14.02 Principles of Macroeconomics, Spring 2000

3 pages

Note: the total number of points is 105!

Problem Set 7
Due: Wednesday, April 26

**Question 1.**(45 points total) Consider an economy characterized by the following aggregate demand equation:

$$y_t = m_t - p_t,$$

where  $m_t$  is the (log) money stock under control of the Fed,  $p_t$  is the price level, and  $y_t$  is output. All variables in this problem are in logs, so  $y = \ln Y$ , etc. Assume that there are two groups of agents in the economy – miners and metal workers. Denote their wages by  $w_t^A$  and  $w_t^B$ , respectively. Also assume that wage contracts are written down for two years: so miners would renegotiate contracts in odd years, and metal workers – in even years. The wage of each group is set for the duration of the contract (2 years). The wage-setting equation is

$$w_{t}^{X} = w_{t+1}^{X} = p_{t} + \alpha y_{t}$$
, where  $X \in \{A, B\}$ 

for the group that has a chance of negotiating its wage in year t (here  $\alpha < 1$ ). Notice that workers setting wage at time t perfectly forecast the price level at the beginning of year t – they use rational expectations. The price setting equation in the economy is

$$p_t = \frac{w_t^A + w_t^B}{2}.$$

- a) (8 points) Define a steady state: this is when  $w_t^A = w_t^B$  and money stock is constant. Consider a steady state in which m=0 (a pure normalization implying that the level of the money stock is 1). Characterize the steady state: what is the steady state level of output? What is the price level? What are the nominal wages  $w^A, w^B$ ? What are the real wages (notice that since we are working with logs, real wages are  $\log(W^X/P)$ )?
- b) (7 points) Suppose that at time t=0, the economy was at the steady state with m=0. Assume that at time t=1, money stock increases unexpectedly to  $\bar{m}>0$  and stays at that level forever. Rational workers know that the increase is going to be permanent. Write down the wage-setting

equation for the group of workers that can negotiate a new contract at time 1 (notice that wages of the other group remain fixed for this year at their previous level!). Combine this with aggregate demand and price-setting to arrive at a closed-form solution for wages of both groups and price level at time 1  $(w_t^A, w_t^B)$ . What happened to output? How did real wages of both groups change? (A formula is required.)

- c) (7 points) As time progresses and metal workers get a chance to renegotiate wages, what happens in period t=2 to wages (nominal and real) of both groups? Check whether the wages of metal workers will be higher than miners' at t=2. What will happen to the price level and the output (remember,  $m_t \equiv \bar{m} > 0$ )? Does the output return to its steady state level? Did the real wages catch up completely to their steady state levels?
- d) (10 points) What will happen in years t > 1? Define  $w_t^L$  and  $w_t^F$  as the wages of "leaders" and "followers", i.e. wages of those groups which respectively set the wage in year t and those who have set their wage at t-1. Notice that

$$w_t^F = w_{t-1}^L$$
.

Provide a recursive formula for  $w_t^L$  for t > 0 (that means  $w_t^L$  as a function of  $w_{t-1}^L$  and  $\bar{m}$  only).

- e) (6 points) Check the expression for  $m_t w_t^L$ . It is a function of  $(m w_{t-1}^L)$ . Given that  $\alpha < 1$ , does it converge anywhere? What is the new steady state in this economy  $(y, p, w^X)$ , real wages). What is the permanent effect of money on output compared to t = 0?
- f) (5 points) We assumed that workers have rational expectations of prices. In fact, notice that they knew that the increase in prices is going to continue. What is responsible for the gradual adjustment of the economy to the steady state?
- g) (2 points) What is the phenomenon that explains why the money stock does or doesn't affect output in this model? Give your best *two-word* answer.

Question 2. (40 points total) Assume that the production function is  $Y = K^{0.6+\varepsilon}N^{0.4}$ . Suppose we start at  $K_0 = 1$ . Depreciation rate is 2%. Saving rate is 15%. There is no population growth, so  $N_t \equiv N$ . The equation for the evolution of capital stock is  $K_{t+1} - K_t = sY_t - \delta K_t$ .

a) (10 points) Define the steady state as the situation where capital stock per capita and output per capita K/N and Y/N are constant. Compute the steady state levels of capital and output per capita for  $\varepsilon = 0$ .

- b) (10 points) Is there a steady state if  $\varepsilon$  equals 0.4? Why? Give your best 4 words answer.
- c) (10 points) Compute the value of consumption per capita in the steady state in part a) ( $\varepsilon = 0$ ). Define the golden rule level of capital. Compute the golden rule saving rate for this economy.
- d) (10 points) Instead of  $Y = K^{0.6+\varepsilon}N^{0.4}$ , assume that  $Y = K^{0.6}H^{\varepsilon}N^{0.4}$ , where H is human capital. Suppose that the economy can achieve a growth rate of its human capital stock of 3%. Depending on  $\varepsilon$  being either 0 or 0.4, how will the steady state capital stock be affected? Should we redefine the concept of the steady state level of capital if  $\varepsilon = 0.4$ ? (Hint: Write Y as  $Y = K^{0.6}(HN)^{0.4}$ , and replace N, which is population, with HN, "effective population" in the steady state's definition.) Compute the steady state capital stock for  $\varepsilon = 0.4$  according to this new definition.

**Question 3.** (5 points each) True, False, Uncertain. Provide a brief explanation.

- a) Suppose you are in charge of designing a disinflation program. If no nominal rigidities are present in the economy, then the optimal path of the money stock should involve a slow decrease for a first few decades to build up credibility and then a drastic and huge cut in money supply, after which money should stop decreasing.
- b) The Lucas critique states that coefficients of the Phillips curve might fluctuate over the business cycle, therefore, one should carefully plan timing for a disinflation to ensure that the reform is enacted when coefficients are most favorable.
- c) It is impossible for economies to grow indefinitely because of the positive depreciation rate.
- d) Nominal rigidities argument implies that people suffer from a form of money illusion they will not realize that price level has changed in response to monetary shock and for some time will continue setting wage contracts with lower real wage. This makes it an important channel whereby nominal money growth leads to increase in real output.