

An Asymmetric Gravitational Wave Propulsion System

Jeffery A. Cameron

Transdimensional Technologies, Inc., Huntsville, Alabama 35801

Abstract

Gravitational wave radiation is generated by the quadrupole moment of matter that is in motion. An analytical model is constructed to investigate the radiation pattern and radiated power of a single resonant vibrator cylinder, as compared to that of a phased linear array. The linear array is then evaluated in terms of phase relationship in order to create an asymmetry in the radiated pattern and hence a directional force. This is compared to the force of a solar sail in the earth orbital plane.

Introduction

Einstein¹ derived the weak-field solution of the gravitational wave in accordance with the general and special theories of relativity. This paper uses this scientific background to describe a revolutionary new approach to propulsion. Programmable laser diodes in conjunction with semiconductor materials will be used to generate a highly directional transverse wave gravitational wave (TWGW) radiator. This asymmetric TWGW radiator will create a directional force through the center of mass of the radiating system, thus forming a propellantless propulsion system.

Theoretical work indicates the ability to generate gravitational wave radiation through the quadrupole moment of matter under stress and strain. The radiation pattern is symmetric about the center of mass, and the direction of the pattern is at right angles to the stress and strain vector. The radiation pattern looks like a torus. The radiated power is very small (10^{-29} watts); however, when a linear sequence of radiators is put together and their patterns are allowed to superimpose, the total radiated power can approach kilowatts. The asymmetry of the radiated pattern is produced by adjusting the phase of the radiators. The resulting power imbalance will produce a force through the center of mass of the radiators. In order to accomplish this task, high peak power laser diodes will be used to photo-acoustically drive thin-film resonators. It becomes imperative to understand the energy flow between the laser diode driving the resonator and the result in gravitational radiation from the radiator.

Theory

In order to establish an analytical background consider the following. Let $\eta_{\mu\nu}$ be a Lorentz

metric, then the Riemannian metric is expressed as

$$g_{\mu\nu} = \eta_{\mu\nu} \sqrt{K} h_{\mu\nu}$$

as a first approximation under the weak-field assumptions, K is Einstein's constant. The potential of the field can be expressed as

$$\phi_{uv} = h_{uv} - \frac{1}{2} \eta_{uv} h$$

resulting in the form for 4-space

$$\square \phi^{a\beta} = 2\sqrt{K} T^{a\beta}$$

with retarded potential solutions of the form

$$\phi^{uv}(x) = (\sqrt{K} / 2\pi) \int T^{uv}(x^\ell, x^0)$$

This form will enable the definition of the energy-momentum complex of the gravitational field in order to evaluate the radiation energy and directivity of the gravitational wave.

The Poynting vector of the gravitational wave can be expressed as

$$U_o^l = \frac{1}{8} (-2\phi^{\rho\sigma,l} \phi_{\rho\sigma,o} + \phi^l \phi_{,o})$$

The derivatives of the potential fields ϕ with respect to time and space coordinates are expressed by the second and third derivatives of the mass tensor.

$$mc^2 \equiv \iiint T^{00}(x', x_G^0) d^3 x'$$

momentum tensor,

$$cP^K = \delta_0 x^{K'} \iiint T^{00}(x', x_G^0) d^3 x'$$

and the quadruple moment tensor,

$$I^{iK} = x^i x^{K'} \iiint T_{00}(x', x_G^0) d^3 x'$$

The derivative forms for the potential fields are substituted into the expression for the Poynting vector, thus giving the expression for the radiated energy per unit time, or power, within a solid angle $d\Omega$, as follows.

$$P_o^\ell = U_o^\ell d\sigma_\ell = \frac{1}{8} \left(\frac{\sqrt{K}}{4\pi} \right)^2 f(\theta, \phi, x_G^o) d\Omega$$

Here the factor

$$f(\theta, \phi, x_G^o)$$

represents the directivity of the gravitational wave radiator.

Of particular interest for this paper is the resonance vibrator, conceptually similar to what Weber^{2,3,4} used as a gravitational wave detector. The resonance vibrator is a cylinder (Figure 1), which is placed under stress and strain.

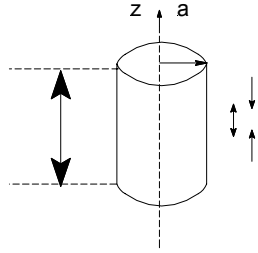


Figure 1: Resonant Vibrator Cylinder

At this point assume the following dynamic variables:

$$\text{Displacement } \varepsilon = A_1 \sin\left(\frac{\omega}{V_s}\right) \cos \omega t$$

$$\text{Particle velocity } v = \frac{d\xi}{dt} = -V_p \sin\left(\frac{\omega}{V_s}\right) z \sin \omega t$$

$$\text{Strain } \varepsilon = \frac{d\xi}{dz} = \frac{V_p}{V_s} \cos\left(\frac{\omega}{V_s}\right) z \cos \omega t$$

$$V_p = A_1 \omega \quad V_2 = \left(\frac{B_s}{\rho}\right)^{\frac{1}{2}}$$

$B_s \rightarrow$ Young's Modulus

The directivity can now be expressed as follows (Figure 2).

$$f \sim \sin^4 \theta$$

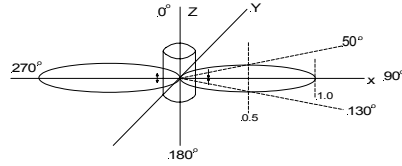


Figure 2: Resonant Vibrator/Gravitational Wave Radiation

The pattern resembles a torus or "donut" mode.

It can be seen that the maximum radiation occurs in the plane perpendicular to the vibrating z axis (Figure 2).

Consider a linear array of resonant vibrators (Figure 3).

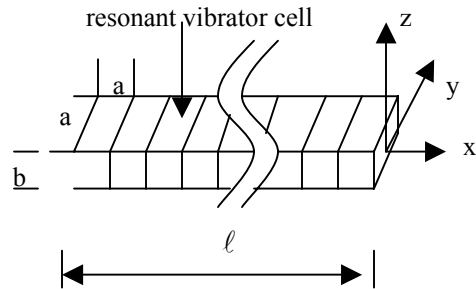


Figure 3: Resonant Vibrator Linear Array

Let high peak power laser radiation from laser diodes be injected along the z axis to induce acoustic stress in the material (Figure 4).

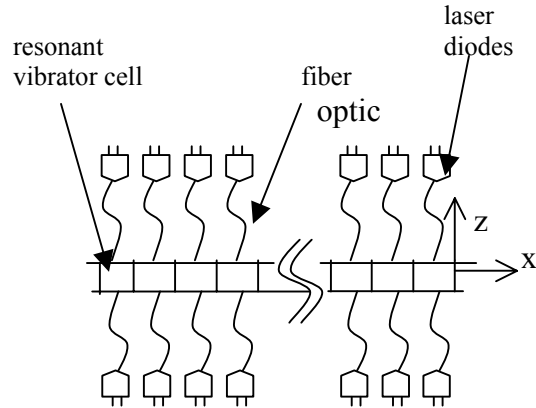


Figure 4: Laser Diodes and Linear Array

The stresses will generate a weak gravitational wave along the x axis. The gravitational wave generated from a number of "cells" along the x

axis can be added in phase. The resultant gravitational "beam" along the x axis is extremely intensified compared to a single resonant vibrator "cell." The linear arrangement will be referred to as a traveling wave (TW)gravitational wave (GW) radiator, or TWGW radiator.

The directivity of the TWGW radiator can be expressed as (Figure 5)

$$f \approx \left(\frac{\sin\left(\frac{\pi\omega}{V_s}(1 - \cos\Theta)\right)}{\left(\frac{\pi\omega}{V_s}\right)(1 - \cos\Theta)} \right)^2 \cos^4 \Theta$$

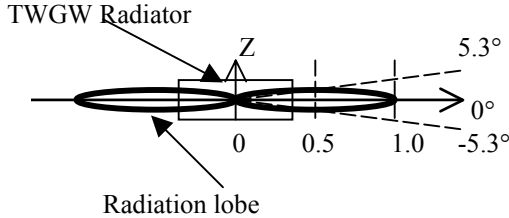


Figure 5: TWGW Radiator Symmetric Radiation Pattern

The graphical result of Figure 5 is based on the geometry of Figure 3 and the constants as follows.

Material: Quartz

$$b = 3 \times 10^{-5} \text{ m}$$

$$a = 3 \times 10^{-3} \text{ m}$$

$$\ell = 0.3 \text{ m}$$

$$\omega = 6.28 \times 10^{12} \text{ radian / sec}$$

(resonant frequency)

$$V_s = 5 \times 10^3 \text{ m / sec (wave speed)}$$

$$\rho_o = 2650 \text{ kg / m}^3 \text{ (density)}$$

As can be seen in Figure 5, the TWGW radiation is very directional. This brings to question what type of power levels could be estimated. To begin, consider the single resonant vibrator cell (Figures 2 and 3). The radiated power can be estimated by the following expression.

$$P_{RV} = (5.7 \times 10^{-2}) G \rho_o^2 \pi^2 a^4 e_m^2 V_s \left(\frac{V_s}{C}\right)^5$$

Where

$$G = 6.67 \times 10^{-11} \frac{\text{Nt} \cdot \text{m}^2}{\text{kg}}$$

(Gravitational Constant)

$$\varepsilon_M = 4 \times 10^{-3} \text{ (Strain)}$$

Then

$$P_{RV} = 2.14 \times 10^{-29} \text{ watt}$$

As anticipated, this is a very small number. However, for the TWGW radiator, the radiated power can be estimated by the following expression.

$$P_{TW} = (7 \times 10^{-5}) G \rho_o^2 a^2 \ell^2 e_m^2 V_s \left(\frac{V_s}{C}\right)^5 \left(\frac{b\omega}{V_s}\right)^6 \Omega$$

Where

$$\Omega(\pm 5.3^\circ) = 2.7 \times 10^{-2} \text{ sr}$$

(Solid angle of the radiation)

Then

$$P_{TW} = 1.66 \times 10^3 \text{ watt}$$

This shows how important the phase relationships between individual resonant vibrators are!

At this point it is important to realize that the radiated gravitational wave carries energy and momentum with it. This is expressed as

$$P = \frac{\varepsilon}{C} = \frac{Pt}{C}$$

Where \underline{P} is the radiated power, t is time, ε is the energy of the gravitational wave, and P is the momentum.

The resulting reaction force on the TWGW structure is expressed as

$$F = \frac{P}{t} = \frac{P}{C}$$

Referring to Figure 5, it can be seen that the radiation pattern is symmetric about the center of the TWGW structure. Therefore any reaction force is balanced! However, consider a variation in the phasing of the laser diodes (Figure 4), where the lobes become asymmetric (Figure 6).

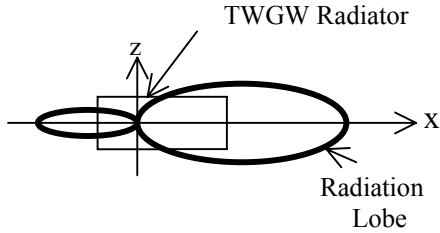


Figure 6: TWGW Radiator Asymmetric Radiation Pattern

This can be accomplished by pulse timing, variation in pulse rate, laser diode intensity, alternate materials, and geometry. Here there is a net force in the direction of the least intense gravitational wave (Figure 7).

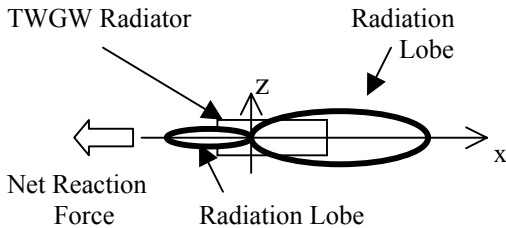


Figure 7: Reaction Force for Asymmetric Radiator TWGW

The reaction force is expressed as

$$F_{net} = F_{RL1} - F_{RL2}$$

$$= \frac{P_{RL1} - P_{RL2}}{C}$$

As an example, let $P_{RL1} = 500$ watt and $P_{RL2} = 3.5 \times 10^3$ watt, then

$$F_{net} = 10 \times 10^{-6} \text{ Nt}$$

The net force, although small, raises the possibility of a propellantless propulsion concept, utilizing current technologies.

Further Investigations

Mathematical analysis indicates that gravitational wave propulsion is possible. Further investigation will entail the following objectives.

- 1) Investigate the efficiency of converting laser light into acoustic stress within TWGW generating material candidates. The acoustic stresses are responsible for generating the quadrupole moment

needed to give rise to TWGW radiation. It will be imperative to address questions on how material uniformity, temperature, geometry, laser pulse width, repetition rate, and wavelength will impact performance.

- 2) Investigate phasing techniques of the TWGW elements in order to create various asymmetries in the radiating pattern. The directivity and intensity of the TWGW radiator is key to the success of generating a net propulsive force. How energy and momentum transports are affected by individual radiator phasing are of utmost importance. The question of whether harmonics distribute energy into other "modes" or are negligible must be considered.

The TWGW system must be analytically modeled as part of the above investigations. The results will lead to study of a test article that will be used to demonstrate the use of the gravitational wave Poynting vector imbalance as a means of generating a propulsive force through the center of mass of the TWGW system. Conceptually a scale version of the system could provide propellantless propulsion into the outer regions of the solar system and/or orbital transfer missions.

Space Flight Application

An interesting example is a comparison of the TWGW system to that of a solar sail at the earth orbital plane. Let the TWGW system be 1000 meters in length. The resulting net propulsive force would be about 61 Nt. For a square solar sail with a perfect reflectivity experiencing a solar flux of 1.3×10^3 watts/m², the required area would be around 1.4×10^7 m² for a force of 61 Nt. This is a perimeter length of approximately 3.7×10^3 m and must be normal to the solar disk to experience the maximum momentum transfer. Also, as the sail increases its distance from the solar disk, the intercepted flux decreases with the square of the distance. The TWGW system is not dependent on the solar disk for operation.

Conclusion

An analytical model has been created to investigate the nature of the radiated TWGW pattern with respect to phase relationships between individual "cell" radiators. Future

models will investigate the magnitude of side lobes or harmonics to determine whether asymmetries cause less than desirable effects. These results will be used for evaluation of the generation of a net propulsive force.

References

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Acknowledgements

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