two time periods, named “today” and “tomorrow.” Consumers trade today and consume tomorrow. The number of bananas available for consumption tomorrow depends on the weather. Let s denote the state of tomorrow's weather: $s = R$ when it rains and $s = N$ when there is no rain. There is a probability of π of it raining tomorrow, and probability $1 - \pi$ of no rain tomorrow.

There are two groups of consumers, 1 and 2. The following utility function describes the preferences of consumer i over consumption of bananas tomorrow:

$$U(x_{i,s}) = -\exp\{-\gamma_i x_{i,s}\}, \text{ for all } i=1,2, \text{ all } s = R, N$$

where $x_{i,s}$ denotes the number of bananas consumed by agent i in state s , and where $\exp(y)$ stands for e^y . Now, assume that when it rains, consumer $i = 1$ gets e_R bananas and nothing when there is no rain. And consumer $i = 2$ gets no bananas when it rains and e_N bananas when there is no rain.

Today, these two groups of consumers can competitively trade two types of claims: claim R at a price of p_R and claim N at a price p_N . Holding one claim R today gives the owner one banana tomorrow if it rains tomorrow and nothing if it doesn't rain. Holding one claim N today gives the owner one banana tomorrow if there is no rain tomorrow and nothing if it rains.

- Before any trade in the claims, how many claims R and N does consumer 1 have? consumer 2?
- What is the total supply of these two types of claims in the economy?
- Explain why γ_i is a measure of how risk averse agent i is.
- Suppose consumers maximize expected utility. State the maximization problems for agent 1 and 2 in determining $x_{i,s}$. (*Hint*: remember that for any variable y , the derivative of $\exp\{f(y)\}$ with respect to y equals $f'(y) \exp\{f(y)\}$.)
- Solve for the equilibrium prices p_R and p_N .
- Show that $p_R / p_N < \pi / (1 - \pi)$ if $e_R > e_N$.

- g. Suppose $\gamma_1 > \gamma_2$ (so that consumer 1 is more risk averse than consumer 2). For which consumer is the quantity $|x_{i,R} - x_{i,N}|$ smaller? Provide an intuitive explanation first.
- h. Prove your claim in part (g).