Passive Error Correction: Trapped Ion Qubits

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and Department of Physics

FOCUS

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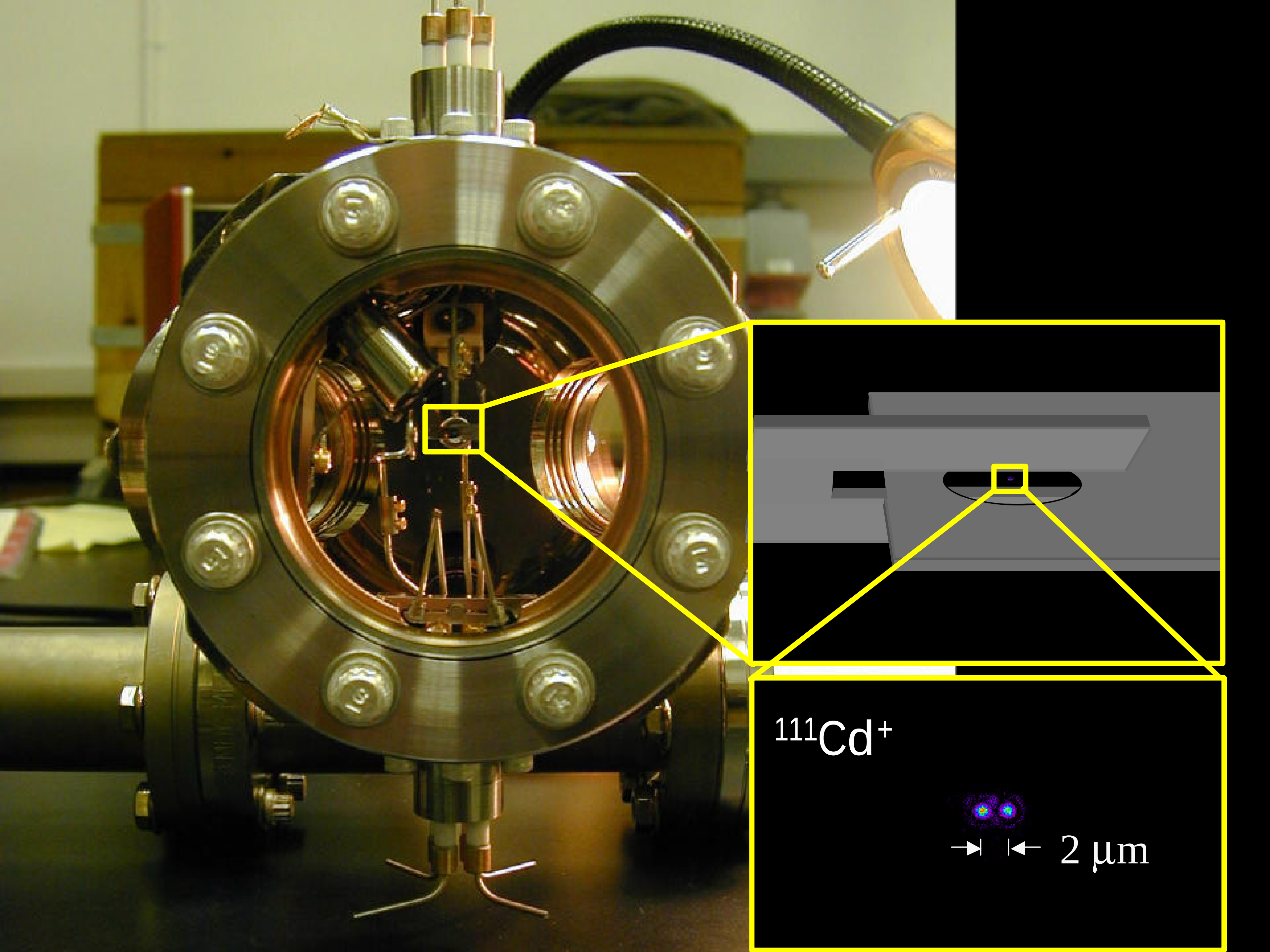
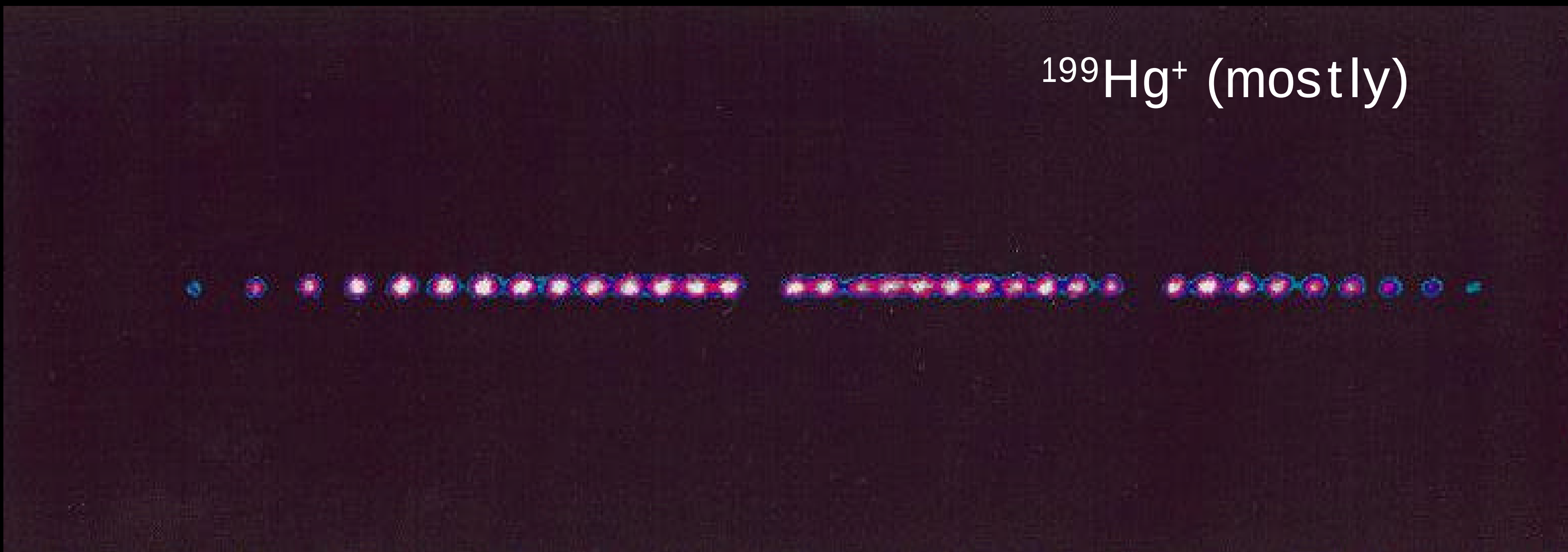
ARDA US Advanced Research and Development Activity

US Army Research Office

US National Security Agency

National Science Foundation

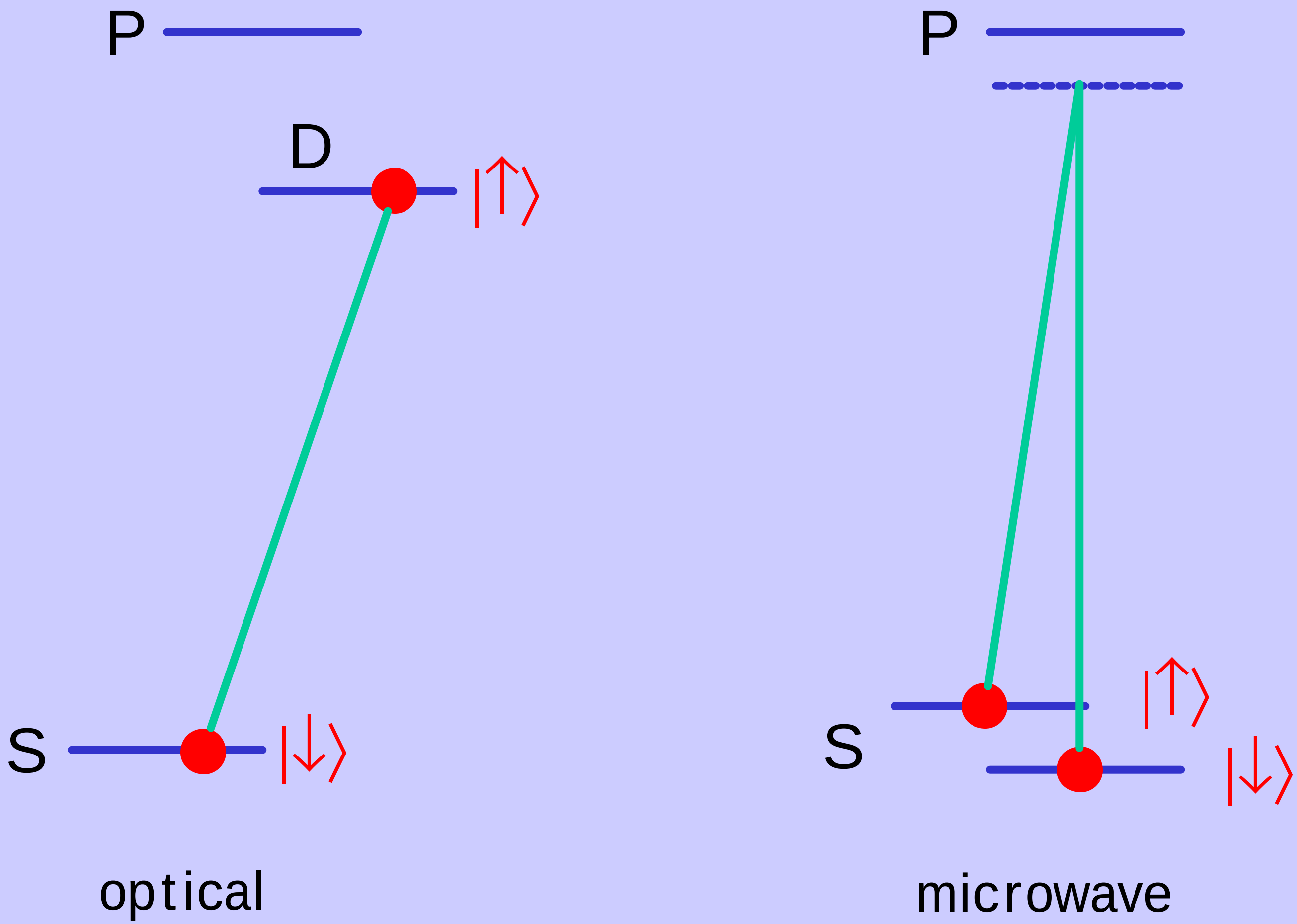

$^{111}Cd^{+}$
2 μm

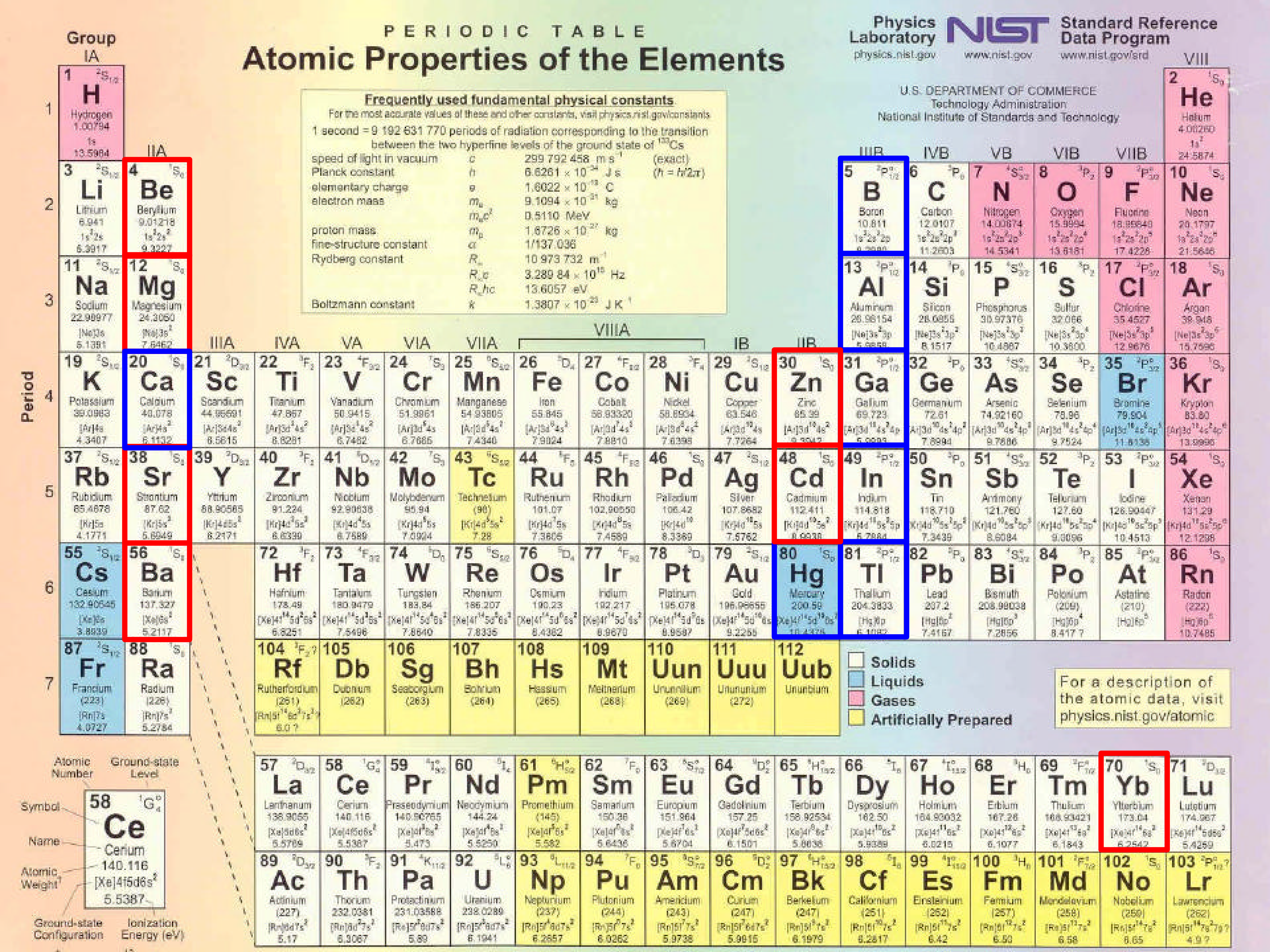


0.3 mm

Bergquist, Itano, Wineland (NIST-Boulder)

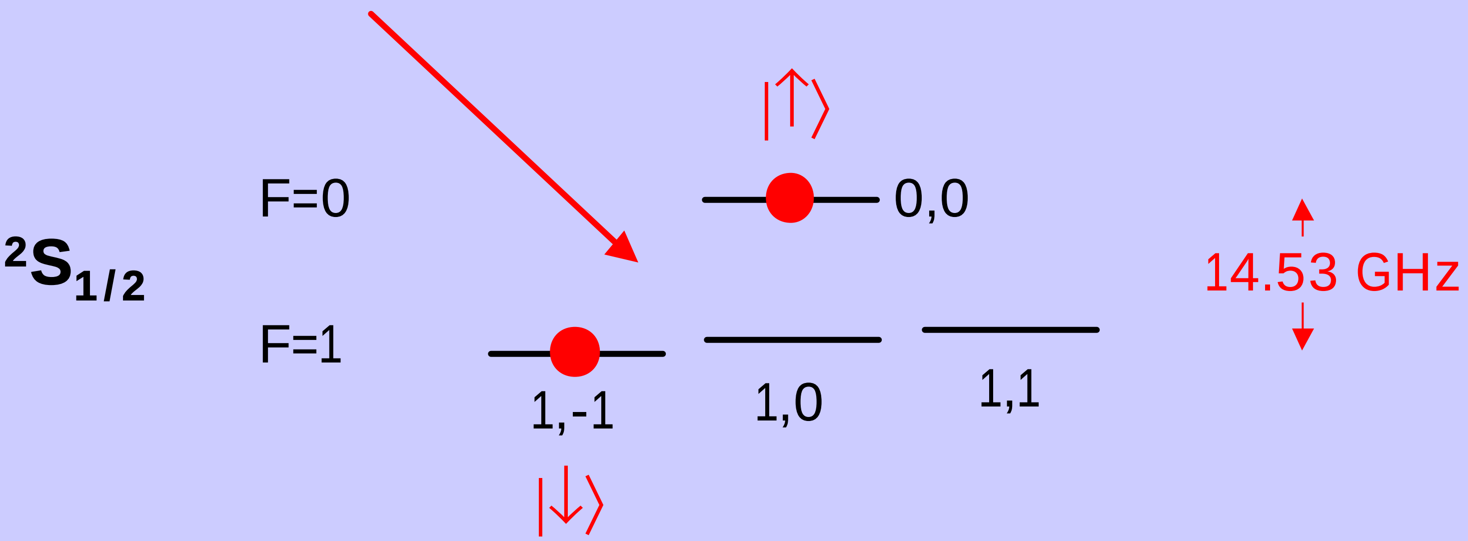
Good two-level systems




Physics Laboratory
physics.nist.gov
NIST
www.nist.gov
Standard Reference Data Program
www.nist.gov/srd
PERIODIC TABLE
Atomic Properties of the Elements
U.S. DEPARTMENT OF COMMERCE
Technology Administration
National Institute of Standards and Technology
Frequently used fundamental physical constants
For the most accurate values of these and other constants, visit physics.nist.gov/constants
1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of 133Cs
speed of light in vacuum c 299 792 458 m s-1 (exact)
Planck constant h 6.6261 × 10-34 J s (ħ = h/2π)
elementary charge e 1.6022 × 10-19 C
electron mass me 9.1094 × 10-31 kg
mec2 0.5110 MeV
proton mass mp 1.6726 × 10-27 kg
fine-structure constant α 1/137.036
Rydberg constant R∞ 10 973 732 m-1
R∞c 3.289 84 × 1015 Hz
R∞hc 13.6057 eV
Boltzmann constant k 1.3807 × 10-23 J K-1
Group
IA
IIA
IIIA
IVA
VA
VIA
VIIA
VIIIA
IB
IIB
IIIB
IVB
VB
VIB
VIIB
VIII
Period
1
2
3
4
5
6
7
1 H Hydrogen 1.00794
2 He Helium 4.002602
3 Li Lithium 6.941
4 Be Beryllium 9.012182
5 B Boron 10.811
6 C Carbon 12.0107
7 N Nitrogen 14.00674
8 O Oxygen 15.9994
9 F Fluorine 18.9984032
10 Ne Neon 20.1797
11 Na Sodium 22.989770
12 Mg Magnesium 24.3050
13 Al Aluminum 26.981538
14 Si Silicon 28.0855
15 P Phosphorus 30.973761
16 S Sulfur 32.066
17 Cl Chlorine 35.4527
18 Ar Argon 39.948
19 K Potassium 39.0983
20 Ca Calcium 40.078
21 Sc Scandium 44.955910
22 Ti Titanium 47.867
23 V Vanadium 50.9415
24 Cr Chromium 51.9961
25 Mn Manganese 54.938049
26 Fe Iron 55.845
27 Co Cobalt 58.933200
28 Ni Nickel 58.6934
29 Cu Copper 63.546
30 Zn Zinc 65.39
31 Ga Gallium 69.723
32 Ge Germanium 72.61
33 As Arsenic 74.92160
34 Se Selenium 78.96
35 Br Bromine 79.904
36 Kr Krypton 83.80
37 Rb Rubidium 85.4678
38 Sr Strontium 87.62
39 Y Yttrium 88.90585
40 Zr Zirconium 91.224
41 Nb Niobium 92.90638
42 Mo Molybdenum 95.94
43 Tc Technetium (98)
44 Ru Ruthenium 101.07
45 Rh Rhodium 102.90550
46 Pd Palladium 106.42
47 Ag Silver 107.8682
48 Cd Cadmium 112.411
49 In Indium 114.818
50 Sn Tin 118.710
51 Sb Antimony 121.760
52 Te Tellurium 127.60
53 I Iodine 126.90447
54 Xe Xenon 131.29
55 Cs Cesium 132.90545
56 Ba Barium 137.327
72 Hf Hafnium 178.49
73 Ta Tantalum 180.9479
74 W Tungsten 183.84
75 Re Rhenium 186.207
76 Os Osmium 190.23
77 Ir Iridium 192.217
78 Pt Platinum 195.078
79 Au Gold 196.96655
80 Hg Mercury 200.59
81 Tl Thallium 204.3833
82 Pb Lead 207.2
83 Bi Bismuth 208.98038
84 Po Polonium (209)
85 At Astatine (210)
86 Rn Radon (222)
87 Fr Francium (223)
88 Ra Radium (226)
104 Rf Rutherfordium (261)
105 Db Dubnium (262)
106 Sg Seaborgium (263)
107 Bh Bohrium (264)
108 Hs Hassium (265)
109 Mt Meitnerium (268)
110 Uun Ununnilium (269)
111 Uuu Unununium (272)
112 Uub Ununbium
Solids
Liquids
Gases
Artificially Prepared
For a description of the atomic data, visit physics.nist.gov/atomic
57 La Lanthanum 138.9055
58 Ce Cerium 140.116
59 Pr Praseodymium 140.90765
60 Nd Neodymium 144.24
61 Pm Promethium (145)
62 Sm Samarium 150.36
63 Eu Europium 151.964
64 Gd Gadolinium 157.25
65 Tb Terbium 158.92534
66 Dy Dysprosium 162.50
67 Ho Holmium 164.93032
68 Er Erbium 167.26
69 Tm Thulium 168.93421
70 Yb Ytterbium 173.04
71 Lu Lutetium 174.967
89 Ac Actinium (227)
90 Th Thorium 232.0381
91 Pa Protactinium 231.03588
92 U Uranium 238.0289
93 Np Neptunium (237)
94 Pu Plutonium (244)
95 Am Americium (243)
96 Cm Curium (247)
97 Bk Berkelium (247)
98 Cf Californium (251)
99 Es Einsteinium (252)
100 Fm Fermium (257)
101 Md Mendelevium (258)
102 No Nobelium (259)
103 Lr Lawrencium (262)
Atomic Number
Ground-state Level
Symbol
Name
Atomic Weight
Ground-state Configuration
Ionization Energy (eV)
58 Ce Cerium 140.116 [Xe]4f5d6s2 5.5387

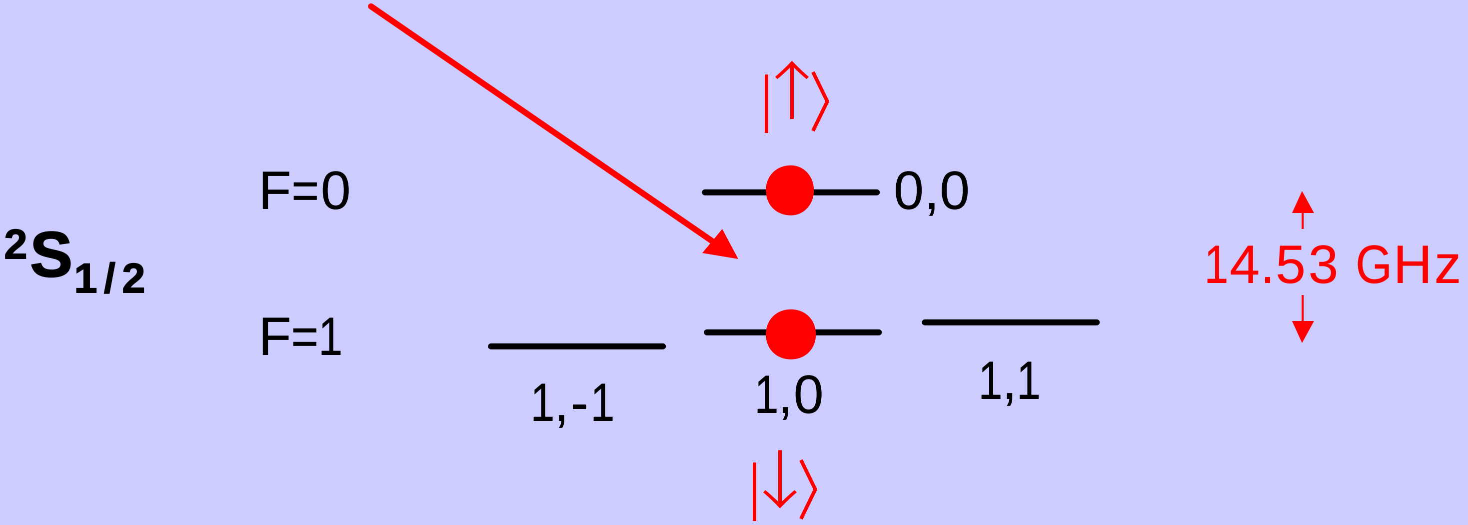
$^{111}Cd^{+}$ ground state atomic structure

$\Delta\nu = 1.4 \cdot B$ MHz (B in Gauss)

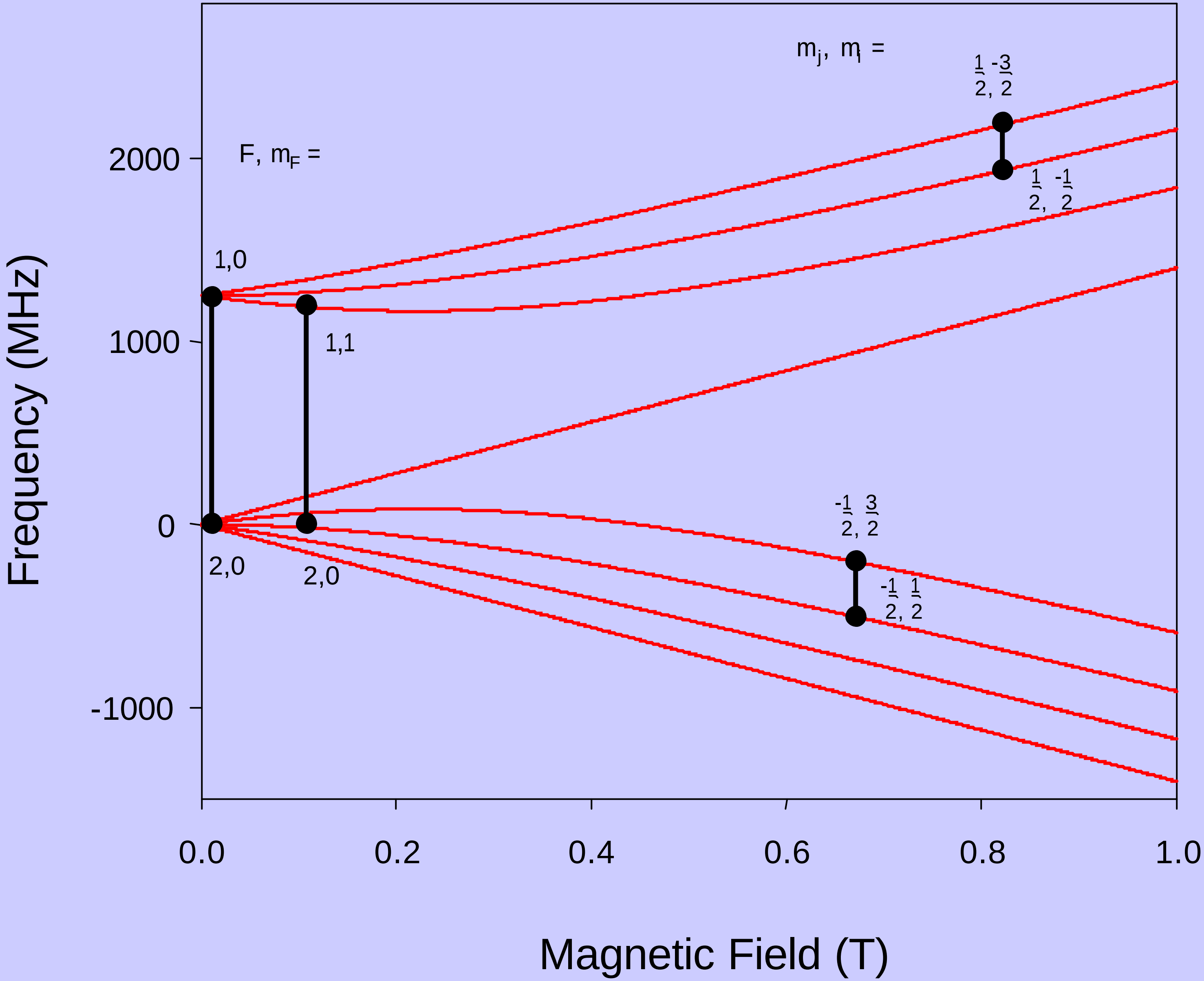


$^{111}Cd^+$ ground state atomic structure

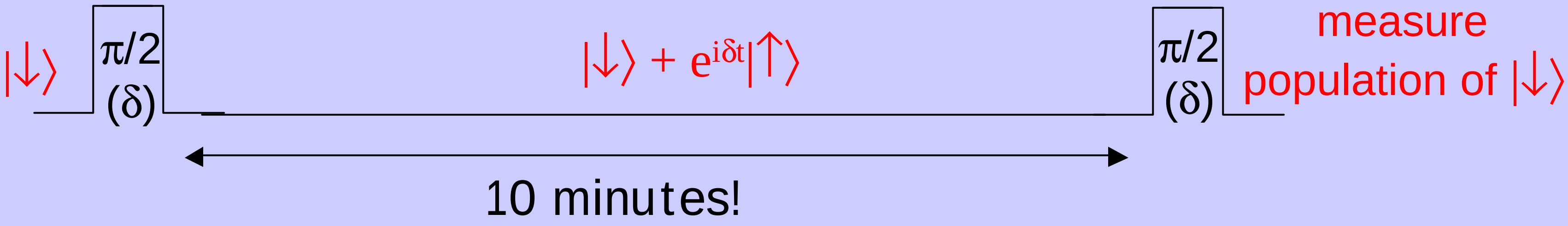
$\Delta\nu = 250 \cdot B^2$ Hz (B in Gauss)

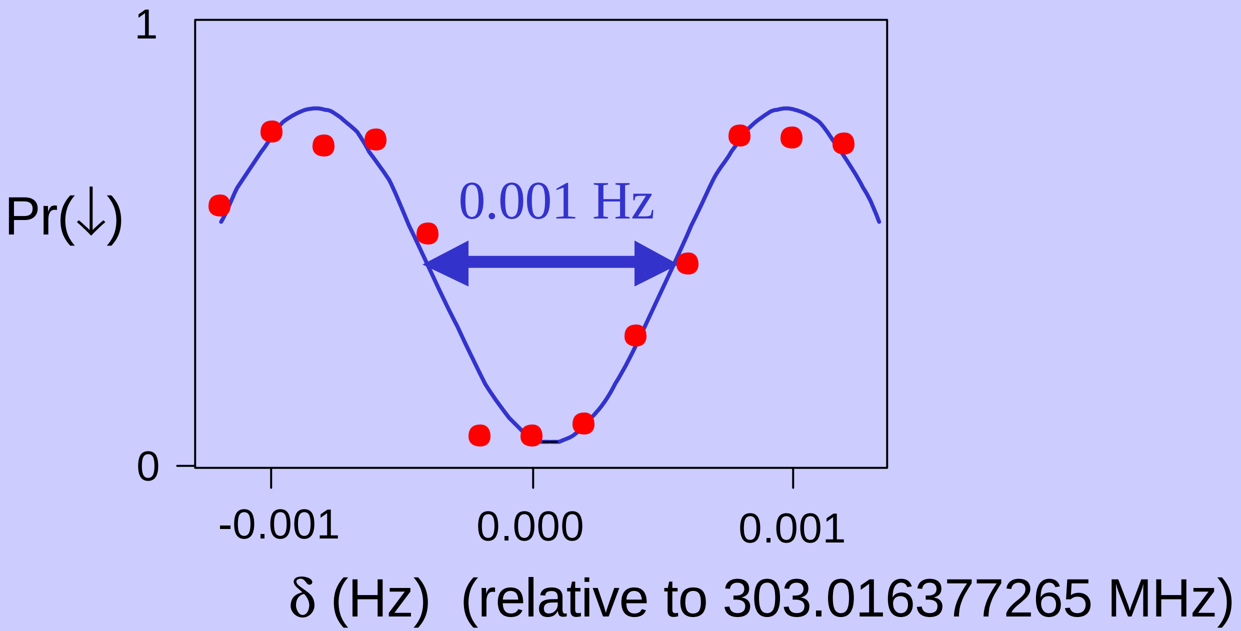


Hyperfine ground states of $^9Be^+$

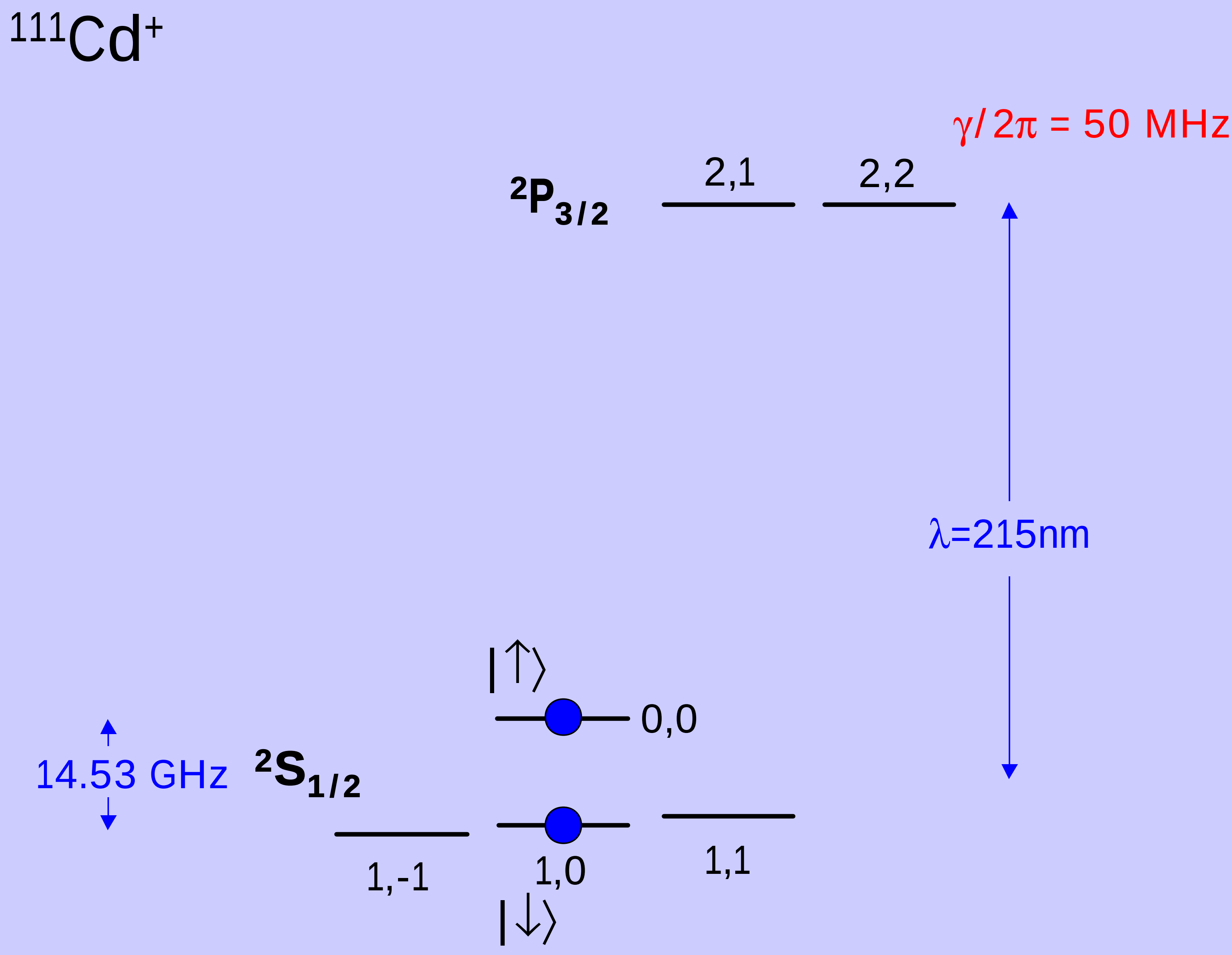


Ramsey spectroscopy on field-independent transition in $^9Be^+$



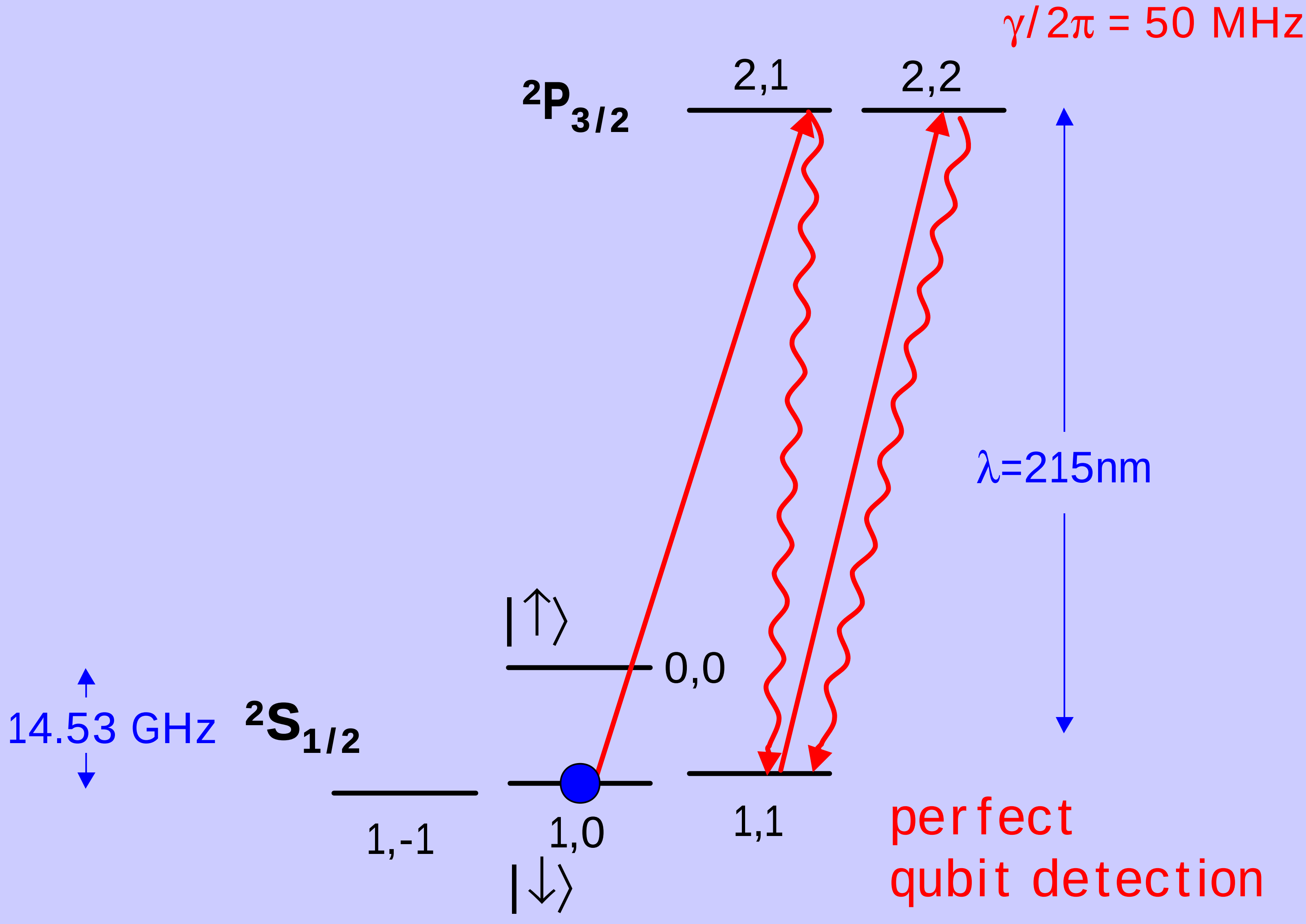


NIST-Boulder, 1989

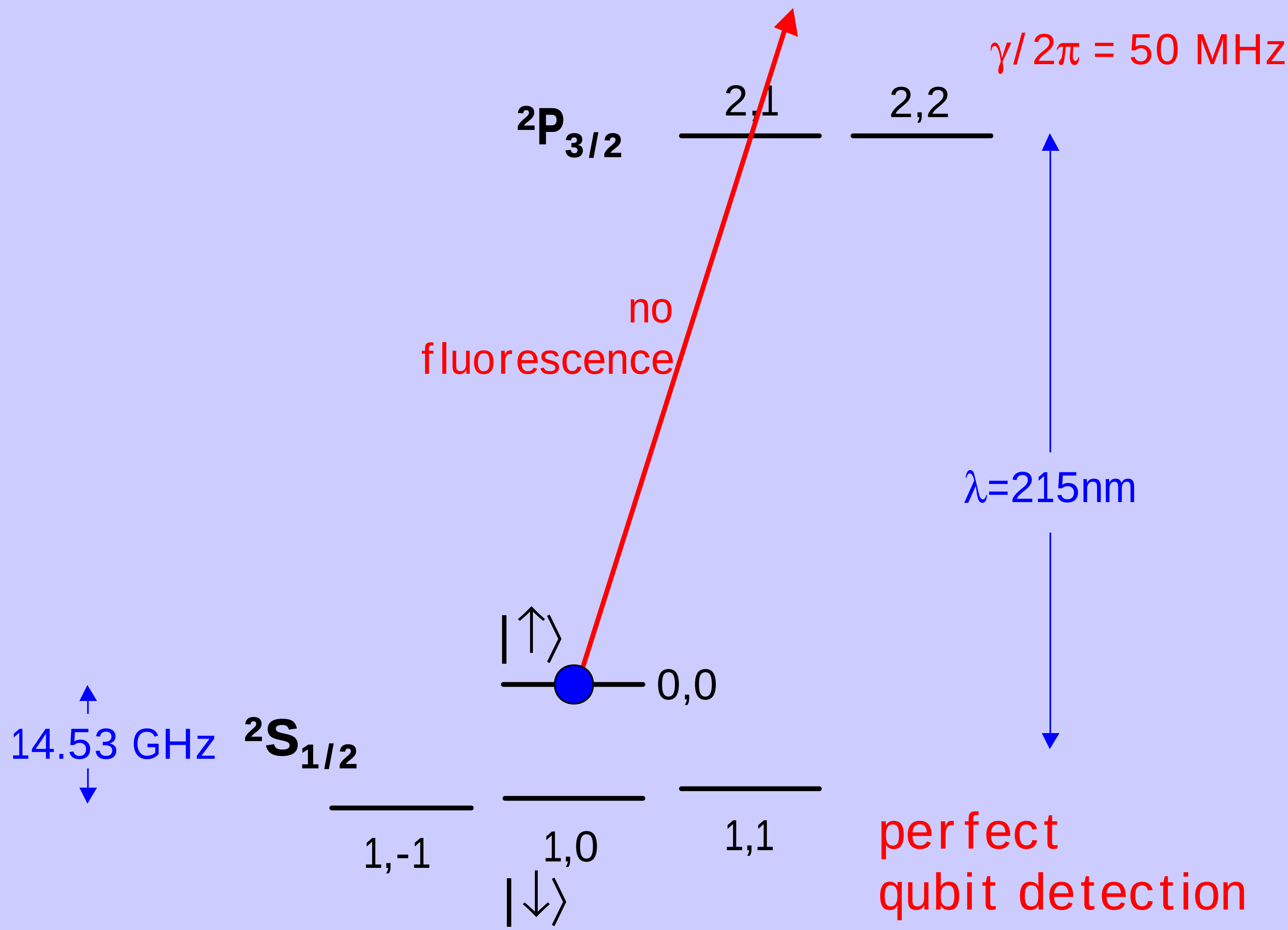


$^{111}Cd^{+}$
$\gamma/2\pi$ = 50 MHz
$^{2}P_{3/2}$
2,1
2,2
λ=215nm
$|\uparrow\rangle$
0,0
14.53 GHz
$^{2}S_{1/2}$
1,-1
1,0
1,1
$|\downarrow\rangle$

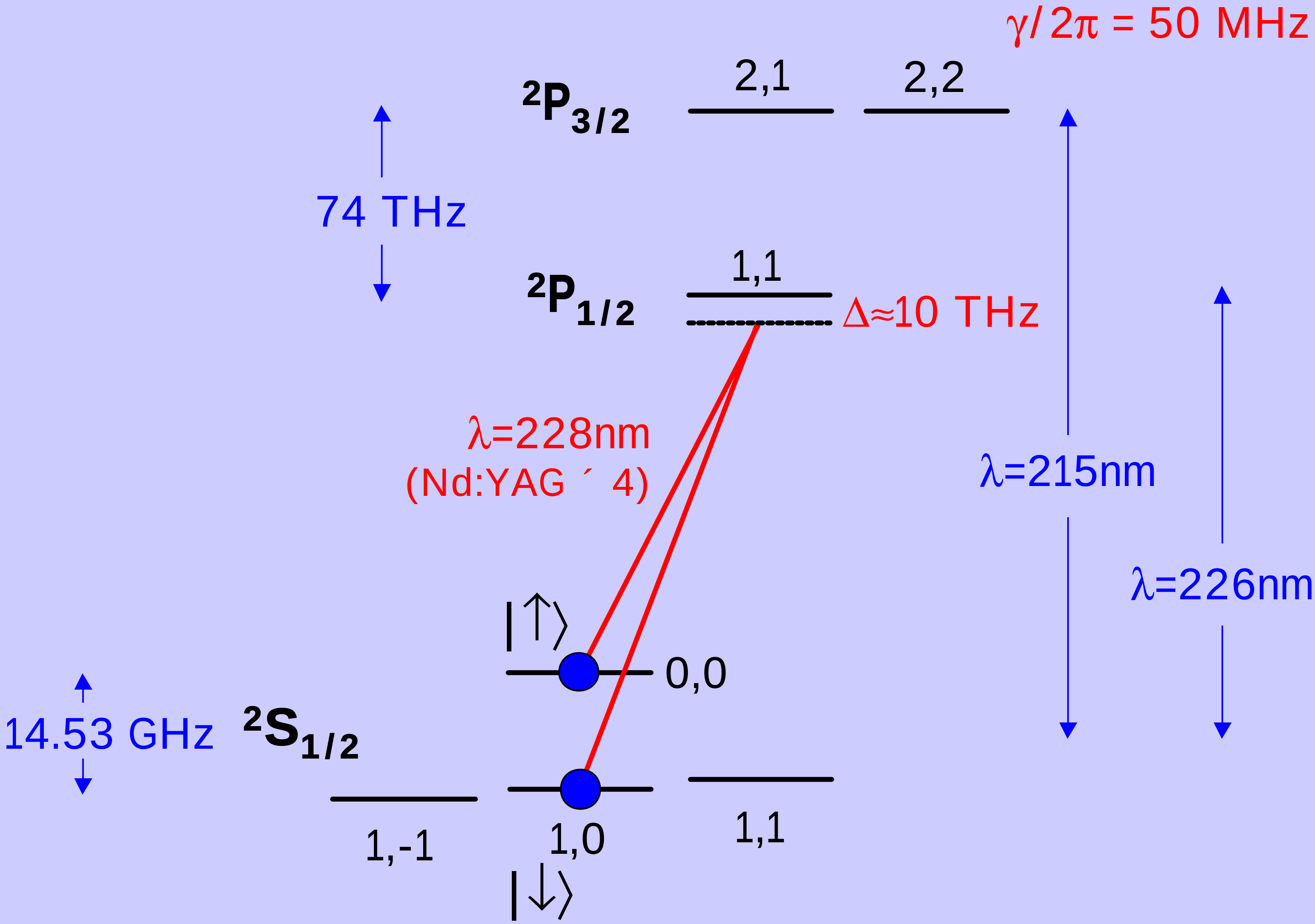
$^{111}Cd^+$ measurement



$^{111}Cd^+$ measurement

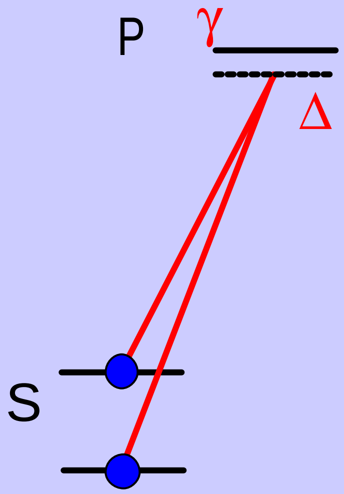


Driving coherent qubit superpositions:
2-field stimulated Raman transitions



Spontaneous emission (off-resonant)

g = resonant Rabi frequency
(S-P transition strength, optical power)
γ = linewidth of S-P transition
Δ = detuning from excited state



- Effective stimulated Raman Rabi frequency

$$\Omega = \frac{g^2}{\Delta}$$

- Spontaneous emission rate

$$\Gamma = \frac{g^2\gamma}{(1+s)(\gamma/2)^2 + \Delta^2}$$

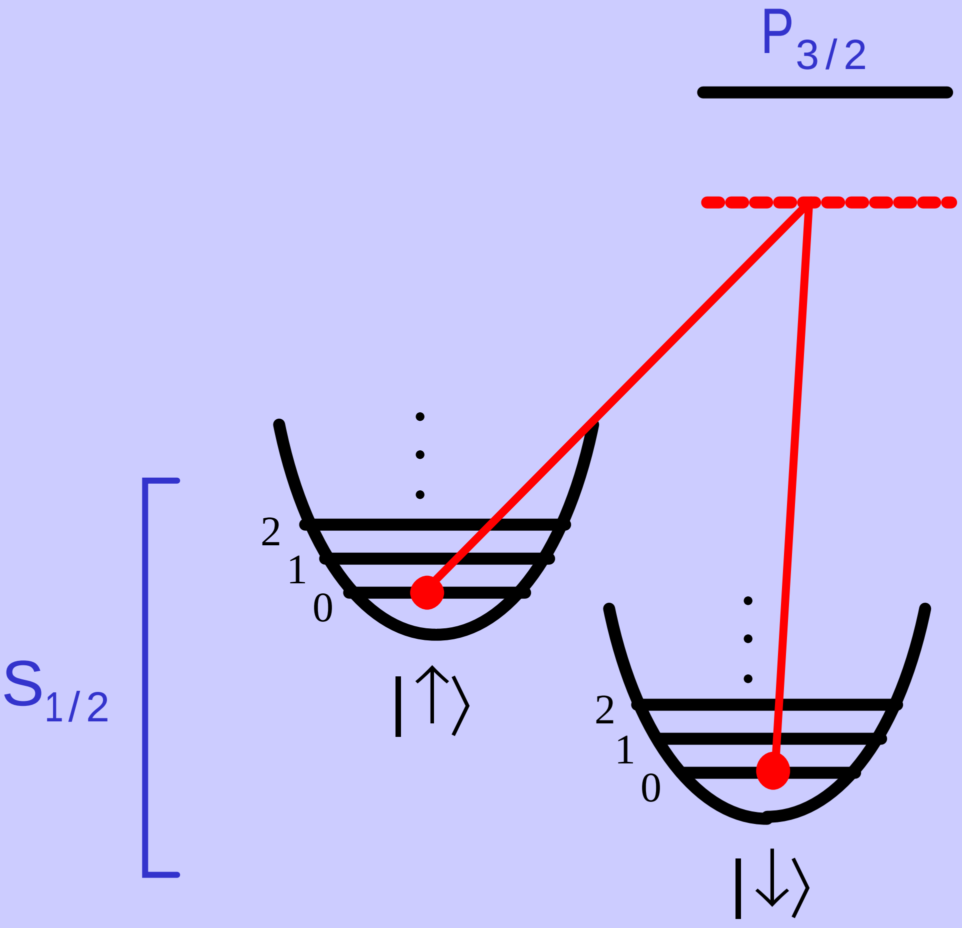
$s = I/I_s$ = saturation parameter = $8g^2/\gamma^2$

$$\cong \frac{g^2\gamma}{\Delta^2} \quad \text{for large detuning } \Delta$$

$$\frac{\Omega}{\Gamma} = \frac{\gamma}{\Delta}$$

probability of spontaneous emission per Rabi cycle
[independent of power for $g \ll \Delta$]

Ion	γ(MHz)	Δ_{fs} (THz)	γ/Δ_{fs}
Be^+	20	0.2	10^{-4}
Ca^+	25	7	$3 \cdot 10^{-6}$
Sr^+	40	20	$1 \cdot 10^{-6}$
Cd^+	50	74	$7 \cdot 10^{-7}$
Hg^+	70	330	$2 \cdot 10^{-7}$

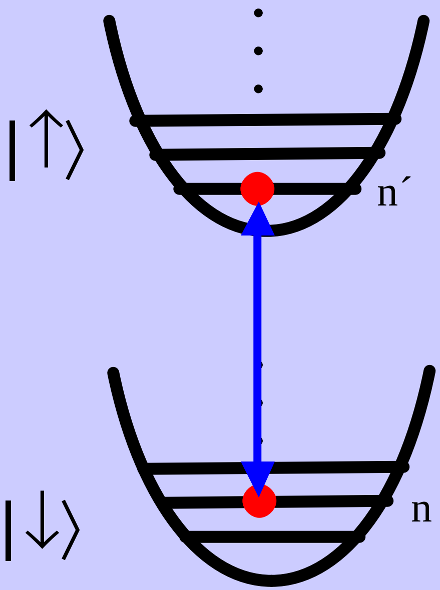


$P_{3/2}$
$S_{1/2}$
2
1
0
$|\uparrow\rangle$
2
1
0
$|\downarrow\rangle$

field-induced spin-motion coupling:
bound-bound transitions in a harmonic molecule

$$\boldsymbol{H} = -m \bullet \boldsymbol{E}(\boldsymbol{x}) = g(s_+ \, e^{i\boldsymbol{k}\bullet\boldsymbol{x} - i\delta t} + s_- \, e^{-i\boldsymbol{k}\bullet\boldsymbol{x} + i\delta t})$$

δ = detuning from free-atom resonance



for $\delta = (n' - n)\,\omega$,

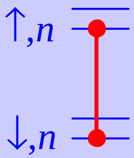
$$\Omega_{n,n'} = \Omega \langle n' |\, e^{\,ih\,(a + a^\dagger)}\, | n \rangle$$

$h = kx_0$ = "Lamb-Dicke" parameter
$= (\delta k) x_0$ for Raman case ($\delta k = k_2 - k_1$)

$x_0 = (\hbar / 2mw)^{1/2}$

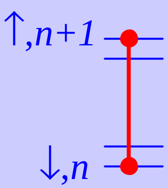
$$\Omega_{n,n'} = \Omega\langle n'|\, e^{ih\,(a+a^\dagger)}\, |n\rangle \;=\Omega\, e^{-\frac{\eta^2}{2}} \sqrt{\frac{n_<!}{n_>!}}\, h^{|n'-n|} L_{n_<}^{|n'-n|}(h^2)$$

“carrier” (δ=0; $n'=n$)



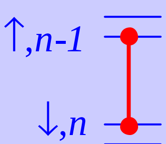
$$\Omega_{n,n} = \Omega\; e^{-\frac{\eta^2}{2}} L_n(h^2) \;= \boxed{\Omega[1} - h^2(n+½) + ½h^4(n^2+¼) + \ldots]$$

“1[st] upper sideband” (δ = +ω; $n'=n+1$)



$$\Omega_{n,n+1} = h\,\Omega e^{-\frac{\eta^2}{2}} \frac{L_n^1(h^2)}{\sqrt{n+1}} \;= \boxed{\Omega[h\sqrt{n+1}} - ½h^3(n+1)^{3/2} + \ldots]$$

“1[st] lower sideband” (δ = –ω; $n'=n–1$)



$$\Omega_{n,n-1} = h\,\Omega e^{-\frac{\eta^2}{2}} \frac{L_{n-1}^1(h^2)}{\sqrt{n}} \;= \boxed{\Omega[h\sqrt{n}} - ½h^3 n^{3/2} + \ldots]$$

LAMB-DICKE LIMIT: $h^2 n << 1$

Lamb-Dicke limit

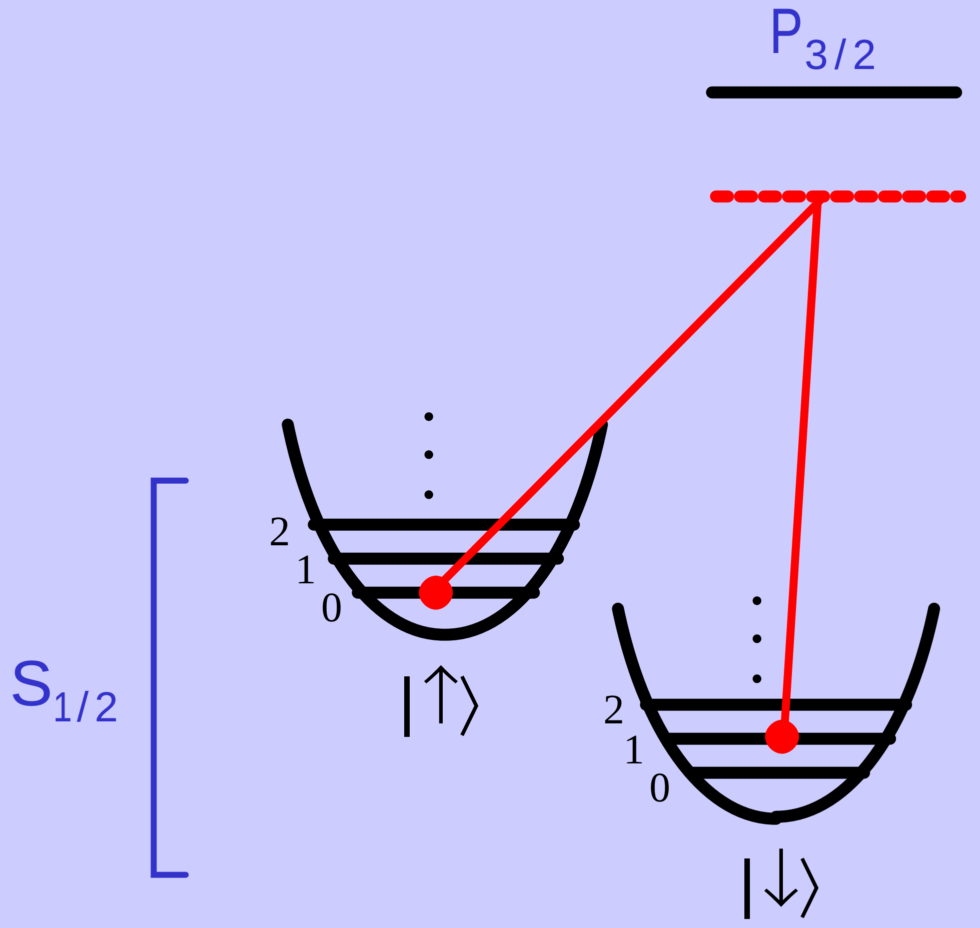
$h^2 n << 1$

…..or…

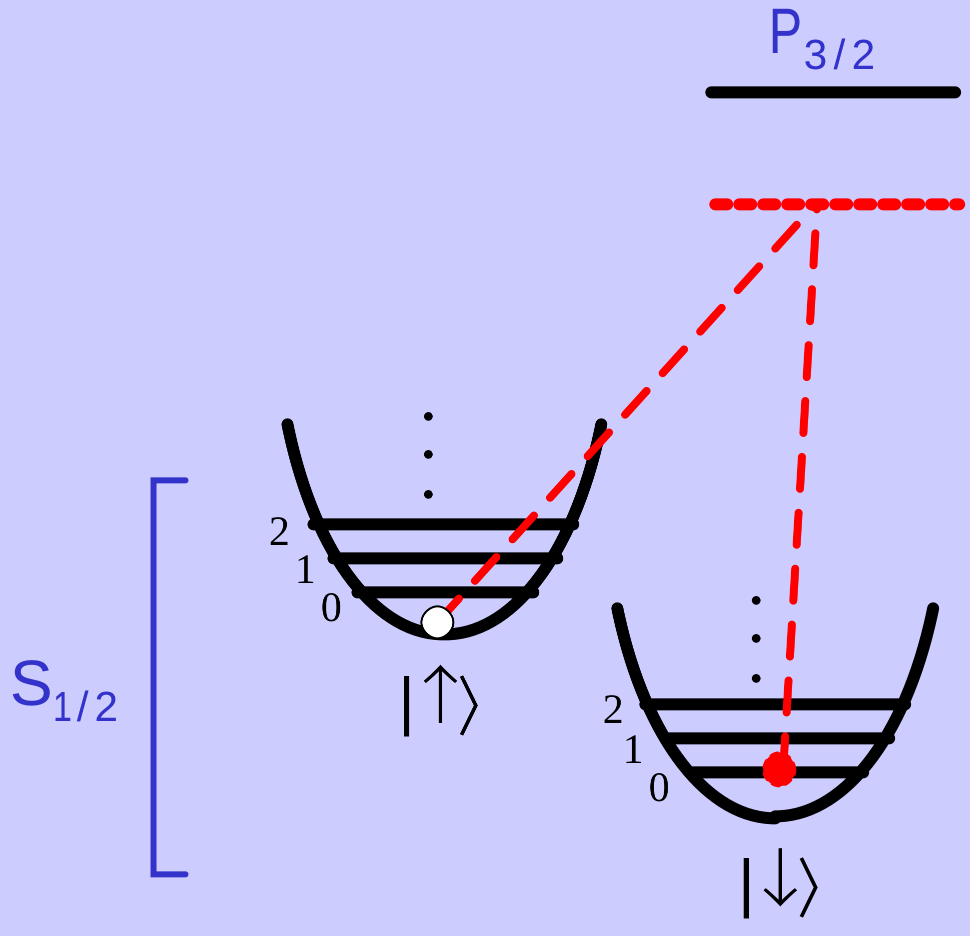
$(kx_0)n << 1$

$(k\langle x \rangle)^2 << 1$

ion wavepacket is confined to much less than $\bar{\lambda}$

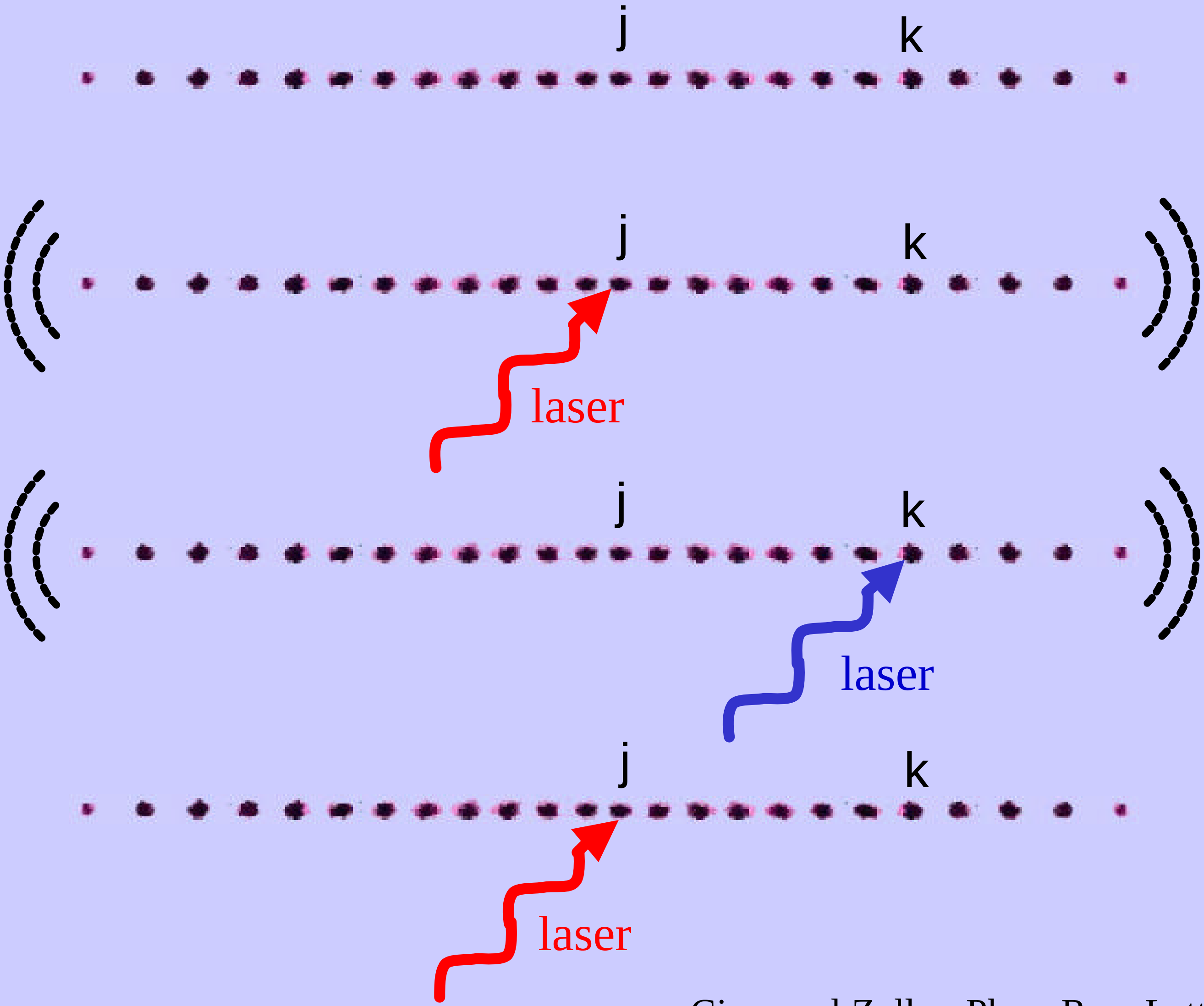


Mapping: $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\,|0\rangle_m \rightarrow |\downarrow\rangle\,(\alpha|0\rangle_m + \beta|1\rangle_m)$



Mapping: $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\,|0\rangle_m \rightarrow |\downarrow\rangle\,(\alpha|0\rangle_m + \beta|1\rangle_m)$

Circa-Zoller Trapped Ion Quantum Computer



laser cool to n=0 ground state

map j^{th} qubit to collective motion

flip k^{th} qubit iff collective motion

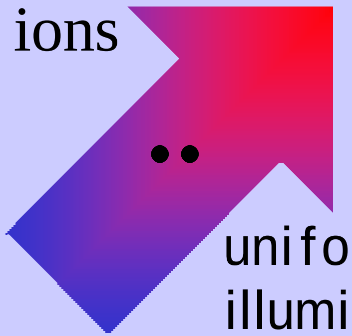
map collective motion back to j^{th} qubit

Cirac and Zoller, Phys. Rev. Lett. **74**, 4091 (1995)

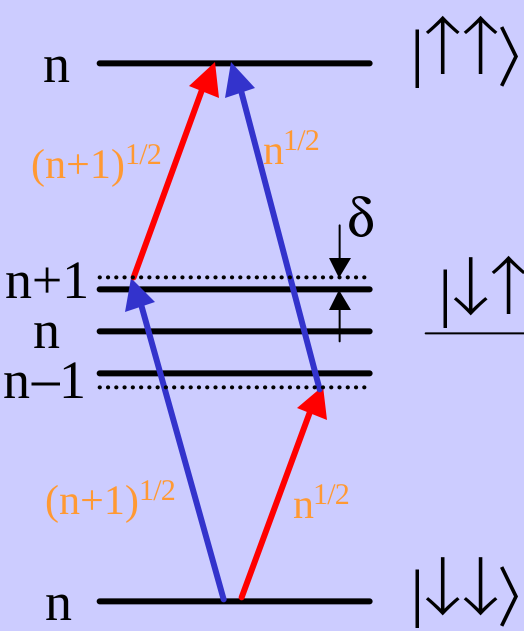
Mølmer-Sørensen Quantum Gate

Mølmer and Sørensen, PRL **82**, 1835 (1999)
Sørensen and Mølmer, PRL **82**, 1971 (1999)

N=2 ions



uniform
illumination



$$\frac{|\downarrow\uparrow\rangle + e^{i\phi}|\downarrow\uparrow\rangle}{\sqrt{2}}$$

$$\Omega_{\downarrow\downarrow,\uparrow\uparrow} = \frac{(\eta g\sqrt{n+1})^2}{\delta} + \frac{(\eta g\sqrt{n})^2}{-\delta}$$

$$= \frac{(\eta g)^2}{\delta}$$

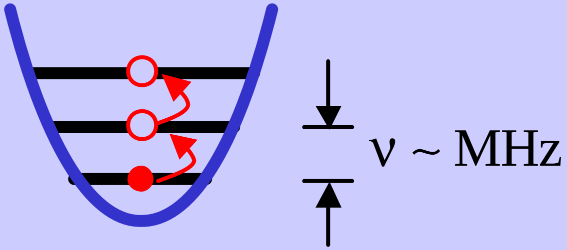
independent of motion !

(within Lamb-Dicke regime)

$$|\downarrow\downarrow\rangle \Rightarrow |\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$$
$$|\downarrow\uparrow\rangle \Rightarrow |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$$
$$|\uparrow\downarrow\rangle \Rightarrow |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$
$$|\uparrow\uparrow\rangle \Rightarrow |\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle$$

- don't need pure state of motion!
  (but must be in LD regime)
- no focusing
- can be as fast as direct
  sideband Rabi freq

Decoherence of C.O.M. motion



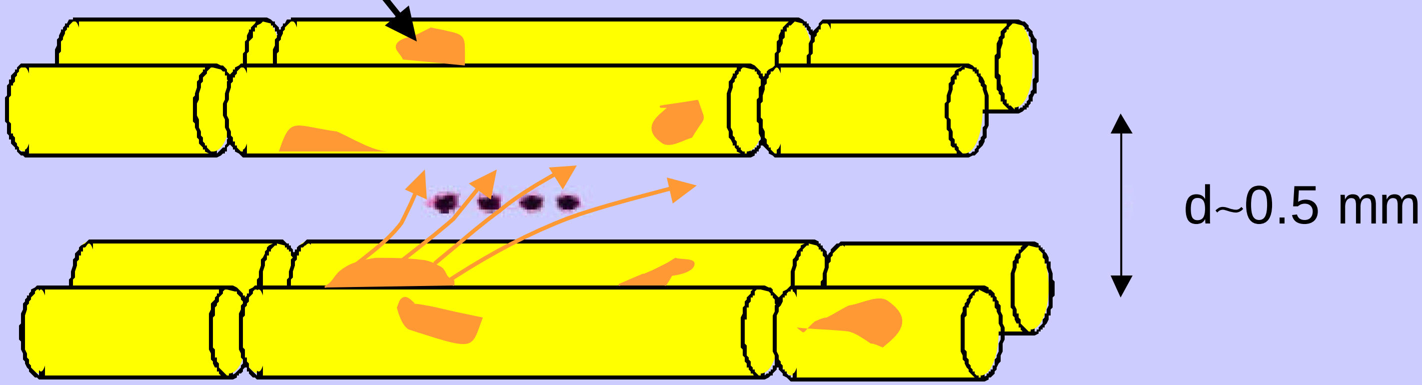
$\gamma = d\langle n\rangle/dt$

expect $\gamma \sim 1\ \text{sec}^{-1}$ (blackbody rad.)

measure $\gamma \sim 10^3 - 10^4\ \text{sec}^{-1}$ (1995)

fluctuating patch
potentials on surface

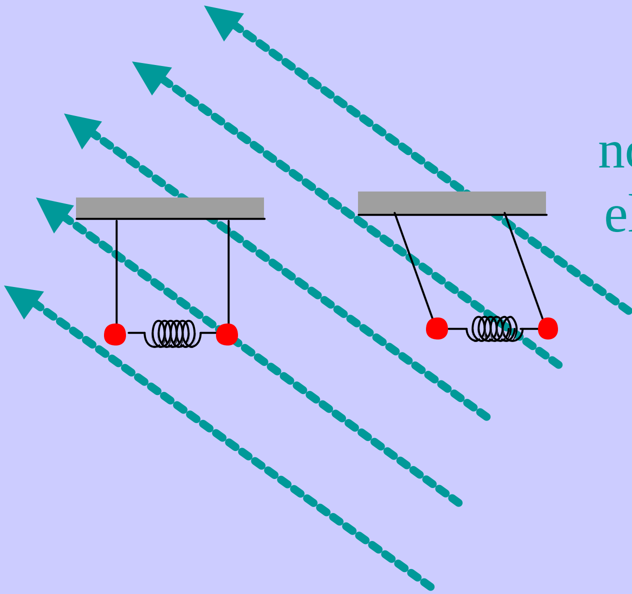
$\gamma \sim 1/d^4$ or even stronger!



USE CLEAN ELECTRODES! Q. Turchette, et. al., PRA **61**, 063418 (2000)

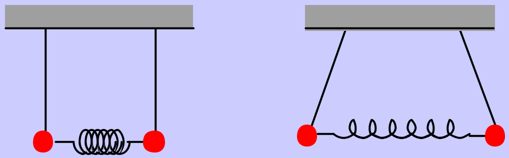
COM mode at ω

$x_{COM} = x_1 + x_2$



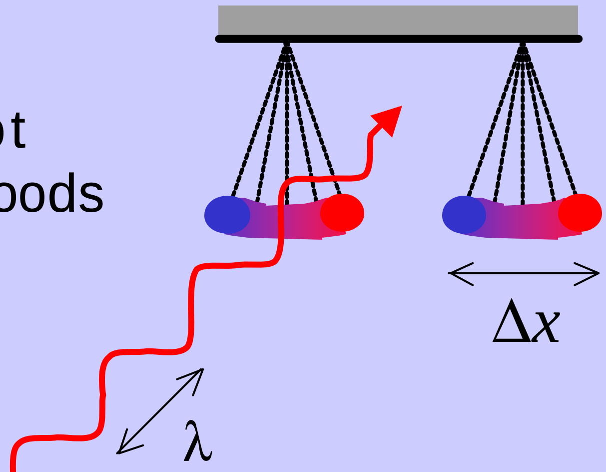
"Stretch" mode at $\omega\sqrt{3}$

$x_{STR} = x_1 - x_2$



A decoherence-free subspace!

... but still not out of the woods



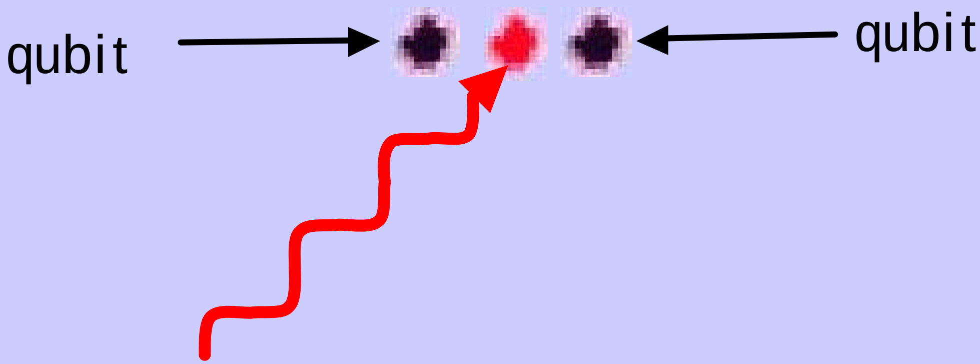
x_{STR} fixed; x_{COM} random

$\Rightarrow \; e^{-(\Delta x/\lambda)^2}$ gate fidelity

Debye-Waller effect

“ion-in-the-middle” sympathetic laser cooling:

quench heating without disturbing internal state
OR symmetric stretch mode

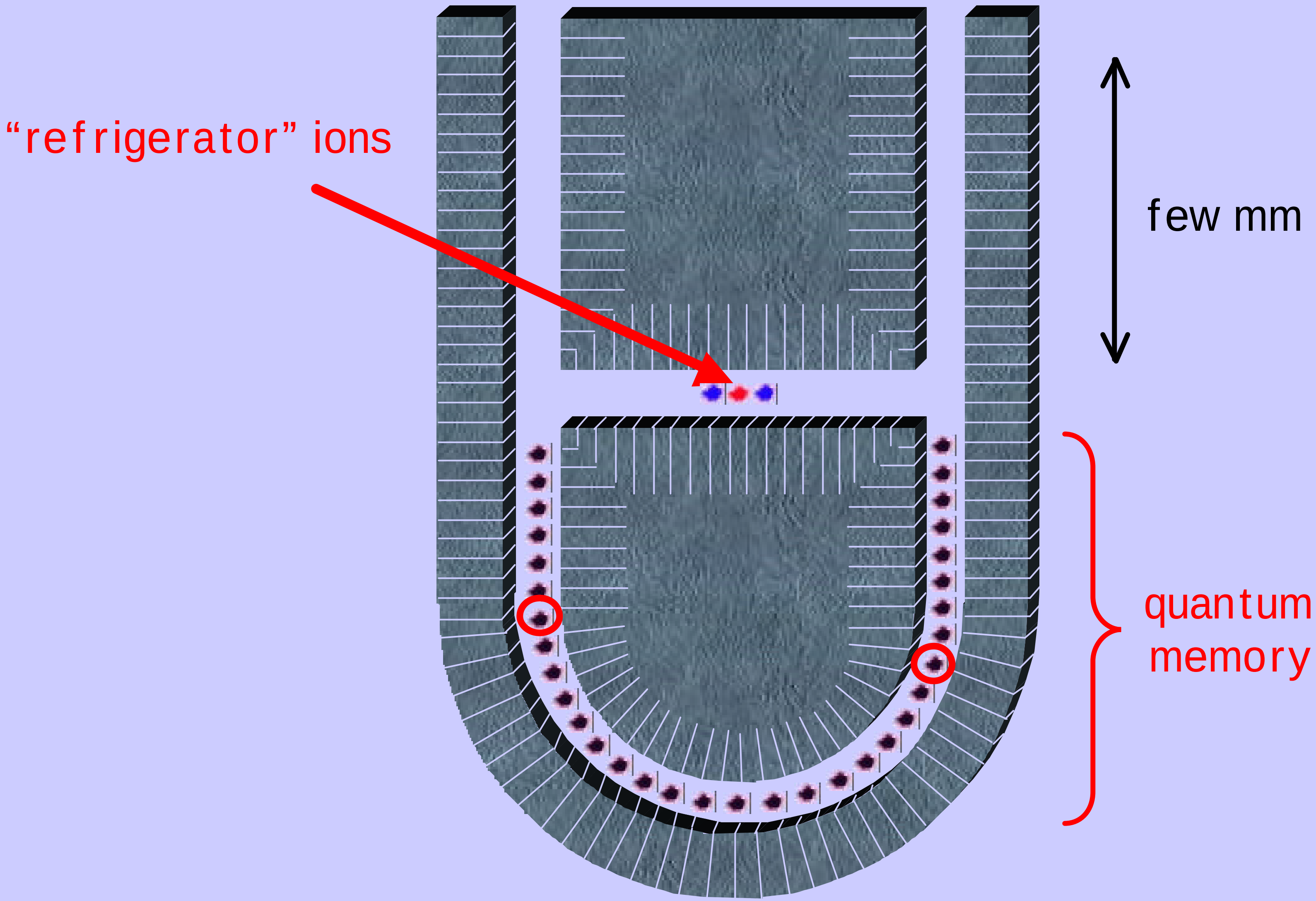


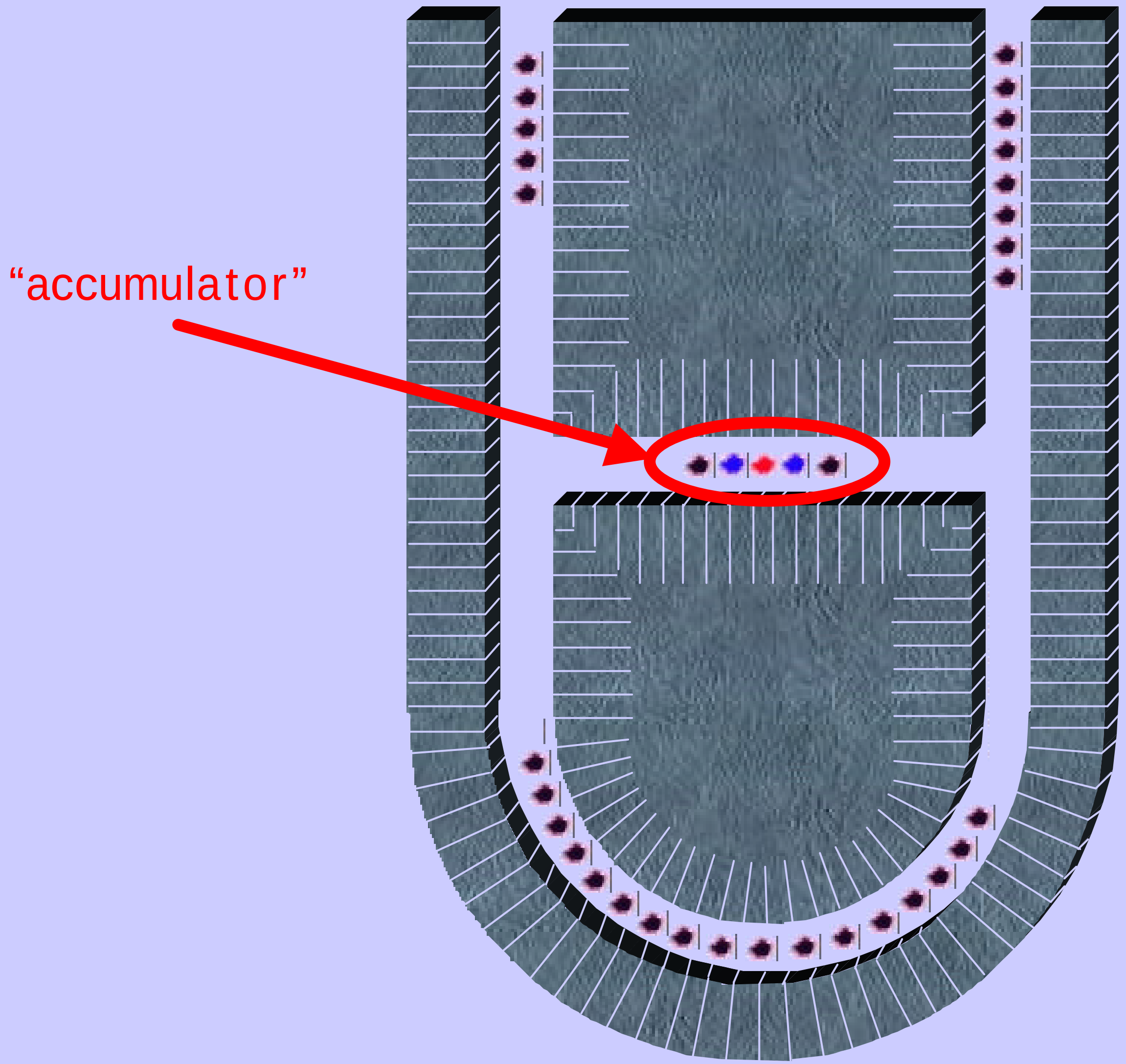
continuous cooling of middle ion
eliminates C.O.M. heating from uniform fields
(tight focusing, or different species)

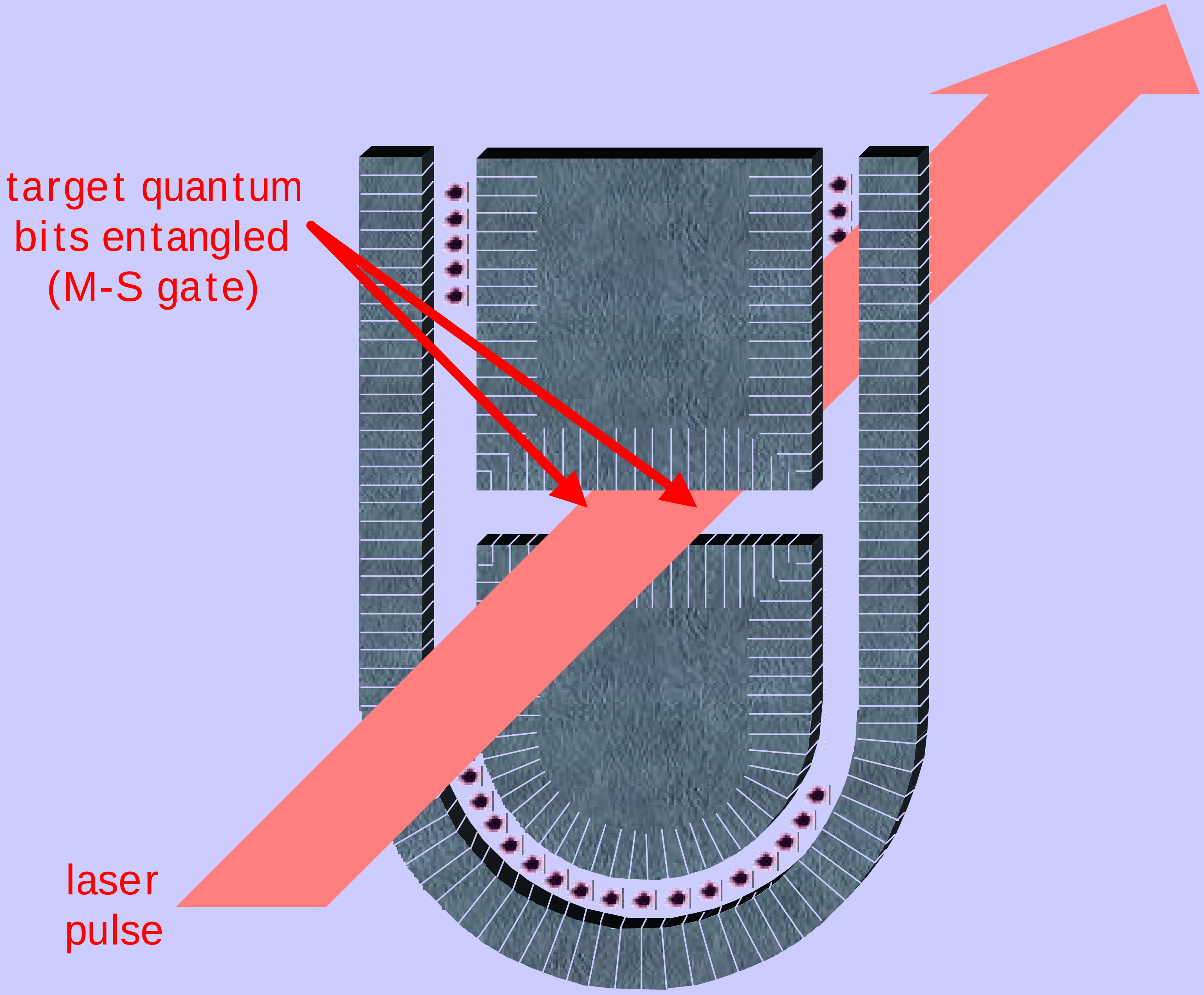
D. Kielpinski, et. al., Phys. Rev. A **61**, 032310 (2000)

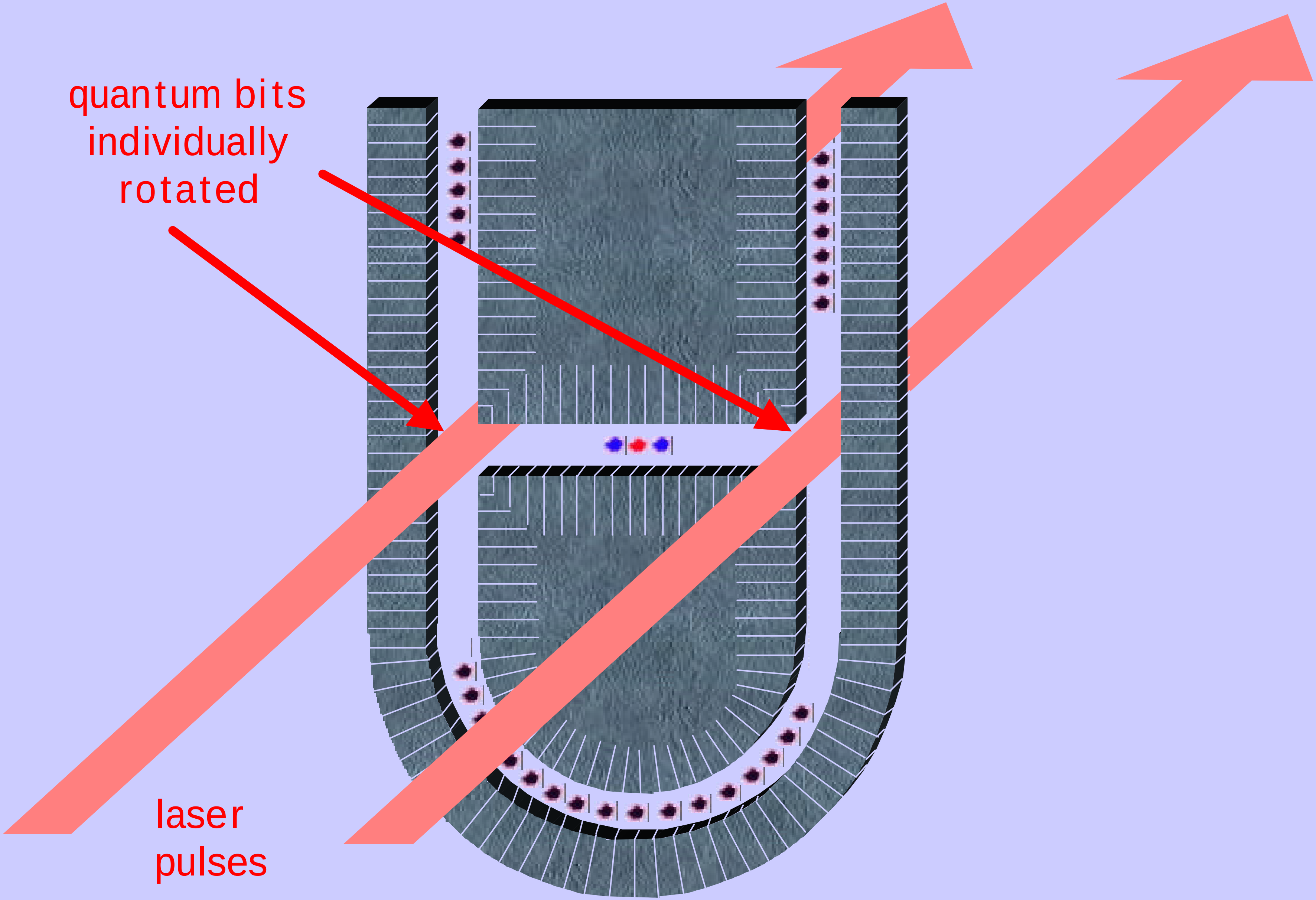
The Quantum CCD

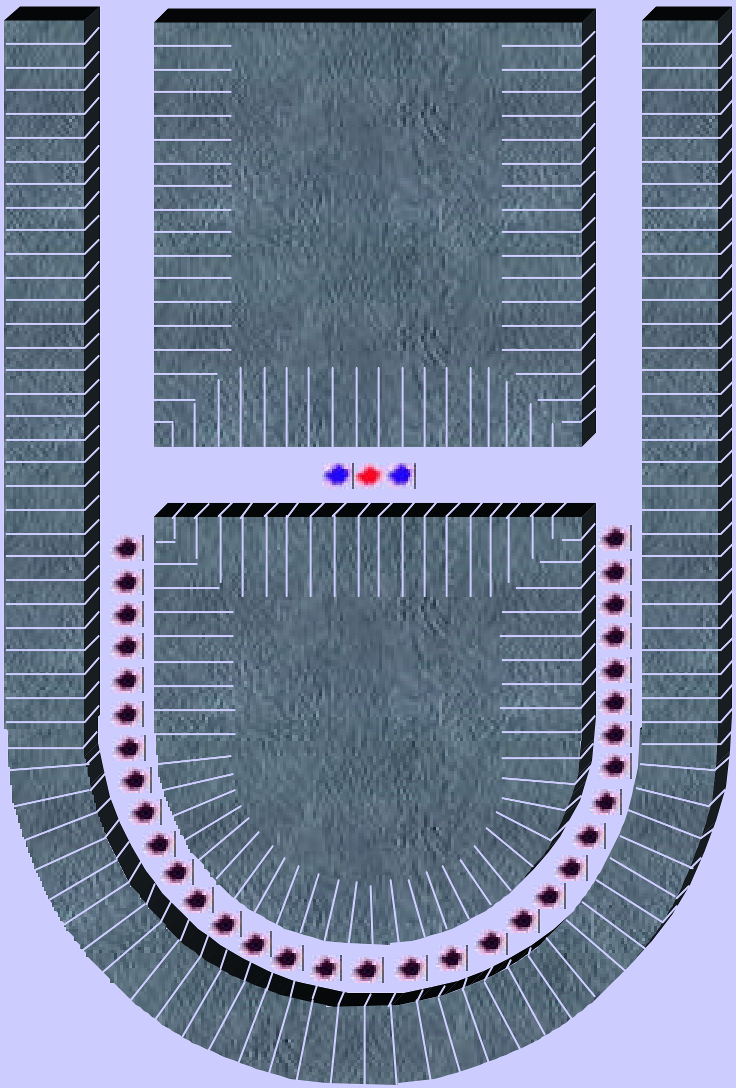
D. Kielpinski, C. Monroe, D. Wineland, *Nature* **417**, 709 (2002)



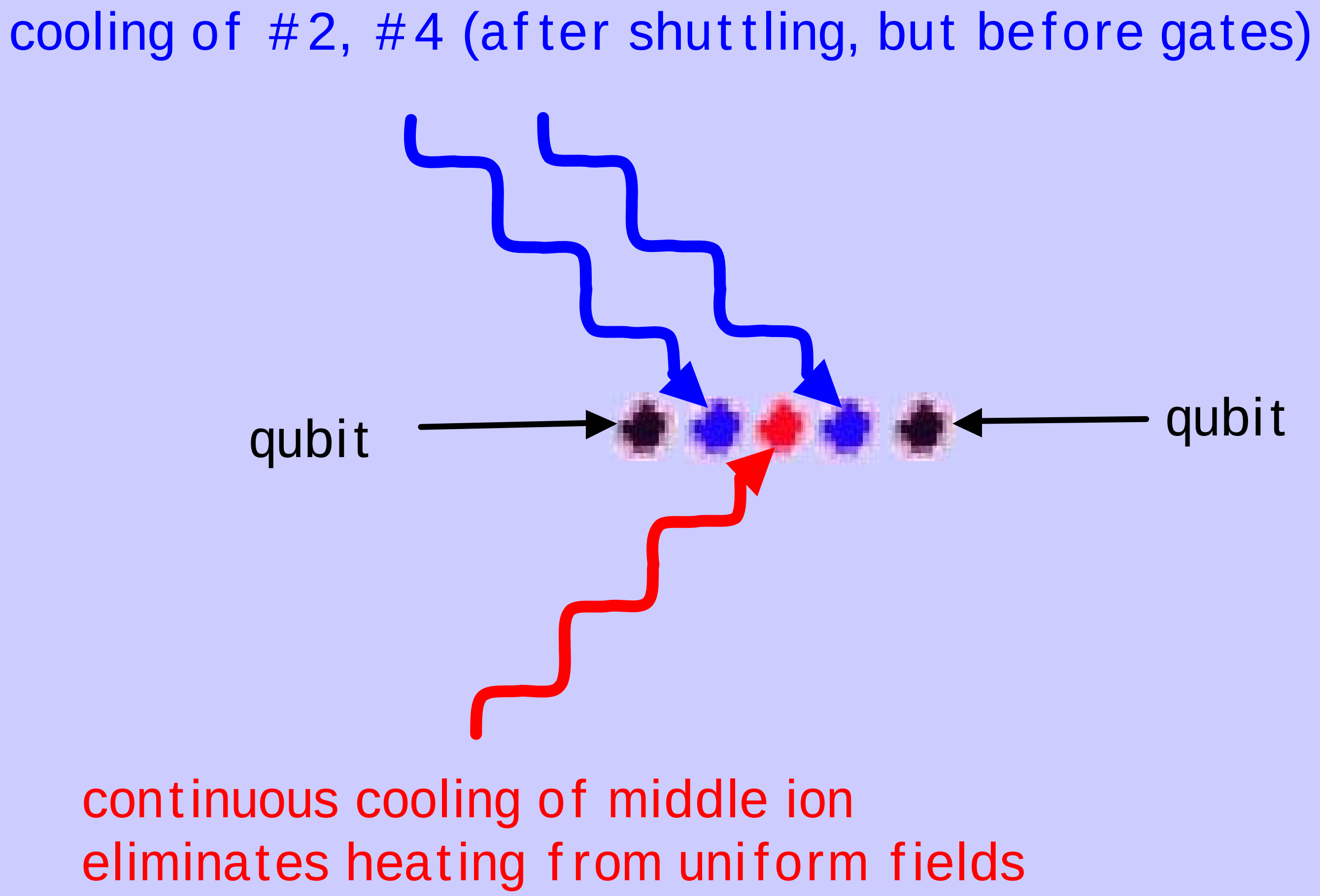

“accumulator”


target quantum
bits entangled
(M-S gate)
laser
pulse


quantum bits
individually
rotated
laser
pulses



Sympathetic Cooling (again): quench qubit motion in between gates



[D. Kielpinski… Phys. Rev. A (2000)]

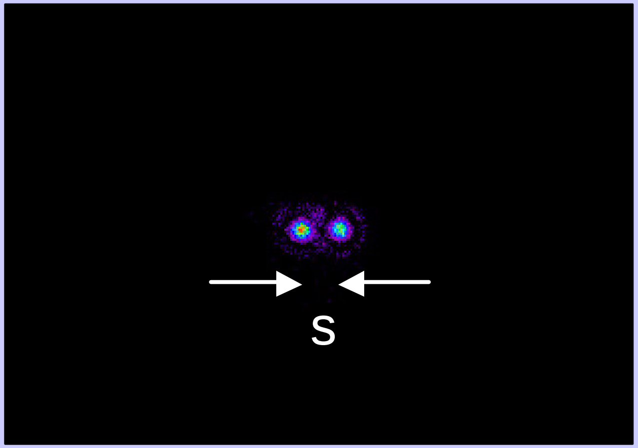
Potential pitfalls in quantum CCD:

- fluctuating magnetic/electric fields
- repeated positioning of ions in accumulator to better than optical wavelength

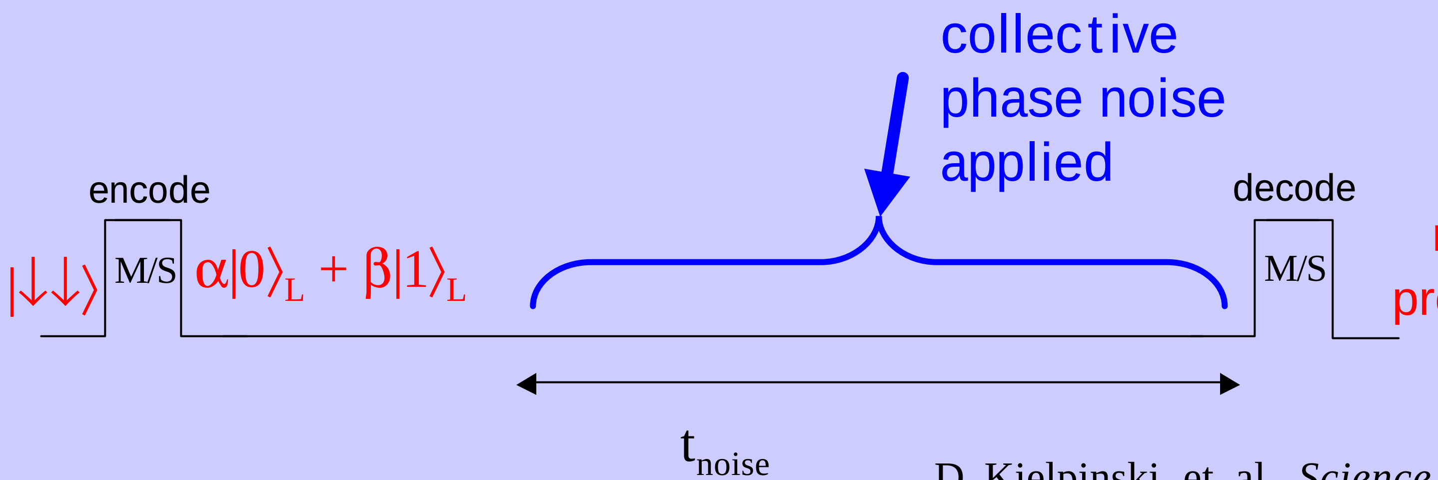
both are “long wavelength” λ_ϕ phase errors

solution... encode in DFS!

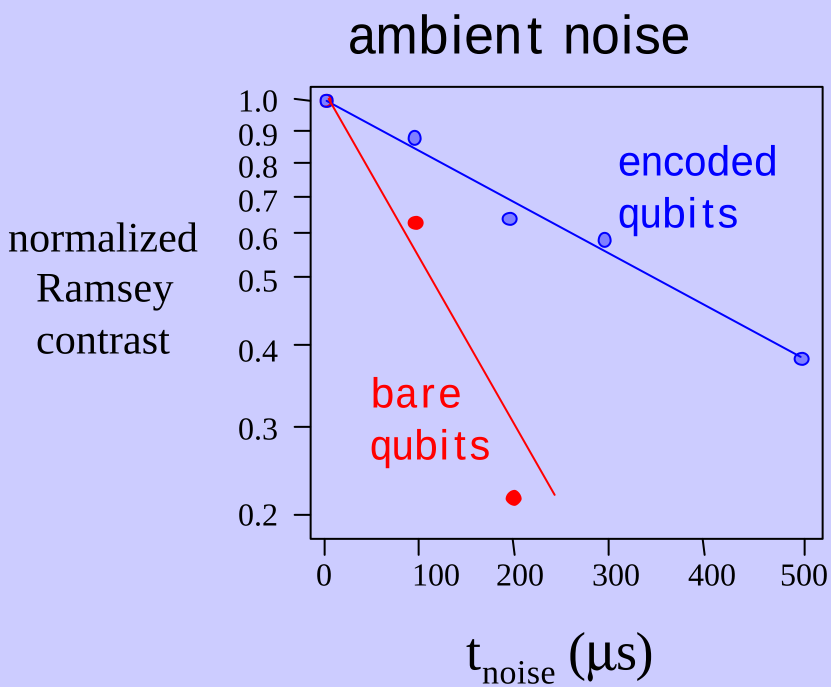
$$|1\rangle_L = |\downarrow\rangle|\uparrow\rangle - i|\uparrow\rangle|\downarrow\rangle$$
$$|0\rangle_L = |\downarrow\rangle|\uparrow\rangle + i|\uparrow\rangle|\downarrow\rangle$$

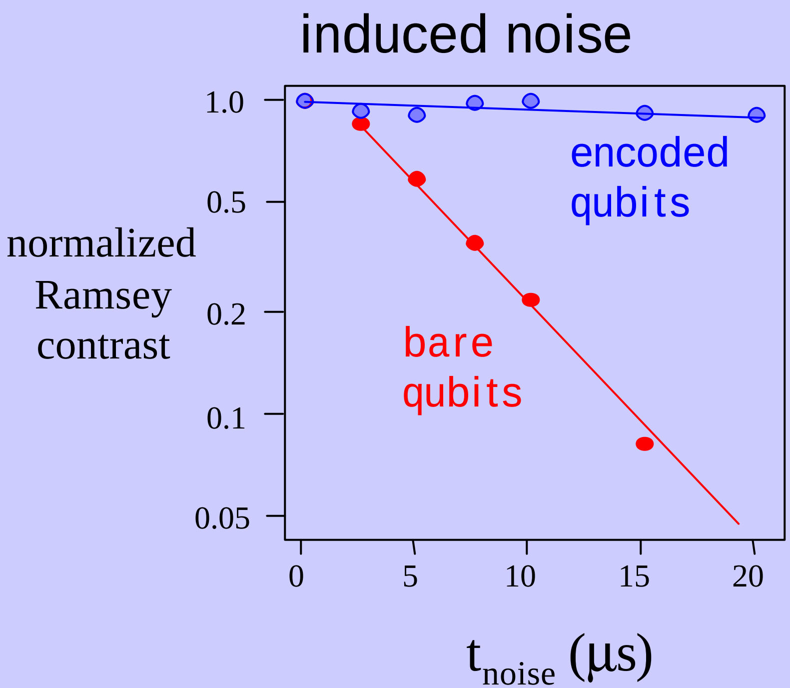


should help when $s \ll \lambda_\phi$



D. Kielpinski, et. al., *Science* **291**, 1013 (2001)



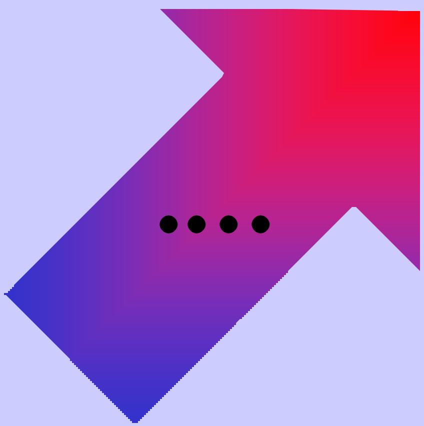


How to compute in DFS?

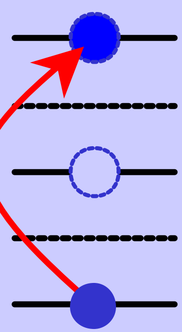
Ans: Molmer/Sorensen gate scalable to arbitrary N!

$$|\downarrow\downarrow\downarrow\cdots\downarrow\rangle \Rightarrow \frac{|\downarrow\downarrow\downarrow\cdots\downarrow\rangle + |\uparrow\uparrow\uparrow\cdots\uparrow\rangle}{\sqrt{2}}$$

e.g., 4 ions (Sackett et. al., Nature 2000)



$|J,J_z\rangle$



$|2,2\rangle = |\uparrow\uparrow\uparrow\uparrow\rangle$

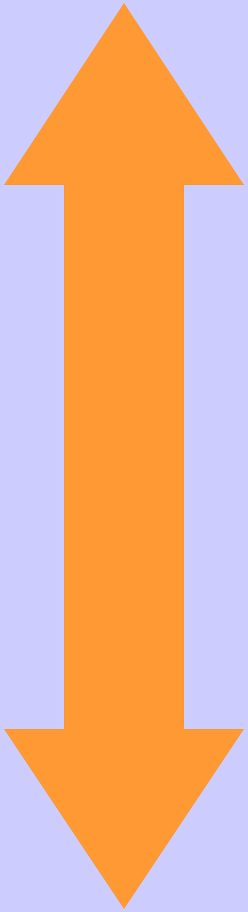
$|2,0\rangle = |\uparrow\uparrow\downarrow\downarrow\rangle + \cdots$

$|2,-2\rangle = |\downarrow\downarrow\downarrow\downarrow\rangle$

Coupling: $H = g\, J_x^2$ flips all *pairs* of spins

Qubit Decoherence Control

passive



active

- trapped ion hyperfine states
- 1st-order magnetic field (and AC Stark) insensitive states
- non-C.O.M. motional modes for multi-ion operations
- Cool to Lamb-Dicke limit to suppress gate decoherence
- sympathetic cooling to quench unwanted motion
- Decoherence-free subspaces
- Bang-bang control
- Error correction