Lecture XX **Economics 202A** **Fall 2002**

I will return your logs on Tuesday. I had thought that I could get them finished but I was interrupted.

There are four more lectures after today:

November 19 – Modigliani-Miller

November 21 - Models of coordination Failure
Real business cycles: Nelson and Plosser

November 26- European unemployment

Thursday, December 5 - Nature of the Labor Market including income distribution

We will have log presentations in the last week and I will make the last lecture on the Thursday of that week optional and will not test you on it on the final.

Romer and Summers: look at Romer chapter to review the logic behind the effect of policy changes on investment, especially when the change in policy occurs after a delay.

Last time I introduced the article by Shiller.

Shiller thought that there was too much variance in stock prices to be consistent with the rather low variance in dividend streams. I gave the reason for that and then at the end of last time I was motivating the reasons why Shiller's first demonstration had problems. Shiller estimates this for detrended stock prices and detrended PDV of dividends. But if each of these series is following a random walk with trend, rather than being a stable random deviation from trend, then there is a high standard error in his estimate of the trend. This induces considerable inaccuracy to his procedure.

As a result Shiller's estimates of

$\sigma(p_t^*)$ *relative* to $\sigma(p_t)$ may be quite *inaccurate.*

Now let's step back and ask why *detrending* was necessary.

In fact if P_t, the *undetrended price* of stocks follows a *random walk* then in fact $\sigma(p_t)$ does not make sense.

p_t is not stationary and therefore it does not necessarily have a variance.

The variance of a variable x_t which follows a random walk does not in fact exist.

At the end of the lecture I was about to review the later paper in which Shiller and Campbell solved for this problem that stock prices and the present discounted value of dividends are not stationary.   I am going to save lecture time by letting you go reserving for the notes the description of this paper that I did not finish at the end of last time.

In this class we will turn to studies which take the view that, in fact, the efficient markets hypothesis is correct. Indeed, there were many studies that seemed to show that the evidence was consistent with the efficient markets hypothesis.

In today's class I am going to review an article by Larry Summers that explains why those tests, which seem to show that the efficient markets hypothesis is correct, are less conclusive than you might think.

First, let me review standard tests of *efficient markets*.

The efficient markets hypothesis is important in being the strongest, most natural evidence in favor of *rational expectations* economics.

If it is true, or not – that is, if it gives a valid description of stock markets – therefore makes some difference as to how we should approach economics more generally.

This leads us to the article by Summers: "Does the Stock Market Rationally Reflect Fundamental Values?"

Most tests of efficient markets have *failed* to reject the efficient markets hypothesis. Because of this *failure to reject* most people think (maybe I should say) *used to think* that the efficient markets hypothesis correctly described the world.

According to Summers the reasons that those tests *failed to reject* the efficient markets hypothesis is that the tests have *Weak Power.*

What do I mean by *Weak Power?*

Even if markets behaved in a way markedly different from efficient markets, still, the tests would fail to reject. They would *fail to reject* the hypothesis that markets are *efficient.*

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Let me go over Summers' article.

Define P_t* as the expected value of future dividends.

NOTE: this notation is *NOT* the same as Shiller's.

In Summers' notation:

$$P_t^* = E\left(\sum_{k=0}^{\infty} \frac{D_{t+k}}{(1 + r)^{k+1}} | \Omega_t \right).$$

One form of the efficient markets hypothesis is that the actual price P_t equals P_t*:

RHS of BB

$$P_t = P_t^*$$

We can do a little bit of algebra on this function to get the usual tests of the efficient markets hypothesis.

Let us derive the standard arbitrage formula that comes out of this.

That formula is that the return on stocks is the return on bonds plus a random error term, or

$$R_t = r_t + e_t$$

$$E(e_t | \Omega_t) = 0.$$

IMPORTANT: ERASE BB except for P_t* formula.

RHS of BB

Analogous to the formula for P_t*, P_{t+1}* is:

$$P_{t+1}^* = E\left(\sum_{k=0}^{\infty} \frac{D_{t+k+1}}{(1 + r)^{k+1}} | \Omega_{t+1} \right).$$

If you stare at P_t* and P_{t+1}* long enough you will see term-by term how they differ:

$$P_t^* (1 + r) - P_{t+1}^* = E (D_t|\Omega_t) + [\![E(\sum_{k=0}^{\infty}\frac{D_{t+k+1}}{(1 + r)^{k+1}}|\Omega_t) - E(\sum_{k=0}^{\infty}\frac{D_{t+k+1}}{(1 + r)^{k+1}}|\Omega_{t+1})]\!]$$

Dividing the LHS and the RHS by P_t* and rearranging terms yields:

$$r = \frac{P_{t+1}^* - P_t^*}{P_t^*} + \frac{D_t}{P_t^*} + \epsilon_t \ ,$$

where ϵ_t = [term in curlicue brackets divided by P_t*].

FOOTNOTE: Being slightly sloppy, forgetting that there might be covariance between the term in [curlicue brackets] and the denominator P_t*. END FOOTNOTE

We thus find (being slightly sloppy) that

$E(\epsilon_t| \Omega_t) = 0$.

The reason we get this relation is – ***as always with rational expectations*** **– that the innovation in expectations between t+1 and t should be uncorrelated with information at t. Otherwise a better forecast would have been made at t.**

Now assume efficient markets:
Suppose that the ***actual price*** **equals the expected discounted dividends.**

So $P_t = P_t^*$.

In that case we can rewrite:

$$r = \frac{P_{t+1} - P_t}{P_t} + \frac{D_t}{P_t} + \epsilon_t \ ,$$

or

$$R_t - r = \epsilon_t$$

where $E (\epsilon_t \mid \Omega_t) = 0$,

and R_t is the standard return on stocks: the capital gain plus the dividend

return per dollar invested.

Of course you should have expected such a result. This is merely an arbitrage equation, which says that with perfect markets, the expected rate of return holding stocks and bonds should be exactly the same.

All we have done with our mathematical operations is to convert the initial arbitrage statement about the equivalence of prices and expected future dividends to a statement about the equivalence of rates of return on stocks and bonds.

Now let's consider the implications of this formula for testing of the efficient markets hypothesis.

A "weak form" test of efficient markets is to regress current excess return, $R_t - r$, on previous excess returns, previous $(R_t - r)$'s, to see if there is no correlation.

For example we might run the regression:

$$R_t - r = \beta (R_{t-1} - r) + e_t.$$

We look at the value of β. If β is not significantly different from zero, we have failed to reject the efficient markets hypothesis.

A "strong form" test of efficient markets, in contrast, is to regress R_t on variables other than lagged R_t that are in the information set Ω_{t-1}.

The conventional wisdom is that weak form tests *fail to reject* the efficient markets hypothesis and that strong form tests *reject* the efficient markets hypothesis.

With this bit of background let's see what Summers does.

ERASE BB

Let me now go over what would happen if we used these tests but we had another model for the behavior of stock prices.
Summers constructs an alternative model.
The alternative model is *quite* different from the *Efficient Markets Hypothesis.*
Yet the usual tests of the efficient markets hypothesis will not reject *efficient markets.*

Let's look at Summers' alternative model and, ultimately, what would happen if you used the efficient markets tests in that model.

What we are going to find is that the usual weak form tests that seem to accept efficient markets have *very* weak power against an interesting alternative.

I will now first spell out what that alternative may be.

As before let P_t* again be the expectation of the present discounted value of dividends so that:

$$P_t^* = E\left(\sum_{k=0}^{\infty} \frac{D_{t+k}}{(1 + r)^{k+1}} | \Omega_t \right).$$

According to Summers' model the price of stocks has two components:

$$p_t = p_t^* + u_t$$

where p_t = (log of) actual price
p_t* = (log of) fundamental price
u_t = an error term (in logs).

But unlike the usual error term, u_t is *serially correlated.*

$$u_t = \alpha\, u_{t-1} + v_t$$

and α is *close* to one so that the deviations of stock prices from fundamentals are said to be slow-moving.

The basic idea here is that the price of the stock has two components.
One component is the price of the *expected value of future dividends.*
***And* to that is added a "fad." The fad is the u_t term.**
It is called a "fad" because it is serially correlated.

This describes the alternative model.
I am going to show:
even when the difference between this model and efficient markets is economically quite significant, weak form tests will accept the efficient markets hypothesis.
Such tests are misleading, because they should have rejected.

There are now three stages of the argument.
The *first* stage establishes the autocorrelation of the returns process.

The *second* stage of the argument obtains the formula for the standard error of the

estimate of a typical weak form estimate of serial correlation.

The *third* stage of the argument establishes reasonable parameter values to be used to test for efficiency.

Let's now begin the first part of the argument.

We want to obtain formulas for the true autocorrelations for this process.

To be consistent with what we later do, let me now go back to note that I wrote:

***lower case* p_t denotes the log of price,**
***and lower case* p_t^* denotes the *log* of the expected future value of dividends.**

POINT

We are now going to see what the returns process looks like.
We need to derive that.
After that we will see the number of observations that it would take to reject the efficient markets hypothesis if there was an economically significant fad.

That is, we will see how many observations we would need to reject efficient markets if u_t is economically large.

This is going to take some time, indeed the rest of the lecture.

So let's begin.

By definition the *actual return* is

$$R_t = \frac{DIVIDEND}{P_t} + \frac{P_{t+1} - P_t}{P_t}.$$

That is the number that is reported in the financial pages every day.

Summers makes two approximations:

$$\frac{DIVIDEND}{P_t} = \frac{DIVIDEND}{P_t^*}$$

and

$$\frac{P_{t+1} - P_t}{P_t} = p_{t+1} - p_t \ .$$

You may question either of these approximations. If you truly hate either one of them you should write a computer program to see whether or not it makes a difference.

And we know

$\mathbf{p_t = p_t^* + u_t}$

So we also know:

$\mathbf{p_{t+1} = p_{t+1}^* + u_{t+1}}$.

This yields as our approximation:

$$R_t \cong \frac{DIVIDEND}{P_t^*} + (p_{t+1}^* - p_t^*) + (u_{t+1} - u_t)$$

$$\cong \frac{DIVIDEND}{P_t^*} + \frac{P_{t+1}^* - P_t^*}{P_t^*} + (u_{t+1} - u_t)$$

From our earlier calculation about fundamental value we know that the first two terms are

$\mathbf{r + \epsilon_t}$

and the last two terms are

$\mathbf{u_{t+1} - u_t}$.

Now to simplify notation define the variable Z_t to be the excess return over bonds:

$\mathbf{Z_t = R_t - r}$,

so that

$$\mathbf{Z_t = \epsilon_t + u_{t+1} - u_t,}$$

and we know that u_t follows the process:

$$\mathbf{u_t = \alpha\, u_{t-1} + v_t.}$$

ERASE BB.

It is now easy to check that Z_t follows the following process:

$$(*) \qquad \mathbf{Z_t = \alpha\, Z_{t-1} + \epsilon_t - \alpha\, \epsilon_{t-1} + v_{t+1} - v_t}$$

How do we know?

Because $\mathbf{Z_t = \epsilon_t + u_{t+1} - u_t}$

$$\mathbf{\alpha\, Z_{t-1} = \alpha\, \epsilon_{t-1} + \alpha u_t - \alpha u_{t-1}}$$

so that

$$\mathbf{Z_t - \alpha\, Z_{t-1} = \epsilon_t - \alpha\epsilon_{t-1} + u_{t+1} - \alpha u_t + u_t - \alpha u_{t-1}}$$

$$\mathbf{= \epsilon_t - \alpha\epsilon_{t-1} + v_{t+1} - v_t.}$$

NOTE to reader: Summers' equation (6) has a *misprint. END NOTE*

Now let's see what tests of efficient markets would reveal if Z_t behaves according to (*).

You can calculate as a homework exercise Summers' formula for the autocorrelation function.

$$\mathbf{\sigma_Z^2 = 2\,(1 - \alpha)\, \sigma_u^2 + \sigma_\epsilon^2}$$

$$\rho_k = -\frac{\alpha^{k-1}\,(1-\alpha)^2\,\sigma_u^2}{2\,(1-\alpha)\,\sigma_u^2 + \sigma_\epsilon^2}.$$

We are now beginning to zero in on our answer.

The usual weak form tests look at ρ_1 – the serial correlation of returns.

The usual tests show that ρ_1 is quite low and not statistically different from zero.
The question now is: what is the standard error of the typical ρ_1 that one would estimate.
That is the second stage of the argument.

We will use these formulas to show something truly astounding.

We will show that the usual tests of the efficient markets hypothesis have *very* low power against Summers' alternative hypothesis.

Just how weak is what is so *astounding*.

ERASE BB

I am going to leave on the blackboard only the formulas for ρ_1 and σ_Z^2 since they are our focus.

LEAVE ONLY ON BB: formula for ρ_1, σ_Z^2. PUT BOX AROUND THEM.

We can construct a table for ρ_1 as a function of

$\sigma_\epsilon^2/\sigma_u^2$ \ α	.75	.90	.95	.995
1.0				
.5				
.25				
.025				

We are going to consider the *power of tests* for realistic values of parameters in this table.

You can already see where the result is going to come from that the power of the standard regression tests for efficient markets is going to be weak.

You would expect α to be a number very close to one if we are using monthly data.
$1 - \alpha$ is the rate of convergence of stock prices to fundamental values. It is the rate of disappearance of the fad.

In turn, a value of $1 - \alpha$ that is close to zero should give a low value of ρ_1.
You can see this by looking at the formula for ρ_1.
In that formula $(1 - \alpha)$ is squared. POINT

Let's examine the consequences of these formulae if we had 50 years of stock data and ran the usual efficient markets tests.
These tests tend to show ρ_1 is not significantly different from zero.

We will compare the standard error of ρ_1 to its true value in our table. In this way we will see whether "acceptances" of the efficient markets hypothesis were warranted. By acceptance of course I mean a failure to reject.

FOOTNOTE: "acceptances" is in quotation marks because an "acceptance" here is actually a failure to reject. END FOOTNOTE

If returns are uncorrelated, so $\alpha = 0$, the formula for the standard error of ρ_1 is approximately $1/\sqrt{n}$, as I shall show, where n is the number of observations.

ρ_1 is the coefficient in the regression:

$$Z_t = \beta Z_{t-1} + e_t$$

and the least squares formula for the standard error of the estimated β is:

$$\text{var}(\beta^{est}) = \sigma_\epsilon^2 \, (X'X)^{-1}.$$

With the null hypothesis that $\beta=0$, then

$$x_2 = \epsilon_2$$

$$x_3 = \epsilon_3$$

$$\vdots$$

$$x_n = \epsilon_n$$

So

$$var(\beta^{est}) = \frac{var(\epsilon)}{\epsilon_2^2 + \epsilon_3^3 + \ldots + \epsilon_n^n}$$

$$\cong \frac{var(\epsilon)}{(n-1)\ var(\epsilon)} = \frac{1}{n-1}.$$

$$var(\beta^{est}) \cong \frac{1}{n-1}.$$

$$var(\rho_1^{est}) \cong \frac{1}{n-1}.$$

Summers says that with 600 obervations:

$$st.dev.(\rho_1^{est}) \cong \frac{1}{\sqrt{597}}.$$

SMALL FOOTNOTE: It is 597 because you lose one observation in forming the Z_t's from the stock prices. You also lose an observation when you run your regression because the first LHS variable for which there is a corresponding Z_{t-1} is $Z_{2.}$ END FOOTNOTE

FURTHER FOOTNOTE: Summers has quite intentionally made a small mistake here. He calculates this standard error on the assumption that there is no fad. This mistake is intentional. He thinks that what he calculates here is probably a good approximation. That could be tested by simulations. I think that he is right that it is a good approximation. END FURTHER FOOTNOTE

So the standard deviation of the estimated ρ_1 with 600 observations, or 50 years of monthly stock data, is

$$\sigma(\rho_1^{est}) \cong .042.$$

This tells us the s.d. of the estimate of ρ_1 with 600 months of data. How does that compare to the actual value of ρ_1?

This is the third stage of the argument. We will now choose reasonable parameters. With these parameters we will see that the actual value of ρ_1 in our table is very small relative to its standard error of estimate.

Let's see by choosing a point in our table.

There are two parameters we might choose fairly easily:

σ_u^2 **and** α.

We then need a value of σ_ϵ, but that will not be very difficult.

Then we get a value of ρ_1.

Let's try on for size $\sigma_u^2 = .09$.

This means that with a normal distribution of u the stock price will be more than 30 percent away from its fundamental value 32 % of the time.

With a normal distribution, 32 % of the time the variable u will be more than one standard deviation from its mean.

Therefore if σ_u is .3, 32 percent of the time the stock will deviate by more than 30 percent from its fundamental value.

Let's also try on for size a value of α.

Let's choose $\alpha = .98$.

With monthly data that means that it takes three years to eliminate one-half of an error in value, since

$$(.98)^{36} = .483213.$$

σ_Z has been recorded as .20 per year by Ibbotsen and Sinquefeld.

So on a monthly basis

σ_Z^2 **per month** $\cong (.20)^2/12 \cong .004$.

Then using the formula for σ_Z^2 he can calculate σ_ϵ^2 .

Knowing σ_Z^2 , α, and σ_u^2 Summers can calculate σ_ϵ^2.

He can do that from the earlier formula:

$$\sigma_Z^2 = 2\ (1 - \alpha)\ \sigma_u^2 + \sigma_\epsilon^2.$$

FOOTNOTE:

$.004 = 2\ (.02)(.09) + \sigma_\epsilon^2$ END FOOTNOTE

Then knowing σ_ϵ^2 he can calculate ρ_1.

Point to formula.

$$\rho_1 = -.009.$$

Previously we calculated the standard error of ρ_1 as .042 with 600 months of data.

So in the sample with 600 months (50 years of data) ρ_1 would be roughly 1/5 th its standard error.

You would not be able to reject the efficient markets hypothesis with the test even though stock prices are more than 30 percent away from their fundamental value about 1/3 of the time.

How large would the sample have to be to reject with probability 50 %.

A rule of thumb is that one rejects the null hypothesis at the 5 % level when the coefficient is double the standard error.

The expected value of the coefficient is -.009.

So the standard error must fall to .004 to get a rejection 50% of the time.

This will occur when there are more than 5,000 years of data, so N = 60,000 months.

In that case

$s.e. = 1/ n^{1/2} = (1/60,000)^{1/2} \cong [\ (1/250)(1/250)]^{1/2} = .004.$