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Optical Implementation of Two Dimensional Bipolar Hopfield Model Neural Network

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ABSTRACT

The Hopfield model with bipolar neural states (-1,1) has an advantage in that the output signal-to-noise ratio is $\sqrt{2}$ times that of the unipolar neural states (0,1). Also, the information storage capacity of the bipolar neural states can be twice that of the unipolar model. It is difficult to represent bipolar quantities in an intensity distribution. A method to achieve full bipolar performance in a single channel optical associative memory is presented in this paper. A two dimensional bipolar Hopfield model optical neural network has been implemented by coding the biased interconnection weights, a distributed background and an input-dependent dynamic threshold on a single mask. Content addressability properties are improved through the introduction of a distributed background. Computer simulations of two dimensional bipolar neural networks have been performed .

Key Words: associative memory, neural networks, image processing, pattern recognition, optical computing.

I. Introduction

A neural network consists of a highly interconnected array of simple processing elements. When the number of processors (neurons) is very large, the massive parallelism, high speed, low crosstalk and high-density connectivity advantages of optics motivate their use in neural networks. The simple optical neural network model requires only two basic operations: vector matrix multiplication and thresholding. Psaltis and Farhat (1985) first demonstrated an optical implementation of Hopfield's neural network model (Hopfield, 1982) based on a vector matrix multiplier. The Hopfield model with bipolar neural states (–1,1) has an output signal to noise ratio $\sqrt{2}$ times that of the Hopfield model with unipolar (0,1) states. Also, the information storage capacity of a bipolar states neural network can be twice that of a unipolar neural network (Hopfield, 1982).

It is difficult to implement bipolar quantities in an intensity distribution. In order to handle both positive and negative quantities of inter connection weights two sets of hardware were required (Psaltis and Farhat, 1985) one for handling positive interconnection weights and the other set for negative weights. In this dual channel hardware system, positive and negative interconnections are processed separately and then combined electronically or optically. The hardware in this system is complex. Jang and Jung (1988) introduced a positive offset to the interconnection weights so as to make them nonnegative and make possible a single channel optical realization. Dynamic thresholding is needed to eliminate this added offset effect caused by the output. To overcome the problem that arises due to an imbalance between the number of ones and of zeros in stored images, a modification of the Hopfield model has been presented, and the result of this modification is a model with bipolar addressing (Oh *et al.*, 1988; Ramachandran and Gunasekaran, 1996).

A method for optical implementation of the bipolar Hopfield algorithm has been proposed by David and Saleh (1990), who used inner products of unipolar data. Wang and Mu (1992) achieved fully bipolar performance in a single channel optical associative memory by coding the biased interconnection weights (a distributed background) and an input-dependent dynamic threshold on a single mask.

In this study, by using an outer-product approach and adding a distributed background to the output of an optical associative memory, a two-dimensional bipolar optical Hopfield model was simulated. We will show that the performance of the bipolar state network is superior to that of the unipolar one. We will also present the optical set up for the optical implementation of the bipolar model.

II. Two Dimensional Hopfield Model

The two dimensional Hopfield model neural network can be summarized as follows. It consists of N^2 mutually interconnected neurons, whose current states are characterized by binary states:

$$\boldsymbol{V} = \begin{pmatrix} V_{11} & \cdots\cdots & V_{1N} \\ V_{21} & \cdots\cdots & V_{2N} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ V_{N1} & \cdots\cdots & V_{NN} \end{pmatrix}$$

with V_{ij} (1 or 0) denoting the state of neuron V_{ij}. A set of M patterns or images $V^{(m)}$, $m = 1, 2, \ldots, M$, each with $N \times N$ elements or pixels, is stored in the network. The stored memory matrix element $\boldsymbol{T}_{ijkl}$ denotes the interconnection strength between neurons ij and kl:

$$\boldsymbol{T}_{ijkl} = \sum_{m=1}^{M} [2V_{ij}^{m} - 1][2V_{kl}^{m} - 1] - M\delta_{ijkl}$$

$$\text{for } 1 \leq i,\ j,\ k,\ l \leq N, \quad \textbf{(1)}$$

where δ_{ijkl} is a Kronecker delta function defined as

$$\delta_{ijkl} = 1 \qquad \text{if } i = k \text{ and } j = l$$

$$= 0 \qquad \text{otherwise.}$$

To iterate the two dimensional neural network, the next state of neuron (i,j) is determined by the current states of the remaining neurons as

$$V_{i,j}(\text{next state}) = 1 \qquad \text{for } U_{ij} \geq 0$$

$$= 0 \qquad \text{for } U_{ij} < 0,$$

where

$$U_{ij} = \sum_{k=1}^{N}\sum_{l=1}^{N} \boldsymbol{T}_{ijkl} V_{kl}. \quad \textbf{(2)}$$

If $\boldsymbol{T}_{ijkl}$ is multiplied by one of the stored images V_{ij}^{mo}, then the product U_{ij}^{mo} is an estimate of the stored image $[2V_{ij}^{mo} - 1]$:

$$U_{ij}^{mo} = \sum_{k=1}^{N}\sum_{l=1}^{N} \boldsymbol{T}_{ijkl} V_{ij}^{mo}. \quad \textbf{(3)}$$

When the input differs from any of the stored images, for example, in the case of an incomplete or partially erroneous version of a stored image, the network updates the neural states in accordance with the following iteration form:

$$V_{ij}(n+1) = f\left|U_{ij}(n)\right|$$

$$U_{ij}(n) = \sum_{k=1}^{N}\sum_{l=1}^{N} \boldsymbol{T}_{ijkl} V_{kl}(n), \quad \textbf{(4)}$$

where $V_{ij}(n)$ and $V_{ij}(n + 1)$ are the values input to the (i,j)th neuron for the nth and the $(n + 1)$th iterations, respectively, and $U_{ij}(n)$ is the output estimate of the (i,j)th neuron for the nth iteration. This process of determining the next state from the current state is repeated until convergence to the stored image is accomplished, which is the nearest neighbour of the input from among the stored images.

III. Two Dimensional Bipolar Hopfield Model

A neural network with bipolar neural states and bipolar interconnections possesses an improved output signal-to-noise ratio, a large convergence radius, high converging speed, and increased storage capacity. For the bipolar Hopfield model, the four dimensional memory interconnection is that shown in Eq. (1). However, the output estimate of the (i,j)th neuron is modified to obtain

$$2U_{ij}(n)\Big|_b = \sum_{k=1}^{N}\sum_{l=1}^{N} T_{ijkl}[2V_{kl}(n) - 1], \quad \textbf{(5)}$$

$$U_{ij}(n)\Big|_b = \sum_{k=1}^{N}\sum_{l=1}^{N} T_{ijkl}\, V_{kl}(n) - \sum_{k=1}^{N}\sum_{l=1}^{N} \tfrac{1}{2} T_{ijkl},$$

$$U_{ij}(n)\Big|_b = U_{ij}(n) - \sum_{k=1}^{N}\sum_{l=1}^{N} \tfrac{1}{2} T_{ijkl}. \quad \textbf{(6)}$$

The second term in the last equation acts as the neuron dependent threshold (TH):

$$V_{ij}(n+1) = f\{U_{ij}(n)\Big|_b\} = \begin{vmatrix} 1 & \text{for } U_{ij}(n) > TH \\ 0 & \text{otherwise.} \end{vmatrix} \quad \textbf{(7)}$$

The process is conducted by means of fixed neuron-dependent thresholding of the resultant output for a unipolar input. That is, a neuron dependent *TH* (*DTH*) gives to the current optical neural nets a bipolar neural property. The distributed threshold may be expressed as

$$DTH = \sum_{m=1}^{M} \left(2V_{ij}^{m} - 1\right)\left(N_0^m - \frac{N^2}{2}\right) - \frac{M}{2}, \quad \textbf{(8)}$$

where N_0^m is the number of ones in the stored image $V_{ij}^{(m)}$.

IV. Optical Implementation of Bipolar Neural Network

Because it is impossible to realize negative values with an optical mask and because adding a background is much easier when using optics, the interconnection weights are biased in order to yield a nonnegative memory matrix:

$$\boldsymbol{T}_{ijkl}\Big|_p = \boldsymbol{T}_{ijkl} + A, \quad \textbf{(9)}$$

where the bias A is the absolute value of the minimum element of the former matrix $\boldsymbol{T}_{ijkl}$. The effect of the bias on the output estimate produces an input dependent uniform background, which can be eliminated by properly adjusting the output threshold level. The expected output estimate of the bipolar Hopfield model is expressed in terms of the unipolar model as

$$\begin{aligned} u_{ij}(n)\Big|_b &= \sum_{kl=1}^{N} \boldsymbol{T}_{ijkl}\left[V_{kl}(n) - \frac{1}{2}\right] \\ &= \sum_{kl=1}^{N} \left\{\boldsymbol{T}_{ijkl} V_{kl}(n) - \frac{1}{2} T_{ijkl}\right\} \\ &= \sum_{kl=1}^{N} \left\{\boldsymbol{T}_{ijkl}\Big|_p V_{ke}(n) - AV_{ke}(n) - \frac{1}{2} T_{ijkl}\right\} \\ &= \sum_{kl=1}^{N} \left\{\boldsymbol{T}_{ijkl}\Big|_p V_{kl}(n) - AV_{kl}(n) - \frac{1}{2} T_{ijkl}\right\} \\ &= \sum_{kl=1}^{N} \boldsymbol{T}_{ijkl}\Big|_p V_{kl}(n) - N_0(n)A - DTH \\ &= \sum_{kl=1}^{N} \boldsymbol{T}_{ijkl}\Big|_p V_{kl}(n) + (\max - DTH) - (N_0(n)A + \max) \\ &= \sum_{kl=1}^{N} \boldsymbol{T}_{ijkl}\Big|_p V_{kl}(n) + BD - TH(n), \end{aligned} \quad \textbf{(10)}$$

where $N_0(n)$ is the number of ones in the input image $V_{kl}(n)$, max is the maximum of *DTH* and $\boldsymbol{T}_{ijkl}\big|_p$ is the biased interconnection strength.

This expression implies that the bipolar associative recall can be accomplished by applying dynamic neuron independent thresholding $TH(n)$ to the resultant output of a unipolar system and a distributed background (*BD*). All the quantities in the above equation are unipolarly expressed, so that computing may be performed in a single-channel optical system. Therefore, unipolar data is used to realize a bipolar-like neural network.

Comparing Eq. (4) with Eq. (10), we can see that the bipolar output estimate consists of extra terms; hence, we have to form an enlarged matrix $\boldsymbol{X}_{ijkl}$.

For $1 \le i,\ j,\ k,\ l \le N$:

$$\begin{aligned} \boldsymbol{X}_{ijkl} &= \boldsymbol{T}_{ijkl}\big|_p, \\ \boldsymbol{X}_{(N+1)jkl} &= A, \\ \boldsymbol{X}_{i(N+1)kl} &= BD, \\ \boldsymbol{X}_{(N+1)(N+1)kl} &= \max. \end{aligned} \quad \textbf{(11a)}$$

For $1 \le i,\ j \le (N+1)$:

$$\begin{aligned} \boldsymbol{X}_{ij(N+1)l} &= A, \\ \boldsymbol{X}_{ijk(N+1)} &= BD, \\ \boldsymbol{X}_{ij(N+1)(N+1)} &= \max, \end{aligned} \quad \textbf{(11b)}$$

The input image will have an additional dimension:

$$\begin{aligned} V_{(N+1)j} &= 1, \\ V_{i(N+1)} &= 1, \\ V_{(N+1)(N+1)} &= 1. \end{aligned} \quad \textbf{(12)}$$

Then, Eq.(10) becomes

$$\begin{aligned} &\sum_{kl=1}^{N} \boldsymbol{T}_{ijkl}\Big|_p V_{ke}(n) + BD - [N_0(n)A + \max] \\ &= \sum_{kl=1}^{N} \boldsymbol{X}_{ijkl} V_{kl}(n) + \sum_{kl}^{N+1} \boldsymbol{X}_{(N+1)(N+1)kl} V_{kl}(n). \end{aligned} \quad \textbf{(13)}$$

This implies that the thresholded output can be obtained by thresholding the multiplication of an $(N+1)\times(N+1)$ dimensional input and $(N+1)^2 \times (N+1)^2$ element matrix, where the $(N+1)$th dimension of the output estimate is used as the threshold. The

$$
V^{(1)} = \begin{matrix} 11000 \\ 11000 \\ 11000 \\ 11000 \\ 11000 \end{matrix} \qquad V^{(2)} = \begin{matrix} 00011 \\ 00011 \\ 00011 \\ 00011 \\ 00011 \end{matrix}
$$

$$
V^{(3)} = \begin{matrix} 11111 \\ 10001 \\ 10001 \\ 10001 \\ 11111 \end{matrix} \qquad V^{(4)} = \begin{matrix} 00000 \\ 01110 \\ 00100 \\ 01110 \\ 00000 \end{matrix}
$$

Fig. 1. Stored patterns with unequal numbers of ones and zeroes.

additional matrix column and row represent the distributed background and the dynamic threshold. It is the $(N+1)$th column of the enlarged memory matrix that distinguishes the implementation of the bipolar Hopfield model from the current optical implementation of the unipolar Hopfield model.

V. Computer Simulations

Computer simulations were performed for stored patterns with unequal numbers of ones and zeroes (Fig.1). Four patterns with the order 5×5 were chosen for computer simulations. The computer simulation results are listed in Table 1, and the convergence properties of the bipolar and unipolar models are compared. The initializing patterns with Hamming distances that are listed in the first column of Table 1 are erroneous versions of the stored patterns. In the other columns, the number denote the stored patterns, the numbers in parantheses denote the numbers of iterations, and O indicates an oscillatory output.

The initializing input patterns were created by successfully switching the states of the corresponding stored patterns. The switching order in one pattern was chosen so that the Hamming distances from the other stored patterns were as large as possible. The switching orders were

$$V^{(1)} = \{(1,3),\ (2,3),\ (3,3),\ (4,3),\ (5,3),\ (1,4),\ (2,4),\ (3,4),\ (5,4),\ (4,4)\},$$

$$V^{(2)} = \{(1,3),\ (2,3),\ (3,3),\ (4,3),\ (5,3),\ (1,2),\ (2,2),\ (3,2),\ (5,2),\ (4,2)\},$$

$$V^{(3)} = \{(2,2),\ (2,4),\ (3,2),\ (3,4),\ (4,2),\ (4,4),\ (2,3),\ (3,3),\ (4,3),\ (5,3)\},$$

$$V^{(4)} = \{(1,2),\ (1,3),\ (1,4),\ (2,2),\ (2,3),\ (2,4),\ (3,3),\ (5,2),\ (5,3),\ (5,4)\}.$$

For example, according to the fourth line in the above switching order, we changed the state of the particular element of the original stored pattern $V^{(4)}$. For a Hamming distance of four, we changed the states of the (1,2), (1,3), (1,4) and (2,2) elements in the stored pattern $V^{(4)}$. Then, this altered $V^{(4)}$ pattern became an initializing erroneous input sent to our bipolar neural network. This erroneous version $V^{(4)'}$ was as follows:

$$
V^{(4)'} = \begin{matrix} 01110 \\ 00110 \\ 00100 \\ 01110 \\ 00000. \end{matrix}
$$

The Hamming distance between $V^{(4)}$ and $V^{(4)'}$ was four. As indicated in the fourth row of Table 1, the unipolar neural network produced an incorrect output, i.e., the third stored pattern $V^{(3)}$ after 9 iterations. On the other hand, the bipolar network con-

Table 1. Comparison of Computer Simulation Results for a Bipolar and a Unipolar System

Hamming distance from $V_i^{(m)}$	Final output pattern (Number of iterations)							
	Bipolar Scheme				Unipolar Scheme			
	m=1	m=2	m=3	m=4	m=l	m=2	m=3	m=4
1	1(1)	2(1)	3(1)	4(1)	3(4)	3(4)	3(1)	4(2)
2	1(1)	2(1)	3(1)	4(1)	3(4)	3(4)	3(1)	4(2)
3	1(1)	2(1)	3(1)	4(1)	3(4)	3(4)	3(1)	O
4	1(1)	2(1)	3(1)	4(1)	3(5)	3(5)	3(1)	3(9)
5	1(1)	2(1)	3(1)	4(3)	3(4)	3(4)	3(1)	3(9)
6	1(2)	2(2)	3(2)	4(3)	3(4)	3(4)	3(1)	3(3)
7	1(1)	2(1)	3(2)	4(2)	3(4)	3(4)	3(2)	3(3)
8	1(3)	2(3)	3(2)	4(3)	3(4)	3(4)	3(2)	3(4)
9	1(3)	2(3)	3(2)	4(5)	3(3)	3(3)	3(2)	3(3)
10	1(3)	2(3)	O	O	3(3)	3(2)	3(2)	3(3)

$V^{(1)}$	=	11111	$V^{(2)}$	=	00000
		11111			00000
		00000			00000
		00000			11111
		00000			11111
$V^{(3)}$	=	00011	$V^{(4)}$	=	11000
		00011			11000
		00011			11000
		00011			11000
		00011			11000

Fig. 2. Stored patterns with equal numbers of ones and zeroes.

verged correctly to the fourth stored pattern $V^{(4)}$ in the first iteration. Thus, the bipolar network increased the speed of convergence. The distributed background which we used improved the storage capacity and content addressability.

We have also tested the bipolar state network using stored patterns with equal numbers of ones and zeroes (Fig. 2). This test also confirmed the increased retrieval capability of the network. Simulation results are listed in Table 2.

The switching orders for these patterns are as follows:

$$V^{(1)} = \{(1,1), (1,5), (2,1), (2,5), (1,2), (3,3), (4,3), (3,1)\},$$

$$V^{(2)} = \{(4,1), (4,5), (5,1), (5,5), (1,2), (2,3), (3,4), (4,2)\},$$

$$V^{(3)} = \{(1,4), (1,5), (5,4), (5,5), (2,4), (1,1), (3,1), (5,3)\},$$

$$V^{(4)} = \{(1,1), (1,2), (5,1), (5,2), (1,5), (2,4), (3,3), (5,5)\}.$$

The unipolar scheme for this input set produced erroneous results. The simulation results prove that the two-dimensional bipolar scheme is far better than the two-dimensional unipolar model.

Table 2. Simulation Results for the Bipolar Scheme with Equal Ones and Zeroes Patterns

Hamming Distance	$V^{(1)}$	$V^{(2)}$	$V^{(3)}$	$V^{(4)}$
1	1(2)	2(2)	3(2)	4(2)
2	1(2)	2(2)	3(2)	4(2)
3	1(2)	2(2)	3(2)	4(2)
4	1(2)	2(2)	3(2)	4(2)
5	1(2)	2(2)	3(2)	4(2)
6	1(2)	2(2)	3(2)	4(2)
7	1(2)	2(2)	3(2)	4(2)
8	1(2)	2(2)	3(2)	4(2)

VI. Optical Setup

Figure 3 shows a schematic diagram for the experimental setup for optical implementation of the Bipolar Hopfield neural network. An area encoding technique can be used to fabricate the enlarged matrix mask. The enlarged matrix elements are represented by transparent rectangles. The area of each rectangle is proportional to the value of the corresponding element of the enlarged matrix. However, in this case, when the number of neurons in the network is large, it is difficult to converge the light that has passed through the mask to a single spot. Also, the multiplier system will not be free of cross talk. Hence, in order to improve the performance of the system, the input is generated as follows: the plane wave illuminates the pattern displayed on the electrically addressed spatial light modulator (SLM). Liquid crystal display devices (LCD) has many advantages, such as low controlling voltage, low switch energy and no radiation. High-speed optical switches such as ferroelectric liquid crystal devices can improve the speed.

The plane wave illuminates the SLM and passes through the mask, which is placed so as to be in contact with SLM. The cylindrical lens converges the light to the detector array. The distance between the mask and the cylindrical lens is short. Each transparent area in the mask is several millimeters in size. The LCD-SLM with a pixel size about one

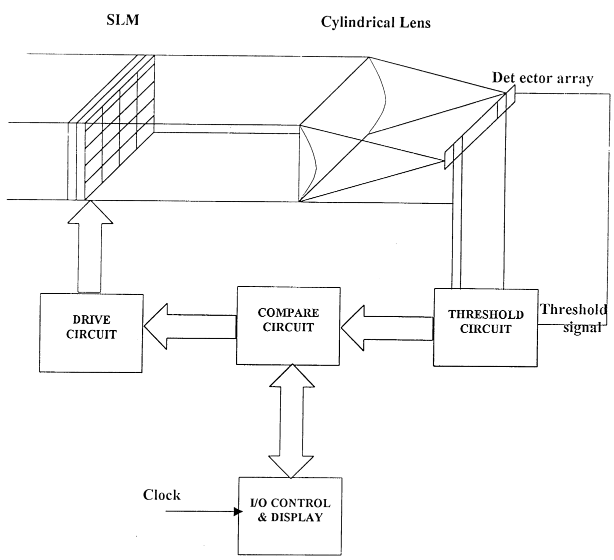

Fig. 3. Optical setup (schematic).

micrometer reduces the light cross talk and diffraction noise.

The electrical signal output of the detector is thresholded in the thresholding circuit, where the (N + 1)th signal acts as a dynamic threshold. The last neuron states stored in the register and the new neuron states are compared to determine whether or not to stop iterating. The input/output control circuit controls the input/output data flow and displays the neuron states stored in the register. When stored in the register, the new neuron states control the driving circuit so that it displays a new input pattern in the SLM. A clock controls the period of time used in one iteration. The period of one iteration is mainly determined by the switching time of LC-SLM.

VII. Conclusion

Computer simulations of two-dimensional bipolar networks have shown that the performance of the bipolar scheme is much better than that of the corresponding unipolar scheme. It is faster in terms of convergence and more accurate when compared to the unipolar scheme, as demonstrated by the tabulated results. The optical set up for implementation of the bipolar two-dimensional Hopfield algorithm has also been presented.

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二維二極狀態之Hopfield類神經網路的較佳應用

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摘　要

Hopfield類神經網路所使用的（−1,1）二極狀態，導致它比一般使用（0,1）二元狀態之類神經網路，在信號與雜信的比例上，具有$\sqrt{2}$: 1的優勢。同時，信息儲存的容量亦具有兩倍的優勢。訊號在緊緻的連續分佈與傳輸之狀況下，欲量化為二極狀態實屬不易。本文提出一種二極量化的方法，可以讓單頻光學結合式記憶體完全以此種方法來達成運算。另外，基於編碼神經元的接地權重連線、分散背景及輸入相依的變動門限值，進而提出二維的二極狀態之Hopfield類神經網路，其中由於分散背景的引出，記憶體所含內容之地址展延性獲得不錯的改進。最後，使用電腦來驗證所提出的二極狀態之Hopfield類神經網路，證明確實具有不錯的效果。