Centrifugation

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Centrifugation

- **When a solid-liquid suspension is rotated in a cylindrical container (bowl) the suspension is subject to a centrifugal force in the radial direction.**
- **Centrifugation is a process by which solid particles are sedimented and separated from a liquid using centrifugal force as a driving force.**
- **Depending on the rotational speed and distance from the axis of rotation, the centrifugal force can be many times greater than the force of gravity, allowing even very small particles or particles slightly denser than the fluid to settle.**

Sludge Thickening and Dewatering

- **In wastewater treatment, sludge is produced from a number of operations such as primary settling, coagulation and flocculation, and biological activated sludge processes.**
- **The sludge produced in these processes may vary greatly in concentration, but it is typically of the order of 1%.**
- **Sludge concentration, typically conducted in stages, is a common way to reduce the amount of sludge to be eventually disposed of. Thickening and dewatering are common operations to concentrate sludges.**

Sludge Thickening

- **Sludge thickening consists of increasing the concentration of a sludge from its original value (typically about 1%) to between 2 and 10%.**
- **Physical methods are typically used, including sedimentation, flotation, centrifugation and cake filtration.**
- **The thickened sludge still contains a significant amount of water and can be pumped.**

Sludge Dewatering

- **Thickened sludge still contains too much water to be economically transported and disposed of.**
- **Sludge dewatering is an operation through which a significant portion of the sludge moisture is removed to produce a semidry cake typically containing solids in concentration ranging between 10 and 40%.**
- **Operations such as cake filtration, centrifugation, and bed drying are commonly employed for this purpose.**

Application of Centrifugation to Waste and Wastewater Treatment

Centrifugation has two main applications, namely:

- **Thickening of sludges and especially activated sludges and chemical slurries. In these cases the sludge produced during primary or secondary treatments is concentrated in its solids content (typically up to 2-10%).**

- **Dewatering of sludges from primary and secondary treatments. In these applications water is removed from sludges that have been already thickened, producing cakes having 15-20% (or even higher) solids concentration.**

Thickening and Dewatering of Sludges via Centrifugation

- **Most centrifugation processes for wastewater treatment operate in a continuous mode.**
- **The performance of centrifuges as sludge thickeners (in which both the clarified liquid and the thickened sludge are fluids) is more easily predicted and quantified than when centrifuges are used as sludge dewatering devices (in which a moist solid or cake must be effectively moved out of the centrifuge as it is formed).**

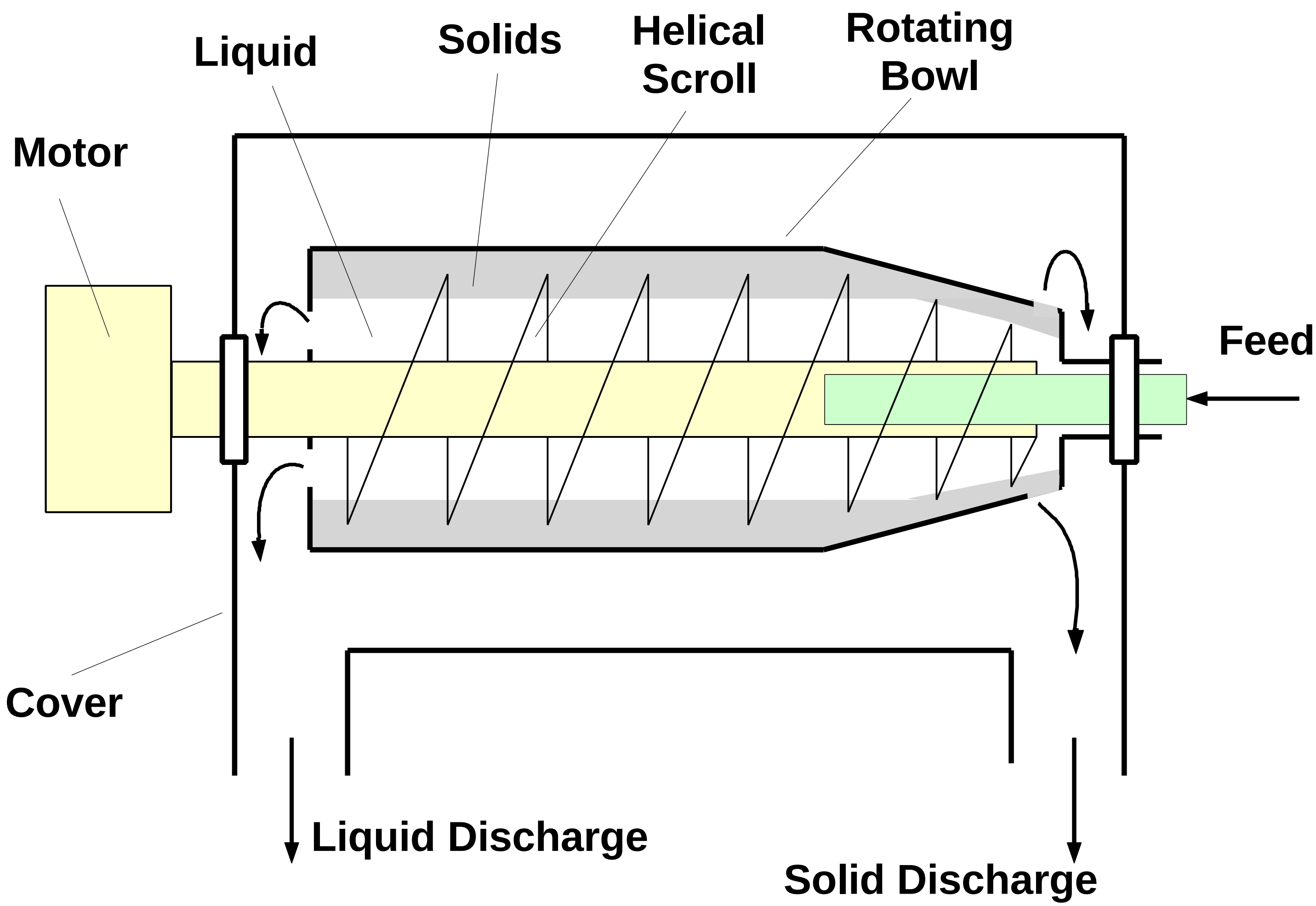
Centrifuges

- **Centrifuges are sedimentation devices in which suspended solids are separated from a liquid under the action of centrifugal forces generated by spinning the internal bowl of the centrifuge.**
- **The resulting settling velocities of the solids can be significantly higher than those generated by gravity forces.**
- **Centrifuges can be thought of as sedimentation vessels operating under high "gravitational" forces.**

Types of Centrifuges Used in Wastewater and Sludge Applications

- **Solid bowl centrifuges (decanters)**
- **Basket centrifuges**
  - **Perforated basket centrifuges**
  - **Imperforated basket centrifuges**
- **Disk bowl centrifuges**

Solid Bowl Decanter



Solid Bowl Centrifuge (or Decanter)

- **A solid bowl centrifuge (or decanter) consists of an elongated cylindrical rotating bowl having a tapered conical end**
- **The bowl length-to-diameter ratio is typically in the range 2.5:1 to 4:1**
- **The suspension is continuously introduced at one end of the bowl and travels parallel to the bowl axis. The clarified liquid is collected at the end of the cylindrical section of the bowl**

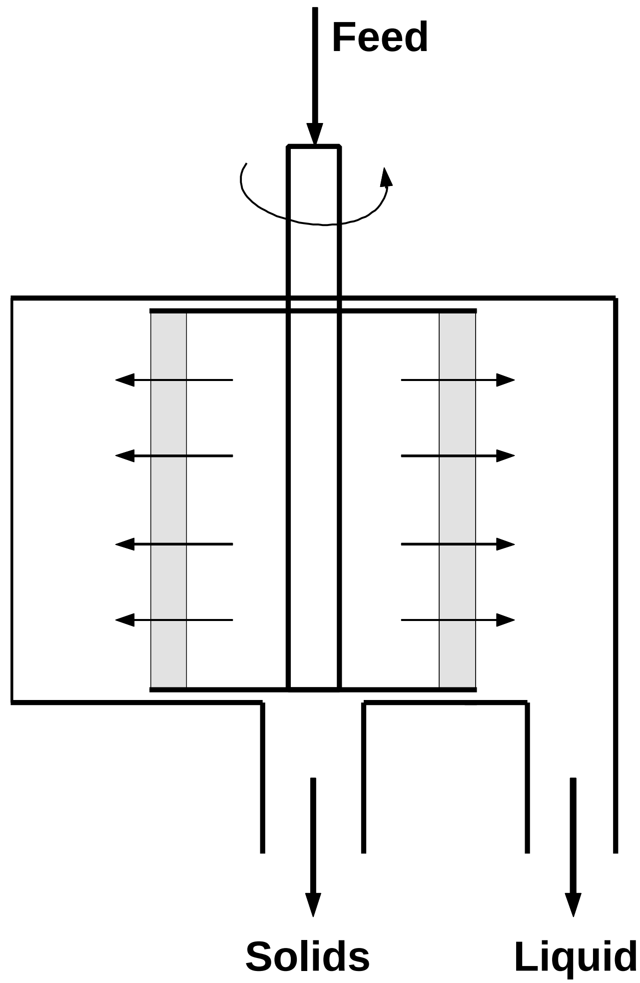
Solid Bowl Centrifuge (or Decanter) (continued)

- **The solids are collected at the end of the tapered section of the centrifuge where the centrifuge forms a *dewatering beach.***
- **The solids move typically countercurrent to the liquid. Co-current centrifuges are also used.**
- **A helical scroll rotating at a slightly different speed than the bowl is used to move the solids toward the tapered end where they are continuously removed through ports.**

Solid Bowl Centrifuge (or Decanter) (continued)

- **Typical rotational speeds in solid bowl centrifuges are in the range 1000 to 2500 rpm.**
- **Higher speeds promote solid separation (settling rate of solids increases with the square of the rotational speed). Maintenance costs increase with speed.**
- **Solid concentration in the cake is typically in the range 4-6% in thickening operations, and 10-35% in dewatering operations.**

Perforated Basket Centrifuge



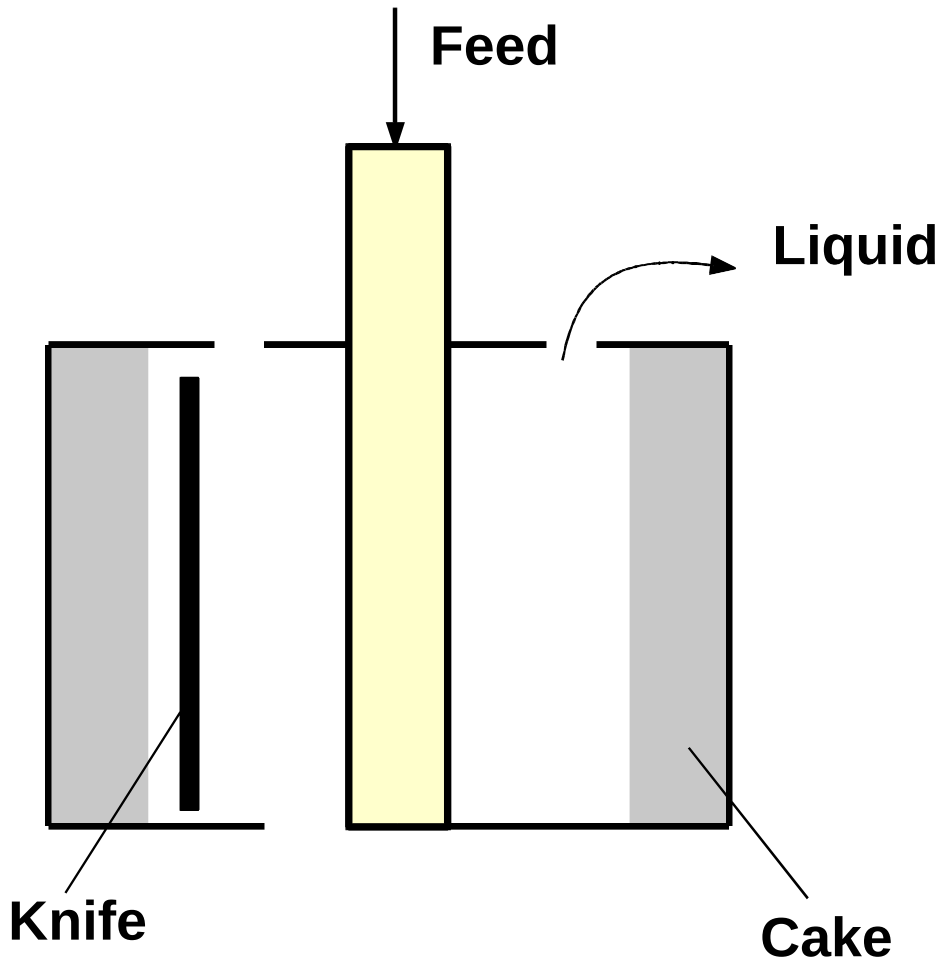
Perforated Basket Centrifuge

- **This type of centrifuge is provided with a rotating perforated basket, which allows a cake of solids to build inside the basket while allowing the liquid to pass through the cylindrical wall of the basket (centrifugal filtration)**
- **The clarified liquid is continuously removed as it emerges from the basket**

Perforated Basket Centrifuge (continued)

- **The solid is intermittently removed with a knife (not rotating with the basket) placed inside the basket, and collected in a vertical chute**
- **Drier cakes are obtained with this type of centrifuge than with other types. Therefore this type of centrifuge is used when the recovery of solids is desirable**

Imperforated Basket Centrifuge



Imperforated Basket Centrifuge

- **This centrifuge consists of an imperforated (solid) basket spinning vertically**
- **The feed is continuously introduced into the centrifuge while the liquid (centrate) is continuously removed from an overflow weir inside the centrifuge**
- **Solids build up during centrifugation forming a cake that must be periodically discharged**
- **After the basket becomes filled with solids the centrifuge slows down and "skimming" (the removal of the top semi-liquid soft cake layer) takes place**

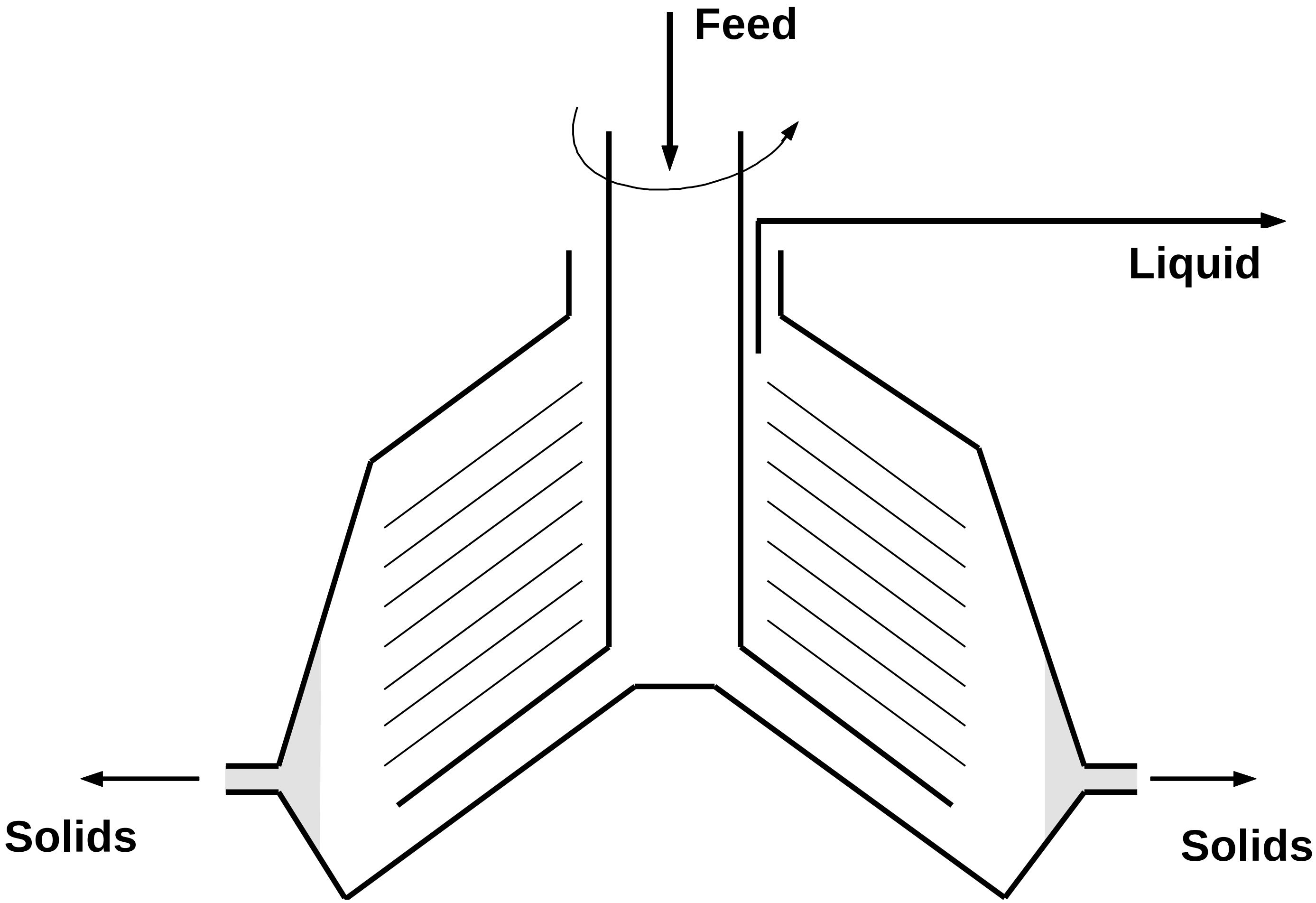
Imperforated Basket Centrifuge (continued)

- **Skimming typically removes 5 to 15% of the bowl solid volume**
- **The bulk of the cake is discharged using a plowing knife moving into the slowly rotating cake**
- **The solid is discharged centrally at the bottom of the centrifuge**
- **Solid accumulation is typically up to 60 to 85% of the maximum available depth**

Imperforated Basket Centrifuge (continued)

- **Automated cycles are possible in which cake unloading takes some 10% of the cycle time.**
- **This type of centrifuge is typically operated at low centrifugal forces and has a relatively low solid handling capacity.**
- **The imperforated basket centrifuge is the only basket centrifuge commonly used for typical sludge dewatering applications.**
- **High solid recovery can be achieved with this centrifuge even without chemical additives.**

Disk Bowl Centrifuge



Disk Bowl Centrifuge

- **In order to leave the centrifuge the liquid must pass between conical plates where the solids can separate after impacting on the plates**
- **The clarified liquid is continuously fed and removed from the center of the centrifuge**
- **The solid cake (thickened sludge) is continuously or intermittently removed from the side of the centrifuge**
- **Only degritted suspension can be fed to this type of centrifuge in order to minimize problems associated with plugging**

Chemical Additions During Centrifugation Operations

- **In sludge thickening operations chemical conditioners (especially polyelectrolytes) are often added to the sludge before feeding it to the centrifuge or directly in the centrifuge.**
- **The additives promote the ability of the suspended particles to flocculate and settle inside the centrifuge, thus increasing solid removal.**
- **Chemical additions promote the removal of fine solids.**

Chemical Additions During Centrifugation Operations

- **Chemical additives can also be used to promote sludge dewatering. However, this is a much more difficult process to describe and predict. Hence, the effectiveness of chemical addition in dewatering processes must be tested in pilot plant experiments.**
- **Cakes with a higher water content are obtained if chemical additions are made during dewatering processes.**

Typical Levels of Polymer Additions for Thickening of Sludges

Type of Sludge	Amount of Polymer Addition (lb dry polymer/ton dry solids)			
	Dissolved Air Flotation	Solid Bowl Centrifuge	Basket Centrifuge	Gravity Belt Filter
Waste Activated	4-10	0-8	2-6	6-14
Anaerobically Digested		8-16		
Aerobically Digested		8-16		

After Metcalf and Eddy, *Wastewater Engineering*, 1991, p. 262

Centrifuge Operations in Wastewater and Sludge Applications

- **Batch**
  - **Perforated basket centrifuge**
  - **Imperforated basket centrifuges**
- **Continuous**
  - **Solid bowl centrifuges (decanters)**
  - **Disk bowl centrifuges**

<u>Centrifuges vs. Vacuum Filters</u>

Centrifuges and vacuum filters are often used for the same sludge treatment purposes.

<u>Advantages of centrifuges over vacuum filters:</u>

- **Lower capital costs;**
- **Lower space requirements;**
- **Capability of treating suspension with poor filtration capability.**

<u>Advantages of vacuum filters over centrifuges:</u>

- **Higher solid concentration in the cake;**
- **Lower solid concentration in the effluent.**

Analysis of Centrifuge Performance

Process Variables Affecting Centrifugation

- **Type of slurry and solid particles contained in it:**
  - **Liquid viscosity**
  - **Liquid density**
  - **Solids concentration**
  - **Particle size distribution**
  - **Surface charge of particles**
  - **Type and/or shape or particles**

Process Variables Affecting Centrifugation (continued)

- **Feed rate of slurry**
- **Agitation speed**
- **Size of centrifuge (distance from rotation axis)**
- **Height of cake**
- **Allowable pressure drop across cake**
- **Mode of operation (batch vs. continuous)**
- **Time at full speed**
- **Depth of skimming**

Possible Cases to Be Examined

- Batch centrifugation (e.g., with imperforated basket centrifuges)
- Continuous centrifugation (e.g., with solid bowl centrifuges)*
- Batch centrifugal filtration (e.g., with perforated basket centrifuges)*
- (Semi-)Continuous centrifugal filtration (e.g., with perforated basket centrifuges)

(*) Cases that will be examined here

Centrifugal Force

If a bowl or basket is rotated the content of the basket is subjected to a radial *centrifugal* force given by:

$$F_e = m\,a_e = m\,r\,w^2$$

where: F_e **= centrifugal force**

m **= mass of object subject to centrifugal force**

a_e **= centrifugal acceleration**

r **= distance of the object from the axis of rotation**

w **= rotational velocity (in radians/unit time)**

Alternative Expression for the Centrifugal Force

The relationship between the rotational speed, *N* expressed in rpm, and the angular velocity, w, is:

$$w = \frac{2\,p}{60} N$$

If the rotational speed is expressed in rpm then the centrifugal force is:

$$F_e = m a_e = m r \left(\frac{2\,p}{60} N \right)^2$$

<u>Centrifugal Acceleration</u>

The acceleration due to the centrifugal force is given by:

$$a_e = r\,\omega^2$$

or

$$a_e = r\left(\frac{2\pi}{60}N\right)^2$$

Relationship Between Centrifugal and Gravity Accelerations: The *g*-Factor

The *g* factor is defined as the ratio of the centrifugal force to the gravity force:

$$g-\text{factor} = \frac{a_e}{g} = \frac{F_e}{F_g} = \frac{r w^2}{g} = \frac{r}{g}\left(\frac{2p}{60}N\right)^2$$

$$g-\text{factor} = \frac{a_e}{g} = \frac{F_e}{F_g} = 0.001118\, r\, N^2$$ **(SI units)**

$$g-\text{factor} = \frac{a_e}{g} = \frac{F_e}{F_g} = 0.000341 r\, N^2$$ **(English units)**

Settling Velocity of Particles Subject to Centrifugal Force

The terminal velocity, v_p, of a spherical particle of diameter D_p and density r_s located in a rotating fluid at a distance r from the rotation axis and moving in laminar flow is:

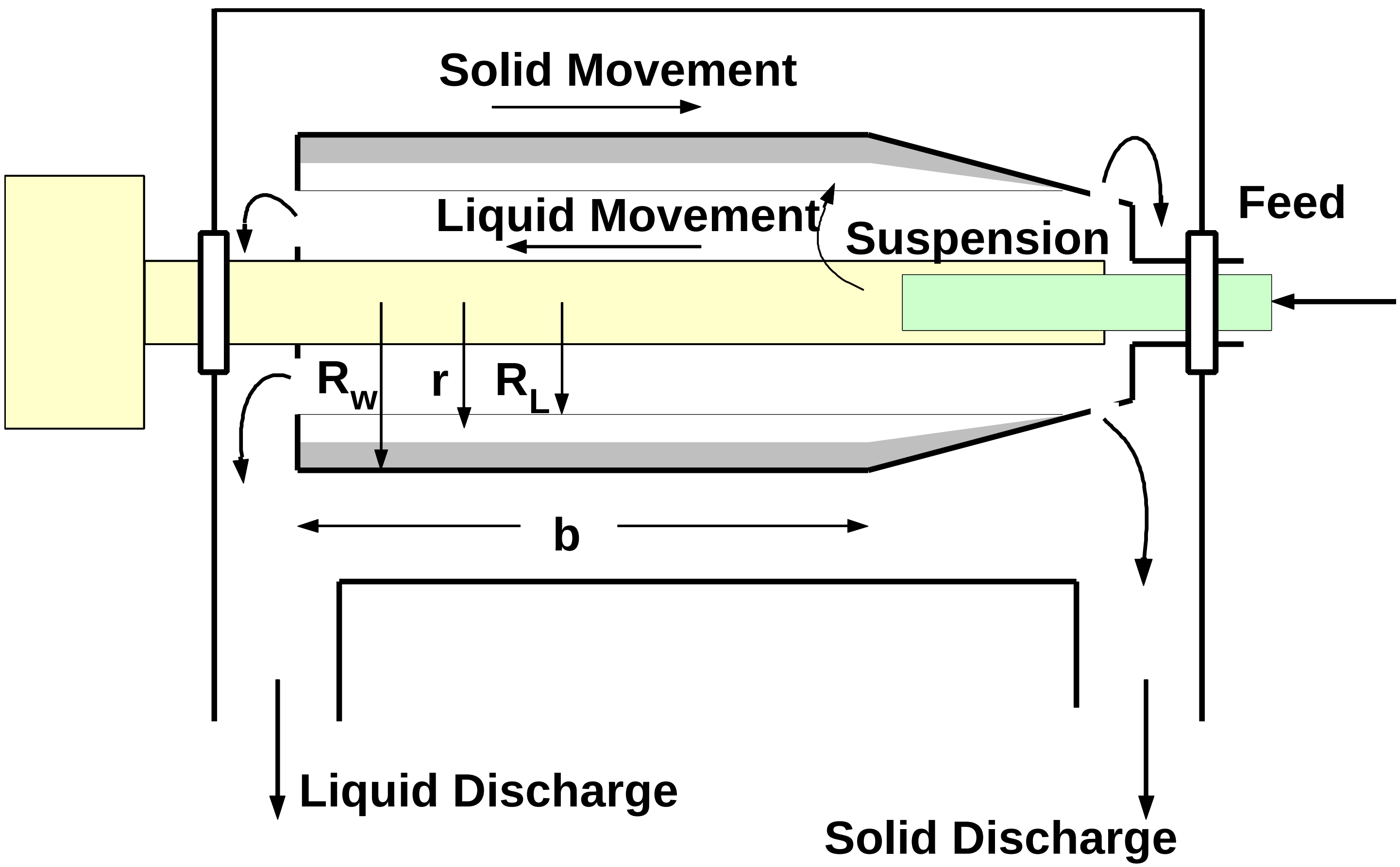
$$v_p = \frac{w^2 r D_p^2 \left(r_s - r_L \right)}{18 m}$$

where:

m **= fluid viscosity**

r_L **= fluid density**

Analysis of Continuous Centrifugation in a Solid Bowl Centrifuge



Nomenclature for Solid Bowl Centrifuges

b **length of the cylindrical part of the centrifuge**

r **generic distance from the axis of rotation**

R_C **radial distance from the axis of rotation to the top layer of the cake**

R_L **radial distance from the axis of rotation to the liquid pool**

R_W **radial distance from the axis of rotation to the centrifuge wall**

Assumptions Made to Model Continuous Centrifugation in a Solid Bowl Centrifuge

- **The feed (suspension) enters at the end of the cylindrical wall of the centrifuge bowl near the tapered end and travels across the centrifuge**
- **If a particles reaches the bowl wall is considered settled**
- **The settled solid particles and the cleared liquid move countercurrent with respect to each other**
- **The settled particles and the cleared liquid are continuously removed at opposite ends of the centrifuge**

Assumptions Made to Model Continuous Centrifugation in a Solid Bowl Centrifuge

- **The particles are assumed to settle independently of each other (Type 1 settling) following Stokes law**
- **Each liquid element remains in the centrifuge for a time equal to the residence time. Hence, the residence time is the time available for the particles to settle**
- **All the particles that have not reached the centrifuge wall within the residence time will exit with the clarified liquid**

Settling Velocity of a Particle Inside a Continuous Solid Bowl Centrifuge

A particle settling inside the bowl centrifuge will have a settling (radial) velocity given by:

$$v_p(r) = \frac{dr}{dt} = \frac{w^2\, r\, D_p^2 \left(r_s - r_L\right)}{18\, m}$$

where $R_L < r < R_w$.

Note that the particle velocity at any time in not constant since as the particle settles its radial position, *r*, changes, and so does v_p.

Relationship Between Time and Distance Traveled by a Particle Inside a Continuous Solid Bowl Centrifuge

The radial distance, *dr*, traveled by a particle during an infinitesimal time interval, *dt*, is:

$$dr = v_p(r)\,dt = \frac{w^2\, r\, D_p^2 \left(r_s - r_L \right)}{18\, m}\, dt$$

This expression can be rearranged and integrated to give:

$$\int_{r_1}^{r_2} \frac{18\, m}{w^2\, r\, D_p^2 \left(r_s - r_L \right)}\, d\,r = \int_{t_1}^{t_2} dt$$

Relationship Between Time and Distance Traveled by a Particle Inside a Continuous Solid Bowl Centrifuge

Upon integration it is:

$$\frac{18\,m}{w^{2} D_p^{2}\left(r_s - r_L\right)} \ln\frac{r_2}{r_1} = t_2 - t_1$$

Residence Time of the Liquid Inside the Continuous Solid Bowl Centrifuge

The residence time, t_o, of the liquid in the bowl is the time an average element of fluid will spend in the centrifuge at steady state conditions:

$$t_o = \frac{V}{Q}$$

where: ***V*** **= volume of centrifuge available to liquid**

Q **= volumetric flow rate of liquid through centrifuge**

Liquid Volume in a Centrifuge

The liquid volume in the centrifuge is determined by the height of the weir over which the liquid is discharged:

$$V = p b \left(R_w^2 - R_L^2 \right)$$

Hence:

$$t_o = \frac{p b \left(R_w^2 - R_L^2 \right)}{Q}$$

Critical Particle Diameter for Settling in a Continuous Solid Bowl Centrifuge

In order for the particle to settle it will have to travel a distance (R_W - R_L), i.e., the distance between the surface of the liquid (where the suspension is added) and the bowl wall, and to do so in the time interval t_o.

Only particles having a diameter larger than a certain critical diameter will be able to cover this distance during the time interval t_o.

Critical Particle Diameter for Settling in a Continuous Solid Bowl Centrifuge

Upon substitution of:

t_1 **with 0**	r_1 **with** R_L
t_2 **with** t_o	r_2 **with** R_w

one gets:

$$\frac{18\,m}{w^2\,D_p^2\left(r_s - r_L\right)}\ln\frac{R_w}{R_L} = t_o = \frac{p\,b\left(R_w^2 - R_L^2\right)}{Q}$$

Critical Particle Diameter for Settling in a Continuous Solid Bowl Centrifuge

From the previous equation the critical particle diameter, D_{po}, can be found to be:

$$D_{po} = \sqrt{\frac{Q}{\pi b\left(R_w^2 - R_L^2\right)} \frac{18\,\mu}{\omega^2\left(\rho_s - \rho_L\right)} \ln\frac{R_w}{R_L}}$$

Particles larger than D_{po} will settle during t_o, particles smaller than D_{po} will not.

Generalized Design Equation for Continuous Solid Bowl Centrifuges

The following equation can be derived from the previous equations, and used as the main design equation for solid bowl centrifuges:

$$Q = \frac{w^2 D_p^2 \left(r_s - r_L \right)}{18\, m} \left[\frac{p\, b \left(R_w^2 - R_L^2 \right)}{\ln \left(\mathrm{R_w} / R_L \right)} \right]$$

Simplified Design Equation for Continuous Solid Bowl Centrifuges

The previous equation can be simplified if one assumes that the distance traveled by the particle in order to settle (equal to the liquid radial height) is small in comparison to the bowl radius, R_w, i.e.:

$$R_w - R_L << R_L \approx R_w$$

Then:

$$\ln\left(\frac{R_w}{R_L}\right) = \ln\left(\frac{R_w - R_L}{R_L} + \frac{R_L}{R_L}\right) =$$

$$= \ln\left(1 + \frac{R_w - R_L}{R_L}\right) \cong \frac{R_w - R_L}{R_L} \cong \frac{R_w - R_L}{R_w}$$

Simplified Design Equation for Continuous Solid Bowl Centrifuges

Assuming that:

$$\ln\left(\frac{R_w}{R_L}\right) \cong \frac{R_w - R_L}{R_w} \quad \textbf{and} \quad R_w + R_L \cong 2R_w$$

the equation:

$$Q = \frac{w^2 D_p^2 \left(r_s - r_L\right)}{18\,m}\left[\frac{p\,b\left(R_w^2 - R_L^2\right)}{\ln\left(R_w / R_L\right)}\right]$$

becomes:

$$Q \cong \frac{w^2 D_p^2 \left(r_s - r_L\right)}{18\,m}\left[p\,b\,R_w \left(R_w + R_L\right)\right] \cong \frac{p\,b\,R_w^2\,w^2 D_p^2 \left(r_s - r_L\right)}{9\,m}$$

Centrifuge Scale-up: the Sigma Value

The design equation:

$$Q = \frac{w^2 D_p^2 \left(r_s - r_L \right)}{18\, m} \left[\frac{p\, b \left(R_w^2 - R_L^2 \right)}{\ln\left(R_w / R_L \right)} \right]$$

can be rewritten as:

$$Q = 2\, \frac{D_p^2\, g \left(r_s - r_L \right)}{18\, m} \left[\frac{w^2}{2g}\; \frac{p\, b \left(R_w^2 - R_L^2 \right)}{\ln\left(R_w / R_L \right)} \right] = v_p\, \Sigma$$

where the parameter S (Sigma value), defined as:

$$\Sigma = \left[\frac{w^2}{2g}\; \frac{p\, b \left(R_w^2 - R_L^2 \right)}{\ln\left(R_w / R_L \right)} \right]$$

is independent of the particle type and size.

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Centrifuge Scale-up: the Sigma Value

For two geometrically similar centrifuges of different sizes (1 and 2) separating the same solid particles from the same fluid it will be:

$$Q_1 = v_p \, \Sigma_1 \qquad \text{and} \qquad Q_2 = v_p \, \Sigma_2$$

To scale up experimental results from a laboratory centrifuge one can use the following scale-up rule:

$$\boxed{\frac{Q_1}{Q_2} = \frac{v_p \, \Sigma_1}{v_p \, \Sigma_2} = \frac{\Sigma_1}{\Sigma_2}}$$

Simplified Expression for the Sigma Value: Simplified Scale-up

Assuming that:

$$\ln\left(\frac{R_w}{R_L}\right) \cong \frac{R_w - R_L}{R_w} \quad \textbf{and} \quad R_w + R_L \cong 2R_w$$

then the Sigma value can be rewritten as:

$$\Sigma \cong \frac{\pi w^2 b R_w^2}{g}$$

i.e.:

$$\frac{Q_1}{Q_2} = \frac{\Sigma_1}{\Sigma_2} \cong \frac{w_1^2 b_1 R_{w1}^2}{w_2^2 b_2 R_{w2}^2}$$

Solid Movement Inside a Continuous Solid Bowl Centrifuge

- **In a continuous solid bowl centrifuge both clarification of the suspension and removal of solids occur simultaneously.**
- **If the settled solids are not transported out at a sufficient rate the pool inside the bowl fills up with solids and no separation occurs.**
- **The settled solids are transported out of a continuous solid bowl centrifuge by the action of a helical screw (scroll) rotating at a slightly different velocity than the bowl.**

Solids Removal in Continuous Solid Bowl Centrifuges

The mass flow rate of solids (moving in the axial direction) removed with the cake at one end of the centrifuge is:

$$W_s = 2\,p\,R_w\left(R_w - R_L\right)\,g_s\,r_s\,Z_s\,\Delta N$$

W_s **= mass flow rate of solid removal**

g_s **= volumetric fraction of solids in the liquid pool**

r_s **= density of solids in the cake**

Z_s **= axial distance between flights of screw (pitch)**

DN **= difference between rotational speed of bowl and that of scroll, in rotations per unit time**

Scale-up of Continuous Solid Bowl Centrifuges Based on Solid Throughput: the Beta Value

If two centrifuges (labeled 1 and 2) are compared in terms of solids removal the following relationship between the mass flow rate of solids can be obtained:

$$\frac{W_{s2}}{W_{s1}} = \frac{2\,p\,R_{w2}\left(R_{w2} - R_{L2}\right)\,g_{s2}\,r_{s2}\,Z_{s2}\,\Delta N_2}{2\,p\,R_{w1}\left(R_{w1} - R_{L1}\right)\,g_{s1}\,r_{s1}\,Z_{s1}\,\Delta N_1}$$

Scale-up of Continuous Solid Bowl Centrifuges Based on Solid Throughput: the Beta Value

Assuming that the fraction of solids is the same in both centrifuges (i.e., $r_{s1} = r_{s2}$) and that the solids fraction is also equal (i.e., $g_{s1} = g_{s2}$), then:

$$\frac{W_{s2}}{W_{s1}} = \frac{R_{w2}\left(R_{w2} - R_{L2}\right) Z_{s2}\,\Delta N_2}{R_{w1}\left(R_{w1} - R_{L1}\right) Z_{s1}\,\Delta N_1} = \frac{b_{c2}}{b_{c1}}$$

This equation defines a second scale-up method based on the *beta value*, i.e., the ratio of solids throughputs for two centrifuges of two different sizes. This method is important to determine the solids removal in full-scale centrifuges based on the results of pilot tests.

Effect of Bowl Diameter Size on Solids Separation

- **Larger bowl diameters (while maintaining the same centrifugal force by slowing down the machine rotation) result in longer retention time within the centrifuge.**
- **This results in higher solids recovery and a wetter cake (i.e., having a lower concentration of solids).**

Effect of Pool Depth on Solids Separation

- **The pool depth is the height of the liquid annulus in the centrifuge.**
- **The pool depth can be adjusted in most centrifuges, even when the centrifuge is in operation.**
- **An increase in the pool depth will result in higher solids recovery (because of the increased residence time), and a wetter cake**

Effect of Feeding Point on Solids Separation

- **The incoming sludge can be fed (at least in some centrifuges) at different points along the axis of the centrifuge.**
- **When the feeding point is closer to the beach a wetter cake will be produced, but the solids content in the centrate will be lower (higher solids recovery) because of the longer time available for solids separation.**

Performance of Solid Bowl Centrifuges

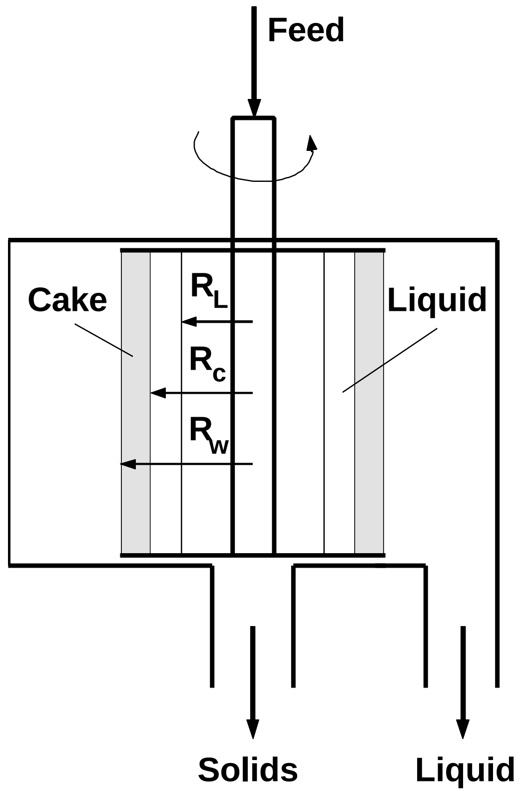
Type of Sludge	Concentration of Solids in Cake (%)	Solid Recovery [No Chemical Addition] (%)	Solid Recovery [Chemical Addition] (%)
Primary	20-40	70-90	75-98
Primary & Activated	15-30	50-80	75-98
Activated	5-15	65-90	85-95
Digested Activated	15-25	70-90	85-95
Lime Softening	35-60	70-90	85-95

After Sundstrom and Klei, *Wastewater Treatment*, 1979, p. 238

Characteristics of Solid Bowl Centrifuges

- **Feed rate range: 1.5-12 L/s (25-200 gal/min).**
- **Specific flow rate range: 3.5-15 m^3/(d KW) (0.5-2 gal/(min hp)).**
- **Rotational speed: 1000-6000 rpm.**
- ***g* factor (ratio of centrifugal force to gravity force): 2000-3000.**
- **Centrifuges with larger pool volumes are more effective at dewatering sludges.**
- **Pool depth (radial height of liquid) can be adjusted in most centrifuges.**

Analysis of Semi-Batch Centrifugal Filtration in a Perforated Centrifuge



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<u>Analysis of Semi-Batch Centrifugal Filtration in a Perforated Centrifuge</u>

Assuming that the cake is <u>not</u> compressible (i.e., a is independent of D*P*) the pressure drop across the cake at any given time is given by:

$$\Delta P(t) = \frac{m\left(a\ X_s V_F(t) + A R_m\right)}{A^2} Q_F(t)$$

where $X_s V_F(t)$ is the amount of solids that have been deposited up to time *t*, and that have formed the cake.

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Analysis of Semi-Batch Centrifugal Filtration in a Perforated Centrifuge

The pressure drop across the cake has to be overcome by the hydraulic head of liquid above the cake. Similarly to the differential pressure head in a gravitational field given by:

$$dP = r\, g\, dz$$

the differential pressure head in a centrifugal field is

$$dP = r\, r\, w^{2}\, dr$$

Analysis of Semi-Batch Centrifugal Filtration in a Perforated Centrifuge

The pressure head available to overcome the pressure drop can be obtained from integration between the radial liquid height, *R_L* and the distance of the vessel wall from the rotation axis:

$$\int_0^{\Delta P} dP = \int_{R_L}^{R_w} r_L \, r \, w^2 \, dr$$

i.e.:

$$\Delta P = r_L \, w^2 \, \frac{\left(R_w^2 - R_L^2\right)}{2}$$

Analysis of Semi-Batch Centrifugal Filtration in a Perforated Centrifuge

Combining the expressions for the pressure drop and the available pressure head one gets:

$$\Delta P(t) = r_L w^2 \frac{\left(R_w^2 - R_L^2\right)}{2} = \frac{m\left(a X_s V_F(t) + A R_m\right)}{A^2} Q_F(t)$$

i.e.:

$$Q_F(t) = \frac{r_L w^2 A^2 \left(R_w^2 - R_L^2\right)}{2 m\left(a X_s V_F(t) + A R_m\right)}$$

Analysis of Semi-Batch Centrifugal Filtration in a Perforated Centrifuge

Note that the area *A* in:

$$Q_F(t) = \frac{\rho_L \omega^2 A^2 \left(R_w^2 - R_L^2\right)}{2\mu\left(\alpha X_s V_F(t) + A R_m\right)}$$

is a function of the radii R_w, R_L and R_c.

In addition, this equation is valid only for a given mass of solids in the cake at a given time. This equation is not in the integral form, i.e., it cannot be used to describe the entire centrifugal filtration process.

Analysis of Semi-Batch Centrifugal Filtration in a Perforated Centrifuge

If the filtration area *A* varies significantly with the radius then it can be shown that the flow rate *Q* can be expressed as:

$$Q_F(t) = \frac{r_L w^2 \left(R_w^2 - R_L^2\right)}{2m\left(\dfrac{a\, X_s V_F(t)}{A_L A_a} + \dfrac{R_m}{A_w}\right)}$$

where: $A_L = 2\,p\,b\left(R_w - R_C\right) \big/ \left[\mathrm{In}\left(R_w / R_C\right)\right]$

$$A_a = p\,b\left(R_w + R_C\right)$$

$$A_w = 2\,p\,b\,R_w$$

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Additional Information and Examples on Centrifugation

Additional information and examples on can be found in the following references:

- **Corbitt, R. A. 1990, *The Standard Handbook of Environmental Engineering*, McGraw-Hill, New York, pp. 5.138-5.139; 6.210-6.211; 6.223-6.229; 9.32-9.34.**
- **Droste, R. L., *Theory and Practice of Water and Wastewater Treatment*, John Wiley & Sons, New York, 1997, pp. 732-735.**
- **Geankoplis, C. J., *Transport Processes and Unit Operations*, 3rd Edition, 1993, Allyn and Bacon, Boston, pp. 828-838.**
- **Haas, C. N. and Vamos, R. J., 1995, *Hazardous and Industrial Waste Treatment*, Prentice Hall, Englewood Cliffs, NJ, pp. 67-69.**

Additional Information and Examples on Centrifugation

- Metcalf & Eddy, 1991, ***Wastewater Engineering: Treatment, Disposal, and Reuse***, **McGraw-Hill, New York, pp. 805-806; 860-864.**

- **Sundstrom, D. W. and Klei, H. E., 1979, *Wastewater Treatment*, Prentice Hall, Englewood Cliffs, NJ, pp. 234-238.**

- **Vesilind, P. A., 1979, *Treatment and Disposal of Wastewater Sludges,* Ann Arbor Science, Ann Arbor, MI, pp. 161-200.**

- **Wentz, C. W., 1995, *Hazardous Waste Management*, Second Edition, McGraw-Hill, New York. pp. 193-194.**