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THE ANGULAR DIAMETERS OF 32 STARS

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SUMMARY

The complete results of the observational programme on single stars carried out with the stellar intensity interferometer at Narrabri are presented. The measurements are analysed to yield the angular diameters of 32 stars in the spectral range O5f to F8. Information is also presented on nine multiple stars.

1. INTRODUCTION

The stellar interferometer at Narrabri Observatory has been used to measure the apparent angular diameters of 32 stars. This programme was started in 1964 June and completed in 1972 February. In a previous paper (**1**) referred to as Paper I, we described the instrument and the observational procedure; in Paper II (**2**) we reported the first results on 15 stars. The present paper gives a complete list of the results including those reported in Paper II with the exception of a preliminary observation (**3**) of α Lyr in 1963.

2. EQUIPMENT

The installation at Narrabri Observatory was described in Paper I. Briefly it consists of two large mosaic reflectors (6·7 m in diameter) mounted on trucks which move around a circular railway track 188 m in diameter. The reflectors are controlled by a computer, assisted by automatic photoelectric guiding, and follow a star in azimuth by moving around the track and in elevation by tilting about a horizontal axis. The separation between the reflectors, the baseline, can be pre-set anywhere within the range 10–188 m and the reflectors move so that this baseline is constant in length and always normal to the direction of the star. At the focus of each reflector the light from the star passes through a narrow-band interference filter and is then focused on to the cathode of a photomultiplier. High-frequency fluctuations (10–100 MHz) in the anode currents of these photomultipliers are carried by cables to a central control building where the time-average of their cross-product, or correlation, is measured in an electronic correlator. The angular diameter of a star is found by measuring this correlation as a function of baseline length.

The installation described in Paper I has remained substantially unchanged throughout the whole programme. However, we have, at various times, introduced improved types of phototube and also modified the circuits of the correlator. As a result the limiting magnitude of the interferometer was increased from $B = +1{\cdot}5$ in 1964 to $+2{\cdot}5$ in 1971 and the stability and reliability of the correlator were greatly improved. A complete list of the various configurations of the equipment is given in the notes to Table I. As noted in Paper II, all these modifications were

TABLE I

Observational data

1	2	3	4		5	6	7
Star	Epoch of observations	Base-line (m)	$\overline{c_\mathrm{N}(d)}$ (Arbitrary units†)	± σ^*	Ob-serving time (h)	Wave-length (Å)	Equip-ment
α Aql	1964 July	9·85	95	14	10·7	4608	b
		14·79	64	13	12·2		
		19·68	39	18	10·9		
		24·6	42	16	12·7		
		29·5	−4	13	14·4		
α Gru	1964 August	9·85	113	22	11·3	4608	b
		26·2	113	25	11·4		
		42·4	72	28	11·3		
		58·2	32	28	11·9		
		73·6	36	26	11·9		
α PsA	1964 September	9·85	103	17	12·4	4608	b
		16·42	76	21	11·0		
		23·0	60	18	11·3		
		29·5	59	20	11·0		
		35·9	42	22	12·6		
		42·4	23	17	12·2		
α Eri	1964 October	9·85	468	40	13·4	4385	a
		18·04	310	49	11·3		
		26·2	226	47	12·7		
		34·3	116	48	11·3		
		42·4	11	45	11·3		
		79·6	−30	98	2·8		
β Cru	1965 June	9·85	286	12	20·1	4385	c
		32·7	194	19	6·9		
		48·8	184	18	7·5		
		64·4	127	19	7·0		
		79·6	120	11·5	24·5		
		94·2	67	17	10·4		
		114·7	20	22	6·5		
		133·2	−8	19	7·3		
		154·3	4·5	7·7	53·9		
α Aql	1965 July	9·85	241	10	29·8	4385	c
		19·68	115	11	29·6		
		24·6	66	15	14·4		
α Lyr	1965 July	9·85	251	8	21·5	4385	c
		19·68	88	8	22·6		
α Gru	1965 August	9·85	361	27	18·4	4385	c
		58·2	128	17	45·8		
α PsA	1965 September	9·85	319	24	16·2	4385	c
		35·9	51	15	41·5		
α Eri	1965 October	9·85	283	12	11·4	4385	c
		32·7	97	9	32·1		
β Ori	1965 October	9·85	252	15	4·6	4385	c
		23·0	98	9·5	10·7		

TABLE I—*continued*

1	2	3	4		5	6	7
Star	Epoch of observations	Base-line (m)	$\overline{c_N(d)}$ ± (Arbitrary	$\sigma^\star$ units†)	Ob-serving time (h)	Wave-length (Å)	Equip-ment
α Car	1865 October	9·58	77	7	5·7	4385	c
		13·85	24	9	4·3		
		14·46	11	10	2·3		
α CMa	1966 February	9·58	129·3	2·6	6·1	4385	c
		12·96	57·9	3·1	3·0		
		18·20	1·7	3·3	3·0		
α Leo	1966 March	9·97	347	19	16·2	4385	c
		45·5	146	22	12·7		
		55·0	42	14	27·9		
β Cru	1966 April	9·97	211	10	11·7	4430	d
		91·2	49	7·5	22·9		
α Pav	1966 August	9·97	248	15	20·6	4430	d
		32·7	177	18	15·4		
		82·5	67	12	38·2		
		100·0	19	36	3·5		
γ Ori	1966 November	9·97	165·0	9·6	15·6	4430	e
		82·5	57·4	7·3	28·6		
ε Ori	1966 December	9·97	139·0	9·5	17·7	4430	e
		99·8	31·4	6·4	38·3		
α CMa	1967 January	9·56	59·7	1·8	2·7	4430	e
		12·93	25·0	1·7	3·7		
ε CMa	1967 January	9·97	142·5	9·1	17·6	4430	e
		67·5	60·8	6·5	42·7		
α CMi	1967 February	9·51	72·6	6·3	13·9	4430	e
		15·40	16·8	3·5	40·4		
ε Sgr	1967 July	9·94	557	42	24·8	4430	f
		42·4	273	72	8·0		
		45·6	110	41	28·3		
		48·8	32	49	17·3		
β CMa	1967 November	9·90	729	43	21·4	4430	f
		121·1	202	58	11·1		
		126·0	354	91	5·0		
		130·9	132	55	12·7		
		135·5	135	69	9·0		
		144·3	145	90	5·1		
κ Ori	1967 December	9·90	811	50	24·7	4430	f
		113·4	216	97	6·4		
		130·9	305	42	36·1		
ζ Ori	1968 December	9·93	366	37	10·5	4430	g
		108·0	235	46	6·5		
		123·6	102	25	21·0		
γ Gem	1969 January	9·93	686	46	17·9	4430	g
		48·8	180	34	31·6		

TABLE I—*continued*

1	2	3	4		5	6	7
Star	Epoch of observations	Base-line (m)	$\overline{c_N(d)}$ (Arbitrary units†)	± $\sigma^\star$	Ob-serving time (h)	Wave-length (Å)	Equip-ment
ζ Pup	1969 February	9·97	650	44	19·6	4430	g
		133·2	217	39	24·1		
		144·3	251	48	19·8		
β Leo	1969 April	9·92	694	49	29·2	4430	g
		48·8	214	34	58·8		
ε Cen	1969 May	9·92	633	41	33·8	4430	g
		144·3	130	29	64·2		
ε Sgr	1969 June	9·91	686	42	18·4	4430	g
		43·2	257	26	47·0		
α CMa	1969 December	9·54	205·9	3·6	5·4	4430	h
		13·07	89·9	2·3	5·6		
η CMa	1970 February	9·88	600	45	25·8	4430	h
		87·0	168	32	55·1		
β Car	1970 March	9·93	586	28	17·9	4430	h
		41·6	171	21	28·5		
γ Crv	1970 May	9·90	612	47	37·5	4430	h
		76·6	285	49	32·5		
		94·2	101	42	44·3		
α Oph	1970 June	9·90	565	47	20·4	4430	h
		42·4	148	30	52·6		
α CMa	1970 December	9·53	257·7	3·3	6·0	4430	h
		13·07	113·4	6·1	1·0		
δ Sco	1971 April	9·90	479	53	15·7	4430	h
		48·8	562	52	23·8		
		121·1	228	31	64·6		
		163·1	−3	65	11·0		
ζ Oph	1971 June	9·90	752	68	29·3	4430	h
		133·2	173	36	88·0		
α Lyr	1971 July	9·62	569	16	7·1	4430	h
		19·59	241	14	9·4		
α Lyr	1971 August	9·95	550	27	3·3	4430	h
		19·59	228	14	11·4		
β Ori	1971 November	9·95	1069	33	19·0	4430	‡
		19·69	631	11	47·6		
α CMi	1972 January	9·56	384	14	12·5	4430	h
		14·81	126	10	25·2		
δ CMa	1972 January	9·62	567	67	16·4	4430	h
		19·77	204	58	23·2		
		26·2	20	83	11·2		

introduced in the intervals between programmes on particular stars and therefore did not affect the measurements of angular size which depend only on the ratio of the correlations at different baselines. On the other hand they did affect the arbitrary scale on which correlation was measured, and to relate one scale to another we made transfer observations on standard stars as discussed in Section 7.2.

3. OBSERVATIONAL PROGRAMME

The interferometer is limited by considerations of signal-to-noise ratio to measuring stars of spectral type earlier than G0 and brighter than $B = +2{\cdot}5$; furthermore, for this particular programme, it was planned to restrict the measurements to single stars or stars with companions at least 2 mag fainter. We therefore chose 34 stars which apparently satisfied these criteria and at the same time represented a reasonable sample of the different spectral types and luminosity classes. Subsequently, our measurements at Narrabri disclosed that seven of the stars (Table IV) are not single and that four of these (β Cen, λ Sco, σ Sgr, δ Vel) have companions which are too bright for our purpose. These four stars were therefore excluded from the programme but we have added to the list the primary stars of two binary systems (α Vir, γ^2 Vel) to make the total 32. These two binaries were observed in separate programmes which have been described elsewhere (4, 5).

It would have been desirable to measure many more stars and this could have been done by increasing the exposures beyond 100 hr to reach stars fainter than $B = +2{\cdot}5$. However, we decided not to work on fainter stars; not only would the exposures have become impracticably long, but there would have been an increasing risk that at very low signal-to-noise ratios minor systematic errors might introduce significant errors into the measured values of angular size.

4. OBSERVATIONAL PROCEDURE

For all the results reported in this paper the observational procedure was the same as that described in detail in Paper I. For convenience we shall summarize here some of the more important features.

Footnotes to Table I

* $\overline{c_N(d)}$ is the normalized, weighted mean correlation from the star in arbitrary units, for a reflector separation d. All the observations have been reduced to the same standard interval. σ is the rms uncertainty in $\overline{c_N(d)}$.

† $\overline{c_N(d)}$ is in arbitrary units which depend on the parameters of the equipment in use at the time. Over the period covered by the observations several modifications were made to the equipment which altered the scale $\overline{c_N(d)}$ (Section 6). There were eight principal configurations of the equipment which are shown by the letters (a)–(h) in column 7. These letters correspond to the use of the following major components:

(a) RCA type 7046 phototubes, 4385 ± 40 Å filters.
(b) RCA type 7046 phototubes, 4608 ± 150 Å filters.
(c) RCA type dev. C 31011 phototubes, 4385 ± 40 Å filters.
Mark I transistorized multiplier.
(d) RCA type dev. C 31011 phototubes, 4430 ± 50 Å filters.
(e) RCA type 8575 phototubes, 4430 ± 50 Å filters.
Mark II transistorized multiplier.
(f) RCA type 8575 phototubes, 4430 ± 50 Å filters.
Mark II transistorized multiplier plus synchronous integrator.
(g) As (f) but with RCA type dev. C 31000 phototubes.
(h) As (f) but with RCA type 8850 phototubes.
(‡) As (h) but with polaroid in front of phototubes.

Before each observing session the delays in the two arms of the interferometer were equalized by measurements on the phototubes, cables and amplifiers. A final check on the whole system was made by observing a bright star and adjusting the relative delay in the two arms of the interferometer to give the maximum signal-to-noise ratio.

Throughout the programme a close check was kept on the gain and zero-drift of the correlator. The correlator was kept running continuously night and day and, between runs on a star, the two phototubes were illuminated by small pea-lamps adjusted in brightness to give the same phototube anode currents as the star. The gain was measured at least once per day and immediately before and after every run on a star. The zero-drift was found by taking a three-day running mean of the correlation recorded with the phototubes illuminated by the pea-lamps.

During observations routine checks were made on the length and orientation of the baseline. As a first precaution we checked, at least once every night, that the actual position of the reflectors on the track agreed with that shown on the control desk; this was done by driving the reflectors to calibrated marks on the track. As a further check the length of short baselines, which are critical for partially resolved stars, were measured directly with a tape measure. Errors in the orientation of the baselines, which introduce differential delays into the light reaching the reflectors, were monitored by recording the indicated azimuths of the two reflectors at least every half hour and comparing them with the true azimuth of the star.

Great care was taken to minimize the effects of any possible systematic errors on the ratio of the correlations observed with different baselines since they might introduce errors into the final values of angular size. For example, to reduce errors which might depend on azimuth and elevation, we carried out the observations of any given star over the same range of hour angles at all baselines. To minimize possible effects due to slow changes in the equipment or in the observing conditions we interleaved measurements at long and short baselines and rejected all observations made when the atmospheric extinction was significantly ($\sim 0{\cdot}2$ mag) higher than normal.

5. DATA REDUCTION

The form in which the observational data are obtained and the method of data reduction have been described in detail in Papers I and II. Briefly, the printed output from the correlator shows the cumulative total of the correlation every 100 s together with the two phototube anode currents averaged over the preceding 100 s.

The first step in reducing the data was to correct the recorded correlation for any zero-drift in the output of the correlator thereby finding the true mean correlation $\overline{c_0(d)}$ from the star when observed with a baseline d. The next step was to normalize this correlation to take account of variations in the light flux received from the star and in the gain of the correlator. Following Paper II, $\overline{c_N(d)}$ the mean normalized correlation per 100 s cycle of the correlator was found from,

$$\overline{c_N(d)} = \overline{c_0(d)}/\mathrm{CAL}(i_{S1}.i_{S2}) \tag{1}$$

where i_{S1}, i_{S2} are the components of the phototube anode currents due to the star alone found by subtracting from the total currents the contributions due to the night sky and moonlight, CAL is the gain of the correlator and all the quantities are averaged over the whole night's run. The rms uncertainty σ in $\overline{c_N(d)}$ is compounded of the uncertainty due to noise in the correlator output and the uncertainty in the zero-drift and, following Paper II, was found from

$$\sigma = \mathrm{CAL}\,.\,\sigma_{\mathrm{STD}}[(i_{\mathrm{T1}}i_{\mathrm{T2}}/p)+(i_1 i_2/q)]^{1/2} \tag{2}$$

where i_{T1}, i_{T2} are the total phototube anode currents averaged over the whole run of p cycles of the correlator on the star; i_1, i_2 are the phototube anode currents averaged over q cycles of the control runs on the pea-lamps; σ_{STD} is the normalized uncertainty in one cycle of the correlator output derived from an analysis of the output fluctuations over very many runs.

The next step was to combine the normalized correlations found on different nights. They are weighted by the inverse square of their uncertainty, so that

$$\overline{c_{\mathrm{N}}(d)} = \sum_r (\overline{c_{\mathrm{N}}(d)}_r/\sigma_r^2)\Big/\sum_r 1/\sigma_r^2 \tag{3}$$

where $\overline{c_{\mathrm{N}}(d)}_r \pm \sigma_r$ represents the normalized correlation observed on the rth night. The uncertainty σ in this final value is given by,

$$\sigma = 1\Big/\sqrt{\sum_r 1/\sigma_r^2}. \tag{4}$$

The final step was to correct the values of mean normalized correlation for the loss of correlation due to errors in the orientation of the baseline which, as noted in Section 4, were recorded every half-hour during observations of a star. From these records the differential time delay in the arrival of the light at the two reflectors was calculated for each star and baseline. A corresponding loss factor was computed as a function of hour angle; the relation between time delay and loss factor was found from measurements on bright stars and differed for each configuration of the equipment. The correlation observed at each baseline was then corrected by a weighted mean loss factor based on the distribution of observing time with hour angle. It should be noted that these corrections were small, only a few per cent, and do not contribute significantly to the overall uncertainty in the final values of correlation.

6. OBSERVATIONAL DATA

Table I is a summary of the observational data on 32 stars listed in chronological order of observation; 10 of the stars have been measured at least twice. Columns 1–3 show the star name, epoch of observation and the baselines in metres; column 4 shows $\overline{c_{\mathrm{N}}(d)}$ the weighted mean normalized correlation (equation (3)) observed at each baseline, with its associated rms uncertainty (equation (4)), expressed in arbitrary units which are not the same for every star since they depend on the particular configuration of the equipment as noted in Section 4; columns 5–6 show the observing time at each baseline and the effective wavelength of the optical system; finally, column 7 shows which of eight different configurations of the equipment were used.

It should be noted that there are some small differences between the values of $\overline{c_{\mathrm{N}}(d)}$ in Table I and those published previously in Paper II. For all the stars, except two (α CMa, α Car), these differences represent minor corrections for errors in the orientation of the baseline (Section 5) which were not taken into account in Paper II. However, for α CMa and α Car the differences are due to the fact that we have restricted the observational data to a narrow range of elevation angles

(35–55°), whereas in Paper II we used data from all elevations. Both these very bright stars have such large angular diameters that they are significantly resolved at a baseline comparable in length with the diameter of the reflectors. As a consequence the normalized correlation at very short baselines depends upon the relative contributions to the total light at the photomultipliers from different parts of the reflectors. Unfortunately, the structure supporting the reflectors is insufficiently rigid and the contributions from different parts of the reflectors changes appreciably with elevation. It follows that the normalized correlation from α CMa and α Car varies with elevation and that at very short baselines this variation may itself be a function of the baseline length. Such a variation would not, of course, be a significant source of error in the final values of angular diameter if it were independent of baseline since the observations at all baselines are carried out over the same range of elevation angles; but for α CMa and α Car the dependence of the effect on baseline might perhaps introduce significant errors and we have therefore restricted the data to a small range of elevations. We have chosen the range (35–55°) because the reflector is adjusted to give its optimum optical performance at 45° and there is experimental evidence to show that there is no significant change of normalized correlation over a range of ± 10° about this optimum angle.

7. RESULTS

7.1 *Introduction*

It was shown by Hanbury Brown & Twiss (**6**) that the normalized correlation $\overline{c_N(d)}$ varies with baseline d as,

$$\overline{c_N(d)} \propto \Delta_\lambda \Gamma_\lambda^2(d) \tag{5}$$

where Δ_λ *is the partial coherence factor* and $\Gamma_\lambda^2(d)$ is the *correlation factor*.

The partial coherence factor Δ_λ takes account of the finite size of the two reflectors which are so large that they partially resolve some of the stars and reduce the correlation at short baselines. In the simple case where the aperture of the reflectors is small compared with the baseline necessary to resolve the star ($\Delta_\lambda \sim 1$) the correlation factor $\Gamma_\lambda^2(d)$ is simply proportional to the square of the modulus of the Fourier transform of the intensity distribution across the light source when it is reduced to an equivalent strip distribution parallel to the baseline (**6**). In the case where a star has a circular disc of uniform intensity it follows that:

$$\Gamma_\lambda^2(d) = \left[\frac{2J_1(\pi\theta_{\mathrm{UD}}\, d/\lambda_0)}{\pi\theta_{\mathrm{UD}}\, d/\lambda_0}\right]^2 \tag{6}$$

where θ_{UD} is the angular diameter of the uniform disc, λ_0 is the mid-band wavelength of the light and it is assumed that the light is, in effect, monochromatic. In the more general case where the angular diameter of a star is large enough to be partially resolved by the individual reflectors, it has been shown (**6**) that,

$$\Delta_\lambda \Gamma_\lambda^2(d) = \frac{1}{A^2}\int\!\!\int\!\!\int\!\!\int \left[\frac{2J_1(\xi)}{\xi}\right]^2 dx_1\, dx_2\, dy_1\, dy_2 \tag{7}$$

where

$$\xi = \frac{\pi\theta_{\mathrm{UD}}}{\lambda_0}\left[(x_1-x_2)^2+(y_1-y_2)^2\right]^{1/2} \tag{8}$$

and (x_1, y_1), (x_2, y_2) represent points on the two reflectors and the integral is taken over the area (A) of each reflector.

The normalized correlation also depends upon whether a star is single or multiple. It was shown in Paper II that, if a star is binary and the angular separation of the two components is completely resolved by the interferometer at the shortest baseline, then the normalized zero-baseline correlation $\overline{c_N(0)'}$ averaged over a range of position angles is reduced relative to a single star $\overline{c_N(0)}$ by the factor,

$$\overline{c_N(0)'}/\overline{c_N(0)} = (I_1^2 + I_2^2)/(I_1 + I_2)^2 \tag{9}$$

where I_1, I_2 are the brightness of the two components. It is simple to extend this analysis to a multiple star with n components and to show that, if the angular separation between all the components is resolved, the zero-baseline correlation is reduced relative to a single star by the factor,

$$\overline{c_N(0)'}/\overline{c_N(0)} = \sum_n I^2 \Big/ \Big(\sum_n I\Big)^2. \tag{10}$$

It follows that if a star yields a correlation which is significantly less than that expected from a single star, then it must be multiple.

7.2 *Method of analysis*

For each star in Table I the angular diameter of the equivalent uniform disc was found by fitting to the observed values of normalized correlation the theoretical curve for $\Delta_\lambda \Gamma_\lambda^2(d)$ given in equation (7). The curve was fitted to the observations by the method of least squares using an iterative computer program. Reasonable initial values were taken for C and θ_{UD}, where C is the normalized correlation at zero-baseline assuming no partial resolution ($\Delta_\lambda = 1$), so that

$$C = \overline{c_N(0)}/\Delta_\lambda \tag{11}$$

and corrections δC and $\delta\theta_{UD}$ were computed to find values of C and θ_{UD} which minimized the weighted squared residuals between the observations and the theoretical curve. The calculation was then repeated using corrected initial values until both $\delta C/C$ and $\delta\theta_{UD}/\theta_{UD}$ were $< 10^{-4}$. The results are shown in Table II.

In estimating the uncertainties in C and θ_{UD} we have taken into account not only the statistical uncertainties in fitting a curve to the normalized correlations but also the possibility that the zero level of the correlator might be systematically displaced by some unknown source of spurious correlation. In Papers I and II we described a variety of special tests which failed to show any spurious correlation, nevertheless, we have used the results of these tests to set upper limits to any such effect. Since writing Papers I and II more tests have been carried out and have lowered the limits significantly. They are now given by

$$\log(\sigma/C) \leq -2{\cdot}71 + 0{\cdot}41\,B + 0{\cdot}07\,B^2 \tag{12}$$

where σ is the uncertainty in the zero-level of the correlator, C is the zero-baseline correlation from the star and B is its apparent blue magnitude. For each star in Table II we have therefore computed the uncertainty in C and θ_{UD} corresponding to an uncertainty in the zero level of the correlator given by equation (12) and combined this with the conventional statistical uncertainty in fitting a theoretical curve to the observations. The final uncertainties in C and $\theta_{\rm UD}$ are shown in Table II.

As they stand the values of C in Table II cannot be interpreted readily because they are in arbitrary units which vary in scale with the different configurations of the equipment. We have therefore reduced them all to the same scale and normalized them by the zero-baseline correlation expected from a single unresolved star. They were first reduced to the same scale by computing from a network of transfer observations a set of scale factors which yielded the best overall agreement between the values of C measured for the same stars but with different configurations. The correlation expected from a single unresolved star was then found by taking the weighted mean $\bar{C}$ for all the stars in Table II and expressing the deviation of each star from this mean in terms of its rms uncertainty σ. All stars with deviations exceeding 3σ were then excluded and the process was repeated until the distribution of the remaining deviations was roughly normal about the weighted mean $\bar{C}$. This value of $\bar{C}$ was then taken to be the best value for the correlation from a single unresolved star and was used to find $C_{\rm N}$ for each star in the list, where

$$C_{\rm N} = C'/\bar{C} \qquad (13)$$

and C' is the measured value of C in Table II expressed on a common scale, and $C_{\rm N}$ is the zero-baseline correlation without partial resolution ($\Delta_\lambda = 1$) normalized by the zero-baseline correlation expected from a single unresolved star. The values of $C_{\rm N}$ found in this way are shown in column 4 of Table III.

The ratio of the true angular diameter of a star ($\theta_{\rm LD}$) to the angular diameter of its equivalent uniform disc ($\theta_{\rm UD}$) depends on the limb darkening law for its atmosphere. The angular diameters of the equivalent uniform discs of the stars listed in Table II must therefore be corrected to find their true angular diameters.

At present limb darkening corrections can only be based on model atmospheres. In Paper I (**1**) a cosine law was adopted and, with appropriate values for the limb darkening coefficient u_λ derived from model atmospheres, the angular diameters were corrected using the relationship between $\theta_{\rm LD}/\theta_{\rm UD}$ and u_λ given by Hanbury Brown & Twiss (**7**). In fact limb darkening is non-linear and, while the cosine law is a reasonable approximation, a more realistic method of correction is used in this paper. The squared modulus of the Fourier transform of the equivalent line intensity distribution across a star as predicted by various model atmospheres was compared with the same function for a uniform disc to find values of $\theta_{\rm LD}/\theta_{\rm UD}$ directly in terms of the parameters $T_{\rm e}$ and $\log g$ of the model. From these results the ratio $\theta_{\rm LD}/\theta_{\rm UD}$ was obtained for each star in Table II using values of $(B-V)_0$ and $\log g$ appropriate to their spectral type and luminosity class. The necessary transformations between $T_{\rm e}$ and $(B-V)$ were those given by Webb (**8**); the intensity distribution across the star was taken from the model atmospheres computed by Carbon, Gingerich & Kurucz (**9**) except for the F8 Ia star δ CMa for which the models computed by Parsons (**10**) were used.

The resulting values of $\theta_{\rm LD}$, the true angular diameters of the stars, are shown in column 6 of Table III. It should be noted that the correction factor from equivalent uniform disc to true angular diameter is small and has a mean value $\theta_{\rm LD}/\theta_{\rm UD} = 1{\cdot}044$. The uncertainties in the adopted values of $\log g$ and in the transformation between $T_{\rm e}$ and $(B-V)_0$ introduce a maximum additional uncertainty, for the adopted models, of $\sim \pm 0{\cdot}5$ per cent into the true angular diameters. In most cases the additional uncertainty is less than this and, in all cases, is negligible compared with the uncertainty in the measurement of $\theta_{\rm UD}$.

7.3 *Explanation of Tables II, III and IV*

In Table II, columns 1–4 show the *Bright star catalogue* (**11**) number, the name of the star, the date and wavelength of observation. Columns 5–8 show the results of fitting a theoretical curve to the observations as described above. Column 5 shows C the normalized correlation at zero-baseline, with the effect of partial resolution removed by putting $\Delta_\lambda = 1$ in equation (9), together with its rms uncertainty; column 6 shows Δ_λ the partial resolution factor; column 7 shows θ_{UD} the angular diameter of the equivalent uniform disc for each observation together with its rms uncertainty; column 8 shows $\bar{\theta}_{\mathrm{UD}}$ the weighted mean value of the angular diameter of the equivalent uniform disc for all observations of the star together with its associated rms uncertainty.

Table III summarizes the results of the whole programme. Columns 1–3 give the *Bright star catalogue* (**11**) number, the name of the star and its spectral classification. Column 4 gives C_{N} the zero-baseline correlation corrected for partial resolution ($\Delta_\lambda = 1$) and normalized by the value expected from a single unresolved

TABLE II

The angular diameters of the equivalent uniform discs and zero-baseline correlations for 30 stars

1	2	3	4	5	6	7	8
B.S.	Name	Year	Wave-length (Å)	$C \pm \sigma$ (Arbitrary units)	Δ_λ	$\theta_{\mathrm{UD}} \pm \sigma$ (10^{-3} seconds of arc)	$\bar{\theta}_{\mathrm{UD}} \pm \sigma$ (10^{-3} seconds of arc)
472	α Eri	1964	4385	523±48	0·987	2·02±0·18	
		1965	4385	315±15	0·989	1·82±0·08	1·85±0·07
1713	β Ori	1965	4385	312±23	0·979	2·56±0·14	
		1971	4430	1291±46	0·982	2·41±0·05	2·43±0·05
1790	γ Ori	1966	4430	168±10	0·998	0·70±0·04	0·70±0·04
1903	ε Ori	1966	4430	141±10	0·999	0·67±0·04	0·67±0·04
1948–9	ζ Ori	1968	4430	374±37	0·999	0·47±0·04	0·47±0·04
2004	κ Ori	1967	4430	811±52	0·999	0·44±0·03	0·44±0·03
2294	β CMa	1967	4430	735±46	0·999	0·50±0·03	0·50±0·03
2326	α Car	1965	4385	240±71	0·890	6·1±0·7	6·1±0·7
2421	γ Gem	1968	4430	726±53	0·994	1·32±0·09	1·32±0·09
2491	α CMa	1966	4385	344±23	0·905	5·65±0·17	
		1967	4430	166±16	0·899	5·84±0·23	
		1969	4430	519±22	0·908	5·57±0·09	
		1970	4430	642±38	0·909	5·54±0·17	5·60±0·15*
2618	ε CMa	1967	4430	145±9	0·998	0·77±0·05	0·77±0·05
2693	δ CMa	1972	4430	793±132	0·966	3·29±0·46	3·29±0·46
2827	η CMa	1970	4430	610±53	0·998	0·72±0·06	0·72±0·06

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TABLE II—*continued*

1	2	3	4	5	6	7	8
B.S.	Name	Year	Wave-length (Å)	$C \pm \sigma$ (Arbitrary units)	Δ_λ	$\theta_{UD} \pm \sigma$ (10^{-3} seconds of arc)	$\bar{\theta}_{UD} \pm \sigma$ (10^{-3} seconds of arc)
2943	α CMi	1967	4430	168 ± 28	0·915	5·32 ± 0·36	
		1972	4430	825 ± 62	0·923	5·05 ± 0·18	5·10 ± 0·16
3165	ζ Pup	1969	4430	651 ± 47	1·000	0·41 ± 0·03	0·41 ± 0·03
3685	β Car	1970	4430	631 ± 33	0·993	1·51 ± 0·07	1·51 ± 0·07
3982	α Leo	1966	4385	371 ± 21	0·994	1·32 ± 0·06	1·32 ± 0·06
4534	β Leo	1969	4430	730 ± 59	0·995	1·25 ± 0·09	1·25 ± 0·09
4662	γ Crv	1970	4430	629 ± 58	0·998	0·72 ± 0·06	0·72 ± 0·06
4853	β Cru	1965	4385	275 ± 11	0·998	0·685 ± 0·029	
		1966	4430	215 ± 10	0·998	0·726 ± 0·034	0·702 ± 0·022
5132	ε Cen	1969	4430	638 ± 45	0·999	0·47 ± 0·03	0·47 ± 0·03
5953	δ Sco	1971	4430	556 ± 46	0·999	0·45 ± 0·04	0·45 ± 0·04
6175	ζ Oph	1971	4430	758 ± 84	0·999	0·50 ± 0·05	0·50 ± 0·05
6556	α Oph	1970	4430	609 ± 57	0·992	1·53 ± 0·12	1·53 ± 0·12
6879	ε Sgr	1967	4430	604 ± 48	0·993	1·52 ± 0·11	
		1969	4430	725 ± 49	0·995	1·31 ± 0·07	1·37 ± 0·06
7001	α Lyr	1965	4385	355 ± 17	0·966	3·26 ± 0·13	
		1971	4430	751 ± 30	0·972	3·01 ± 0·10	
		1971	4430	728 ± 48	0·971	3·04 ± 0·12	3·08 ± 0·07
7557	α Aql	1964	4608	120 ± 22	0·975	2·96 ± 0·43	
		1965	4385	308 ± 17	0·976	2·76 ± 0·14	2·78 ± 0·13
7790	α Pav	1966	4430	241 ± 14	0·998	0·77 ± 0·05	0·77 ± 0·05
8425	α Gru	1964	4608	121 ± 19	0·997	0·97 ± 0·20	
		1965	4385	372 ± 29	0·997	0·98 ± 0·07	0·98 ± 0·07
8728	α PsA	1964	4608	104 ± 16	0·992	1·63 ± 0·28	
		1965	4385	367 ± 30	0·996	2·07 ± 0·14	1·98 ± 0·13

* The formal statistical uncertainty in θ_{UD} for α CMa is ±0·07 (±1·2 per cent). We cannot be certain that all possible systematic effects have been completely eliminated at this level of precision and to cover this we have increased the uncertainty in $\bar{\theta}_{UD}$ for α CMa to ±0·15.

Column 1, number of star in *Catalogue of bright stars* (11); column 2, name of star; column 3, year of observation; column 4, effective wavelength of observation; column 5, normalized zero-baseline correlation with no partial resolution ($\Delta_\lambda = 1$) in arbitrary units; column 6, partial resolution factor; column 7, equivalent uniform disc angular diameter with rms uncertainty; column 8, weighted mean equivalent uniform disc angular diameter with rms uncertainty.

TABLE III

The uniform disc angular diameters, true angular diameters and normalized zero-baseline correlations for 32 stars

1 B.S.	2 Name	3 MK	4 $C_N \pm \sigma$ (normalized)	5 $\bar{\theta}_{UD} \pm \sigma$ (10^{-3} seconds of arc)	6 $\theta_{LD} \pm \sigma$ (10^{-3} seconds of arc)
472	α Eri	‡B3 Vp	0·98 ± 0·05	1·85 ± 0·07	1·92 ± 0·07
1713	β Ori	†B8 Ia	0·98 ± 0·08	2·43 ± 0·05	2·55 ± 0·05
1790	γ Ori	†B2 III	1·03 ± 0·07	0·70 ± 0·04	0·72 ± 0·04
1903	ε Ori	†B0 Ia	0·86 ± 0·07	0·67 ± 0·04	0·69 ± 0·04
1948	ζ Ori	**O9·5 Ib	0·60 ± 0·06	0·47 ± 0·04	0·48 ± 0·04
2004	κ Ori	†B0·5 Ia	1·18 ± 0·09	0·44 ± 0·03	0·45 ± 0·03
2294	β CMa	*B1 II–III	1·07 ± 0·08	0·50 ± 0·03	0·52 ± 0·03
2326	α Car	F0 Ib–II	0·75 ± 0·22	6·1 ± 0·7	6·6 ± 0·8
2421	γ Gem	**A0 IV	1·17 ± 0·09	1·32 ± 0·09	1·39 ± 0·09
2491	α CMa	A1 V	0·91 ± 0·06	5·60 ± 0·15	5·89 ± 0·16
2618	ε CMa	**B2 II	0·89 ± 0·06	0·77 ± 0·05	0·80 ± 0·05
2693	δ CMa	**F8 Ia	0·93 ± 0·18	3·29 ± 0·46	3·60 ± 0·50
2827	η CMa	†B5 Ia	0·99 ± 0·09	0·72 ± 0·06	0·75 ± 0·06
2943	α CMi	F5 IV–V	0·98 ± 0·10	5·10 ± 0·16	5·50 ± 0·17
3165	ζ Pup	†O5 f	1·04 ± 0·08	0·41 ± 0·03	0·42 ± 0·03
3207	γ^2 Vel	§WC8 + O9 I	—	0·43 ± 0·05	0·44 ± 0·05
3685	β Car	A1 IV	1·01 ± 0·06	1·51 ± 0·07	1·59 ± 0·07
3982	α Leo	**B7 V	1·12 ± 0·07	1·32 ± 0·06	1·37 ± 0·06
4534	β Leo	**A3 V	1·17 ± 0·10	1·25 ± 0·09	1·33 ± 0·10
4662	γ Crv	B8 III	0·97 ± 0·10	0·72 ± 0·06	0·75 ± 0·06
4853	β Cru	‡B0·5 III	0·88 ± 0·03	0·702 ± 0·022	0·722 ± 0·023
5056	α Vir	*B1 IV	—	0·85 ± 0·04	0·87 ± 0·04
5132	ε Cen	‡B1 III	1·02 ± 0·07	0·47 ± 0·03	0·48 ± 0·03
5953	δ Sco	†B0·5 IV	0·75 ± 0·07	0·45 ± 0·04	0·46 ± 0·04
6175	ζ Oph	**O9·5 V	1·01 ± 0·12	0·50 ± 0·05	0·51 ± 0·05
6556	α Oph	**A5 III	0·94 ± 0·09	1·53 ± 0·12	1·63 ± 0·13
6879	ε Sgr	A0 V	1·02 ± 0·06	1·37 ± 0·06	1·44 ± 0·06
7001	α Lyr	†A0 V	0·99 ± 0·04	3·08 ± 0·07	3·24 ± 0·07
7557	α Aql	A7 IV, V	0·94 ± 0·06	2·78 ± 0·13	2·98 ± 0·14
7790	α Pav	‡B2·5 V	1·01 ± 0·07	0·77 ± 0·05	0·80 ± 0·05
8425	α Gru	‡B7 IV	1·11 ± 0·08	0·98 ± 0·07	1·02 ± 0·07
8728	α PsA	†A3 V	1·02 ± 0·08	1·98 ± 0·13	2·10 ± 0·14

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Column 1, number of star in *Catalogue of bright stars* (**11**); column 2, name of star; column 3, MK spectral classification; column 4, zero-baseline correlation with no partial resolution ($\Delta_\lambda = 1$) normalized by the value expected from a single unresolved star; column 5, weighted mean equivalent uniform disc angular diameter with rms uncertainty; column 6, true angular diameter allowing for effects of limb darkening (see Section 7.2).

TABLE IV

Multiple stars

B.S.	Name	$C_N \pm \sigma$*	Remarks
1948/9	ζ Ori	0·60 ± 0·06	
3207	γ^2 Vel	—	See (5)
3485	δ Vel	0·65 ± 0·06	
4853	β Cru	0·88 ± 0·03	
5056	α Vir	—	See (4)
5267	β Cen	0·47 ± 0·02	
5953	δ Sco	0·75 ± 0·07	
6527	λ Sco	0·48 ± 0·08	
7121	σ Sgr	0·54 ± 0·07	

* Zero-baseline correlation with effects of partial resolution removed ($\Delta_\lambda = 1$) normalized by value expected from a single unresolved star.

star; σ is the rms uncertainty and includes the uncertainty in the original measurements, the uncertainty in the scale factor used to reduce this measurement to a common scale and the uncertainty in the correlation expected from a single star. Column 5 is reproduced from Table II and gives $\bar{\theta}_{UD}$ the weighted mean angular diameter of the equivalent uniform disc. Column 6 gives θ_{LD} the true angular diameter of the star allowing for the effects of limb darkening (Section 7.2). We have included in Table III the angular diameters of the primary components of two binary systems (α Vir, γ^2 Vel) which were measured in comparatively elaborate programmes reported previously (**4**, **5**). In the original analysis of these two stars no account was taken of the effects of minor errors in the orientation of the baseline. We have therefore reviewed this analysis and find that in both cases the only significant effect is to change the angular diameter of the primary star by a few per cent; the values shown for the primary components of α Vir and γ^2 Vel in Table III include this correction and therefore differ slightly from the previously published (**4**, **5**) values.

Table IV is a list of stars which we have found to give significantly less correlation than expected from a single star and they must therefore be multiple. Columns 1–2 give the *Bright star catalogue* (**11**) number and name of the star; column 3 gives C_N the zero-baseline correlation corrected for partial resolution ($\Delta\lambda = 1$) and normalized by the value expected from a single unresolved star.

8. DISCUSSION

The results of the whole programme of measuring the angular diameters of single stars are summarized in Table III. For 27 of the stars the measured value of C_N does not differ significantly ($< 3\,\sigma$) from unity and is therefore consistent with a single star. An upper limit to the brightness of any companion star can be evaluated for each individual star from equations (9) and (10). For most of the stars in the Table $C_N > 0{\cdot}85$ which means that any companion stars are likely to be at least 2·5 mag fainter. It follows that any correlation due to such a companion would be less than 1 per cent of the correlation from the bright star itself, consequently any error in the angular size of these 27 stars, due to the presence of a companion, would be negligibly small. We are therefore justified in treating the listed values of θ_{LD} for these stars as the angular diameters of single stars.

We note that among the 27 stars in Table III which we have treated as single there are two unresolved astrometric binaries (**12**), α Oph and γ Gem. For α Oph the observed value of C_N is 0·94 ± 0·09 and, taking 3 σ as the lower limit, it is therefore unlikely that any companion is less than 1·5 mag fainter. The astrometric and spectroscopic data on γ Gem have been discussed by Beardsley (**13**) who interprets them to show that γ Gem may be a triple system with the two major components forming a binary with a period of 12·9 yr, a maximum angular separation of 0″·3 and Δm = 1·6 mag. Our observed value of C_N is 1·17 ± 0·09 which, again taking 3 σ as a lower limit, is consistent with a binary star with $\Delta m \geq 3$ mag. Although this result is apparently inconsistent with the model put forward by Beardsley it must be noted that our interpretation of C_N assumes that the angular separation of the components is fully resolved by the interferometer and, at the particular epoch of our observations, this may not have been the case.

However, five of the stars in Table III (ζ Ori, γ^2 Vel, β Cru, α Vir, δ Sco) are clearly not single and have been included in Table IV as multiple stars. ζ Ori is listed (**11**) as a triple system with a difference of about 2 mag between the brightest components. The observed value of C_N (0·60 ± 0·06) shows that the brightest component of the system is itself a multiple star and the simplest interpretation is that it is a binary star with $\Delta m \simeq 2$ mag. Nevertheless, we may certainly interpret θ_{LD} in Table III as the angular diameter of the brightest single star in ζ Ori because it is simple to show that the contributions of the companion stars to the total correlation are so small that their effects on θ_{LD} are negligible in comparison with the observational uncertainty (σ) shown in the Table. γ^2 Vel and α Vir are well-known multiple stars and have been discussed in detail elsewhere (**4, 5**); the values of θ_{LD} shown for them represent the angular diameters of their brightest components. The data on β Cru show clearly that it is a multiple star and this has already been pointed out by Popper (**14**) on the basis of our earlier results; the value of C_N (0·88 ± 0·03) is consistent with a binary star with $\Delta m \simeq 2{\cdot}9$ mag. Again the measured value of angular diameter refers to the brighter component since it can be shown that any effect due to the companion would be negligibly small. δ Sco has not been listed previously as a multiple star but the observed value of C_N (0·75 ± 0·07) is consistent with a binary star with $\Delta m \simeq 1{\cdot}9$ mag. Again the angular diameter refers to the brighter component since any effect of the companion on θ_{LD} would be small, much smaller than the observational uncertainty (σ) in Table III. It follows that we are justified in treating all 32 angular diameters in Table III as the angular diameters of single stars.

There are four stars (δ Vel, β Cen, λ Sco, σ Sgr) in Table IV which do not appear in Table III. These stars were possible candidates for our observing list but were rejected because they were found to have bright companions. δ Vel is listed (**11**) as having a faint double companion, however, the measured value of C_N (0·65 ± 0·06) shows that the bright component must itself be multiple; the simplest interpretation is that it is a binary with $\Delta m \simeq 1{\cdot}3$ mag and since this would produce an appreciable uncertainty in angular size of the brightest component it was rejected from our list. β Cen is well known to exhibit a variable radial velocity and has been discussed by Breger (**15**) and also by Shobbrook & Robertson (**16**). The value of C_N (0·47 ± 0·02) is consistent with a binary star with two equally bright components. λ Sco was reported by Slipher (**17**) to be a spectroscopic binary and by Shobbrook & Lomb (**18**) to be a β CMa variable; the value of C_N (0·48 ± 0·08) confirms that it is multiple and indicates that it is a binary with components of about equal brightness. σ Sgr has not been reported previously as a multiple star but the value of C_{N} ($0{\cdot}54 \pm 0{\cdot}07$) is consistent with a binary star with components of roughly equal brightness ($\Delta m \simeq 0{\cdot}6$ mag).

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NOTE ADDED IN PROOF

We omitted to note that the values for ζ Pup in Tables I, II and III differ slightly, but not significantly, from those published previously (19). These differences represent minor corrections for errors in the orientation of the baseline (Section 5).