Artless innocents and ivory-tower sophisticates: Some personalities on the Indian mathematical scene*

M. S. Raghunathan

At the beginning of the 20th century, science was still an esoteric pursuit of reclusive intellectuals. The quiet revolution in the academic worlds of Göttingen, Copenhagen, Cambridge and Paris of the early decades exploded into global awareness of science with Hiroshima. With the *Sputnik*, science soared even higher in public esteem and many a scientist, became a public figure, a celebrity, an icon for the youth. Physicists dominated this celebrity parade, but there were chemists and biologists in fair number. In this context of hype about science and adulation for the scientist, I think the mathematician is described best by a Tamil proverb: he is the hapless fellow who brought home a copper vessel after taking part in a raid on Kubera's Alakapuri! Many great names in mathematics are entirely unfamiliar to people outside the scientific community. This article is about some mathematicians who have contributed significantly to mathematics in the 20th century and, more importantly, have had considerable influence on mathematics in India. Some of them are probably not very well known even within the scientific community.

A word about the choice of personalities that I have made: they are all men who figured a good deal in the mathematical lore that I was brought up on at the Tata Institute of Fundamental Research; and that lore has its bias.

I begin – inevitably – with Srinivasa Ramanujan, the best known Indian mathematician, who is reckoned among the greatest mathematical intellects of the twentieth century. The romantic story of the passage of the poor clerk in the Madras Port Trust to the portals of the ivory towers of Cambridge and the subsequent tragedy of genius cut-off in its prime by illness is well-known, and so I will not dwell on it.

To many mathematicians, Ramanujan's thought processes have an element of mystery about them. Hardy (who was responsible for Ramanujan becoming known internationally), would have none

S. Ramanujan

of that, but Littlewood, Hardy's close friend and collaborator certainly thought so. Mark Kac, another famous name, describes him as a 'magician rather than a genius'. Bruce Berndt, a mathematician who has now spent two decades unravelling Ramanujan's notebooks, says – and I quote – 'I still don't understand it all. I may be able to prove it, but I do not know where it comes from and where it fits into mathematics'. Some of Ramanujan's Indian contemporaries were sure that he was deeply religious and were ready to believe even in divine intervention.

We may not have a clear idea about the nature of Ramanujan's thought processes. There is however one trait of his which emerges clearly from contemporary writings about him: he was entirely artless and was modest to a fault: his near total lack of feelings of self-importance is indeed striking. He does not seem to have had a true measure of his own extraordinary talents, despite Hardy playing 'Jambavan to his Hanuman'.

There is in fact the episode of his attempted suicide in England that suggests periods of somewhat fragile self-esteem. It happened when the depression caused by his illness was compounded by family problems and Trinity's racist rejection of his nomination to its Fellowship (Trinity did appoint him a Fellow later). Ramanujan threw himself before a train, but luckily a guard brought the train to a halt in time. To prevent his arrest for attempted suicide, Hardy attempted some white lies: he told the Police that Ramanujan was a Fellow of the Royal Society (at that point of time Ramanujan was only a candidate; he was elected a month later) and that Fellows of the Royal Society could not be arrested (which was, of course, nonsense)! The Police were not taken in, but nevertheless did not press the charge.

G. H. Hardy

Ramanujan's insights continue to influence mathematical developments to this day. His collected works have been a source of inspiration to many outstanding mathematicians of the 20th century.

$$q\prod_{n=1}^{\infty}(1-q^n)^{24} = \sum_{n=1}^{\infty} t(n)q^n$$

The Ramanujan *t* function is a function on positive integers defined by the above identity obtained by formal expansion of the left hand side.

The *t* function

*Based on a public lecture delivered at the Annual Meeting of the Indian Academy of Sciences, held at Chandigarh in 2002.

His work on what is now called the Ramanujan tau function, which evoked only a moderate response at that time, later proved to be profound and central to what is called the Theory of Modular Forms. Hecke, a great German mathematician who was one of the architects of the theory was born in the same year – 1887 – as Ramanujan; it does seem a great pity that the two never met. A conjecture of Ramanujan on the tau function was settled in 1974 by Pierre Deligne, a leading mathematician of our era.

One of the most fruitful techniques applied successfully to diverse problems in Number Theory is known as the Circle Method. This originates from the joint work Ramanujan did with Hardy on what is known as the partition function.

Ramanujan's notebooks are a treasure house of beautiful formulae and identities, set down without details. Providing proper proofs to these has been a challenge and at the same time, a public service to the mathematical community. Experience indicates that there is likely to be much more to many of these results than the formal beauty which by itself makes them attractive. It has been said that in the matter of formal manipulations Ramanujan has no equals in the history of mathematics, other than Euler and Jacobi.

A partition of a positive integer n is an expression of n as a sum

$$n = n_1 + n_2 + \ldots + n_r,$$

each n_i a positive integer with $0 < n_1 \leq n_2 \leq \ldots \leq n_r$. The number of different ways of writing n as a sum of other positive integers is denoted by $p(n)$.

Clearly, $p(n)$ is a function of n. This function on the set of positive integers is called the *partition function*.

The partition function

Hardy is much less of an enigma even to us in India, than Ramanujan himself. He was, of course, one of the major mathematical figures of the twentieth century and, as the Ramanujan story shows, he was a wonderful human being. He was also exceptionally articulate and a gifted writer of English prose. His little book *A Mathematician's Apology* giving his view of his profession, makes delightful reading. Hardy was very much the ivory-tower intellectual, and the book is not so much an 'apology' as an emphatic affirmation of his belief in the irrelevance of social relevance in the pursuit of pure science. He has also written an account of the Bertrand Russell affair – the turmoil in Cambridge caused by Russell's unorthodox views and conduct during the war years 1914–18.

Hardy was an ardent cricket fan; so ardent indeed that he calibrated excellence in any field by cricketing greats: the highest accolade was to be in the 'Bradman class'! Interesting people were people who had 'spin' in them.

Hardy was an outstanding analyst as well as a number theorist; his work in analysis has in some ways been more influential than that in number theory. He was the leading expert on Infinite Series; his book *Divergent Series* was a classic of that period. He disapproved of the way mathematics was taught in Britain in his days and sought to reform the same. His book *Pure Mathematics*, was an attempt in that direction. It was the first analysis book in English written in the European spirit, and proved a great success.

K. Ananda Rau

A not-so-well-known contemporary of Ramanujan was K. Ananda Rau, a foundation member of Indian Academy of Sciences, Bangalore. His career had an altogether different kind of trajectory from Ramanujan's. He was born in Madras in 1893 (six years after Ramanujan) into relative opulence and, after a brilliant academic career of the normal kind through school and college in Madras, went to Cambridge in 1914 and became a student of Hardy's. As a student, he won the coveted Smith Prize. He returned to Madras after completing his studies in 1919 and was immediately appointed as Professor of Mathematics at Presidency College, Madras. Ananda Rau was an outstanding analyst who worked a great deal on summability, an area in which Hardy was a leading figure. A theorem named after Ananda Rau figures in Hardy's book *Divergent Series*. He was from all accounts an inspiring teacher, held in great respect and affection by his students, many of whom went on to become fine mathematicians themselves; some among them were leaders on the Indian scene and I will be speaking about them. Ananda Rau retired from service in 1948 at the then mandatory age of 55, but remained mathematically active for more than a decade after that. He died in 1966.

Ananda Rau and Ramanujan

Ananda Rau got to know Ramanujan in Cambridge and he has this to say about his illustrious colleague: 'In his nature he was simple, entirely free from affectation with no trace whatever of his being selfconscious of his abilities'.

Another figure of importance in Indian mathematics of that time was Vaidyanathaswamy, also from Madras. He too went to England, not Cambridge though, to work with some British mathematicians, but that was after some years as a research scholar at the University of Madras.

I should like to emphasise that in his case as well as in the case of Ananda Rau, their career decisions were taken well before the Ramanujan story broke out. Also most of Vaidyanathaswamy's mathematical interests were far removed

R. Vaidyanathaswamy

from those of Ramanujan's and he was among the earliest to venture into areas such as Symbolic Logic, Lattice Theory and Topology that were not British favourites of his times. After his return from England in 1925, he spent a year in Benares and joined Ananda Rau in Madras. The two men joined forces to create a lively and congenial atmosphere for mathematics students in Madras. Madras University administration, in its infinite wisdom, kept him as a Reader his retirement in 1952. After retirement, he worked for a few years at the Indian Statistical Institute in Calcutta and later at Sri Venkateswara University in Tirupati.

Vaidyanathaswamy seems to have been cast in the mould of traditional Indian scholarship. He was a keen student of Aurobindo's philosophy and studied *Vedic* texts in-depth, offering his own interpretations.

Number theory, Ramanujan's area of interest, naturally had many adherents in the country; but work that had a truly great impact came only in the mid-thirties, and it came from S. S. Pillai. Pillai was born in 1901 in the Tirunelveli district of Tamil Nadu. His mother died within a year of his birth and he was brought up by his father with the help of an aged woman-relative. He made good progress at school, but tragedy struck again; his father passed away when he was in the final year of school. His talents had earned him the lasting affection of a teacher, Sastriar, who stepped in to help him continue his education. Pillai also secured scholarships and completed the B A degree at Maharaja's College at Trivandrum and moved to Madras where he become a student of Ananda Rau for whom he cherished life-long affection and respect. He soon proved himself to be a researcher of the first rank and after taking his Ph D at Madras, took up his first job at Annamalai University; he later moved to Trivandrum then again to Calcutta and finally back to Madras University.

S. S. Pillai

It is in the thirties at Annamalai University that Pillai's talents were in full bloom and he cracked a problem that was engaging some of the finest minds. David Hilbert had shown that for every integer $k > 0$, there is a smallest integer $g(k) > 0$, such that every positive integer can be expressed as a sum of $g(k)$ k^{th} powers. Pillai's work centred on the exact determination of $g(k)$. He achieved the complete determination for $k \geq 7$, a superb achievement by any reckoning. He went on a little later to tackle the even more difficult case $k = 6$. However, a controversy over priorities involving the American mathematician L. E. Dickson was a cause for some distress to Pillai and his Indian colleagues. Pillai published his great papers in Indian journals which did not have a wide circulation; nevertheless, recognition did come eventually for these outstanding contributions, but tragedy struck once more before he could savour his success. On 31 August 1950, Pillai died in an air-crash over Egypt; he was on his way to the US – his first trip abroad – to spend a year at the Institute for Advanced Study in Princeton, where he had been invited. The news was received with great shock by the many mathematicians assembled at Harvard, where Pillai was to participate in the International Congress of Mathematicians before going on to Princeton. Pillai's work on the 'Waring' problem – the determination of the $g(k)$ – is a piece that has given him a permanent place in the history of mathematics.

The determination of $g(k)$ for all k has now been completed, the case $k = 4$ was the one that defied mathematicians the longest, till about 10 years ago when another Indian, R. Balasubramanian in collaboration with two Frenchmen, Deshouillers and Dress settled the matter. Pillai had numerous other important contributions as well. There is little doubt that Pillai would have achieved a great deal more if his life had not been cut short so abruptly.

Hilbert's Theorem (1909): *Given a positive integer k, there is a positive integer r such that every positive integer n is the sum of r k^{th} powers of integers.*

In other words, every positive integer n is equal to $n_1^k + n_2^k + \cdots + n_r^k$ for suitable non-negative integers n_1, n_2, … , n_r. The *same r* works for every n. There is evidently a minimal such r for a given k; this minimal r is denoted by $g(k)$.

Waring Problem: Determine $g(k)$.
$g(1) = 1$ (Obvious)
$g(2) = 4$ (Lagrange)
$g(3) = 9$ (Wieferich and Kempner, 1912)
Solved by Pillai for $k \geq 6$ (1936)
$g(5) = 37$ (Chen, 1936)
$g(4) = 19$ (Balasubramanian, Deshouillers and Dress, 1986)

The Waring problem

'Pillai', in the words of Prof. K. Chandrasekharan (of whom too I will speak) 'was a person of genuine modesty and remarkable simplicity. He possessed that rare quality among intellectuals – intellectual honesty – which endeared him to his friends, but lost him many material advantages'. 'He was', says Chandrasekharan, 'unsophisticated in a peculiar sense'.

In the early part of the twentieth century, British mathematics held sway over us. It did produce some beneficial results. But in the twenties and thirties, the most exciting developments in mathematics were taking place in Paris and Göttingen, not Cambridge. These developments seem to have had no serious immediate impact on the Indian scene. Summability, an area of great interest to Hardy was pursued by many Indians, including Ananda Rau and many of his students and they made very substantial contributions. But the subject continued to be pursued by many Indians long after it had ceased to offer exciting challenges. So there was an element of truth in the impish comment: ‘Hardy spoilt many Indian mathematicians; but of course Ramanujan was much too great to be spoilt!’ That brings me to the man who made that statement – André Weil.

Andr. Weil

André Weil was one of the greatest mathematicians of the 20th century. He was a colourful personality with a powerful sense of humour and was not averse to using it to cause discomfiture. He spent two years 1930–32 in India as a Professor of Mathematics at Aligarh Muslim University. He tells the amusing story of how this came about in his autobiography. Syed Ross Masood, the then Education Minister to the Nizam of Hyderabad was vacationing in Europe when he was appointed Vice-Chancellor of the Aligarh Muslim University, founded by his grandfather Syed Ahmed Khan. He decided to cut short his stay in Paris and return home, but he wanted to recruit a suitable person for a chair in French Culture at the university, before he left Europe. Towards this end, he met with the famous French Indologist Sylvain Levy, who summoned Weil to their presence. Weil, who had just completed his doctorate in mathematics as a student at the famed Ecole Normale Superieure in Paris, was in touch with Sylvain Levy because of his interest in India; among other things, Weil had been keenly following a course of lectures on Kalidasa’s *Meghadutam* given by Levy at the Sorbonne. The interview was brief; Masood offered the chair on the spot and Weil accepted it. But six months later, when the formal offer came, it was a chair in mathematics, not French Culture that was offered! Weil says that he never found out when exactly Masood came to know of his mathematical credentials!

Weil packed his bags and left for Aligarh to take up the headship of the Mathematics Department there, armed with powers to hire and fire faculty – he was all of 23! The contrast between the supreme confidence of the Normalien (as students of the Ecole Normale Superieure are known) and the diffidence of the clerk from the Port-Trust is striking, but it would hardly surprise the sociologist. Weil promptly sacked one of his three colleagues, removed another temporarily and let the third continue, only to regret that decision soon. Weil spent two years in Aligarh during which he made many friends, not all of whom were academics. He had a good measure of the generally poor quality of mathematics in the country, but was pleasantly surprised to come across some talented young people of promise. He recruited one of them, T. Vijayaraghavan, a student of Hardy, to the vacant post in the department; this, despite the fact that Vijayaraghavan lacked formal qualifications. The two struck up a lasting friendship. Apart from their common interest in mathematics, Vijayaraghavan with his scholarship in Tamil and Sanskrit, could cater to Weil’s interest in Indian culture. Weil also came to know other Indian mathematicians like Kosambi and Chowla. He seems to have travelled a good deal. That included a trip to Trivandrum to participate in an Annual Meeting of the Indian Mathematical Society. On his way to Trivandrum, in Chennai he made the acquaintance of Ananda Rau and Vaidyanathaswamy. His experiences in Trivandrum provide some amusement. Younger men at the meeting had to devise ingenious ways of reconciling some of the older colleagues’ insistence on observing caste taboos with the possible sensitivities of the *mlecha* present among them.

Vijayaraghavan was four years Weil’s senior in age. He did well enough in school, but at college his performance was not very good by the usual criteria. This was because, like Ramanujan, he had become interested in serious mathe-

T. Vijayaraghavan

matics and found the curricular material unexciting. Luckily for him, a real mathematician – Ananda Rau – who could recognize talent that the examination system was incapable of detecting, was at the helm of affairs and he could secure admission to the B A (Honours) course in University. The parallel with Ramanujan continued: he sent Hardy his researches and eventually in 1925 went to Oxford (Hardy had moved there) to work with him. It is not surprising that many in Madras looked upon him as a spiritual successor to Ramanujan. All through, Ananda Rau had been a great help and Vijayaraghavan would apparently recall with great pleasure his meetings with his teacher. Vijayaraghavan’s was a fine mind and he was soon publishing excellent papers in Analysis and related areas. He worked on what are known as Tauberian theorems and produced work of truly high quality.

Vijayaraghavan was a keen problem-solver and had no great fascination for building theories or acquisition of extensive scholarship. He was always on the lookout for interesting problems and was quite happy to get to know them from the knowledgeable. Weil, on the other hand, developed theories and for him problem-solving while important, was secondary: with the right theory the solutions to problems will assuredly fall out.

Physically too, the two were a study in contrast. Weil was slim and fit. He enjoyed walking a great deal and used to call it (in the Macarthy days) his ‘un-american’ activity. Vijayaraghavan’s was a portly frame which reflected correctly his sedentary lifestyle.

Weil’s initial cordial relationship with Masood did not survive for long. His independent spirit came into conflict with the system in which the Vice-Chancellor was a demi-god; and Masood's perception of the University as a family legacy did not help. Towards the end of his second year in Aligarh, Weil went on a short vacation to Europe (where in fact he exerted himself to acquire books for the Aligarh library). He returned to find himself summarily dismissed. His friend Vijayaraghavan had quit and moved to Dacca in protest when Masood, in Weil's absence, offered him the professorship from which he planned to oust Weil. Weil returned to Paris after a brief stay with Vijayaraghavan in Dacca. During that stay, Sarvapalli Radhakrishnan, the then Vice-Chancellor, invited him to take up a position at Andhra University. Weil was attracted by the offer but eventually declined when Radhakrishnan was unable to meet his demand for a free hand to run the department.

Vijayaraghavan later moved to Madras to head the then newly formed Ramanujan Institute. He died in 1955 at the relatively young age of 53. Chandrasekharan has this to say of Vijayaraghavan: 'No one who knew him intimately – as a working mathematician, as a genial host or as an affectionate father – could fail to say here was an intellectual of whom his country could be proud.'

'Vijayaraghavan loved lecturing, and was a lucid, effective and sometimes brilliant lecturer, especially on mathematical topics which were of immediate interest to him.'

'It was a pet saying of his that one could not claim that one knew a theorem, unless one could give not less than three different proofs of it, of which at least one proof was one's own.'

It is hardly surprising that he was on occasions hoist with his own petard!

Weil's influence on Indian mathematics during the Aligarh sojourn cannot be considered as greatly significant, although some individuals like Vijayaraghavan would have benefited from his presence. It is some thirty years later that his mathematics had a big impact in this country: after all, Aligarh happened at the very beginning of Weil's career as a mathematician, and the researches that were to wield influence came later. Two streams of mathematics of outstanding quality which emerged from TIFR in the sixties had their origins in Weil's works: moduli of vector bundles and rigidity phenomena in discrete groups. These topics are, of course, pursued at many centres, but Indian mathematicians have been at the forefront of the developments in these areas. In 1994, Weil received the Inamouri Prize, a prestigious international award given to outstanding achievement across fields by a philanthropic Japanese Foundation. An Indian, my colleague S. Ramanan of the Tata Institute, was extended the privilege of speaking on Weil's work on vector bundles at the award function held in Kyoto.

P. Deligne

As I had mentioned before, Weil was one of the great figures of 20th century mathematics. He revolutionized the area of interface of Number Theory and Algebraic Geometry, and set the agenda for research in the area for several decades. I mentioned that Deligne proved Ramanujan's conjecture on the tau function. This was achieved by first establishing some conjectures of Weil and showing that Ramanujan's conjecture was a consequence of the Weil conjectures. Apart from his research, Weil was deeply involved in other forms of promotion of mathematics. He was one of a small number of mathematicians who formed the 'Bourbaki' group. The Bourbakis set about writing a series of books which have been of great value to the mathematical community. The Bourbaki group also began organizing seminars in Paris on diverse topics to help disseminate current research, and this activity is continuing to this day.

Indian thought certainly had great influence on Weil. In his autobiography, he says that the only religious ideas that appealed to him were those to be found in Hindu philosophical thought. During the Second World War, Weil refused to do military service and ironically he cited the *Bhagavad Gita* (whose ostensible aim was to get Arjuna to fight) to justify his stand: his true *dharma* was the pursuit of mathematics and that was what he should be doing, not assisting in the war effort, however just the cause!

For all his fascination for India, Weil never went back to India till 1968. That year, he was in India to lecture at an international conference hosted by TIFR. In Bombay, TIFR put him up at the Taj hotel, but as we discovered later, our hospitality simply could not match what was offered to him elsewhere in the country. In Delhi, he stayed at the Rashtrapathi Bhavan as a guest of the then president Zakir Husain, a friend from the Aligarh days! And it was the Raj Bhavan in Kolkata again – Weil had known the governor Dharam Veera as a civil servant

Zakir Husain

in Aligarh! In Chennai, he met Rajaji who cut him short when he was trying to remind him about their previous meeting in 1931 with, 'Oh yes, I remember you very well. You are the French professor who was in Aligarh and I find that your English is as bad as ever!' Weil had an excellent command of English, but spoke the language – inevitably – with a French accent. The ivory-tower was no doubt Weil's natural habitat, but it would appear that he could, if he wanted to, come out and give a lesson or two to Dale Carnegie!

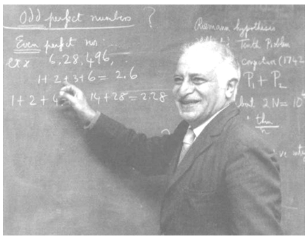
I cannot resist the temptation to add a few words about my own personal encounters with the great man. I spent the academic year 1966–67 at the Institute for Advanced Study in Princeton, where Weil had become a permanent member in 1960. He was, of course, by then a much admired legendary figure, but with a reputation for acerbic wit that discouraged personal contact. I had a routine daily encounter with him though. I would go down to a lounge, where tea was

Rajaji

served, to read the *New York Times*. Weil would turn up soon after and pace up and down. It was evident that he wanted the newspaper and after holding out for a few brave minutes, I would give up the paper before I was really done with it, and he would promptly pick it up!

On one occasion at a party, I found myself in a corner with Weil. I do not know why, but I felt obliged to start a conversation and said something about D. D. Kosambi, mathematician turned historian. Weil responded with, 'Young man, I find that people who know nothing about Kosambi want to talk about him! Let me tell you this: he was one of the finest intellects to come out of your country.' I was, of course, petrified, but he went on to talk about Kosambi and many other things at length. I slowly recovered and the rest of the evening spent with him turned out to be quite pleasant.

Sarvadaman Chowla, I expect will be somewhat better known in this part of the country than some of the others I have mentioned. Sarvadaman was born in Cambridge, England where his father Gopal Singh, who was himself a mathematics professor, was visiting in 1907. He completed schooling in Lahore and took his degree from Government College, Lahore, an institution that counts among its alumni, many illustrious names; among them Chowla himself, of course. In 1929, Gopal Singh accompanied his son to Cambridge where Sarvadaman was to pursue his mathematical researches under Littlewood. For young Chowla, the joy of entering the hallowed precincts was marred by the sudden death of his father in Paris on his way back to India. Despite this sad beginning to his doctoral studies, Chowla performed very well in Cambridge and secured his doctorate in 1931. He returned to India the same year to take up a professorship at St. Stephens College, Delhi. There he met and married a student, Himani Mazumdar, who till her death in 1970 was to take good care of all practical matters, much of which Chowla was far from adept at handling. Their only daughter is also a mathematician.



S. Chowla

From Delhi, Chowla moved successively to Banaras Hindu University, Andhra University in Waltair, and finally returned to teach at Government College at Lahore. He fled Lahore with his family in 1947 and after a short stay in Delhi, went to the Institute for Advanced Study in Princeton. He was never to return to India. In 1949, he accepted a Professorship at the University of Kansas at Lawrence; in 1952, he moved to Boulder, Colorado and finally to Pennsylvania State University from where he retired in 1976. He continued to be mathematically active, spending several years in Princeton and later at Lawrence, where he collaborated with a former student Mary Cowles. He died in 1995. Chowla's first paper appeared in 1925 and his last in 1986. Six decades of active mathematics is quite exceptional indeed.

Chowla's enthusiasm for mathematics was unbounded, and he had few outside interests. People who knew him, found him a friendly, good-humoured person who was modest to a fault. He had many students and believed in offering only broad guidance and letting them work largely on their own. He was all the same very successful in transferring to them his own excitement over mathematics. One of his students referred to him as a 'perpetual ambassador for number theory'. His collaborators included many illustrious names; let me mention a few: Bambah, Pillai, Ramachandra and C. R. Rao from India; Erdos, Selberg, Shimura from outside. Chowla's name is identified with a number of deep mathematical results, bearing testimony to his high standing in the field.

Another major figure in Indian mathematics I want to talk about is Minakshisundaram, and I will refer to him as Minakshi, as he was known to his friends. He was born in Trissur, Kerala in 1913. He had his early education in Madras and took his B A (Honours) degree from Loyola College, Madras. He became a research scholar at Madras University under Ananda Rau. This naturally led him to work on Tauberian theorems and he produced work of very high quality in the area. However, in 1937–38, he came once again under the influence of his erstwhile teacher at Loyola College, Rev Father Racine.

I will say more about Minakshi presently, but I want to take time out to speak about Father Racine, a remarkable Frenchman whose contribution to mathematics in India is extraordinary.

Father Racine was born at Tomay-Charente in France in 1897. He enlisted for active service in the First World War in 1916 and was demobilized three years later after an ankle injury that left a limp for the rest of his life. He then entered the Jesuit order and was ordained a priest in 1929. He spent four years studying mathematics in Paris and obtained a doctorate in 1934. He was sent to India to work at St. Joseph's College, Tiruchirappalli. He moved to Loyola College, Madras in 1939 and stayed there till his death in 1976, nine years after his retirement in 1967.

Rev. Fr. C. Racine

Father Racine had worked with Élie Cartan and Hadamard, both legendary figures in mathematics. He counted André Weil and Henri Cartan (another famous mathematician and Élie Cartan's son) among his friends. With this background, Racine naturally had an excellent perspective on mathematics, which he brought to India with him. He began weaning some Indian mathematicians away from traditional Cambridge-inspired areas and Minakshi was his first big success; and there was a galaxy of brilliant students to follow; the list would occupy substantial space in any 'who's who' of Indian mathematics. To mention a few names: K. G. Ramanathan, C. S. Seshadri, M. S. Narasimhan, Raghavan Narasimhan, C. P. Ramanujam.

Father Racine was apparently not an exciting speaker. Students found his classroom lectures difficult to follow. His French accent combined with what amounted to mumbling to the blackboard, made things worse. It was, however, outside the classroom that his influence was decisive. He was remarkably good at spotting talent and then encouraging it. He liked talking informally to his students, especially the talented ones and gave them invaluable advice in their career decisions. Mathematical activity was by no means Father Racine's sole preoccupation. Apparently, he was a spiritual adviser to the Jesuit community of the college and was engaged in resolving personal problems for the Catholic laity around him. The French government conferred on him the coveted 'Legion d'honeur' in 1962. In all the 42 years he spent in India, he made only two trips to France; yet he remained very much a Frenchman. But there can be little doubt that he loved India more than France. I was not privileged to be his student, but remember with pleasure the one long and informal meeting I had with him in the company of my teacher M. S. Narasimhan. He was an excellent example – by no means unique – of the coexistence of the cassock with a lively disposition.

Let me get back to Minakshisundaram. As I said, Minakshi, under the influence of Father Racine moved away from summability – though he retained a love for the subject to the end of his life – into more modern areas; specifically he turned his attention to Differential Equations. He took his D Sc degree at the Madras University in 1940 with a thesis on nonlinear parabolic differential equations. He then found himself hopelessly stranded without a job. Father Racine helped him earn some money by arranging for him to coach a few students, till he was appointed a lecturer in the Department of Mathematical Physics of Andhra University in Waltair.

He was arguably the most gifted Indian mathematician of his generation. His work on the eigen-values of the Laplacian on Compact Riemannian manifolds was of the highest quality and has had a lasting impact. His best work, some of it in collaboration with a Canadian, Pleijel, was carried out in Princeton at the Institute for Advanced Study during 1946–48, when he was visiting there. The Princeton visit materialized, thanks to the efforts of Marshall Harvey Stone, an American mathematician of the first rank, who visited India in 1946 and made more trips later. Stone was also responsible for many other Indians visiting the institute in Princeton. That institute has certainly had a big role – even as Cambridge in an earlier epoch – in shaping Indian mathematics.

S. Minakshisundaram

Minakshisundaram returned to India after a two-year stay in Princeton. Soon after his return, Minakshisundaram was promoted to a professorship at Andhra University. In 1950, he spent a few months at the Tata Institute, which resulted in an excellent book *Typical Means*, co-authored by Chandrasekharan. That year, he also visited Princeton again and the next year too, he was in the US on a short visit. He made yet another trip to the US in 1958 and on his way back to India, he gave one of the prestigious invited half-hour addresses at the International Congress of Mathematicians at Edinburgh.

As years went by, he found the university milieu stifling for creative work. He had problems getting along with some colleagues, whose mathematical credentials were nowhere near his. Minimal contact with really first-rate minds was all that he needed to produce work of quality, but the university was unable to provide him that. He was eventually appointed professor at the then newly formed Institute of Advanced Study in Simla and was very happy to move there – there was isolation still, but at least in other respects the prospects were pleasanter. He embarked on writing a book on Spectral Theory, but unfortunately died in 1968 without completing it. Minakshisundaram's work of the forties came to the fore again around the time of his death and was an important component in a new approach to the Atiyah–Singer Index theorem – one of the great theorems of the 20th century. Another Indian, V. K. Patodi was destined to play a big role in this development.

M. H. Stone

I now move on to Harish-Chandra, the greatest Indian mathematician since Ramanujan.

Harish-Chandra (Harish to his friends) was born in October 1923 in Kanpur. His father Chandrakishore Mehrotra was an engineer with the Railways. After schooling in Kanpur, he went to Allahabad to pursue higher studies in physics. There, he came under the influence of

K. S. Krishnan. Harish held Krishnan in great affection and esteem throughout his life. He was a brilliant student and in the final examinations secured 100% marks in one of the papers – the examiner was C. V. Raman. After taking his Masters degree, he went to Bangalore to pursue research in physics with Homi Bhabha; and Harish has some joint papers with Bhabha from that period. In Bangalore, he made acquaintance with Raman, who apparently enjoyed taking long walks with the young man from Allahabad. He took up lodgings with the Kales – G. T. Kale was librarian at the Institute of Science and his wife, a Pole, had been Harish's French teacher in Allahabad. Lalitha, Kales' daughter, would apparently attempt to distract the serious student with her lively pranks. She was to become his wife some years later and pampered him through life, not letting practical everyday matters distract him from his work.

Harish–Chandra

Homi Bhabha arranged for him to go to Cambridge, to work with the legendary Dirac. While in Cambridge, he made a trip to Europe visiting, among other places, Zurich. At Zurich, he was at a lecture by the formidable Wolfgang Pauli and had the gumption to point out that the great man was wrong on some point in the lecture. Instances like this enhanced further his credentials as a scientist, already established by his scientific work.

Dirac moved to the Institute for Advanced Study in Princeton for a year-long visit and he took Harish-Chandra along with him. Harish's physics was highly mathematical and motivated him to study infinite dimensional representations of certain special Lie groups.

In Princeton, he met Chevalley, a great French mathematician – one of the Bourbaki group – and interaction with Chevalley led to his abandoning physics altogether and taking up mathematics. He was apparently not entirely comfortable with physics, even as he admired physicists for their 'sixth sense'. Nevertheless, his mathematical work did owe much to his physics background. Infinite dimensional representation theory of Lie groups which he embarked on, was by no means a central area of mathematics in the early fifties. On the other hand, quantum mechanics had led physicists to study infinite dimensional representations of the Lorentz group. Over a decade of intense single-minded devotion to the subject Harish-Chandra, with his powerful ideas, almost single-handedly brought the subject of representation theory from the periphery of mathematics to centre stage.

Harish-Chandra moved to Columbia University after his stint as a visiting member at the Institute for Advanced Study. He was prolific and effected the transformation of the representation theory of Lie groups, that I mentioned just now, during the Columbia years. In 1955–56 he took a year off from Columbia to visit the Institute for Advanced Study again. He spent 1957–58 in Paris on a Guggenheim Fellowship. Harish-Chandra found many exciting things happening in Paris and wanted to continue there for another year. Columbia's Dean however expressed his displeasure over the request for an extension of the stay in Paris, coming as it did so soon after two leaves of absence so close together. Harish sought advice from André Weil, a good friend, who happened to be in Paris at that time. Weil, for whom dean-baiting was a welcome diversion, told Harish-Chandra to not only insist on being given leave from Columbia, but demand a pay-raise as well. Weil told Harish that he was too good a mathematician for Columbia to afford losing him. Harish-Chandra took Weil's advice and found that Columbia had the good sense to retain him on his terms!

Harish-Chandra was a serious candidate for the Fields Medal in 1958. There may be some people among the readers who do not know what the Fields Medal is. Let me say a few words about it. The medal, named after a Canadian mathematician who left a small legacy for it, is regarded by the mathematical community as the highest form of recognition extended to achievement in mathematics. Up to four medals are awarded at the International Congress of Mathematicians which takes place once in four years. It is considered the equivalent of the Nobel in prestige (the Nobel is not available for mathematics). At 10,000 Swiss Francs, the cash award that goes with the medal is a pittance though, compared to the Nobel. There is another important aspect in which the Fields Medal is different from the Nobel Prize: the prize is given only to a mathematician who is under 40 at the time of the International Congress.

C. Chevalley

As I said, Harish-Chandra was a serious candidate for the medal and the grapevine has it that he was passed up only as result of the intellectual prejudices of the Committee's chair. That year, two medals were awarded, one to the British mathematician, Roth and the other to the Frenchman Thom. Carl Ludwig Siegel, the Chairman of the committee was undoubtedly one of the great mathematicians of the 20th century, but he was intolerant of mathematical styles and philosophy different from his own; and in his reckoning Harish-Chandra's mathematics was of the degenerate Bourbaki kind, of which he disapproved. Ironically enough, in the opinion of the Bourbakis themselves, Harish-Chandra was the true successor of Siegel as a mathematician. Harish-Chandra not getting the Fields Medal is no reflection on his mathematics; on the other hand, it shows how problematic it is to devise good mechanisms for deciding awards and prizes.

In 1963, Harish-Chandra was invited to become a permanent member of the Institute of Advanced Study in Princeton and in 1968, he was named IBM von

We are all familiar with the real number system. Now, mathematicians have invented a whole lot of other number systems known as p-adic systems – one, in fact, for each prime p. We know that real numbers can be expressed as decimals, the decimal expansion extending possibly to infinitely many terms involving negative powers of 10. With p-adic numbers it goes the reverse way: every p-adic number has an expansion in powers of the prime p; now the terms involving positive powers of p possibly extending to infinity. In the p-adic system, higher and higher positive powers of p get closer and closer to zero! At first encounter one finds them quite weird, but eventually the discomfort disappears with familiarity. As there are infinitely many primes, there are infinitely many such number systems. They are of great interest in number theory. They behave very differently from the real number system, but resemble each other in many ways. So, in a way, the real number system is the odd one out, even though it is the only one which we can visualize and corresponds closely to our intuition; the p-adic systems are, if anything, counter-intuitive. That is the background to be kept in mind.

On one occasion, I was at a lecture at the Princeton Institute on p-adic Lie groups. The speaker was constantly referring to 'compact unipotent groups'. At some point, a member of the audience interrupted with 'I do not understand – compact unipotent groups are trivial'. Weil who was also in the audience reacted immediately with, 'Oh, Harish always thinks of the pathological case of real numbers!' and the audience burst out laughing. Harish was Harish-Chandra, the foremost expert on the representation theory of real Lie groups and, beyond doubt, the greatest Indian mathematician since Ramanujan.

Two years after that episode, which clearly indicated his lack of familiarity with p-adic groups, Harish-Chandra was at the forefront of research in representation theory of p-adic groups!

Neumann Professor of Mathematics. He was a Fellow of both the Indian Academy of Sciences and the Indian National Science Academy. He was elected to the Royal Society in 1973 and later to other academies as well. He gave invited plenary addresses at the International Congress of Mathematicians in Amsterdam in 1954 and in Moscow in 1966. For a man of his accomplishments, he received relatively few awards and honours though.

I had the privilege of a little personal acquaintance with Harish-Chandra. During the year I spent in Princeton, I met and talked to him several times. Unfortunately, I never interacted with him mathematically, but had numerous interesting conversations about mathematics. He made a strong impression on me and came through as an intense person with an austere outlook. He had strong views on mathematics and the mathematical scene in India, and was forthright in expressing them. After that year in Princeton, my meetings with him were far and few between. In October 1983, I was again in Princeton to participate in a conference to honour Armand Borel (the only mathematician who has collaborated with Harish-Chandra and a colleague of his at the Institute for Advanced Study). On the last day of the conference, I was at a luncheon party hosted by Harish-Chandra and his gracious wife (Lalitha had become Lily in Princeton) – he had turned sixty just the previous week. Late that evening, I was to learn that Harish-Chandra who had been at his lively best in the afternoon had died early in the evening. The week that began as a celebration of an outstanding mathematical intellect ended sadly, mourning the departure of another.

So far I have talked about men who were considerably senior to me in age. I now come to two outstanding names who were my contemporaries: C. P. Ramanujam and Vijay Kumar Patodi. When I joined TIFR as a student in 1960, Ramanujam was already there, still a student, yet to take his degree. He had, however, a formidable reputation as a versatile and deep scholar and an original mind. Expectations from him were such that there was a mild air of disappointment that he had not yet (at 22) come out with some creative work. He himself seemed to be in doubt about his abilities. But, 1962 set at rest all anxiety on this count. He came up with a very nice result on cubic forms over algebraic number fields. And shortly thereafter, he gave a brilliant and remarkably simple proof of a much sought after result related to the Waring problem which I mentioned in connection with Pillai. It was a question that had interested Siegel greatly. After these initial forays into number theory, Ramanujam moved into algebraic geometry, where his contributions were again highly original and are valued greatly.

Many of us, his fellow students learnt a great deal of mathematics from Ramanujam and some of that in areas quite different from his own research interests. His scholarship was stupendous and his enthusiasm for mathematics was infectious. Learning from him was an exhilarating experience. He kept odd hours and many of our learning sessions with him would go late into the night. Ramanujam was widely read outside mathematics as well.

Ramanujam was without doubt one of the finest mathematical intellects to come out of India, but he was far from satisfied with his own achievements. Unfortunately, he came to be tormented as early as 1964 by an illness which was diagnosed as schizophrenia, with severe depression which interfered badly with his work. The malady would surface periodically, but in between, in periods of lucidity he would come up with beautiful results. Eventually, he ended his life in October 1974 during one of his bouts of depression. He was 36.

C. P. Ramanujam

Ramanujam was mostly at TIFR during his short career, but visited several other mathematical centres. In fact, he took up a Professorship at Chandigarh in 1965, intending to stay there permanently. He was immensely liked by his colleagues and reciprocated their warmth, but illness struck and he left Chandigarh after just eight months. A six-month visit to France on an invitation from the prestigious Institute des Hautes Étude Scientifiques had also to be cut short for similar reasons. He later spent some time at Warwick University in England and also at the University of Genoa in Italy. His colleagues in Genoa have fond remembrances of him and a lecture hall in the Mathematics Department there was named after him after his death. At his request, he was relocated in Bangalore by TIFR and he spent the last year of his life there.

Just two years after the loss of Ramanujam, in December 1976, Indian mathematics suffered another big blow. Vijay Kumar Patodi, a star still on the rise was cruelly cut-off at the age of thirty-one. Patodi graduated from Banaras Hindu University in 1966 and after a year as a Research Scholar at Bombay University joined TIFR, to continue working for a doctorate. He had been taking courses at TIFR even while he was student at the university and it was obvious to his teachers that he was exceptional. Patodi absorbed rapidly a lot of deep and difficult mathematics. Towards the end of 1969, he came across a paper of McKean and Singer and quietly started working on a conjecture stated in the paper. The paper made use of the profound work of Minakshisundaram on the Laplacian on compact manifolds. The main thrust of the conjecture was to offer an alternative approach to the classical Gauss–Bonnet theorem. A few months later, Patodi settled the conjecture and the announcement came as a total surprise to everyone at TIFR. The paper appeared in 1971 in the *Journal of Differential Geometry*, and it was instantly big news. Patodi received many invitations and visited Singer at MIT and later Atiyah at the Institute for Advanced Study. He developed further the ideas and techniques of his first paper, to obtain many interesting results; and all this culminated in a collaboration with Atiyah and Bott, where they were able to give a proof of the famous Atiyah–Singer index theorem by the heat equation techniques suggested in the McKean–Singer paper.

V. K. Patodi

In the midst of his great professional success, there was unfortunately terrible news. He was informed by his doctor that he had a serious medical condition and the prognosis was far from optimistic. Even while fighting hard to keep his illness at bay, Patodi kept at his mathematics and was producing excellent work. The end came in December 1976, shortly before a planned kidney transplant. In the short period of six years, despite frequent interruptions because of his illness, Patodi produced eleven papers of superb quality. Some of these papers appeared in print, posthumously.

K. Chandrasekharan

All the people I have spoken about so far are no more. The next person I am going to talk about, K. Chandrasekharan, is living – in retirement in Zurich, Switzerland. He was born in 1920 and was initiated into mathematics by – who else – Ananda Rau in Madras. He went to the Institute for Advanced Study in Princeton to do postdoctoral work. Chandrasekharan worked in number theory and summability – like many others of his generation. His mathematical achievements are of the first rank, but his even greater contribution to Indian mathematics, I think, lies elsewhere. He was an extraordinarily gifted organizer and administrator of science – in the Bradman class, if we use Hardy's terminology. Homi Bhabha visited Princeton in 1949 when Chandrasekharan was there and offered him a position at the Tata Institute of Fundamental Research. There is a story about this which I have not attempted to authenticate, but it rings true.

Nehru and KC

Chandrasekharan was taking a walk with the great von Neumann when they saw Bhabha walking with Einstein at a distance. von Neumann asked Chandrasekharan if it was true that he was planning to move to Bombay to work at Bhabha's institute. When Chandrasekharan responded in the affirmative, von Neumann said, 'That man is as good a physicist as any, but do not let him intimidate you – stand up to him', or words to that effect. It would appear that KC as Chandrasekharan was known, followed that advice – differences of opinion with Bhabha seem to have been among the causes that led to his leaving TIFR in 1965 and move to Zurich.

I should like to tell you another anecdote relating to KC, of which I have first-hand knowledge. This again goes back to my Princeton visit in 1966–67. I once accompanied a friend with expensive tastes to a clothing shop in Princeton. My friend ordered a suit for himself, while I acquired a scarf (which cost me 16 dollars – in 1966, mind you!). The shopkeeper who kept up a conversation right through asked us if we knew, von Neumann and we said that we knew of him. Then he asked us if we knew Chandrasekaran and we told him that we did indeed know him; at which point he said, 'They are the only two gentlemen from that Institute who knew how to dress!'

In the decade and a half that he spent at TIFR, Chandrasekharan transformed the fledgling School of Mathematics there into a centre of excellence, respected the world over. Chandrasekharan initiated a very successful programme of recruitment and training of Research Scholars at TIFR; the programme continues to this day, along the same lines that he set down. He was uncompromising in insisting on high standards. In his Princeton days, he became acquainted with many of the leading mathematicians of the world and put these contacts to excellent use. Herman Weyl gifted his collection of *Mathematische Annalen* volumes to the TIFR, thanks to Chandrasekharan. With his unusual abilities, he was able to persuade many leading mathematical figures to visit the Tata Institute and deliver courses of lectures over a period of two months and more, to his carefully selected Research Scholars. Among the many distinguished men who visited the Tata Institute in the fifties, two names stand out: L. Schwartz (a Fields medallist) and C. L. Siegel. They both had tremendous influence on the way mathematics evolved at TIFR. Schwartz persuaded many of his colleagues and students to visit the Tata Institute. Research Scholars were asked to write notes for the lectures given and these lecture notes enjoy a great reputation in the mathematical community to this day.

Chandrasekharan was an excellent judge of mathematics even in areas outside his own specialization, and responded quickly to the achievements of his wards. Equally, non-performance at the high level he had set had no place at TIFR. Chandrasekharan managed to instill in the students at TIFR, strong commitment to hard work without their losing their romantic attachment to mathematics. One important reason for his success was the freedom he gave the students to work on what they pleased. The visitors gave them exposure to different mathematical areas, many of them far removed from KC's own interests, and students were encouraged to pursue whatever caught their fancy. Rev Father Racine from Madras whom Chandrasekharan knew from his Madras days and whom he held in great respect, provided him with a steady stream of talent.

Chandrasekharan was ably assisted in his endeavours by K. G. Ramanathan. A student of Racine's, Ramanathan too was in Princeton for postdoctoral work. There, he came under the influence of E. Artin (another of 20th century's major figures) and even more, Siegel. Through him, many at TIFR were exposed to some of Siegel's profound works, and this is very much reflected in the work that came out of TIFR in the sixties and seventies. Ramanathan was also responsible (during the seventies and eighties) for building a small but highly competent group in Partial Differential Equations – an area of great importance in applications. The group was located in the campus of the Indian Institute of Science in Bangalore, with a view to foster interaction with that institution. Ramanathan passed away in 1992.

Chandrasekharan's influence went well beyond Indian mathematics. For some 24 years from the mid-fifties, he was a member of the Executive Committee of the International Mathematical Union (IMU). He also served as the Secretary for two terms and as President for one term. His initiatives on this committee were numerous and valued greatly. He was responsible for the IMU sponsoring the International Mathematical Colloquium held once every four years at the Tata Institute starting 1956. These meetings have been on diverse topics in mathematics; topics that are of current interest internationally and to which Indian mathematicians have contributed substantially. They have been a great success over the years and an invitation to the meeting is considered prestigious.

I should like to mention a personal experience in this connection. In 1964, Tata Institute held a Colloquium on 'Differential Analysis' and the organizing committee headed by Chandrasekharan extended an invitation to me to give a talk. A few weeks before the colloquium, I was told that I should rehearse my lecture before KC in his office. My teacher Narasimhan was also present at the rehearsal. Chandrasekharan's own mathematical interests had little to do with the subject of my talk; nevertheless, he listened to me patiently for more than an hour, interjecting now and then to tell me how I should present something and generally giving me tips on lecturing. I had in those days, a reputation as a poor speaker (which I hope does not hold now) but as it turned out, thanks to Chandrasekharan, I gave a lecture that was received very well indeed.

In the fifties, Chandrasekharan held the editorship of the *Journal of the Indian Mathematical Society*. During this period, several great papers appeared in the journal thanks to Chandrasekharan's abilities at persuading some of the great names in the field to publish there.

We at the Tata Institute certainly owe a great deal to Chandrasekharan and are grateful for the great start he gave us.

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