Quasi-local contribution to the gravitational self-force

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Matched Expansions

- Poisson and Wiseman, Capra 1.
- Green's function:

$$G^{\text{bare}}_{\text{ret}} = G^{\text{dir}}_{\text{ret}} + G^{\text{tail}}_{\text{ret}}.$$

- Self-force(Quinn & Wald; Mino, Sasaki & Tanaka):

$$f^{\alpha}_{\text{self}}(x) = -4\mu^2 \int_{-\infty}^{+\infty} \nabla^{\alpha}_{x} G^{\text{tail}}_{\text{ret}}(x, x')\rho(x')d\tau'.$$

- Can isolate $G^{\text{tail}}_{\text{ret}}$ easily near particle.
- Can calculate $G^{\text{bare}}_{\text{ret}}$ by mode sum farther from particle (where sum converges more rapidly).

- Equivalent expression:

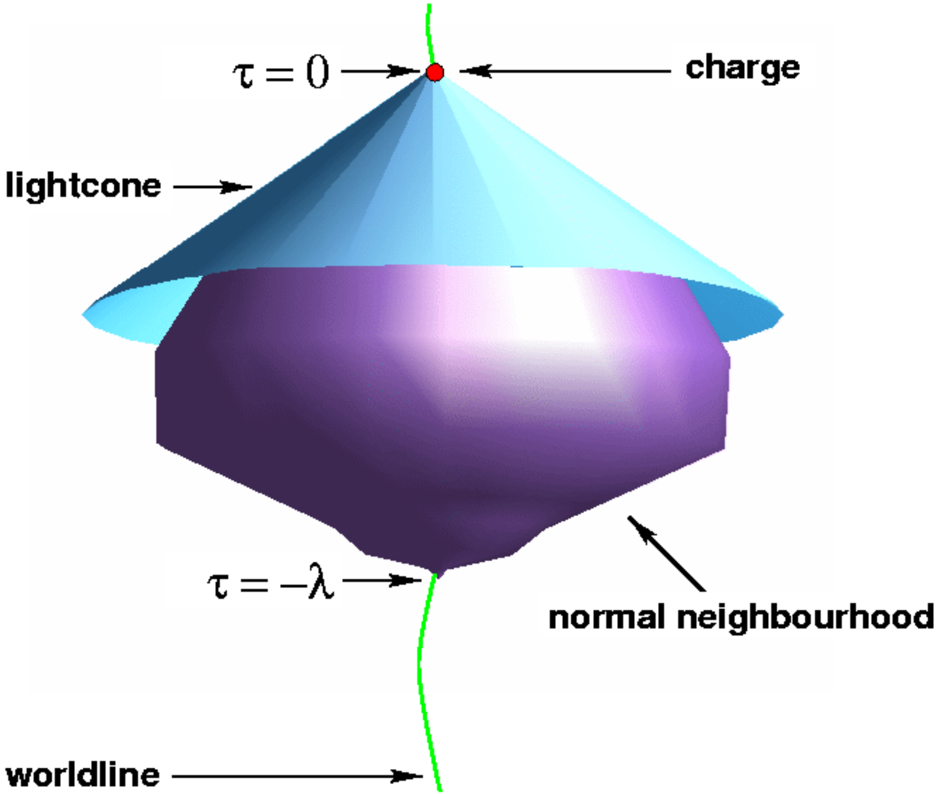
$$f^{\alpha}_{\text{self}}(x) = -4\mu^2 \left[\int_{-\infty}^{\lambda} \nabla^{\alpha}_{x} G^{\text{bare}}_{\text{ret}}(x, x')\rho(x')d\tau' + \int_{\lambda}^{0} \nabla^{\alpha}_{x} G^{\text{tail}}_{\text{ret}}(x, x')\rho(x')d\tau' \right].$$

- Hadamard form (normal neighbourhood):

$$G^{\text{bare}}_{\text{ret}}(x, x') = \frac{1}{4\pi}\left[U(x, x')\delta(\sigma(x, x')) + V(x, x')\Theta(\sigma(x, x'))\right].$$

- Identification:

$$\begin{aligned} G^{\text{dir}}_{\text{ret}} &= \frac{1}{4\pi} U(x, x')\delta(\sigma), \\ G^{\text{tail}}_{\text{ret}} &= \frac{1}{4\pi} V(x, x')\Theta(\sigma), \end{aligned}$$



$\tau = 0$
charge
lightcone
$\tau = -\lambda$
normal neighbourhood
worldline

Expansion of $V(x,x')$

- In literature (Allen, Folacci and Ottewill, PRD38, 1988.)
- Normal coordinate expansion of $V(x,x')$:

$$\begin{aligned} V_{\alpha\beta\mu\nu} &= v^0_{\alpha\beta\mu\nu} + v^0_{\alpha\beta\mu\nu\rho}\sigma^{;\rho} + (\frac{1}{2}v^0_{\alpha\beta\mu\nu\rho\tau} + v^1_{\alpha\beta\mu\nu}g_{\rho\tau})\sigma^{;\rho}\sigma^{;\tau} \\ &\quad +(\frac{1}{6}v^0_{\alpha\beta\mu\nu\rho\tau\lambda} + v^1_{\alpha\beta\mu\nu\lambda}g_{\rho\tau})\sigma^{;\rho}\sigma^{;\tau}\sigma^{;\lambda} + \mathcal{O}[(\sigma^{;\rho})^4]. \end{aligned}$$

- Quasi-local self force:

$$\begin{aligned} f^\alpha &= -\mu^2 (C^\alpha{}_{\xi\eta\beta} C_\gamma{}^{\xi\eta}{}_\delta\, u^\beta u^\gamma u^\delta \lambda^2 + \\ &\quad [?(C^2)^{;\alpha} \\ &\quad +(?C_{\beta\xi\eta\gamma} C_\delta{}^{\xi\eta}{}_\epsilon{}^{;\alpha} + ?C^\alpha{}_{\xi\eta\gamma} C_\delta{}^{\xi\eta}{}_{\epsilon;\beta} + ?C_{\beta\xi\eta\gamma} C^{\alpha\xi\eta}{}_{\epsilon;\beta})\, u^\beta u^\gamma u^\delta u^\epsilon]\lambda^3 \\ &\quad +\mathcal{O}[\lambda^4]. \end{aligned}$$