Spring 1996
14.01 Exam 1
Professor Jeffrey Harris
Instructions

1. This is a closed book exam. You may use calculators. On the cover of each exam booklet, please write:

   - The number of the question you are answering
   - Your name and ID number
   - The name of your instructor (List your Friday recitation instructor if you go to Professor Harris's lectures)
   - The time your class meets

2. In case you have forgotten your instructor's name or the correct spelling, here is a list with class times.

   Andres Almazan (MWF 1)

   Meghan Busse (MWF 9, 10)

   Marci Castillo (MWF 9)

   Giovanni Dell'Ariccia (F 10, 11)

   Harrison Hong (MWF 12, 1)

   Mike Lii (F 10, 12)

   David Riker (MWF 11, 1)

   Charlotte Wojcik (MWF 11, 12)

3. There are <u>four</u> questions on the exam. Answer all questions. You have 2 hours to complete the exam. The exam will be graded out of a total of 100 points.

4. You may find it useful to remember that the equation of a line with slope m which passes through the point (x_1, y_1) is $y = m(x - x_1) + y_1$, and that $\frac{\partial \ln x}{\partial x} = \frac{1}{x}$.

5. Please do not start working on the exam until the proctors announce you may begin. Please hand in your exams promptly when the proctors announce you must cease work.

Place your answer to this question in booklet #1

Question #1 (20 points)
True/False/Uncertain

Please evaluate each of the 5 statements as either true, false, or uncertain. Each statement is worth 4 points. Credit is determined by the quality of your explanation. No credit will be given for answers lacking an adequate explanation.

a) In the space of just six months, weekly sales of coffee rose from 1 million to 2 million pounds, while the price of coffee remained unchanged at $4 per pound. Therefore, the supply curve for coffee must have shifted outward.

b) Subsidies that encourage farmers to produce more peanuts will create surplus in the peanut market.

c) If the supply curve for product X is perfectly inelastic with respect to price, shifts in the demand curve for product X will not change the equilibrium quantity of product X.

d) If managers and salesmen are perfect complements in production, then a decline in the wages of salesmen has no effect on the firm's hiring of managers.

e) Consider a firm that can vary the levels of all of its inputs and faces unchanging input prices. If the firm's production function exhibits constant returns to scale, then it cannot have increasing marginal costs.

Answers to Question 1 on first midterm

- (a) FALSE. The statement is that supply *must* have shifted outward. To argue that it is false, you just need to state a counter-example: If the supply curve is flat, i.e., perfectly elastic, then supply does not shift to get to the new equilibrium.

- (b) FALSE. Subsidies will shift the peanut supply curve to the right, leading to a higher equilibrium quantity at a lower equilibrium price. There will not be a surplus unless there are price controls on peanuts. A subsidy is not a price control. (Some people interpreted surplus to refer to the effect on producer surplus rather than surplus in the sense of excess demand; if the argument was coherent and correct, then credit was awarded).

- (c) TRUE. This is easily illustrated in a graph in which the supply curve is perfectly vertical. (People that got this wrong typically had *horizontal* supply curves).

- (d) FALSE. In the production theory that we have studied, workers are factors of production, as are capital and electricity, for example. A decrease in the price of salesmen will give the cost-minimizing firm an incentive to hire more of them. Because salesmen and managers are perfect complements, the increase in salesmen must be matched in fixed proportions with an increase in managers. (Some people talked about an "income effect" shifting the firm up to a higher indifference curve: this is the general idea, though you should be careful not to confuse the consumer theory, which deals in budget constraints and indifference curves, and the production theory, which deals in isocost lines and isoquants).

- (e) TRUE. If the firm can vary all inputs, there are constant returns to scale, and factor prices are constant, marginal costs must be constant. 10% more output requires 10% more inputs under constant returns to scale. The cost of producing $Q = f(L, K)$ units of output is $wL + rK$. The cost of producing λQ units of output is $w(\lambda L) + r(\lambda K) = \lambda(wL + rK)$ for all λ. So doubling output costs exactly twice as much with fixed w and r. Costs are increasing linearly in output, and so marginal costs are that constant slope.

Place your answer to this question in booklet#2

Question #2 (25 points)

The US government has had long-standing policies to protect the US sugar market from foreign competition. Suppose that it is now considering changing those policies. Before making any decisions, however, it would like to know some things about the sugar market. The government discovers that in the US the equilibrium price for sugar is $500 per ton, the equilibrium quantity is 10,000 tons, and that at this equilibrium point, the price elasticity of demand is -2.0 and the price elasticity of supply is 1.5.

(3 points) a) Find the linear demand and supply curves consistent with the above information.

(3 points) b) Calculate consumer surplus in this market.

Use the information in the following hypothetical news article to answer parts c) and d).

> The Senate this week appointed a four-member committee to evaluate the President's plan to change government policies protecting sugar.
>
> For decades, US producers of sugar have been protected from competition from foreign producers, primarily Cuba and other Caribbean nations, by restrictions that ban sugar imports. Consumer advocacy groups, as well as manufacturers of soft drinks, candy, and other prepared foods claim that the restrictions raise the price of sugar in the US to 25 cents a pound, more than twice the price in the rest of the world. Manufacturers have responded by using part corn syrup (a sweetener made from corn) and part sugar, although the price of corn syrup is also higher than the price of sugar in the rest of the world. Senate leaders indicate that the committees first priority will be to decide whether the import restrictions in fact should be removed.
>
> The committee is made up of four members. Senator Beete of Idaho represents the agricultural southern part of the

state, where sugar beets (which are refined into sugar) are one of the major cash crops. Senator Maize of Iowa also represents an agricultural constituency, but one in which corn is the major crop. Senator Havana of Florida represents an area of Florida heavily populated by Caribbean immigrants, many of whom have friends and family in their native countries who grow sugar cane. Senator Koke of Georgia represents Atlanta. (See related article on page 12, Coca-Cola Company of Atlanta Makes Donation to Senator Koke's Reelection Campaign.)

(3 points) c) Would you expect the equilibrium price of sugar to rise or fall if the import restrictions were lifted? Would the equilibrium quantity rise or fall? What would happen to the amount of sugar produced by US producers?

(16 points) d) Write several sentences for each of the four senators describing what you would expect his or her position to be. For each senator identify

- what market he or she cares about
- what is likely to happen in that market if the import restrictions are lifted
- whether he or she will support or oppose lifting the restrictions

SOLUTION TO QUESTION 2

(a) We are looking for two expressions of the form:

$$\begin{aligned} Q^d &= \alpha + \beta P \\ Q^s &= \gamma + \delta P \end{aligned}$$

By definition:

$$E_d = \frac{dQ^d}{dP} \cdot \frac{P}{Q^d}$$

and

$$E_d = \frac{dQ^s}{dP} \cdot \frac{P}{Q^s}$$

Using given information:

$$\beta = \frac{dQ^d}{dP} = E_d \cdot \frac{Q^d}{P} = -2 \cdot \frac{10000}{500} = -40$$

$$\delta = \frac{dQ^s}{dP} = E_s \cdot \frac{Q^s}{P} = 1.5 \cdot \frac{10000}{500} = 30$$

To find the remaining two parameters, α and γ simply substitute:

$$10000 = \alpha + (-40 \cdot 500) \Rightarrow \alpha = 30000$$

and similarly:

$$10000 = \gamma + 30 \cdot 500 \Rightarrow \gamma = -5000$$

Conclusion:

$$\begin{aligned} Q^d &= 30000 - 40P \\ Q^d &= -5000 + 30P \end{aligned}$$

- **Grades:**
  - Confusion between slopes and elasticities was severely penalized (no credit)

(b) Consumer surplus is in this case (because of the linearity of the demand function) the area of the triangle ABC (see attached figure figsolq2b.ps):

$$CS = \frac{(P^0 - P^*) \cdot Q^*}{2} = \frac{(750 - 500) \cdot 10000}{2} = \$1,250,000$$

where: P^0 is the price that would make $Q^d = 0$.

- **Grades:**
  - If you missed part (a) (you did not find the right demand function) but you computed CS according to the demand function you derived in (a) I gave you full credit.
  - Other minor mistakes were penalized with partial credit.

(c) Opening U.S. sugar market to foreign seller translates into a shift in the supply curve of sugar (from S to S' in the attached figsolq2c.ps). According to that graph, Q^o represents the quantity in the market under the prohibition, Q^* the equilibrium quantity after the lifting of sanctions and Q^{US} the quantity that U.S. sugar producers can sell after lifting the prohibition. Prices P^* and P^o are respectively the equilibrium prices without and with prohibitions.

From the graphics figsolq2c.ps it is immediate to observe that:

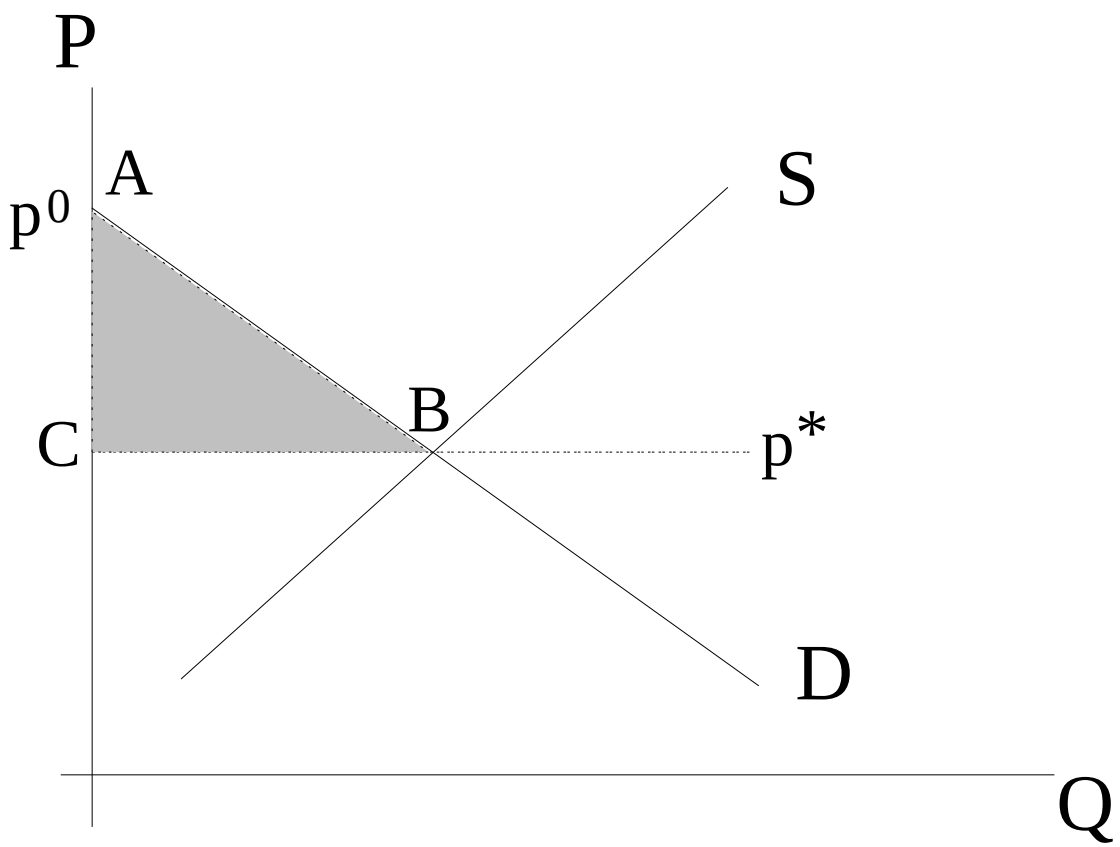
$$Q^* > Q^o > Q^{US}$$

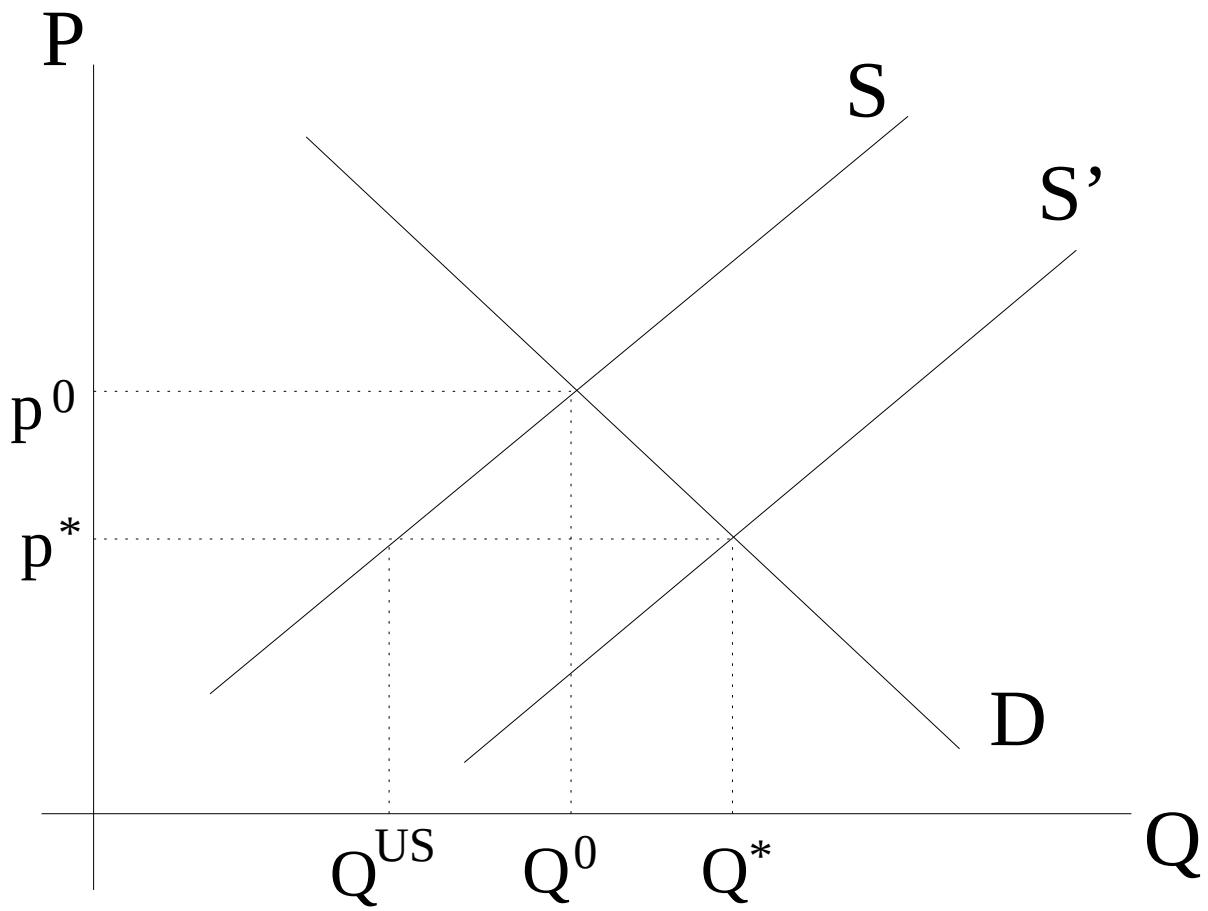
and also that

$$P^* < P^o$$

- **Grades:**
  - Verbal explanations (if sufficiently clear) received full credit.
  - Mention of the results without any other explanation received half the grade.

(d) The following table summarizes the answers


P
A
S
p0
B
C
p*
D
Q


P
S
S’
p0
p*
D
Q
QUS
Q0
Q*

Senator	**Market**	**Position**	**Reason**
Beete	US sugar market (supply side)	Oppose	(A)
Maize	US corn market (supply side)	Oppose	(B)
Havana	World sugar market (supply side)	Support	(C)
Koke	Sweetener market (demand side)	Support	(D)

(A) Lost profits for sugar farmers. US sugar producers sell less at lower price. (see (c)).

(B) Lost profits for corn farmers. The demand of corn shifts in. They sell less at less price.

(C) More profits for Caribbean farmers. Demands shifts out. Caribbean Farmers sell more at a higher price.

(D) Decrease in price of sweeteners⇒decrease cost of production of soda⇒ increase of profits for soda manufacturers.

- **Grades:**
  - I was very generous in this part. I divided the total grade in four parts (one for each senator); four points are the full grade for each of them. I was not particularly demanding about how precise you defined the market they care about, instead I focused more on what the reasons that you offered to support why their respective constituencies could gain or loose.
  - Graphical analysis was valued. To get full credit you have to analyze senator's positions in terms of shocks to supply and demand. In case of Senator Koke, to refer to the effect in production costs was also required.

Place your answer to this question in booklet#3

Question #3 (30 points)

Rachel and Ross decide to go to the neighborhood pizza parlor one evening (they are not in the mood for coffee). As they are seated, the restaurant manager tells them that he has some bad news. The restaurant has run out of all toppings except mushrooms and peppers. Rachel and Ross are too tired to go anywhere else, so they decide to order.

The menu indicates that the price of mushrooms is \$1 per topping unit, while the price of peppers is \$2 per topping unit. Rachel and Ross total up their money and decide they would like to spend \$16 on the pizza.

Rachel's utility function defined over mushrooms (M) and peppers (P) is

$$U(M,P) = \frac{3}{4}lnM + \frac{1}{4}lnP$$

where ln is the natural log function.

(3 points) a) What is Rachel's marginal rate of substitution between P and M? (Assume throughout this problem that on a diagram M is on the x-axis and P is on the y-axis.)

Assume for parts (b) and (c) that Rachel can decide on the mushroom and pepper combination according to her utility function, and is in control of the total funds (the \$16).

(3 points) b) What is Rachel's budget constraint?

(5 points) c) Solve for Rachel's utility maximizing combination of mushrooms and peppers.

Ross also has a utility function defined over mushrooms (M) and peppers (P),

$$U(M,P) = 200M + 200P$$

(5 points) d) Now assume Ross has control over the total funds and can choose his preferred combination. Solve for Ross's optimal combination of

mushrooms and peppers.

As Rachel and Ross are arguing over whose optimal combination to put on the pizza, the restaurant manager stops at their table to deliver their drinks. He tells them that since there are only two toppings available, he has decided to let them have the peppers for only \$1 per topping unit.

(2 points) e) Using Rachel's utility function, determine whether peppers are a normal or inferior good for Rachel. Explain your answer.

(3 points) f) Illustrate on a diagram the substitution and income effects of the change in the price of peppers using your answer to part (e). Do NOT solve for Rachel's new optimizing combination.

(3 points) g) What happens to Ross's utility maximizing combination after the price change has occurred?

(3 points) h) Using your results from part (g), explain why Rachel and Ross have no trouble deciding on a mushroom and pepper combination after the price of peppers changes.

(3 points) i) How might your answer to part (h) change if Ross's utility function were $U(M, P) = (M + P)^2$ instead?

Question 4

a) Rachel's MRS = (MU_M/MU_P) = (3/4M)/(1/4P) = 3P/M
The MRS is not the same thing as the price ratio. MRS will equal the price ratio at the utility maximizing point, but at other points on the indifference curve this is not necessarily true. You got up to 2 points for demonstrating you knew what MRS is and 1 point for the correct answer.

b) Budget Constraint: $I = P_M M + P_P P \Rightarrow 16 = M + 2P$

c) Rachel's optimizing bundle will be at

$$MRS = P_M/P_P$$

This tells us that

$$3P/M = 1/2$$

So

$$6P = M$$

Plug into the budget constraint

$$M + 2P = 16 \Rightarrow 6P + 2P = 16 \Rightarrow P = 2$$

Thus,

$$P^* = 2 \quad \& \quad M^* = 12$$

You got up to 3 points for setting up the problem correctly, and 2 points for the correct answer.

d) Here the combination that maximizes Ross' utility is $M^* = 16$ and $P^* = 0$. This is a corner solution. It is not true that any combination is as good as any other. Compare Ross' utility from 16 mushrooms with any other bundle that he can possibly purchase. Ross' MRS = (1 pepper/1 mushroom) for all points on his indifference curve. The price ratio is (1 pepper/2 mushrooms). Ross is willing to accept 1 mushroom for 1 pepper, but the market is willing to give him 2 mushrooms for 1 pepper, so he is willing to get as many mushrooms as he can. But, since he cannot consume negative amounts of peppers, he is constrained to consume at most 16 mushrooms which is his entire budget. You got up to 3 points depending on how you set up your approach to the problem, and up to 2 points more for the correct answer. Some people put that Ross' would buy nothing but mushrooms but did not state how many mushrooms he would buy. That cost people a point.

e) If you calculate Rachel's demand function you would get:

$$P^* = I/4P_P$$

Since $dP^*/dI > 0$, peppers are a normal good. A normal good is a good where consumption rises as income rises keeping all prices constant. Thus, you cannot conclude a good is normal just because consumption increases as the price drops, or that utility increases as you consume more of the good. Consumption of inferior goods can increase as the price drops. Also, utility increases with consumption even for inferior goods. You got a

point for stating the good was normal and another point for giving a correct reason.

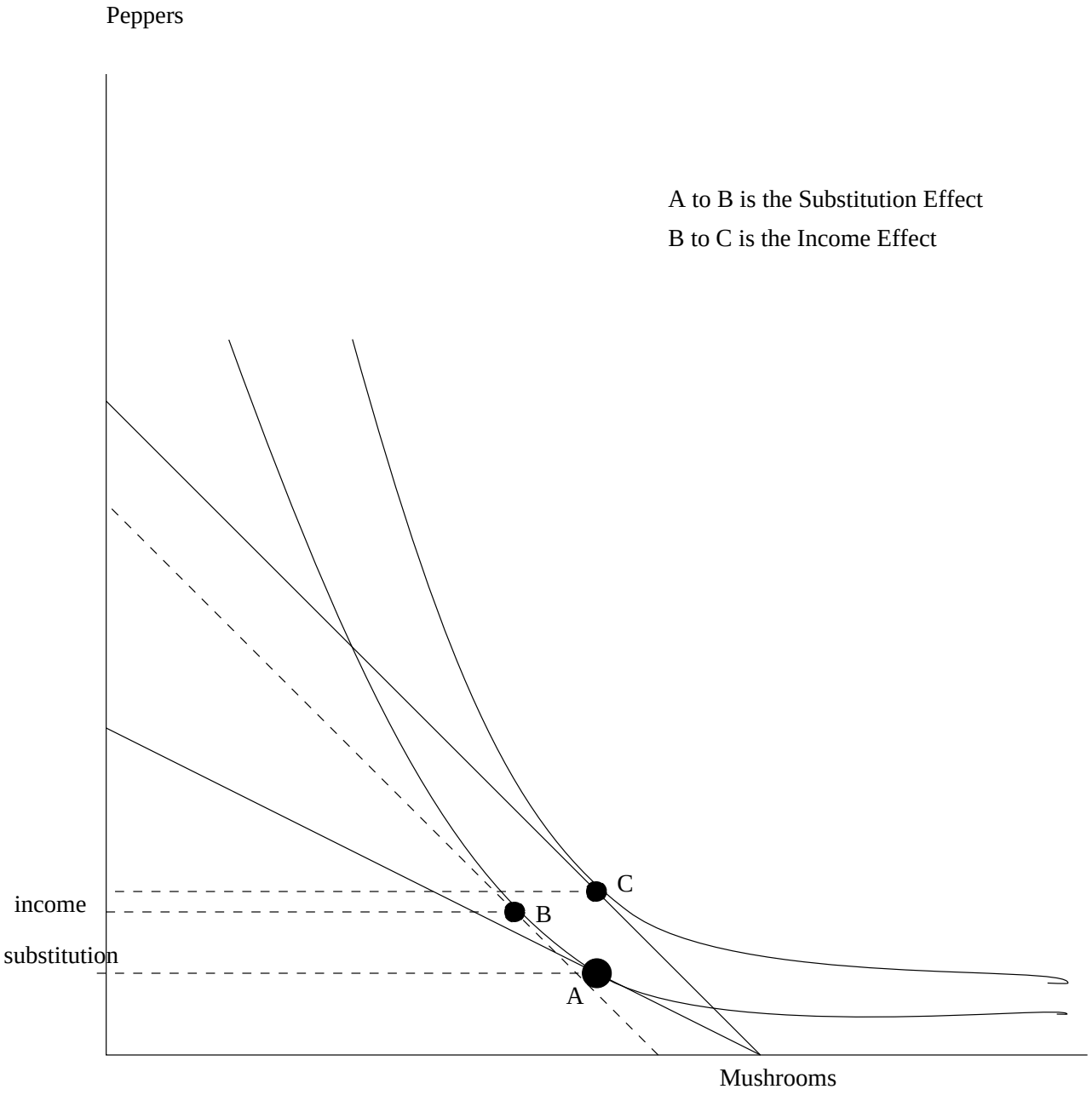
f) 1 point for correct budget lines. 1 point for correct indifference curves. 1 point for the correct points and labels. See diagram.

g) The price ratio is now 1 which is equal to Ross' MRS=1. Thus, Ross will be indifferent between any combination of peppers and mushrooms as long as his entire budget is spent. You may have lost a points, even if you said Ross is indifferent, if you said this was the same as before in part d.

h) Since Ross is indifferent, he will be fine with whatever Rachel chooses.

i) The MRS for this utility function is just the same as the original, so the answer will not change. Also, this utility function is just an order preserving transformation of the original, so they represent the same preferences. This utility function has exactly the same indifference curves as the original.

PROBLEM 3f)


Peppers
A to B is the Substitution Effect
B to C is the Income Effect
C
B
A
income
substitution
Mushrooms

Place your answer to this question in booklet#4

Question #4 (25 points)

J.R. owns the Southfork Oil Refinery, which manufactures gasoline using two inputs, petroleum and electricity, according to the production function

$$Q = P^{\frac{1}{4}} E^{\frac{1}{2}}$$

where Q is the number of gallons of gasoline that can be produced in a day using P gallons of petroleum and E kilowatt hours of electricity. Electricity costs \$0.10 per kilowatt hour and petroleum costs \$1.25 per gallon.

(5 points) a) Suppose J.R. decides that Southfork should produce 5,000 barrels of gasoline a day. In order to produce this much gasoline at minimum cost how much petroleum and how much electricity should Southfork use?

(2 points) b) How much will it cost Southfork to produce 5,000 barrels using the input quantities you found in part a)?

There is *only one* supplier from whom Southfork Oil Refinery can buy electricity: Dallas Edison Electric Company. Suppose that Dallas Edison wants to be sure how much electricity its industrial customers will buy over the next year, so is requires Southfork to sign a legally binding contract to purchase *exactly* 250,000 kilowatt hours a day for the next year. Dallas Edison still charges the same amount: \$0.10 per kilowatt hour, or a total of \$25,000 per day. (This means that no matter how much it chooses to produce, Southfork *must* buy the electricity every day for the next year.)

(3 points) c) Given the new contract, how long does the short run last for Southfork Oil? How do you know?

(2 points) d) Under the contract, what is Southfork's (short run) production function?

(2 points) e) Give an expression for the number of barrels of petroleum (P) Southfork will use to produce Q barrels of gasoline.

(3 points) f) Under this new contract, what is Southfork's (short run) cost function (as a function of output)?

(3 points) g) Under the contract, would Southfork ever choose to produce if it were making losses? Would it ever have done so under the old system? Explain.

(5 points) h) If Southfork produces 5,000 barrels of gasoline, what will its costs be under the contract (i.e. using the cost function in part f))? To produce other amounts of gasoline, will it cost Southfork more or less under the contract than it did when it bought variable amounts of electricity? Why?

14.01 Spring 1996

Exam 1, Question 4 answers

a) The cost minimizing use of inputs is where the MRTS equals the input price ratio; namely, where

$$\text{MRTS} = \frac{MP_P}{MP_E} = \frac{\frac{1}{4}P^{-\frac{3}{4}}E^{\frac{1}{2}}}{\frac{1}{2}P^{\frac{1}{4}}E^{-\frac{1}{2}}} = \frac{E}{2P} = \frac{P_P}{P_E} = \frac{1.25}{0.1} \text{ or, simplifying, where } E = 25P.$$

The cost minimizing use of inputs to produce $Q = 5000$ must satisfy this and $5000 = P^{\frac{1}{4}}E^{\frac{1}{2}}$. Plugging $E = 25P$ into this we get $5000 = P^{\frac{1}{4}}(25P)^{\frac{1}{2}} = 5P^{\frac{3}{4}}$. Solving for P gives us **P = 10,000**. And therefore **E = 250,000**.

b) The general form of the cost function is $C = P_P P + P_E E$. Plugging in the input prices and the P and E we solved for in part a) tells us that the cost of producing 5000 gallons is $(1.25)(10,000) + (0.1)(250,000) =$ **37,500**.

c) The definition of the short run is the time interval over which at least one input is fixed. In this case, the short run lasts **1 year** because under the contract with Dallas Edison, Southfork Oil is committed to use a fixed amount of electricity every day for 1 year.

d) Since the amount of electricity is fixed at 250,000 kilowatt hours per day in the short run, the short run production function will be $Q = P^{\frac{1}{4}}(250,000)^{\frac{1}{2}} = 500P^{\frac{1}{4}}$.

e) We can easily derive an expression for the number of gallons of P used to produce Q gallons of gasoline from the production function found in part d): $P = \left(\frac{Q}{500}\right)^4$.

f) Using the same general format as part b), we can solve for the short run cost function as

$$C = P_P P + P_E E = (1.25)\left(\frac{Q}{500}\right)^4 + (0.1)(250,000) = 1.25\left(\frac{Q}{500}\right)^4 + 25,000.$$

g) If the price of gasoline fell such that Southfork was making losses, but it was losing less than $25,000 per day, it would still choose to produce and make losses. Before the contract, it would have stopped production if it were making any losses at all. This is because under the contract, Southfork has a fixed cost; thus, it would choose to produce if it were losing less that it would if it produced nothing (and still had to pay the fixed cost.) Without having signed the contract (i.e. without a fixed cost) Southfork would have produced nothing rather than lose money.

h) If Southfork produces 5,000 gallons of gasoline its costs will be $1.25\left(\frac{5000}{500}\right)^4 + 25,000 = \mathbf{37,500}$. Notice that the contract fixes electricity at the optimal amount of electricity for producing 5000 gallons of gasoline, and therefore the cost of producing 5000 gallons in the short run (when electricity is fixed) will be the same as the cost of producing 5000 gallons in the long run (when Southfork can choose the level of both of its inputs.) To produce any other amount of gasoline, however, it will cost Southfork more under the contract (i.e. in the short run) than not under the contract (i.e. in the long run) because for other levels of output, 250,000 kilowatt hours may not be the cost minimizing amount of electricity to use.