Interview with Shiing Shen Chern

This article is based on an interview with Shiing Shen Chern conducted on March 4, 1998, by Allyn Jackson, with mathematical help from Dieter Kotschick.

Shiing Shen Chern is one of the greatest living geometers. He was born on October 28, 1911, in Jia Xin, China. His father had a degree in law and worked for the government. When Chern was a youngster, China was just starting to establish Western-style colleges and universities. He entered Nankai University at the age of fifteen and was drawn to physics but, finding himself clumsy with experimental work, eventually settled on mathematics. In 1930 he entered the graduate school of Tsinghua University, where there were a number of Chinese mathematicians who had obtained Ph.D.s in the West. Among these was Guangyuan Sun (Dan Sun), who had been a student of E. P. Lane at the University of Chicago. Some twenty years later, Chern became Lane's successor at Chicago. In 1932 Wilhelm Blaschke, a mathematician from the University of Hamburg, visited Tsinghua, and his lectures had a great influence on Chern.

Notices: *After your studies in China, you decided to get a doctorate in the West.*

Chern: I was given a fellowship to come to the West by Tsinghua University in 1934, after one year of assistantship and three years in the graduate school. I decided Europe was a better place than the United States. The normal thing to do was to come to the United States, but I was not interested in Princeton or Harvard.

Notices: *Why not?*

Chern: Not so good. I wanted to be a geometer. The United States did not have the type of geometry I wanted to work on, so I went to Europe. At that time, I think I had the advantage that although I was a beginning student, I had some ideas about what I wanted, about the mathematical situation in the world, who are the good mathematicians, where are the best centers. My evaluation could have been wrong, but I had my ideas. And I decided to go to Hamburg. In fact, it was a very good choice. At the end of the nineteenth century the center of science was Germany, including mathematics. And the center of mathematics in Germany was Göttingen, with Berlin and Munich not far behind. And Paris, of course, was always a center.

I graduated from Tsinghua graduate school in 1934. In 1933 Hitler took power in Germany, and there was great movement in German universities. The Jewish professors were removed, and so on, and Göttingen collapsed. And Hamburg became a very good place. Hamburg was a new university founded after the First World War. It was not so distinguished, but the math department was excellent. So I went there at the right time.

It was in Hamburg that Chern first came into contact with the work of Elie Cartan, which had a profound influence on Chern's approach to mathematics. At that time, Erich Kähler, a Privatdozent at Hamburg, was one of the main proponents of Cartan's ideas. Kähler had just written a book, the main theorem of which is now known as the Cartan-Kähler theorem, and he organized a seminar in Hamburg. On the first day of the seminar all of the full professors—Blaschke, Emil Artin, and Erich Hecke—attended.

Chern: [The seminar] looked like a kind of celebration. The classroom was filled, and the book had just come out. Kähler came in with a pile of the books and gave everybody a copy. But the subject was difficult, so after a number of times, peo-

Allyn Jackson is senior writer and deputy editor of the Notices. *Her e-mail address is* `axj@ams.org`. *Dieter Kotschick is professor of mathematics at the University of Munich.*

ple didn't come anymore. I think I was essentially the only one who stayed till the end. I think I stayed till the end because I followed the subject. Not only that, I was writing a thesis applying the methods to another problem, so the seminar was of great importance to me. I even came to see Herr Kähler after the seminar. A lot of times we had lunch together. There was a restaurant near the institute, and we had lunch together and talked about all kinds of things. My German was very limited, and Herr Kähler did not speak English at that time. Anyway, we got along. So, as a result, I finished my thesis very quickly.

Everybody knew that Elie Cartan was the greatest differential geometer. But his writings were very difficult. One reason is that he uses the so-called exterior differential calculus. And in our subject of differential geometry, where you talk about manifolds, one difficulty is that the geometry is described by coordinates, but the coordinates do not have meaning. They are allowed to undergo transformation. And in order to handle this kind of situation, an important tool is the so-called tensor analysis, or Ricci calculus, which was new to mathematicians. In mathematics you have a function, you write down the function, you calculate, or you add, or you multiply, or you can differentiate. You have something very concrete. In geometry the geometric situation is described by numbers, but you can change your numbers arbitrarily. So to handle this, you need the Ricci calculus.

Chern had a three-year fellowship, but finished his degree after only two years. For the third year, Blaschke arranged for Chern to go to Paris to work with Cartan. Chern did not understand much French, and Cartan spoke only French. On their first meeting, Cartan gave Chern two problems to do. After some time they happened to meet on the stairs at the Institut Henri Poincaré, and Chern told Cartan he had been unable to do the problems. Cartan asked Chern to come to his office to discuss them. Chern thereafter came regularly to Cartan's office hours, which often attracted a large number of visitors who wanted to meet with the famous mathematician. After a few months, Cartan invited Chern to meet with him at his home.

Chern: Usually the day after [meeting with Cartan] I would get a letter from him. He would say, "After you left, I thought more about your questions..."—he had some results, and some more questions, and so on. He knew all these papers on simple Lie groups, Lie algebras, all by heart. When you saw him on the street, when a certain issue would come up, he would pull out some old envelope and write something and give you the answer. And sometimes it took me hours or even days to get the same answer. I saw him about once every two weeks, and clearly I had to work very hard. This lasted for a year, till 1937, and then I went back to China.

Photography by Peg Skorpinski.

Shiing Shen Chern

When he returned to China, Chern became a professor of mathematics at Tsinghua, but the Sino-Japanese War severely limited his contact with mathematicians outside China. He wrote to Cartan about his situation, and Cartan sent a box of his reprints, including some old papers. Chern spent a great deal of time reading and thinking about them. Despite his isolation Chern continued to publish, and his papers attracted international attention. In 1943 he received an invitation from Oswald Veblen to come to the Institute for Advanced Study in Princeton. Because of the war it took Chern a week to reach the United States by military aircraft. During his two years at the Institute, Chern completed his proof of the generalized Gauss-Bonnet Theorem, which expresses the Euler characteristic of a closed Riemannian manifold of arbitrary dimension as a certain integral of curvature terms over the manifold. The theorem's marriage of the local geometry to global topological invariants represents a deep theme in much of Chern's work.

Notices: *What among your mathematical works do you consider the most important?*

Chern: I think the differential geometry of fiber spaces. You see, mathematics goes in two different directions. One is the general theory. For instance, everybody has to study point set topology,

everybody has to study some algebra, so they get a general foundation, a general theory that covers almost all mathematics. And then there are certain topics which are special, but they play such an important role in application of mathematics that you have to know them very well, such as the general linear group, or even the unitary group. They come out everywhere, whether you do physics or do number theory. So there is the general theory, which contains certain beautiful things. And the fiber space is one of these. You have a space whose fibers are very simple, are classical spaces, but they are put together in a certain way. And that is a really fundamental concept. Now, in fiber spaces the notion of a connection becomes important, and that's where my work comes in. Usually the best mathematical work combines some theory with some very special problems. The special problems call for development in the general theory. And I used this idea to give the first proof of the Gauss-Bonnet formula.

The Gauss-Bonnet formula is one of the important, fundamental formulas, not only in differential geometry, but in the whole of mathematics. Before I came to Princeton [in 1943] I had thought about it, so the development in Princeton was in a sense very natural. I came to Princeton and I met André Weil. He had just published his paper with Allendoerfer[1]. Weil and I became good friends, so we naturally discussed the Gauss-Bonnet formula. And then I got my proof. I think this is one of my best works, because it solved an important, a fundamental, classical problem, and the ideas were very new. And to carry out the ideas you need some technical ingenuity. It's not trivial. It's not something where once you have the ideas you can carry it out. It's subtle. So I think it's a very good piece of work.

Notices: *One of your other most important works was the development of characteristic classes.*

Chern: The characteristic classes—they are not that impressive. Characteristic classes are very important, because these are the fundamental invariants of fiber spaces. Fiber spaces are very important;

[1] *This paper presented a proof of Gauss-Bonnet that was less desirable than Chern's in that it relied on the fact that a Reimannian manifold can be locally isometrically embedded in a Euclidean space. Chern's proof used only intrinsic properties of the manifold.*

therefore, the characteristic classes come up. But it did not take me that much thought. They come up often, even the characteristic class c_1 comes up, because in electricity and magnetism you need the notion of complex line bundles. And the complex line bundles lead to c_1, which comes up in Dirac's paper on quantum electrodynamics. Of course, Dirac did not call it c_1. When c_1 is not zero, that's related to the so-called monopole. So characteristic classes are important in the sense that they come up naturally in concrete problems, fundamental problems.

Notices: *When you first developed the theory of Chern classes in the 1940s, were you aware of Pontryagin's work and the fact that the Pontryagin classes of a real bundle could be recovered from the Chern classes of its complexification?*

Chern: My main idea is that you should do topology or global geometry in the complex case. The complex case has more structure and is in many ways simpler than the real case. So I introduced the complex Chern classes. I read the Pontryagin papers, but the real case is much more complicated. I didn't see his full papers, but I think he made some kind of announcement in *Doklady* in English. I learned from Hirzebruch the relations between Chern classes and Pontryagin classes.

Chern classes can be expressed in terms of the curvature, in terms of the local invariant. I was mainly interested in the relations between local properties and global properties. When you study spaces, what you can measure are the local properties. It's very remarkable that some local properties are related to the global properties. The simplest case of the Gauss-Bonnet formula is that the sum of angles of a triangle is 180 degrees. It shows up already in very simple cases.

Notices: *You are seen as one of the main exponents of global differential geometry. Like Cartan you have worked with differential forms and connections and so on. But the German school, of which Wilhelm P. A. Klingenberg is one of the exponents, does global geometry in a different way. They don't like to use differential forms, they argue with geodesics and comparison theorems, and so on. How do you see this difference?*

Chern: There is no essential difference. It's a historical development. In order to do, say, geometry on manifolds, the standard technique was Ricci calculus. The

Bay Laurels: Endowed Professorship Honors Chern

S. S. Chern is the recipient of many international honors, including six honorary doctorates, the U.S. National Medal of Science, Israel's Wolf Prize, and membership in learned academies around the world. He has also received a more homegrown honor, the dream-turned-reality of an appreciative student of 30 years ago, who grew up in the Bay Area.

When Robert Uomini would buy his 10 tickets for the California State Lottery, he had an unusual "what if I win?" fantasy: He wanted to endow a professorship to honor S. S. Chern. While an undergraduate at U.C. Berkeley in the 1960s, Uomini was greatly inspired by a differential geometry course he took from Chern. With Chern's support and encouragement, Uomini entered graduate school at Berkeley and received his Ph.D. in mathematics in 1976. Twenty years later, while working as a consultant to Sun Microsystems in Palo Alto, Uomini won $22 million in the state lottery. He could then realize his dream of expressing his gratitude in a concrete way.

Uomini and his wife set up the Robert G. Uomini and Louise B. Bidwell Foundation to support an extended visit of an outstanding mathematician to the U.C. Berkeley campus. There have been three Chern Visiting Professors so far: Sir Michael Atiyah of the University of Cambridge (1996), Richard Stanley of the Massachusetts Institute of Technology (1997), and Friedrich Hirzebruch of the Max Planck Institute for Mathematics in Bonn (1998). Jean-Pierre Serre of the Collège de France will be the Chern Visiting Professor for 1999.

The foundation also helps to support the Chern Symposium, a yearly one-day event held in Berkeley during the period when the Chern Visiting Professor is in residence. The March 1998 Symposium was co-sponsored by the Mathematical Sciences Research Institute and was expanded to run for three days, featuring a dozen speakers.

—A. J.

fundamental problem was the form problem, which was solved by Lipschitz and Christoffel, particularly Christoffel. And the Christoffel idea went to Ricci, and Ricci has his book on the Ricci calculus. So all these people, including Hermann Weyl, learned mathematics through the Ricci calculus. Tensor analysis had such an important role, so everybody learned it. That's how everybody started in differential geometry, with tensor analysis. But somehow in certain aspects, the differential forms should come in. I usually like to say that vector fields is like a man, and differential forms is like a woman. Society must have two sexes. If you only have one, it's not enough.

After spending 1943–45 at the Institute in Princeton, Chern returned to China for two years, where he helped to build the Institute of Mathematics at the Academia Sinica. In 1949 he became a professor of mathematics at the University of Chicago and in 1960 moved to the University of California, Berkeley. After his retirement in 1979 he continued to be active, and in particular helped launch the Mathematical Sciences Research Institute in Berkeley, serving as its first director from 1981 to 1984. Chern has had forty-one Ph.D. students. This number does not count the many students he has had contact with on his frequent visits to China. Because of the Cultural Revolution in China, the country lost many talented mathematicians and the tradition of mathematical research almost died out. Chern did much to regenerate this tradition. In particular, he was instrumental in starting the Nankai Institute for Mathematics in Tianjin, China, in 1985.

Notices: *How often do you get back to China?*

Chern: In recent years I go back every year, usually staying one month or longer. I started the institute at Nankai, and the most important thing is to get some good young people who will stay in China. In this respect we have been successful. Our new faculty includes Yimin Long (dynamical systems), William Chen (discrete mathematics), Weiping Zhang (index theory), and Fuquan Fong (differential topology). There are other very good young people. I think the main obstruction to the progress of mathematics in China is the low pay. By the way, the International Mathematical Union has chosen Beijing for the next International Congress of Mathematicians.

Notices: *Do you think that will be a big boost for mathematics in China?*

Chern: Oh, yes. But I think what I am worried about is that there will be too many mathematicians in China.

Notices: *It's a large country; maybe they need a lot of mathematicians.*

Chern: I think they don't need too many mathematicians. China is a large country, so naturally it has a lot of talent, particularly in the smaller places. For instance, there's the International Mathematical Olympiad for the high school students, and China generally does very well. In order to achieve well in competitions like this, the students need training, and as a result other topics could be ignored. Now the parents in China want their children to know more English, go into business, and make more money. And these exams don't give money. One year I think they just did less of this training, and China immediately dropped. What do you do for a country with 1.2 billion people? It means that the standard of living cannot be very high, if you have any social justice.

In 1934, when Chern chose to go to Germany for graduate study, geometry was a peripheral subject in the United States. By the time of his retirement in 1979, geometry counted as one of the most vibrant specialties on the U.S. mathematical scene. Much of the credit for this transformation goes to Chern. Still, he is modest about his achievements.

Chern: I don't think I have big views. I only have small problems. In mathematics a lot of concepts and new ideas come in, and you just ask some questions, you try to get some simple answer, and you want to give some proofs.

Notices: *That's how you get your ideas, just observing things?*

Chern: Yes, in most cases you don't have an idea. And in even more cases your ideas don't work.

Notices: *You're describing yourself as a problem solver, rather than somebody who builds a theory.*

Chern: I think the difference is small. Every good mathematician has to be a problem solver. If you are not a problem solver, you only have vague ideas, how can you make a good contribution? You solve some problems, you use some concepts, and the merit of mathematical contributions, you probably have to wait. You can only see it in the future.

It is very difficult to evaluate a mathematician or a part of mathematics. Like the concept of differentiability. Some time ago, twenty or thirty years ago, a lot of people just didn't like differentiability. I heard a lot of people who told me personally, "I'm not interested in any mathematics with a notion of differentiability." These are the people who tried to make mathematics simple. If you exclude notions involving differentiability you could exclude a lot of mathematics. But it's not enough. Newton and Leibniz, they should play a role. But it's interesting because there are ideas in mathematics which are controversial.

Notices: *Can you give some examples of controversial ideas in mathematics?*

Chern: One thing is that some of the papers nowadays are too long. Like the classification of finite simple groups. Who is going to read 1,000 pages of proof? Or even the proof of the 4-color problem. I think people have to make mathematics interesting.

I think mathematics won't die soon. It will be around for some time, because there are a lot of beautiful things to be done. And mathematics is individual. I don't believe you can do mathematics with a group. Basically, it's individual. So it's easy to carry on. Mathematics does not need much equipment. It's not like other sciences. They need more material support than mathematics. So our subject will last for some time. Now, human civilization, I don't know how long it will last. That's a very much bigger problem. But mathematics itself, we will get along for some time.

At the age of eighty-six Chern continues to do mathematics. In recent years he has been especially interested in Finsler geometry, which he discussed in a Notices *article two years ago ("Finsler geometry is just Riemannian geometry without the quadratic restriction", September 1996, pages 959–963).*

Chern: Finsler geometry is much broader than Riemannian geometry and can be treated in an elegant way. It will be the subject of the basic course on differential geometry within the next ten years in many universities.

I have no difficulty in mathematics, so when I do mathematics, I enjoy it. And therefore I'm always doing mathematics, because the other things I cannot do. Like now, I am retired for many years, and people ask me if I still do mathematics. And I think my answer is, it's the only thing I can do. There is nothing else I can do. And this has been true throughout my life.


物理几何是一家
共同携手到天涯
黑洞單極窮奧秘
纖維連絡織錦霞
進化方程孫立異
对偶曲率瞬息空
疇算竟有天人用
拈花一笑不言中

Physics and geometry are one family.
Together and holding hands they roam
to the limits of outer space.
Black hole and monopole exhaust
the secret of myths;
Fiber and connections weave to interlace
the roseate clouds.
Evolution equations describe solitons;
Dual curvatures defines instantons.
Surprisingly, Math. has earned
its rightful place
for man and in sky;
Fondling flowers with a smile -- just wish
nothing is said!

A poem written by Chern. Provided courtesy of T. Y. Lam.