AIAA 2003–0210

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41st Aerospace Sciences Meeting and Exhibit
January 6–9, 2003
Reno, Nevada

AERODYNAMIC SHAPE OPTIMIZATION OF WINGS INCLUDING PLANFORM VARIATIONS

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This paper describes the formulation of optimization techniques based on control theory for aerodynamic shape design in inviscid compressible flow modelled by the Euler equations. The design methodology has been extended to include wing planform optimization. A model for the structure weight has been included in the design cost function to provide a meaningful design. A practical method to combine the structural weight into the design cost function has been studied. Results of optimizing a wing-fuselage of a commercial transport aircraft show a sucessful trade of planform design, leading to meaingful designs. The results also support the necessity of including the structure weight in the cost function.

INTRODUCTION

WHILE aerodynamic prediction methods based on CFD are now well established, and quite accurate and robust, the ultimate need in the design process is to find the optimum shape which maximizes the aerodynamic performance. One way to approach this objective is to view it as a control problem, in which the wing is treated as a device which controls the flow to produce lift with minimum drag, while meeting other requirements such as low structure weight, sufficient fuel volume, and stability and control constrains. Here we apply the theory of optimal control of systems governed by partial differential equations with boundary control, in this case through changing the shape of the boundary. Using this theory, we can find the Frechet derivative (infinitely dimensional gradient) of the cost function with respect to the shape by solving an adjoint problem, and then we can make an improvement by making a modification in a descent direction. For example, the cost function might be the drag coefficient at a fixed lift, or the lift to drag ratio. During the last decade, this method has been intensively developed, and has proved to be very effective for improving wing section shapes for fixed wing planform.[1,2,7,8,11–14]

In this work we report on recent improvements in the adjoint method, and also consider its extension to planform design. It is well known that the induced drag varies inversely with the square of the span. Hence the induced drag can be reduced by increasing the span. Moreover, shock drag in transonic flow might be reduced by increasing sweep back or increasing the chord to reduce the thickness to cord ratio.

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Consequencely an optimization which considers only the pressure drag would lead to a wing with excessive span and sweep back. In order to produce a meaningful optimization problem, we therefore include a simple structure weight model based on the span, sweep back, and taper.

MATHEMATICAL FORMULATION

In this work the equations of steady flow

$$\frac{\partial}{\partial x_i} f_i(w) = 0$$

where w is the solution vector, and $f_i(w)$ are the flux vectors along the x_i axis are applied in a fixed computational domain, with coordinates ξ_i, so that

$$R(w, S) = \frac{\partial}{\partial \xi_i} F_i(w) = \frac{\partial}{\partial \xi_i} S_{ij} f_j(w) = 0$$

where S_{ij} are the coefficients of the Jacobian matrix of the transformation. Then geometry changes are represented by changes δS_{ij} in the metric coefficients. Suppose one wishes to minimize cost function of a boundary integral

$$I = \int_{\mathcal{B}} \mathcal{M}(w, S)\, d\mathcal{B}_\xi + \int_{\mathcal{B}} \mathcal{N}(S)\, d\mathcal{B}_\xi$$

where the integral of $\mathcal{M}(w, S)$ could be an aerodynamic cost function, e.g. drag coefficient, and the integral of $\mathcal{N}(S)$ could be a structural cost function, e.g. wing weight. Then one can augment the cost function through Lagrange multiplier ψ as

$$I = \int_{\mathcal{B}} \mathcal{M}(w, S)\, d\mathcal{B}_\xi + \int_{\mathcal{D}} \psi^T \mathcal{R}(w, S)\, d\mathcal{D}_\xi + \int_{\mathcal{B}} \mathcal{N}(S)\, d\mathcal{B}_\xi$$

A shape variation δS causes a variation

$$\delta I = \int_{\mathcal{B}} \delta\mathcal{M}\, d\mathcal{B}_\xi + \int_{\mathcal{D}} \psi^T \frac{\partial}{\partial \xi_i} \delta F_i\, d\mathcal{D}_\xi + \int_{\mathcal{B}} \delta\mathcal{N}\, d\mathcal{B}_\xi$$

The second term can be integrated by parts to give

$$\int_{\mathcal{B}} n_i \psi^T \delta F_i d\mathcal{B}_\xi - \int_{\mathcal{D}} \frac{\partial \psi^T}{\partial \xi_i} \delta F_i \, d\mathcal{D}_\xi.$$

Now, choosing ψ to satisfy the adjoint equation

$$(S_{ij} \frac{\partial f_j}{\partial w})^T \frac{\partial \psi}{\partial \xi_i} = 0$$

with appropriate boundary conditions depending on the cost function, the explicit dependence on δw is eliminated, allowing the cost variations to be expressed in terms of δS and the adjoint solution, and hence finally in terms of the change $\delta \mathcal{F}$ in a function $\mathcal{F}(\xi)$ defining the shape.

Thus one obtains

$$\delta I = \int \mathcal{G} \delta \mathcal{F} \, d\xi = \langle \mathcal{G}, \delta \mathcal{F} \rangle$$

where $\mathcal{G}$ is the infinite dimensional gradient (Frechet derivative) at the cost of one flow and one adjoint solution. Then one can make an improvement by setting

$$\delta \mathcal{F} = -\lambda \mathcal{G}$$

In fact the gradient $\mathcal{G}$ is generally of a lower smoothness class than the shape $\mathcal{F}$. Hence it is important to restore the smoothness. This may be effected by passing to a weighted Sobolev inner product of the form

$$\langle u, v \rangle = \int (uv + \epsilon \frac{\partial u}{\partial \xi} \frac{\partial v}{\partial \xi}) \, d\xi$$

This is equivalent to replacing $\mathcal{G}$ by $\bar{\mathcal{G}}$, where in one dimension

$$\bar{\mathcal{G}} - \frac{\partial}{\partial \xi} \bar{\mathcal{G}} \frac{\partial \bar{\mathcal{G}}}{\partial \xi} = \mathcal{G}, \quad \bar{\mathcal{G}} = \text{zero at end points}$$

and making a shape change $\delta \mathcal{F} = -\lambda \bar{\mathcal{G}}$.

IMPLEMENTATION

COST FUNCTION FOR PLANFORM DESIGN

In order to design a high performance transonic wing, which will lead to a desired pressure distribution, and still maintain a realistic shape, the natural choice is to set

$$I = \alpha_1 C_D + \alpha_2 \frac{1}{2} \int_{\mathcal{B}} (p - p_d)^2 dS + \alpha_3 C_W \tag{1}$$

with

$$C_W = \frac{\mathcal{W}_{wing}}{q_\infty S_{ref}} \tag{2}$$

where

C_D	=	drag coefficient,
C_W	=	normalized wing structure weight,
p	=	current surface pressure,
p_d	=	desired pressure,
q_∞	=	dynamic pressure,
S_{ref}	=	reference area,
$\mathcal{W}_{wing}$	=	wing structure weight, and
α_1, α_2, α_3	=	weighting constants.

A practical way to estimate $\mathcal{W}_{wing}$ is to use the so-called Statistical Group Weights Method, which applies statistical equations based on sophisticated regression analysis. For a cargo/transport wing weight, one can use[3]

$$\mathcal{W}_{weight} = 0.0051 (W_{dg} N_z)^{0.557} S_w^{0.649} A^{0.5} (t/c)_{root}^{-0.4} (1 + \lambda)^{0.1} cos(\Lambda)^{-1.0} S_{csw}^{0.1} \tag{3}$$

where

A	=	aspect ratio,
N_z	=	ultimate load factor; = 1.5 x limit load factor,
S_{csw}	=	control surface area (wing-mounted),
S_w	=	trapezoidal wing area,
t/c	=	thickness to chord ratio,
W_{dg}	=	flight design gross weight,
Λ	=	wing sweep, and
λ	=	taper ratio at 25 % MAC.

In addition, if the wing of interest is modeled by five planform variables such as root chord (c_1), mid-span chord (c_2), tip chord (c_3), span (b), and sweepback(Λ), as shown in Figure 1, it can be seen from the wing weight formula (3) that the weight is estimated to vary inversely with $cos(\Lambda)$, where Λ is the wing sweep.

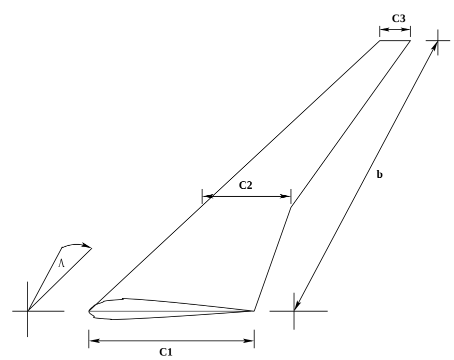

Fig. 1 Modeled wing governed by five planform variables; root chord (c_1), mid-span chord (c_2), tip chord (c_3), span (b), and sweepback(Λ).

Here if the sweepback is allowed to vary and α_3 is chosen to be sufficiently large in the cost function (1), we should expect the optimization to decrease the sweep back angle at the cost of an increase in shock drag.

A change of span effects the wing weight of the wing weight formula (3) through changes of the trapezoidal wing area (S_w), the aspect ratio (A),and the wing-mounted area (S_{csw}). From the wing weight formula (3), an increase of span will cause S_w, A, and S_{csw} to increase, resulting in an increase of wing weight. Since induced drag varies inversely with the square of span, if the span is allowed to vary and α_3 is chosen to be sufficiently large, it is expected the optimizer to reduce the span at the cost of an increase in drag.

Variations of c_1, c_2, and c_3 effect the wing weight of the wing weight formula (3) via variations of aspect ratio (A), wing-mounted wing area (S_{csw}), trapezoidal wing area (S_w), and thickness to chord ratio (t/c). The effect of an individual chord change on the wing weight is plotted in Figure 2. Figure 2 shows that the wing weight of the Statistical Group Weights Method is a linear function of chord length. And because the slope varies along the span location, the change of chord at different span location will effect the wing weight differently. With the same change in chord length, the decrease of the mid-span chord (c_2) tends to give more weight reduction than the others.

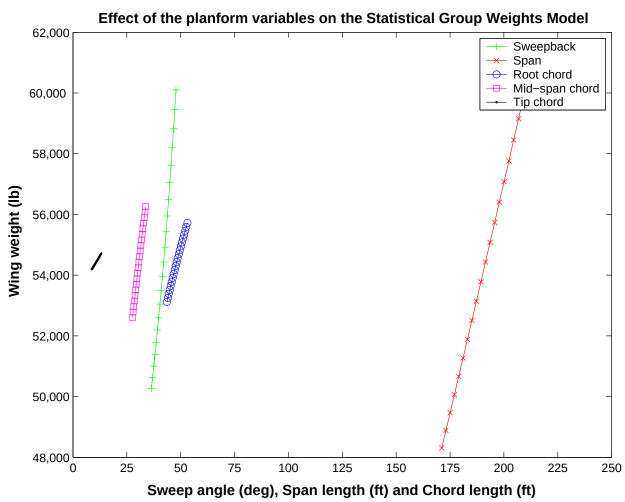

Fig. 2 Effect of sweepback(Λ), span (b), root chord(c_1), mid-span chord(c_2), and tip chord(c_3) on the Statistical Group Weights Method

If either c_1, c_2, or c_3 is allowed to vary, and again α_3 is chosen to be sufficiently large in the cost function (1), the optimizer would be expected to reduce the chord length with a consequent increase in shock drag.

In these ways the inclusion of a weight estimate in the cost function should prevent the optimization from leading to an unrealistic wing planform, and lead to a good overall performance.

AERODYNAMIC GRADIENT CALCULATION FOR PLANFORM VARIABLES

Gradient information can be computed using a variety of approaches such as the finite-difference method, the complex step method,[10] and the automatic differentiation.[4] Unfortunately, their computational cost is still proportional to the number of design variables in the problem. In an optimum transonic wing design, suppose one chooses mesh points on a wing surface as the design variables, which is an order of 1000 or more, it is impractical to calculate the gradient using the methods mentioned earlier. In our planform optimization, the design variables are points on the wing surface plus the planform variables. To evaluate the aerodynamic gradient with respect to the planform variables, since the number of planform variables (five in this study) is far less than that of the surface optimization, one could calculate the gradient by the finite-difference method, the complex step method or the automatic differentiation. However, the cost for the gradient calculation will be five times higher. A more efficient approach is to follow the adjoint formulation.

Consider the aerodynamic contribution of the cost function (1)

$$\delta I = \int\limits_{\mathcal{B}} \delta\mathcal{M}\, d\mathcal{B}_\xi + \int\limits_{\mathcal{D}} \psi^T \delta R\, d\mathcal{D}_\xi$$

This can be split as

$$\delta I = \left[I_w\right]_I \delta w + \delta I_{II}$$

with

$$\delta\mathcal{M} = [\mathcal{M}_w]_I \delta w + \delta\mathcal{M}_{II}$$

where the subscripts I and II are used to distinguish between the contributions associated with variation of the flow solution δw and those associated with the metric variations δS. Thus $[\mathcal{M}_w]_I$ represents $\frac{\partial\mathcal{M}}{\partial w}$ with the metrics fixed. Note that δR is intentionally kept unsplit for programming purposes. If one chooses ψ as ψ^*, where ψ^* satisfies

$$(S_{ij}\frac{\partial f_j}{\partial w})^T \frac{\partial\psi^*}{\partial\xi_i} = 0,$$

then

$$\begin{aligned}
\delta I(w,S) &= \delta I(S) \\
&= \int\limits_{\mathcal{B}} \delta\mathcal{M}_{II}\, d\mathcal{B}_\xi + \int\limits_{\mathcal{D}} {\psi^*}^T \delta R\, d\mathcal{D}_\xi \\
&\approx \sum_{\mathcal{B}} \delta\mathcal{M}_{II}\Delta\mathcal{B} + \sum_{\mathcal{D}} {\psi^*}^T \Delta\bar{R} \\
&\approx \sum_{\mathcal{B}} \delta\mathcal{M}_{II}\Delta\mathcal{B} + \sum_{\mathcal{D}} {\psi^*}^T \left(\bar{R}|_{S+\delta S} - \bar{R}|_S\right),
\end{aligned}$$

where $\bar{R}|_S$ and $\bar{R}|_{S+\delta S}$ are volume weighted residuals calculated at the original mesh and at the mesh perturbed in the design direction.

Provided that ψ^* has already been calculated and $\bar{R}$ can be easily calculated, the gradient of the planform variables can be computed effectively by first perturbing all the mesh points along the direction of interest. For example, to calculate the gradient with respect to the sweepback, move all the points on the wing surface as if the wing is pushed backward and also move all other associated points in the computational domain to match the new location of points on the wing. Then re-calculate the residual value and subtract the previous residual value from the new value to form $\Delta\bar{R}$. Finally, to calculate the planform gradient, multiply $\Delta\bar{R}$ by the costate vector and add the contribution from the boundary terms.

This way of calculating the planform gradient exploits full benefit of knowing the value of adjoint variables ψ^* with no extra cost of flow or adjoint calculations.

CHOICE OF WEIGHTING CONSTANTS

The choice of α_1 and α_3 greatly effects the optimum shape. An intuitive choice of α_1 and α_3 can be chosen by considering the problem of maximizing range of an aircraft. The simplified range equation can be expressed as

$$R = \frac{V}{C}\frac{L}{D} log \frac{W_1}{W_2}$$

where

C = Specific Fuel Consumption,
D = Drag,
L = Lift,
R = Range,
V = Aircraft velocity,
W_1 = Take off weight, and
W_2 = Landing weight.

If one takes

$$\begin{aligned} W_1 &= W_e + W_f = \text{fixed} \\ W_2 &= W_e \end{aligned}$$

where

W_e = Gross weight of the airplane without fuel,
W_f = Fuel weight,

then the variation of the weight can be expressed as

$$\delta W_2 = \delta W_e.$$

With fixed $\frac{V}{C}$, W_1, and L, the variation of R can be stated as

$$\begin{aligned} \delta R &= \frac{V}{C}\left(\delta\left(\frac{L}{D}\right) log \frac{W_1}{W_2} + \frac{L}{D}\delta\left(log \frac{W_1}{W_2}\right)\right) \\ &= \frac{V}{C}\left(-\frac{\delta D}{D}\frac{L}{D} log \frac{W_1}{W_2} - \frac{L}{D}\frac{\delta W_2}{W_2}\right) \\ &= -\frac{V}{C}\frac{L}{D} log \frac{W_1}{W_2}\left(\frac{\delta D}{D} + \frac{1}{log \frac{W_1}{W_2}}\frac{\delta W_2}{W_2}\right) \end{aligned}$$

and

$$\begin{aligned} \frac{\delta R}{R} &= -\left(\frac{\delta C_D}{C_D} + \frac{1}{log \frac{W_1}{W_2}}\frac{\delta W_2}{W_2}\right) \\ &= -\left(\frac{\delta C_D}{C_D} + \frac{1}{log \frac{C_{W_1}}{C_{W_2}}}\frac{\delta C_{W_2}}{C_{W_2}}\right). \end{aligned}$$

If we minimize the cost function defined as

$$I = C_D + \alpha C_W,$$

where α is the weighting multiplication, then choosing

$$\alpha = \frac{C_D}{C_{W_2} log \frac{C_{W_1}}{C_{W_2}}}, \qquad (4)$$

corresponds to maximizing the range of the aircraft.

DESIGN CYCLE

The design cycle starts by first solving the flow field until at least 4 orders of magnitude drop in the residual. The flow solution is then passed to the adjoint solver. Second, the adjoint solver is run to calculate the costate vector. Iteration continues until at least 4 orders of magnitude drop in the residual. The costate vector is passed to the gradient module to evaluate the aerodynamic gradient. Then, the structural gradient is calculated and added to the aerodynamic gradient to form the overall gradient. The steepest descent method is used with a small step size to guarantee that the solution will converge to the optimum point. The design cycle is shown in Figure 3.

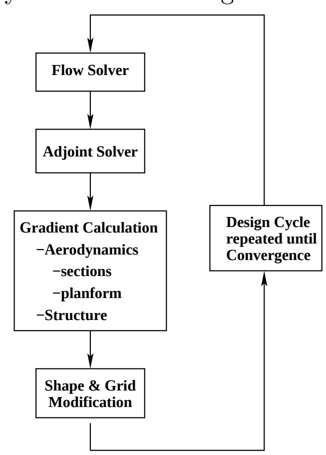

Fig. 3 Design cycle

FLOW SOLVER AND ADJOINT SOLVER

The flow solver and the adjoint solver chosen in this work are codes developed by Jameson.[5–7,11] The flow solver solves the three dimensional Euler equations, by employing the JST scheme, together with a multistep time stepping scheme. Rapid convergence to a steady state is achieved via variable local time steps, residual averaging, and a full approximation multi-grid scheme. The adjoint solver solves the corresponding adjoint equations using similar techniques to those of the flow solver. In fact much of the software is shared by the flow solver and adjoint solver.

RESULTS

VALIDATION OF AERODYNAMIC GRADIENT WITH RESPECT TO PLANFORM VARIABLES

To verify the accuracy of the aerodynamic gradient with respect to planform variables calculated by employing the adjoint method, we compare the planform gradients using adjoint and finite-difference methods. For the purpose of comparison, calculations are done at a fixed angle of attack to eliminate the effect of pitch variation on the gradient. The case chosen is the Boeing 747-200 wing fuselage combination at Mach 0.87, and wing angle of attack 2.3 degrees. The computational mesh is shown in Figure 6.

The aerodynamic gradient with respect to the planform variables is calculated using both the adjoint and the finite-difference methods. A forward differencing technique is used for the finite-difference method with a moderate step size of 0.1% of planform variables to achieve both small discretization error and small cancellation error. The flow and adjoint solvers are run until both solutions are fully converged.

To verify the result more generally, different geometries are used in this comparison. Each new geometry is generated sequentially by allowing section changes of the current geometry. Figure 4 shows the planform gradient comparison. It can be seen that the results from both methods match each other very well, independent of the geometry. This result indicates that the adjoint method provides an accurate planform gradient while reducing the computational cost by a factor of five.

Figure 5 shows the effect of the step size used in the gradient calculation. The results indicate that the step size almost has no effect on the adjoint gradient, showing an additional advantage of the adjoint gradient over the finite difference gradient. It is not surprising that the finite-difference gradient is dependent on the step size, since it depends on recalculating the flow for each shape perturbation.[9]

REDESIGN OF THE BOEING 747 WING

We present a result to show that the optimization can successfully trade planform parameters. The case chosen is the Boeing 747-200 wing fuselage combination at Mach 0.87 and a lift coefficient $C_L = 0.42$. The computational mesh is shown in Figure 6.

In this test case, the Mach Number is moderlately higher than the current normal cruising Mach number of 0.85. We allowed section changes together with variations of sweepback, span, root chord, mid-span chord, and tip chord. Figure 7 shows a baseline calculation with the planform fixed. Here the drag was reduced from 101.3 counts to 88.8 counts (12.3% reduction) in 8 design iterations with relatively small changes in the section shape.

Figure 8 shows the effect of allowing changes in sweepback, span, root chord, mid-span chord, and tip chord. The parameter α_3 was chosen according to formula 4 such that the cost function corresponds to maximizing the range of the aircraft. Here in 8 design iterations the drag was reduced from 101.3 counts to 88.4 counts (12.7% reduction), while the normalize structure weight was slightly reduced from 0.03215 to 0.03211 (0.1% reduction). This test case shows a good trade off among the planform variables to achieve an optimal performance for a realistic design. The optimizer reduces the sweepback to get a reduciton in the structure weight, while increases the length of span and chords to achieve a reduction in drag. Though the reduction of sweepback causes drag to increase, the reduction of drag from the increase in span and chord length is superior, resulting an overall drag reduction. This is also true for the improvement of the overall structure weight; the weight reduction from decreasing of sweepback is more than the increase of weight due to the increase of span and chord length.

We present another test case to show the trade off between the aerodynamic cost function and the structure cost function. In this test case, the parameter α_3 was chosen to be large enough that the cost function is optimized by reducing the structure weight of the aircraft. Here in 30 design iterations the drag was reduced from 101.3 counts to 90.2 counts (11.0% reduction), while the normalized structure weight was reduced from 0.03215 to 0.03042, (5.4% reduction). As a result of the trade between wing weight reductions and increased drag, the overall drag reduction was less than in the previous figures. These results verify the feasibility of including planform effects in the optimization. Figure 10 shows the optimized shape of B747-200 obtained from the planform optimizor.

It should be noted that in the presence of viscous effects, an optimal design may be significantly different. The steep adverse pressure gradient in the outboard region of the wing in Figure 9 could cause a separation and bufflet. In a more realistic design, viscous effects certainly need to be considered.

CONCLUSION

An aerodynamic design methodology for planform optimization has been developed and validated. To realize meaningful designs, a model for the strucutral weight is included in the design cost function. The results of optimizing a wing-fuselage of a commercial transonic transport aircraft has highlighted the importance of the structural weight model. It is the trade between the structural cost function and the aerodynamic cost funtion that prevents an unrealistic result and leads to a useful design. Hence a good structure model can be critical in an aerodynamic planform optimization.

Currently, the structural model is a function of the planform variables but does not depend on the aerodynamic loading. A more realistic model to estimate the structure weight should be used for more practical designs. Also, in the current study, viscous effects have been neglected. It is easy to envision that viscous effects can greatly alter flow phenomena over the wing, leading to different optimum designs. Hence, to realize a practical optimization tool, the Navier-Stokes equations with an appropriate turbulence model should be used. In order to produce a design that works well over a range of operating conditions, the optimization procedure should also allow for multiple design points. The necessary extensions to the software are currently in process, and the results will be presented in a future conference.

ACKNOWLEDGMENT

This work has benefited greatly from the support of the Air Force Office of Science Research under grant No. AF F49620-98-1-2002.

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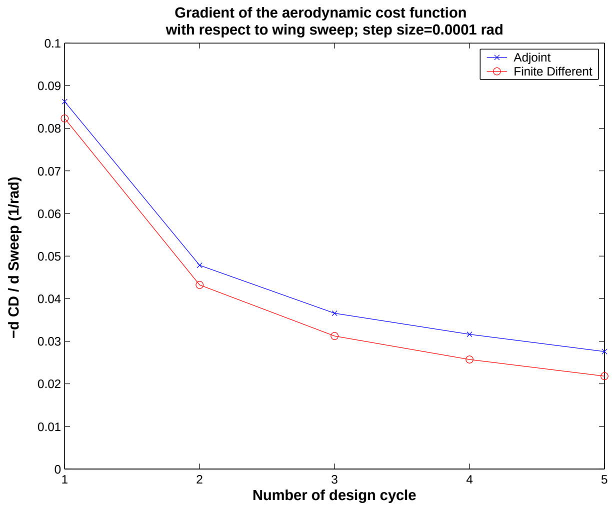
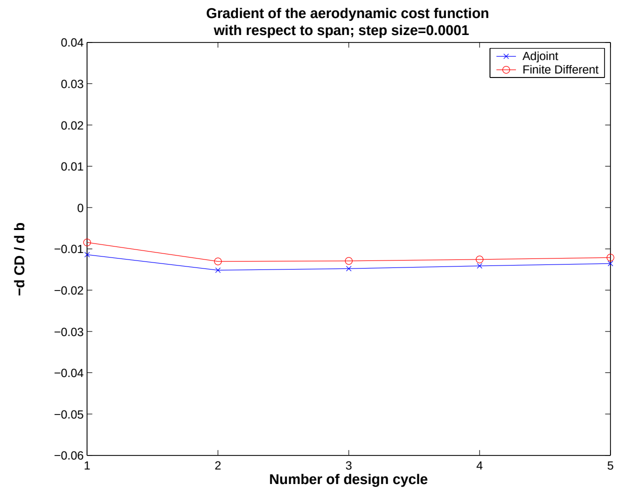
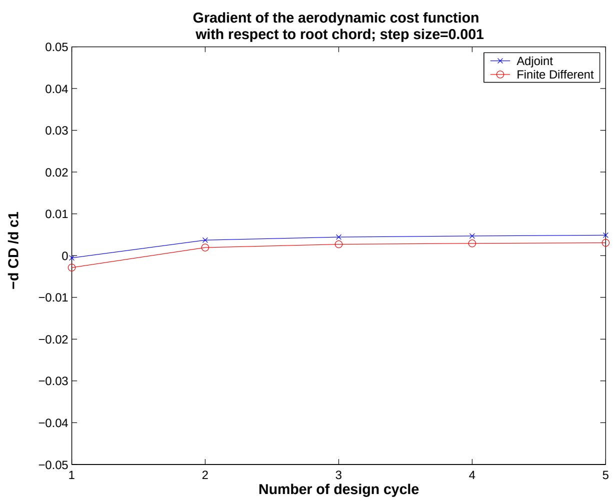


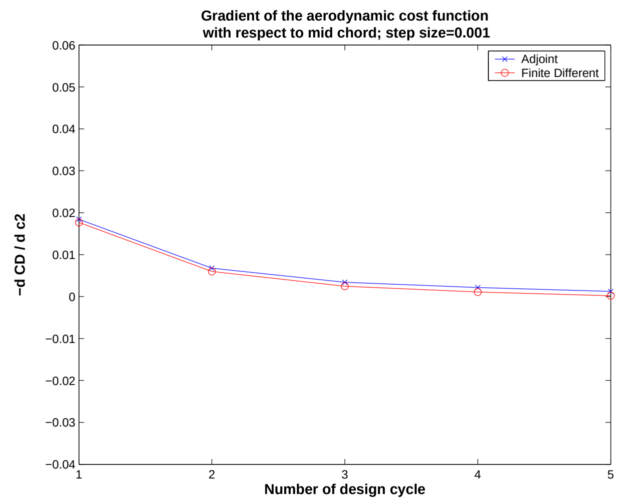
a) Sweepback Angle Λ.



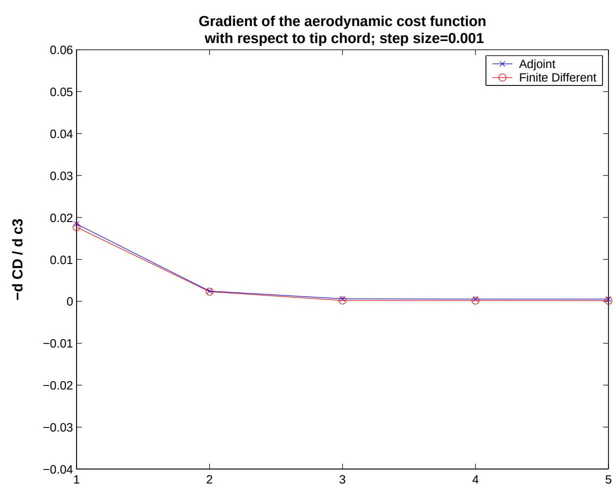
b) Span b.



c) Root chord c_1.



d) Mid-span chord c_2.



e) Tip chord c_3.

Fig. 4 Comparison between adjoint and finite-difference gradients with respect to planform variables. Test case: Boeing 747-200, wing-body configuration, M_∞=.87, fixed α=2.3 degrees.

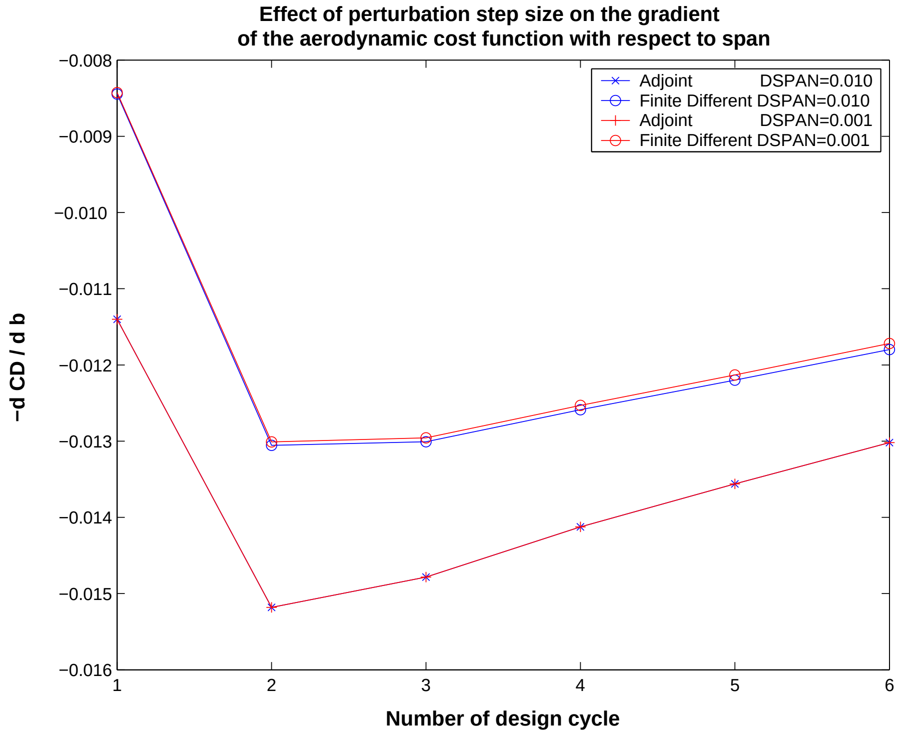


Fig. 5 Effect of purterbation step size on the aerodynamic span gradient. Test case: Boeing 747-200, wing-body configuration, M_∞=.87, fixed α=2.3 degrees.

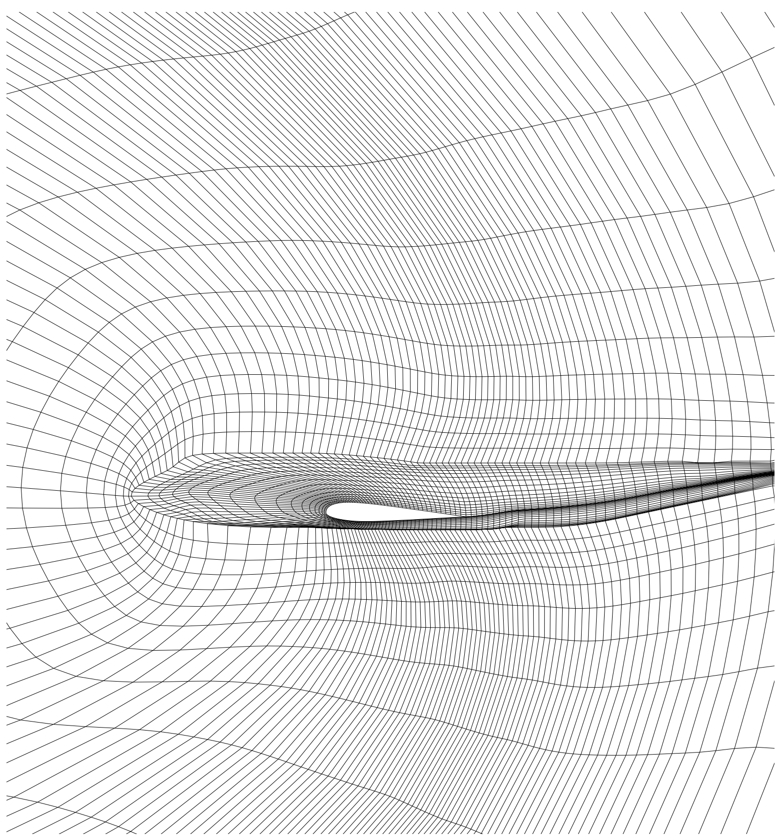


Fig. 6 Computational Grid of the B747-200 Wing Fuselage

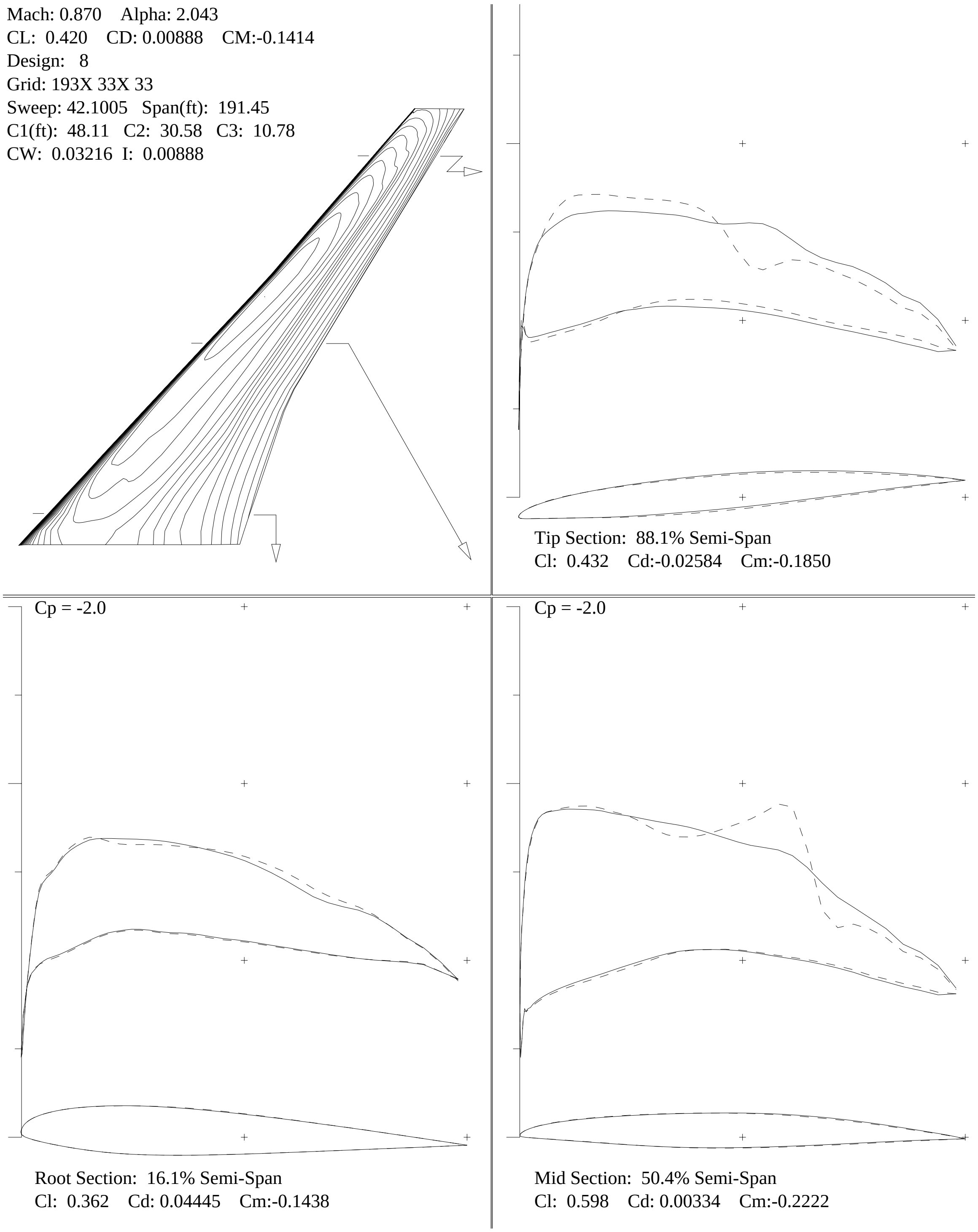


Fig. 7 Redesign of Boeing 747-200, fixed planform

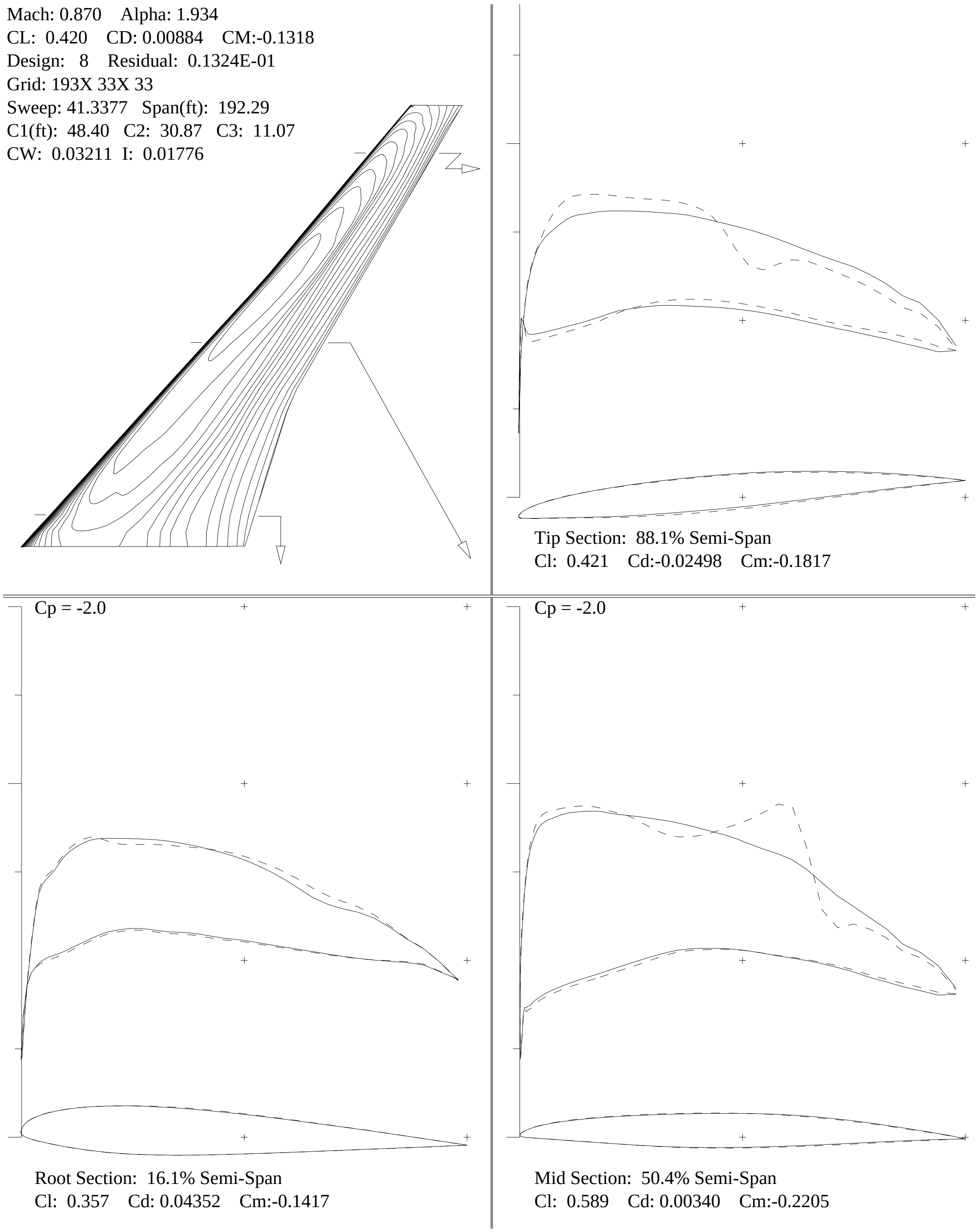


Fig. 8 Redesign of Boeing 747-200, variable planform and maximizing range

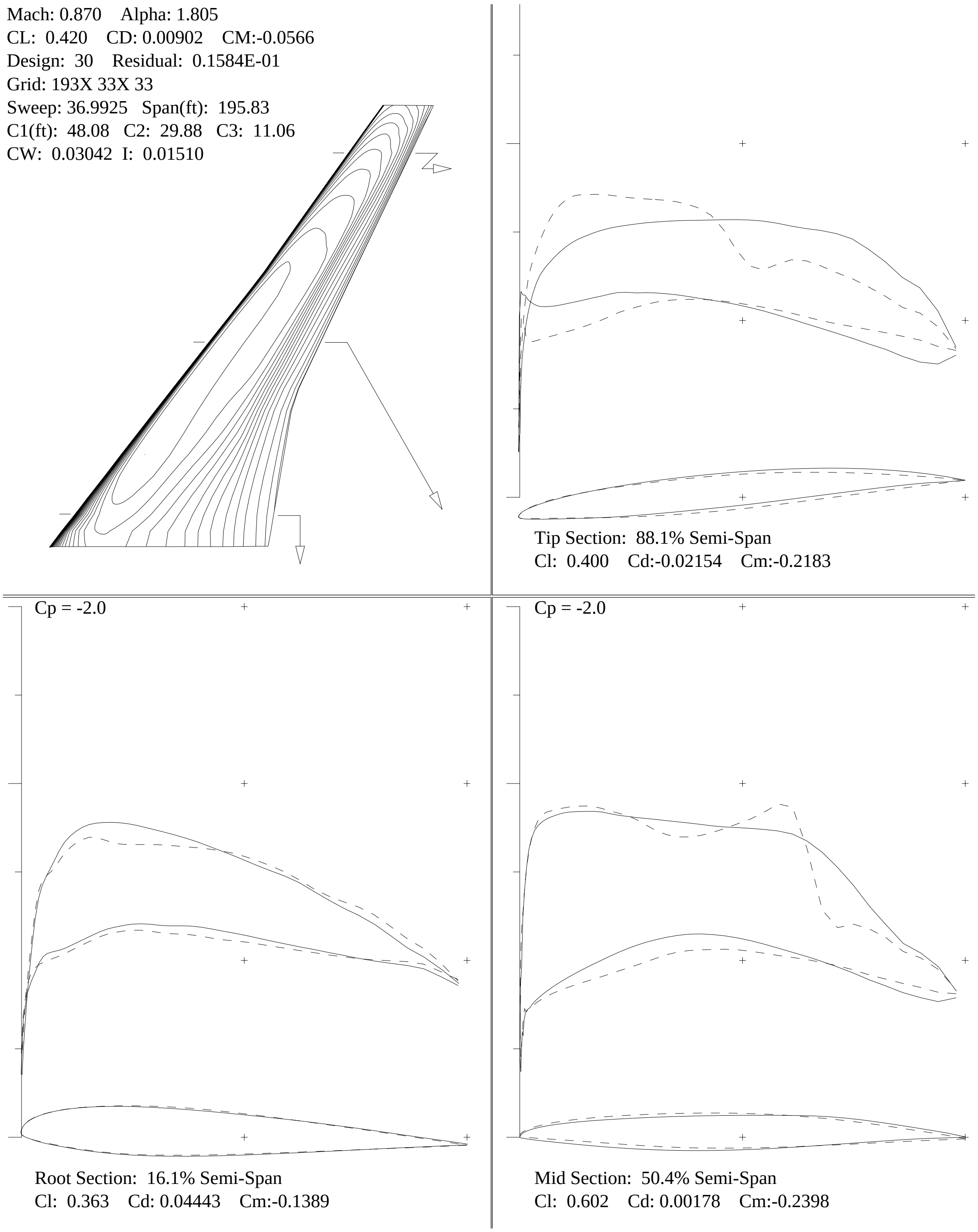
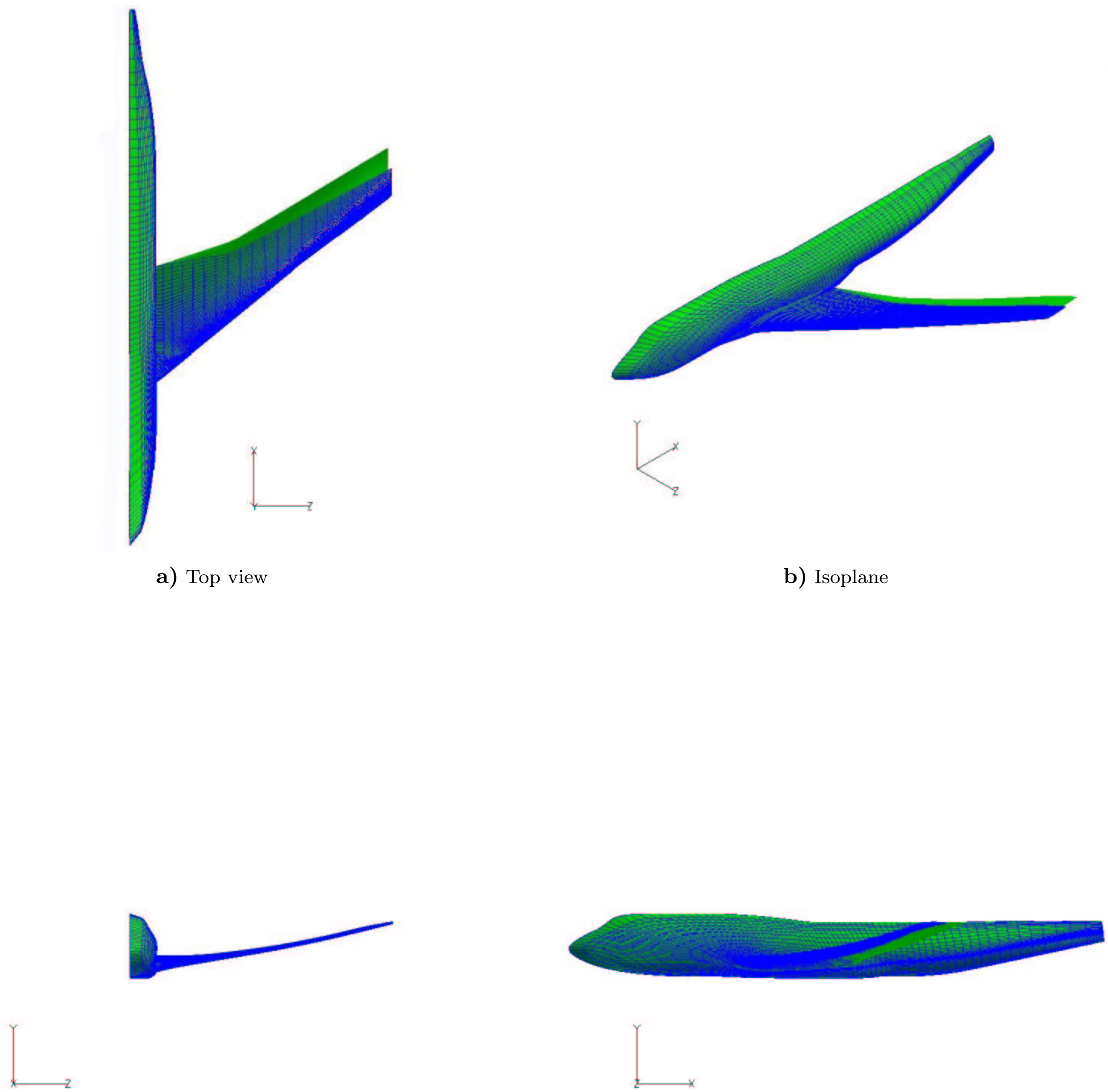


Fig. 9 Redesign of Boeing 747-200, variable planform and optimizing structure weight



a) Top view

b) Isoplane

c) Front view

d) Side view

Fig. 10 Superposition of the baseline geometry and the weight optimized geometry of Boeing 747-200, The baseline geometry is represented by a solid plot. The optimixed geometry is represented by a mesh plot.