

PROBLEM SET FOUR

DUE 19 MARCH

You will legibly write both your full name and section on your completed assignment.

From Chapter 8 (These questions should complement the material presented on 15-17 March)

Problem 1. Consumption Theory and Liquidity Constraints

Assume you have just entered the labor force in 1999 as a college graduate at an annual starting salary of \$50,000. You expect your after-tax real income to grow by five percent each year over the next 45 years that you will be working. You will also expect to start receiving annual Social Security benefits starting at age 65 from the government equivalent to \$75,000 in 1999 dollars. The real value of these benefits will not change over time, and you expect to live 20 years after retirement. The long-term real interest rate is currently three percent.

- What is the present-discounted value of your lifetime wealth?
Hint: if $V = 1+a+a^2+\dots+a^{N-1}$ then $V = (1-a^N)/(1-a)$.
- If you wanted consumption to be constant over your next 65 years, how much should you consume in each year?
- How much are you saving in the first year? Assume your credit history is sufficiently short that you are unable to borrow for your first five years in the labor market. When can you finally smooth consumption over the rest of your life?
- In your first year of working, the government decides to cut your taxes by \$10,000 for 1999 only but then reverse them to normal for the rest of your life. What happens to current consumption in the case that you can borrow? What does this imply about the effectiveness of one-time changes in taxes or government spending on output?
- What happens if the one-time tax cut occurs when you cannot borrow? What does this imply about the presence of liquidity constraints and the effectiveness of fiscal policy.

Problem 2. Investment Theory, Fluctuations, and Irreversibility

Assume that output (Y) is produced by combining capital (K) and labor (N) in using Cobb-Douglas technology so we have $Y = K^b(AN)^{1-b}$. The real cost of capital is $r+\delta$ and real cost of labor is w . Firms choose capital (K) to maximize profits each period.

- Write the optimal level of capital (K^*) as a linear function of Y . Solve for the level of output corresponding to the optimal level of capital.
- Assume firm's capital stock is always at the optimal level so $K = K^*$. Write replacement investment (investment to replace depreciated capital) as a function of lagged output. Write net investment (ΔK) as a function of the change in output (ΔY). Finally, write total investment (I) as the sum of net investment and replacement investment.
- Assume initially $N = 100$, $A = 1$, $b = 0.6$, and the real interest rate is 5 percent while capital depreciates by 10 percent each year. Solve for the current level of output and capital stock.
- Assume the growth rate of technology is 3 percent each year while population growth is zero. What is the growth rate of output and the rate of investment (I/K) in this economy?

- e. Assume the growth rate of technology speeds up to 5 percent each year. What now is the growth rate of output and the rate of investment (I/K) in this economy? Are these permanent changes?
- f. Alternatively assume the interest rate in the economy increases to 5.5 percent. What now is the growth rate of output and the rate of investment (I/K) in this economy? Are these permanent changes?
- g. Now assume that investment is irreversible in the sense that net investment is never negative. Firms behave as follows: if $(1-\delta)K < K^*$ invest until $K = K^*$, but if $(1-\delta)K > K^*$ don't invest at all. What can you say will be true of the level capital on average for these firms? Describe a better policy (in terms of profits) to guide firm investment decisions.
- h. What does this example illustrate about the presence of irreversibility and the reluctance of firms to invest? In particular, if you repeated parts e and f above under the constraint of irreversibility, would investment be less responsive? Does your answer depend on whether or not these are permanent or temporary changes?