Affine versus Metric Gravitation Parity Test

Alan M. Schwartz
organiker@lycos.com
15 May 2004

Abstract
Gravitation theories are antisymmetric or symmetric to parity transformation. Test mass quantitative parity divergence varies from normalized CHI=0 (achiral) to CHI=1 (perfect divergence). Parity Eötvös experiments are proposed contrasting chemically and macroscopically identical space group $P3_121$ and $P3_221$ α-quartz single crystal test masses. A pure geometric model and calculation of a 4.44×10^{17} atom crystal both yield CHI$\geq 1-(1.77 \times 10^{-16})$ for full scale inquiry. It is shown that test mass net divergent property amplitude is 520 times those of composition experiments. Equivalence Principle parity violation is allowed to $\sim 10^{-11}$ difference/average, suggesting measurable failure of the weakest general relativity postulate.

PACS numbers: 04.80.Cc, 11.30.Er

I. INTRODUCTION

Galileo's 1638 universality of free fall, Newton's 1687 invariant proportionality of mass and weight, and Einstein's 1907 elevator *Gedankenexperiment* embody the Weak Equivalence Principle (EP): All local test masses are postulated to fall identically in vacuum regardless of composition or internal structure; inertial and gravitational masses are fundamentally indistinguishable. Mass is an inert marker or an anonymous abstract distortion.

The Weak EP assumes a homogeneous gravitation field is a local approximation around a given world-point. Stronger EP statements are contingent upon the Weak EP [1]. Orbiting, massive, extended, degenerately dense, electromagnetically polarized, physically spinning... bodies exhibit post-Newtonian gravitoelectric and gravitomagnetic effects [2] external to this proposal.

Metric theories postulate the EP and spacetime curvature immediately follows. They deny a gravitation stress-energy tensor given a Minkowski vacuum free-fall reference frame in which constant and uniform non-zero gravitation vanishes. General Relativity (GR) forbids additional geometric tensor fields (i.e., sitting over the tangent bundle) including special directions picked out by extra vector fields. GR forbids special foliations, etc., independent of whether they have nice pullbacks and pushforwards under the diffeomorphism group. Local physics must retain full SO(3,1) symmetry as such or at least asymptotically. All GR predictions are observed within experimental error. Metric theories are symmetric to parity transformation.

Affine/teleparallel theories postulate spacetime torsion. They do not postulate the EP and can violate it [3]. They may contain a parity antisymmetric gravitation stress-energy pseudotensor [4] including parity and spin as EP violation tests. This singular disjoint prediction of metric versus affine gravitation is testable.

A chiral body is not superposable upon its mirror image. Chirality only requires a causal and orientable spacetime manifold. Chirality then arises from coordinate-free Hodge duality, equivalent to a pseudoscalar field (Levi-Civita tensor). The Weak force breaks parity without coordinates or reference frames, using the coordinate-invariant vector triple product. Geometric parity inverts all coordinates without spatial bias. It is more limiting ("direct symmetry index" in Section IV) than chirality plus 180° rotation in 3-space, certainly if rotation is discontinuous. True chiral systems exist in two distinct enantiomorphic states interconverted by space inversion but not by time reversal combined with any proper spatial rotation [5]. Chirality in 3-space is an emergent phenomenon requiring at least four non-coplanar points. The ratio of emergent scale to experimental scale is consequential (Section V).

Chemically and macroscopically identical test masses permitting only extremal quantitative parity divergence are demonstrated to be a new EP test. Practical examples allow a bench-scale large amplitude challenge of the weakest GR postulate. Multiple parts-per-trillion EP violation is allowed.

II. SYMMETRIES AND PROPERTIES

An axiomatic system is only as robust as its axioms. Euclid's Fifth Postulate (Playfair's Axiom: Only one line can be drawn parallel to a given line through an exterior point) fails in hyperbolic and elliptic geometries. Symmetries through strong mathematical correspondences elicit physics vulnerable to geometric assault.

"Different" test masses contrast properties derived from symmetries through Noether's theorem. Noether's theorem requires continuous symmetries or approximation by a finite or countably infinite number of independent infinitesimal generators (Taylor expansion) consistent with smooth Lie groups. Other dependencies, given a larger infinite number of generators (GR and the Bianchi identities), are acceptable.

Parity is the only external symmetry having no continuous or summed infinitesimal approximation. It is excluded from Taylor expansions, smooth Lie groups, and Noether's theorem. Invariance of a linear differential operator under a discrete symmetry requires a partial differential equation invariant under reflection to possess solutions that are also invariant. If **G** is the Hermitian generator of nontrivial unitary operator **U** (e.g., parity), then if **U** commutes with Hamiltonian **H**, then so does **G**: [**H**,**G**]=0. If **U** commutes with **H** it is a symmetry and a conserved quantity. Any system initially in an eigenstate of **U** evolves over time to other eigenstates having the same eigenvalue. Parity the symmetry couples to parity the property without Noether's theorem. Table I lists fundamental test mass properties.

TABLE I. Independent physical symmetries generate EP-testable properties. Gauge transformation is "gt."

Symmetry	Invariance	Conserved property
proper orthochronous Lorentz	translation in time	energy
	translation in space	linear momentum
	rotation in space	angular momentum
discrete	P, coordinates' inversion	spatial parity
	C, charge conjugation	charge parity
	T, time reversal	time parity
	CPT	product of parities
internal	U(1) gt	electric charge
	U(1) gt	lepton number
	U(1) gt	hypercharge
	$U(1)_Y$ gt	weak hypercharge
	U(2) [not U(1)xSU(2)]	electroweak force
	SU(2) gt	isospin
	$SU(2)_L$ gt	weak isospin
	PxSU(2)	G-parity
	SU(3) "winding number"	baryon number
	SU(3) gt	quark color
	SU(3) (approximate)	quark flavor
	S((U2)xU(3)) [not U(1)xSU(2)xSU(3)]	Standard Model

EP tests must consider four fundamental symmetries and their coupled properties: parity, mass-energy, linear and angular momenta. Properties coupled to internal symmetries (e.g., charge conjugation) transform fields amongst themselves leaving physical states (translation, rotation) invariant to at least first order by definition - a local gauge transformation always exists to make the local gauge-field vanish. Two vector potentials differing only by a gauge transformation give the same field. EP tests opposing properties coupled to internal symmetries are default nulls.

Connes interpreted the standard model in non-commutative geometry - gauge symmetry underlain by an algebra [6]. Differential geometry disregards whether the underlying space is discrete or a continuum, including principle bundles, gauge groups and Yang-Mills fields - particularly, the Z_2 group of the parity operator P. The corresponding gauge boson for left-right symmetry is then... the Higgs. The Yang-Mills gauge Lagrangian is the Higgs sector of the Standard Model Lagrangian. Mass is coupled to left-right symmetry.

The Lorentz, Poincaré, or translation group builds gauge theory GR (diffeomorphisms being gauge transformations). The last is teleparallel gravitation in parallelizable manifolds. The Poincaré group gives teleparallel treatments of non-parallelizable manifolds. The Lorentz part of the gauge field (flat; Lorentz force acceleration would be inversely dependent on mass) pulled back to the tangent bundle becomes the Weitzenböck connection; the translation group (cotetrad field) gauge field total curvature is Weitzenböck connection torsion. The Lorentz part can be gauged away for parallelizable manifolds, leaving translation group gauge theory (only locally true in the general case).

Poincaré group gauge theory can be equivalent to Einstein-Cartan gravitation theory [7]. Einstein-Cartan theory operates in Riemann-Cartan spacetime U^4. A curvature and a torsion tensor can be obtained on U^4.

1) If the torsion tensor vanishes, V^4 pseudo-Riemannian spacetime obtains (metric);
2) If the curvature tensor vanishes, A^4 Weitzenböck spacetime obtains (affine/teleparallel);
3) If both tensors vanish, M^4 Minkowski spacetime obtains.

A teleparallel gravitation stress-energy pseudotensor antisymmetric to parity transformation constructs volume integrals for total gravitation four-momentum and total angular momentum. It obtains by comparing vectors at different points of spacetime - a coframe field - unlike GR. When the coframe field changes the pseudotensor changes (not gauge-invariant; not covariant under general coordinate transformations) [8]. This defines an integral energy-momentum as a redistribution of energy between material and gravitation (coframe) fields obeying an exact conservation law. The Lagrangian for GR can arise from the coframe field only and be antisymmetric to parity transformation. Extremal parity pair test masses violate the EP.

Cosmic segregated parity bodies do not exist. Resolved organic enantiomers in differential enthalpy of racemization or combustion studies do not meet geometric criteria for extremal parity pair masses. No prior observation constrains EP inertial versus gravitational parity mass divergence below ~10 parts-per-trillion difference/average. Metric gravitation field theory could experimentally fail a parity challenge.

III. THE EÖTVÖS EXPERIMENT

An Eötvös torsion pendulum is the most sensitive accessible EP test. A symmetric ~6 cm diameter rotor is vertically suspended from a long minimal fiber. It holds two balanced sets of 180°-opposed test masses and has tangential plane mirrors acting as one arm of a long interferometer. The isothermal rig under hard vacuum is multiply isolated from mechanical and field disturbances. As the Earth gravitationally free falls around the sun and inertially rotates about its axis the two accelerations' phase angle undergoes diurnal modulation. A precision turntable allows higher frequency phase-locked detection. If the different test masses do not fall identically a time-varying torque is imposed and rotation occurs until balanced by fiber torsion, causing a signal in the otherwise nulled interferometer. A typical torsion constant of 0.03 dyne-cm/radian allows detection sensitivity of ± 3 nanoradians (~0.18 nm perimeter rotation, an atomic diameter). EP composition tests null to 5×10^{-13} difference/average [9].

Geocenter orbital acceleration varies from 0.6133 cm/sec^2 (perihelion) to 0.5737 cm/sec^2 (aphelion), averaging 0.5930 cm/sec^2 (one astronomical unit). Given World Geodetic System 1984 data, 44.952º latitude affords a maximum 1.6929 cm/sec^2 horizontal component of Earth's spin at sea level. Small imposed accelerations demand large contrasted property concentration and divergence for EP tests.

Composition test mass properties are small fractions of total rest mass. Their net difference is smaller still. Prior EP tests employed <0.2 mass-% net property concentrations. Essentially 100% of extremal parity pair rest mass is active in net divergence - certainly nuclear mass and arguably position-averaged non-valence electrons' mass - as presented in Table II.

TABLE II. Test mass EP-active mass versus rest mass.

Property	Fraction of rest mass
enantiomorphic crystal	99.9775% Te
geometric parity divergence*	99.9726% SiO_2
nuclear binding energy (low Z)	0.76% ^{4}He
neutron versus proton mass	0.14%
electrostatic nuclear repulsion	0.06%
electron mass	0.03%
unpaired spin mass	0.005% ^{55}Mn**
nuclear antiparticle exchange	0.00001%
Weak Force interactions	0.0000001%

* (nuclear mass)/(atomic mass) weighted for isotopic abundances [10].
**Modeled as the aligned undecatiplet.

Every EP composition test has nulled within experimental error. A calculated large-amplitude property based in geometry rather than composition is welcome. Parity Eötvös experiments will achieve the historical standard of net output, and may exceed it.

IV. QUANTITATIVE GEOMETRIC PARITY DIVERGENCE

EP testing with extremal opposite parity test masses requires a quantitative measure of parity divergence. Invert an atomically thin, pliable, unstretchable left glove into a right glove. Few paths pass through a globally achiral intermediate possessing at least one of an S_1 mirror plane, S_2 point of inversion, or S_n improper axis. Required is a continuous function including extreme opposite values and zero without the necessity of passing through zero as the extremes interconvert.

Optical chirality is progressive rotation of the plane of linearly polarized light with propagation through a medium. Silver thiogallate, $AgGaS_2$ with non-polar achiral tetragonal space group I-42d, has immense optical chirality: 522°/millimeter along [100] at 497.4 nm [11]. α-Quartz in parity pair space groups $P3_121$ and $P3_221$ has no measurable optical chirality 56.16° from crystallographic [0001] [12]. Optical chirality does not measure geometric chirality.

The International Union of Crystallography defines a crystal as "any solid having an essentially discrete diffraction diagram" including periodic, quasiperiodic, and modulated lattices; incommensurate misfit or composite structures, and polytypes. All atom coordinates within a periodic crystal can be calculated within experimental error given unit cell space group, axis lengths and angles, and unique fractional coordinates of contents. Only periodic crystals are considered herein.

Quantitative party divergence describes a coordinate set with finite inertia. Determine the center of mass of a set of N coordinates (e.g., the atoms comprising a single crystal spherical ball) . Invert all coordinates through that origin to create the opposite parity set. Coincide the centers of mass. Rotate one set through its Eulerian angles until the global sum of squared distances between each pair of corresponding points in the parity pair is minimal. CHI is the normalized quantitative parity divergence of the set [13]. CHI is globally minimum for all rotations (R) and translations (t) for all correspondences (P) permitted by the colors and/or graph as in Eq. (1):

$$\mathrm{CHI} = d[(\mathrm{Min}_{\{P,R,t\}} D^2)/4T] \qquad (1)$$

where d is the Euclidean dimension, D^2 is the sum of N squared-distances between the set and its parity inversion for a fixed pairwise correspondence with coincident centers of mass, and T is the geometric inertia of the set. CHI varies between zero (achiral; exactly superposable inversions) and one (perfect parity divergence) inclusive. CHI is a continuous function of coordinates only - independent of translation, scale, or size (but not aggregation; see Section V). One value exists for the set and its parity inversion. Published QCM (quantitative chirality measure) software given coordinates calculates CHI and associated diagnostics.

A d-dimensional set containing $N \geq d+2$ points can be continuously transformed into its mirror image without passing through an achiral intermediate [14]. CHI for unperturbed left and right gloves will be identical and in the interval (0,1] (the gloves are chiral). CHIs for all possible intermediates will be in the interval [0,1], then return to the unperturbed glove value with no necessity for or prohibition of passing through zero.

QCM begins with enumeration of graph automorphisms in concentric layers of the array starting at its origin. "Direct symmetry index" DSI, the normalized minimized sum of N squared-distances between the vertices of the d-set and the permuted d-set, measures set similarity to self. DSI>0 beyond a few contained unit cells disqualifies extremal parity pair test masses.

"Correspondences" COR includes the identity element but is not a count of group theory symmetry elements. COR>1 beyond a few contained unit cells disqualifies extremal parity pair test masses. Assigning different atom labels (SiO_2) does not default assign different graph theory point colors. QCM numeric outputs are independent of input file structure format, atom connectivity (if any), and list ordering. All points are assigned unit weight because composition is EP-inert. Organic compounds whose lattice chiralities only arise from atom labels are unsuitable.

QCM as supplied analyzes up to 15,000 atoms. If DSI=0 and COR=1 obtain through ~1100 atoms (a mainframe CPU-day), specific code can generate atom coordinates within successive lattice radii and then CHI values for $\sim 10^8$ atoms/second in a personal computer [15]. Overlapping benchmarks are traceable to QCM.

CHI is theoretically coupled to the number of points in a growing lattice volume, Eq. (2):

$$\log(1\text{-CHI}) = (-2/3)[\log(\text{atoms})] + \text{intercept} \qquad (2)$$

CHI given DSI=0 and COR=1 is a connection between eigenvalues, special functions, and their representation theory with solid angles, and exponentials of fractions of pi [16] at a characteristic scale. The intercept is now modeled as the smallest solid angle subtended by the vertex angle Φ of a polyhedron (the supplement of its dihedral angle) defined by three consecutive atoms within the screw axis helix, Eq. (3),

$$\log(1\text{-CHI}) = -2[\log(\text{radius, Å})] + [\pi(180-\Phi)/60] - \pi \qquad (3)$$

The overall CHI of a self-similar lattice can increase or decrease with growth. An intrinsically chiral lattice ($3_1 3_2$ $4_1 4_3$ $6_1 6_5$ or $6_2 6_4$ screw axes) has CHI>0 even if its formula units are achiral [17]. 2_1 4_2 6_3 screw axes are each simultaneously left- and right-handed screws. These or no screw axes decrease CHI with growth even if formula units are intensely chiral.

A crystallographic space group is a group of automorphisms with a bounded fundamental region. Chiral crystal structures (65 Sohncke space groups of 230 3-space periodic crystallographic space groups) as such are insufficient for constructing parity pair test masses. Space groups must be parity pairs (11 pairs of enantiomorphic space groups in the 65; italicized in Table III) of which three pairs contain both senses of screw axes in conflict and five pairs contain 2_1 screw axes. These 16 enantiomorphic space groups are poor candidates for parity Eötvös experiments. Even given a lattice from the three pairs of fully qualified space groups, a crystal structure must give DSI=0 and COR=1 in QCM for successively larger radii from a few unit cells to ~1100 atoms contained.

TABLE III. Six fully qualified, 43 invalid, and 16 deficient Sohncke space groups.

Status	Space groups					
fully qualified	$P3_1$	$P3_112$	$P3_121$			
	$P3_2$	$P3_212$	$P3_221$			
zero intrinsic	P1	P2	$P2_1$	C2	P4	$P4_2$
lattice chirality	I4	P3	P6	$P6_3$		
2_1 4_2 6_3 screw	$P222_1$	$P2_12_12$	$P2_12_12_1$	$C222_1$	C222	F222
axes only	I222	$I2_12_12_1$	$P4_222$	$P4_22_12$	I422	$P6_322$
C_n axes only	P222	P422	$P42_12$	P312	P321	P622
opposite-sense	$I4_1$	$I4_122$	R3	R32	P23	F23
screw axes	I23	$P2_13$	$I2_13$	P432	F432	I432
	$F4_132$	$P4_132$	$I4_132$	$P4_132$	$P6_2$	$P6_222$
				$P4_332$	$P6_4$	$P6_422$
same-sense +	$P4_1$	$P4_12_12$	$P4_122$	$P6_1$	$P6_122$	
2_1 screw axes	$P4_3$	$P4_32_12$	$P4_322$	$P6_5$	$P6_522$	

V. REDUCTION TO PRACTICE

Testing for parity-antisymmetric gravitation requires opposed identical composition and macroscopic form test masses such that:

1) ~100% of rest mass is experimental active mass.
2) Parity divergence calculation is *ab initio* from coordinate input only.
3) Opposed masses are maximally parity divergent: DSI=0, COR=1, CHI>$1\text{-}10^{-10}$
4) A self-similar test mass has a sub-nanometer emergent scale gaplessly accumulating in-phase to multiple centimeter dimensions and gram masses (a periodic single crystal).
5) Quality single crystals suitable for test mass fabrication can be grown.
6) CHI resists decrease given sparse noise: impurities, vacancies, interstitials, dislocations, mosaicity.
7) Both resolved parities are available absent a ferroelectric phase; allotropy, polymorphism, magnetic inclusions, amorphous volumes, disinclinations; electrical twinning (Dauphiné twins), and optical twinning (Brazil twins).
8) The parity Eötvös experiment runs in unmodified composition Eötvös experiment apparatus with unchanged protocols.
9) Net output is unconstrained by prior observations.

X-ray (scattered by electrons) and neutron (scattered by nuclei) α-quartz diffraction structures are identical within experimental error. Rest mass has consistent coordinates, listed in Table IV.

TABLE IV. Matching coordinates of $P3_221$ α-quartz atoms and nuclei at 298°K.

Diffraction method	Unit cell axes, Å a,b	c	Volume, nm^3	Fractional coordinates x/a	y/b	z/c		Smallest helix vertex angle
x-ray	4.9137	5.4047	0.11301	0.4697	0.0000	1/6	Si	110.56°
[18]				0.4133	0.2672	0.2855	O	
neutron	4.9134	5.4052	0.11301	0.4701	0.0000	1/6	Si	110.53°
[19]				0.4136	0.2676	0.2858	O	

Inhomogeneity (atom or not atom), unit cell anisotropy (unique c-axis), and dependence of CHI upon inertial moments give substantial fluctuations around a uniform trend as test mass radius increases. Explicit calculation of CHI for increasing radius single crystal spherical balls of either $P3_221$ α-quartz or $P3_121$ tellurium[20] given Eq. (1) and (2) give predicted slopes, consistent intercepts, and observed standard deviations of one log(1-CHI) unit about the best fit lines (σ = 0.9970 for all plots).

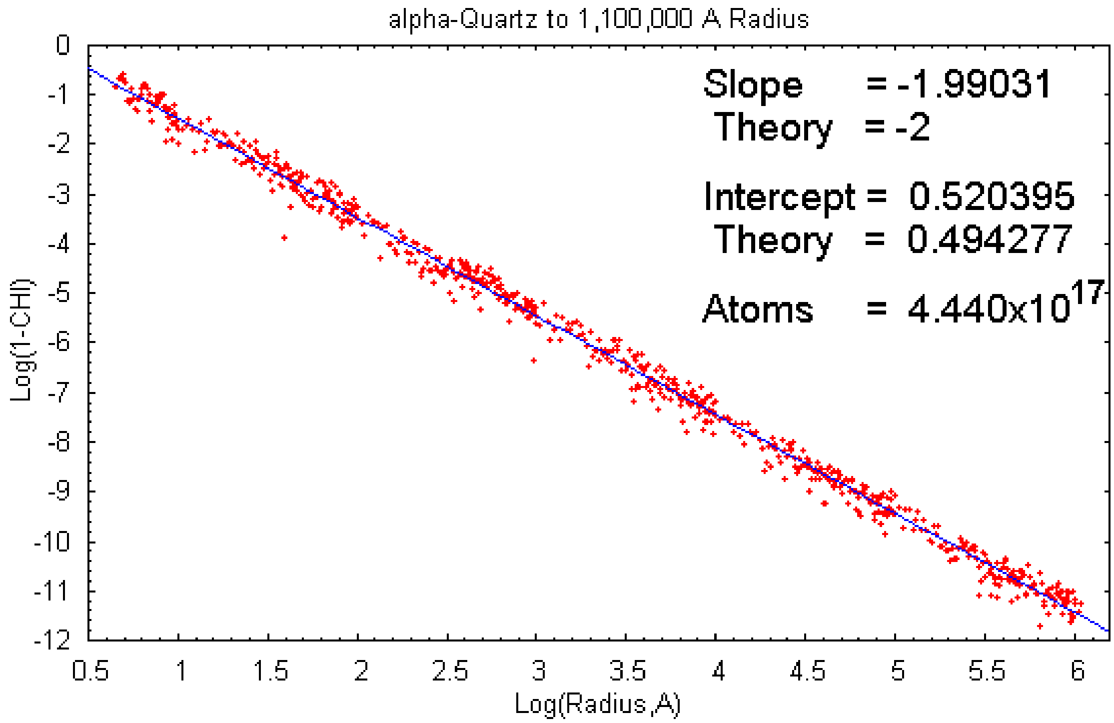

FIG. 1a. Approach to CHI=1 by α-quartz (711 samples) by radius.

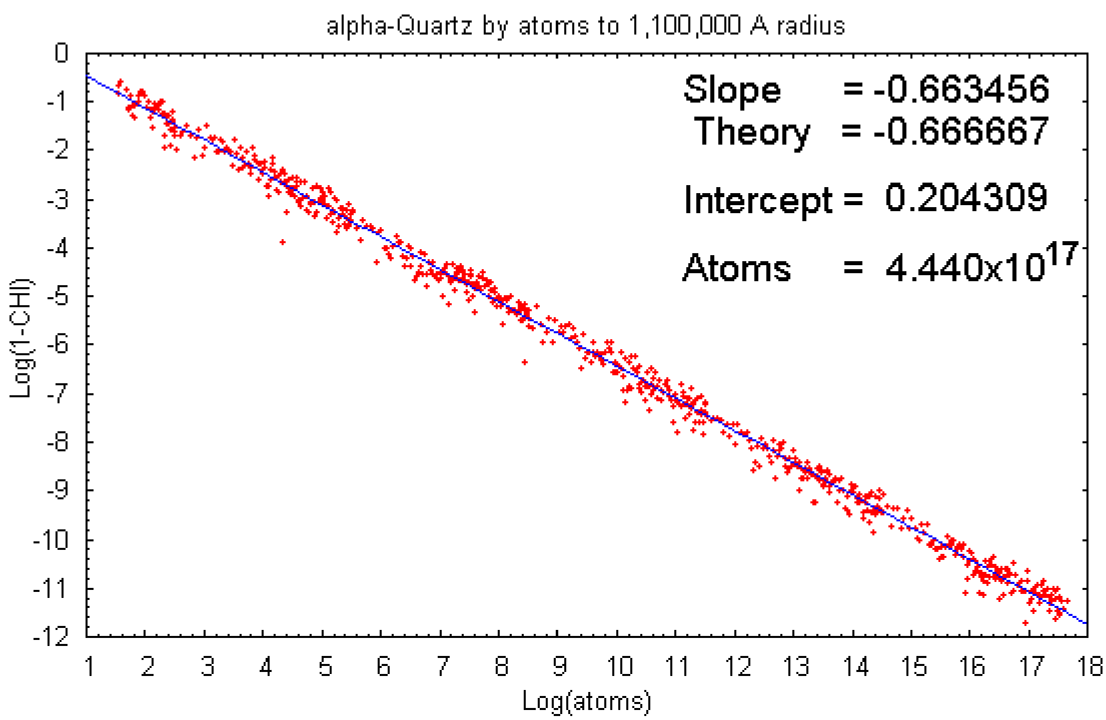

FIG. 1b. Approach to CHI=1 by α-quartz (711 samples) by atoms.

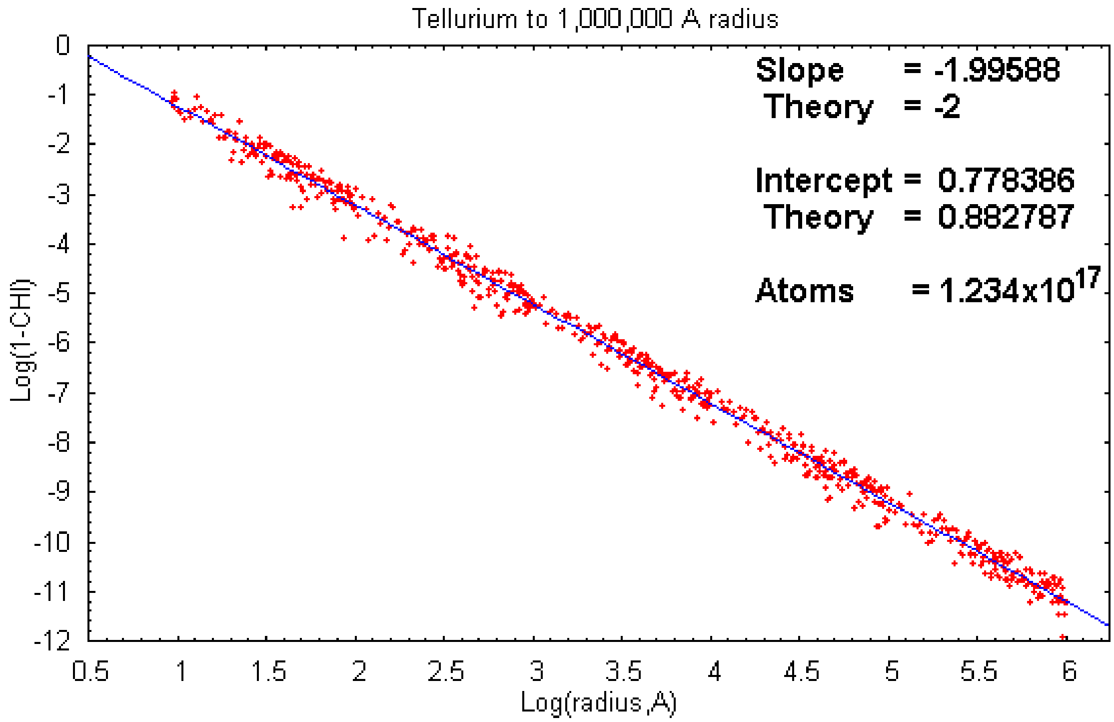

FIG. 2a. Approach to CHI=1 by tellurium (659 samples) by radius.

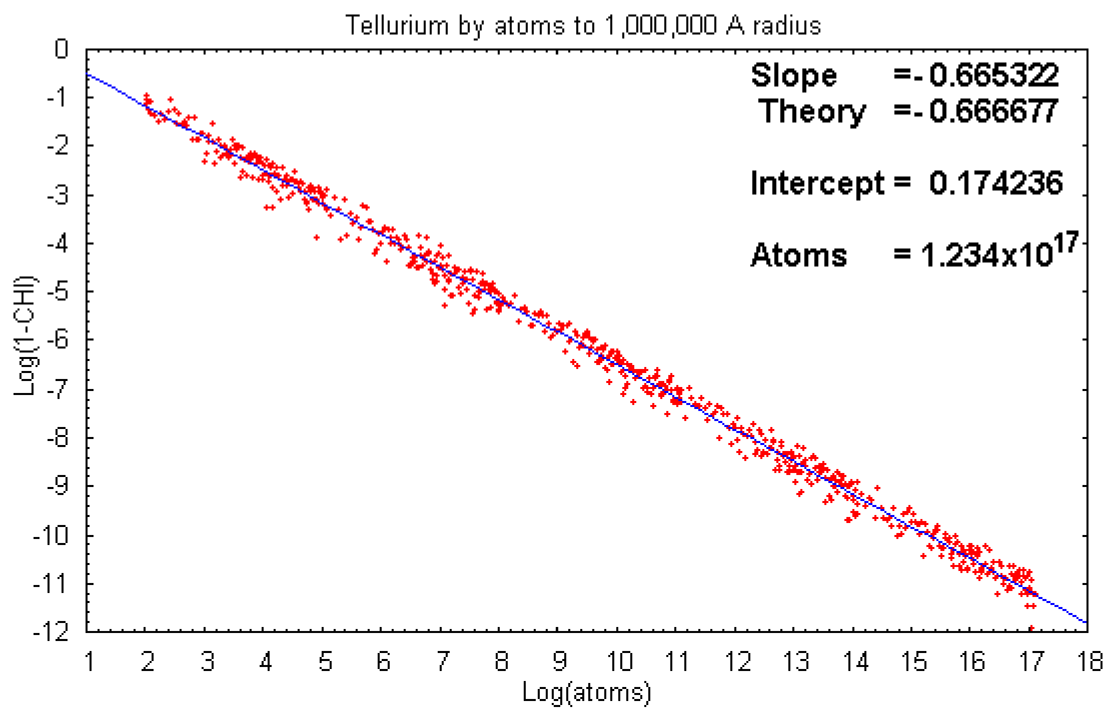

FIG. 2b. Approach to CHI=1 by tellurium (659 samples) by atoms

TABLE V. Modeled and graphic fits of log(1-CHI) versus log(radius).

Crystal lattice	Lattice volume/atom nm^3	Smallest vertex angle	Modeled intercept (-2 slope)	Graphic intercept slope	$(1\text{-CHI})\ x10^{15}$ 1 cm	3 cm diameters
α-quartz	0.01256	0-Si-O	0.49428	0.52040	1.248	0.1387 model
$4.4401x10^{17}$ atoms		110.56º		-1.99031	1.574	0.1768 graph
tellurium	0.03394	Te-Te-Te	0.88279	0.78386	3.054	0.3393 model
$1.2342x10^{17}$ atoms		103.14º		-1.99975	2.472	0.2702 graph

Given Eq. (3) and limiting $\Phi=0°$, log(1-CHI) = -2[log(radius)] + 2π, generated by qualified lattice space groups without atoms participating. Three-centimeter diameter parity test masses will have minimal CHI>1-($8.53x10^{-11}$). Maximally prolate atomic helices with limiting $\Phi=180°$ generate log(1-CHI) = -2[log(radius)] - π. Three-centimeter diameter test masses can have maximal CHI<1-($3.21x10^{-20}$). Overall structural perfection is important, but sparse noise does not corrupt CHI. Calculated $4.3x10^{15}$ nm^3 volumes of α–quartz possess parity divergence deeply asymptotic to CHI=1. A test mass configuration whose parity divergence arose from macroscopic form would be inert in a parity Eötvös experiment.

Polarized spin Eötvös experiments tested for spacetime torsion [21]. Ferrimagnet Dy_6Fe_{23} at -1ºC has no external magnetic field and net 0.4 unpaired electrons/formula unit, or 97 nanograms of (unpaired spins)/g test mass. Measurable output requires a 50,000 metric ton cylinder 20 meters in diameter and length [22].

An evolving 2.2°K Eötvös balance would classically oppose Be and Mg (nuclear binding energies)/baryon [23]. Neutron and proton mass equivalents [24] weighted for magnesium isotopic abundance give a very small difference/average active mass fraction,

p = 938.271998 MeV
n = 939.565330 MeV
Be = 6.462844 MeV/baryon binding energy
Mg = 8.265129 MeV/baryon binding energy
[Mg - Be]/[(17.3202n+16p)/33.3202] = 0.001919

One-centimeter diameter tellurium parity pair single crystal test masses have CHI = 1-($2.47x10^{-15}$). Three-centimeter diameter α-quartz test masses have CHI = 1-($1.77x10^{-16}$). (Active mass)/(rest mass) > 0.9997. Parity Eötvös experiments have a 521-fold active mass fraction advantage over this composition Eötvös experiment and are 10^7 times more favorable than the Dy_6Fe_{23} study.

α-Quartz perfection is characterized by a near-IR absorbance ratio, EIA Standard 477-1 (ANSI/EIA-477-A-90), JIS C 6704, and IEC 60758. O-H stretch intensity at 3410, 3500 or 3585 cm^{-1} is compared with baseline at 3800 cm-1 for a given thickness of Y-cut crystal cored normal to a prismatic face. A-grade α-quartz, perhaps swept to minimize dislocations, is desirable.

A non-null parity Eötvös experiment in α-quartz would be followed by two hemi-parity experiments. Amorphous fused silica test masses would oppose either $P3_121$ or $P3_221$ α-quartz test masses. As with a single shoe fitted to both feet, one expects unequal diastereotopic interactions when a single chiral target (spacetime, by prior demonstration) is separately challenged by both enantiomorphs.

VI. CONCLUSION

Parity Eötvös experiments test the Equivalence Principle with an unexamined external symmetry-coupled physical property. They compare symmetry of the laws with symmetry of the states. Chemically and macroscopically identical, crystallographically enantiomorphic single crystal α-quartz bodies are extremal parity pair test masses of unprecedented net divergent property amplitude and allowed EP violation magnitude. Reproducible net output falsifies metric gravitation. Somebody should look.

ACKNOWLEDGMENTS

Michel Petitjean (quantitative chirality theory and QCM), John Osborn (crystallography), John W. Hooper, Jon Wright (support software), Eugeniu M. Popescu (field theory); CHI calculation (SiO_2: Roy Culley; John E. Scott for cluster programming; Te: Siddhartha Mukherjee, Steve Thompson, Robert Newson).

[1] Sean M. Carroll and George B. Field, "Einstein equivalence principle and the polarization of radio galaxies," Phys. Rev. D **43** 3789 (1991)
[2] Jian Qi Shen, "On some weak-gravitational effects," /gr-qc/0305094; Z.B. Zhou, J. Luo, Q. Yan, Z.G. Wu, Y.Z. Zhang, and Y. X. Nie, "New upper limit from terrestrial equivalence principle test for extended rotating bodies," Phys. Rev. D **66** 022002 (2002)
[3] R. Aldrovandi, J.G. Pereira, and K.H. Vu, "Gravitation without the equivalence principle," /gr-qc/0304106
[4] J.W. Maluf, J.F. da Rocha-Neto, T.M.L. Toribio, and K.H. Castello-Branco, "Energy and angular momentum of the gravitational field in the teleparallel geometry," Phys. Rev. D **65** 124001 (2002); V.C. de Andrade, L.C.T. Guillen, and J.G. Pereira, "Gravitational Energy-Momentum Density in Teleparallel Gravity," Phys. Rev. Lett. **84** 4533 (2000)
[5] Laurence D. Barron, "Optical activity and time reversal," J. Mol. Phys. **43** 1395 (1981)
[6] T. Schücker, "Forces from Connes' geometry," /hep-th/0111236, especially reference [50] and references therein.
[7] Richard T. Hammond, "Torsion gravity," Rep. Prog. Phys. 65(5) 599 (2002)
[8] Yu. N. Obukhov and J.G. Pereira, "Metric-affine approach to teleparallel gravity," Phys. Rev. D **67** 044016 (2003); Yakov Itin, "Energy–momentum current for coframe gravity," Class. Quant. Grav. **19** 173 (2002); V.C. de Andrade, L.C.T. Guillen and J.G. Pereira, "Teleparallel gravity: an overview," /gr-qc/0011087; Soumitra Sengupta, "Parity violation in a gravitational theory with torsion: A geometrical interpretation," Pramana – J. Phys. **53**, (1999)
[9] S. Baeβler, B.R. Heckel, E.G. Adelberger, J.H. Gundlach, U. Schmidt, and H.E. Swanson, "Improved test of the equivalence principle for gravitational self-energy," Phys. Rev. Lett. **83** 3585 (1999); Kenneth Nordtvedt, "Lunar Laser Ranging - a comprehensive probe of post-Newtonian gravity," /gr-qc/0301024
[10] P. Möller, J.R. Nix, W.D. Myers, and W. J. Swiatecki, "Nuclear Ground-State Masses and Deformations," Atomic Data Nucl. Data Tables **59** 185 (1995)
[11] J. Etxebarria, C.L. Folcia, and J. Ortega, "Origin of the optical activity of silver thiogallate," Appl. Cryst. **33** 126 (2000)
[12] G. Szivessey and C. Münster, "Über die Prüfung der Gitteroptik bei aktiven Kristallen," Ann. Phys. (Leipzig) **20** 703 (1934)
[13] Michel Petitjean, "On the root mean square quantitative chirality and quantitative symmetry measures," J. Math. Phys. **40** 4587 (1999); "Chirality and symmetry measures: A transdisciplinary review," Entropy **5** 271-312 (2003)
[14] P.G. Mezey, "Rules on chiral and achiral molecular transformations," J. Math. Chem. **17** 185 (1995)
[15] QCM into FastCHI by Matthew Francey; into 80-bit precision BigCHI by Anthony J. Lapen; into parallelized CHIpir by John E. Scott.
[16] Personal communication, Prof. Penelope Smith, Mathematics Dept., Lehigh University.
[17] Howard D. Flack, "Chiral and achiral crystal structures," Helv. Chim. Acta **86** 905 (2003)
[18] K. Kihara, "An X-ray study of the temperature dependence of the quartz structure," Eur. J. Mineralogy **2** 63 (1990)
[19] A.F. Wright and M.S. Lehmann,"The structure of quartz at 25 and 590 degrees C determined by neutron diffraction," J. Solid State Chem. **36** 371 (1981)
[20] C. Adenis, V. Langer, O. Lindqvist, "Reinvestigation of the structure of tellurium" Acta Cryst. C **45**(6) 941 (1989)
[21] Rogers C. Ritter, Charles E. Goldblum, Wei-Tou Ni, George T. Gillies, and Clive C. Speake, "Experimental test of equivalence principle with polarized masses," Phys. Rev. D **42** 977 (1990)
[22] L.C.Garcia de Andrade, "Spin-polarised cylinders and torsion balances to test Einstein-Cartan gravity?" /gr-qc/0102020
[23] Riley Newman, "Prospects for terrestrial equivalence principle tests with a cryogenic torsion pendulum," Class. Quantum Grav. **18** 2407 (2001)
[24] K. Hagiwara, et al., "Review of Particle Physics," Phys. Rev. D **66** 010001 (2002)