14.02 Principles of Macroeconomics, Spring 2000
4 pages

Problem Set 7
Answers

1. a) In the steady state, $w^A = w^B = p$ from the price setting equation. (notice that we drop time subscripts since time is immaterial in the steady state). Then wage setting implies $p = p + \alpha y \Rightarrow y = 0$ in steady state. Finally, from (AD),

$$0 = y = m - p$$

so $p = m = const$ in the steady state. (Log) real wages are $w^X - p = 0$.

Variable:	p	w^A	w^B	y	$w^A - p$	$w^B - p$
Steady state value:	m	m	m	0	0	0

b) At time $t = 0$, $m = p = w^A = 0$. At time $t = 1$, miners can negotiate their wages, while metal workers are stuck with previous period wages:

$$\begin{aligned} w_1^A &= p_1 + \alpha y_1 \\ w_1^B &= w_0^B = 0 \end{aligned}$$

Combining this with

$$\begin{aligned} y_1 &= m_1 - p_1, \\ p_1 &= \frac{w_1^A + w_1^B}{2} \end{aligned}$$

yields

$$\begin{aligned} w_1^A &= \frac{w_1^A + w_1^B}{2} + \alpha \left(m_1 - \frac{w_1^A + w_1^B}{2} \right) \\ &= (1-\alpha) \frac{w_1^A + w_1^B}{2} + \alpha \bar{m} \Rightarrow \\ \frac{(1+\alpha)}{2} w_1^A &= \frac{(1-\alpha)}{2} w_1^B + \alpha \bar{m} \\ w_1^A &= \frac{(1-\alpha)}{(1+\alpha)} w_1^B + \frac{2\alpha\bar{m}}{(1+\alpha)} = \frac{2\alpha\bar{m}}{(1+\alpha)} \end{aligned}$$

The price level is

$$p_1 = \frac{1}{2} \frac{2\alpha\bar{m}}{(1+\alpha)} + \frac{1}{2} \cdot 0 = \frac{\alpha\bar{m}}{(1+\alpha)}$$

Note that $p_1 < \bar{m}$ (since $\alpha < 1$). For output,

$$y_1 = \bar{m} - \frac{\alpha\bar{m}}{(1+\alpha)} = \frac{\bar{m}}{(1+\alpha)} > y_{\mathrm{ST}}$$

Real wages:

$$\begin{aligned} w_1^A - p_1 &= \frac{\alpha \bar{m}}{(1+\alpha)} \\ w_1^B - p_1 &= -\frac{\alpha \bar{m}}{(1+\alpha)} \end{aligned}$$

Real wages of miners rose above steady state, those of metal workers dropped.

c) $t = 2$: it is now metal workers who have their turn:

$$\begin{aligned} w_2^A &= w_1^A = \frac{2\alpha \bar{m}}{(1+\alpha)} \\ w_2^B &= (1-\alpha)\frac{w_2^A + w_2^B}{2} + \alpha \bar{m} \\ &= \frac{(1-\alpha)}{(1+\alpha)} w_2^A + \frac{2\alpha \bar{m}}{(1+\alpha)} \\ &= \frac{(1-\alpha)}{(1+\alpha)} \frac{2\alpha \bar{m}}{(1+\alpha)} + \frac{2\alpha \bar{m}}{(1+\alpha)} \\ &= \frac{4\alpha \bar{m}}{(1+\alpha)^2} \\ p_2 &= \frac{2\alpha \bar{m}}{(1+\alpha)^2} + \frac{\alpha \bar{m}}{(1+\alpha)} = \frac{(3+\alpha)\alpha}{(1+\alpha)^2} \bar{m} \end{aligned}$$

Note that $p_2 < \bar{m}$. Real wages of metal workers rise doe to renegotiated contract, those of miners fall due to price hike. Since $p_2 < \bar{m}$ we are still not at the steady state. (Log) output is also still above the steady state value of zero: $y = \bar{m}\frac{1-\alpha}{(1+\alpha)^2}$.

d) Similarly to b) and c),

$$\begin{aligned} w_t^L &= (1-\alpha)\frac{w_t^L + w_t^F}{2} + \alpha \bar{m} \rightarrow \\ w_t^L &= \frac{(1-\alpha)}{(1+\alpha)} w_t^F + \frac{2\alpha \bar{m}}{(1+\alpha)} \\ &= \frac{(1-\alpha)}{(1+\alpha)} w_{t-1}^L + \frac{2\alpha \bar{m}}{(1+\alpha)} \end{aligned}$$

e)

$$\begin{aligned} \bar{m} - w_t^L &= \bar{m} - \left(\frac{1-\alpha}{1+\alpha} w_{t-1}^L + \frac{2\alpha \bar{m}}{(1+\alpha)}\right) = \frac{1-\alpha}{1+\alpha}\bar{m} - \frac{1-\alpha}{1+\alpha} w_{t-1}^L \\ &= \frac{1-\alpha}{1+\alpha}\left(\bar{m} - w_{t-1}^L\right) \underset{t\to\infty}{\rightarrow} 0 \end{aligned}$$

The distance converges to zero, in fact, exponentially. In the new steady state, $p = \bar{m} = w^A = w^B$. Output $y = 0$. Real wages are back to zero. There is no permanent effect of money on output.

f) The fact that they were bound by two-year contracts and could not renegotiate wages instantly upon learning about the monetary shock. *Overlapping* contracts are crucial for the phenomenon.

g) `Nominal rigidities`.

2. Assume that the production function is $Y = K^{0.6+\varepsilon} N^{0.4}$. Suppose we start at $K_0 = 1$. Depreciation rate is 2%. Saving rate is 15%. There is no population growth, so $N_t \equiv N$. The equation for the evolution of capital stock is $K_{t+1} - K_t = sY_t - \delta K_t$.

a) Net investment:

$$
\begin{aligned}
sY\left(\frac{K_t}{N_t}\right)-\delta\frac{K_t}{N_t} &= s\frac{K_t^{0.6}N_t^{0.4}}{N_t}-\delta\frac{K_t}{N_t}\\
&= s\left(\frac{K_t}{N_t}\right)^{0.6}-\delta\frac{K_t}{N_t}=0\Rightarrow\\
s\left(\frac{K_t}{N_t}\right)^{0.6} &= \delta\frac{K_t}{N_t}\Rightarrow \text{in the steady state,}\\
\frac{K}{N} &= \left(\frac{s}{\delta}\right)^{\frac{5}{2}}\\
\frac{Y}{N} &= \left(\frac{s}{\delta}\right)^{\frac{5}{2}\cdot 0.6}=\left(\frac{s}{\delta}\right)^{\frac{3}{2}}
\end{aligned}
$$

Substituting numbers,

$$
\begin{aligned}
\frac{K}{N} &= \left(\frac{s}{\delta}\right)^{\frac{5}{2}}=\left(\frac{0.15}{0.02}\right)^{\frac{5}{2}}=154.05\\
\frac{Y}{N} &= \left(\frac{0.15}{0.02}\right)^{\frac{3}{2}}=20.54
\end{aligned}
$$

b) Try to solve the steady state condition as above:

$$
\begin{aligned}
sY\left(\frac{K_t}{N_t}\right)-\delta\frac{K_t}{N_t} &= s\frac{K}{N^{0.6}}-\delta\frac{K}{N}=0\Rightarrow\\
sN^{0.4} &= \delta
\end{aligned}
$$

As long as $N\neq\left(\frac{\delta}{s}\right)^{2.5}$ this equation has no solution. Thus, there is no steady state. Why: the production function with $\varepsilon>0$ exhibits NON-DECREASING RETURNS TO CAPITAL *or* INCREASING RETURNS TO SCALE (both answers valid).

c) Consumption:

$$
\frac{C}{N}=(1-s)\frac{Y}{N}=0.85\cdot 20.54=17.459
$$

Golden rule steady state level of capital is the level that maximizes steady state consumption per capita.

Golden rule:

$$
\begin{aligned}
\max_s\left\{(1-s)\frac{Y}{N}\right\} &= \left\{(1-s)\left(\frac{s}{\delta}\right)^{\frac{3}{2}}\right\}\\
\text{FOC} &:\\
\frac{d}{ds}\left\{(1-s)\left(\frac{s}{\delta}\right)^{\frac{3}{2}}\right\} &= 0\Rightarrow\\
(1-s)\frac{3}{2\delta}\left(\frac{s}{\delta}\right)^{\frac{1}{2}}-\left(\frac{s}{\delta}\right)^{\frac{3}{2}} &= 0\\
(1-s)\frac{3}{2\delta}\left(\frac{s}{\delta}\right)^{\frac{1}{2}} &= \left(\frac{s}{\delta}\right)^{\frac{3}{2}}\\
(1-s)\frac{3}{2}s^{1/2} &= s^{3/2}\\
\frac{(1-s)}{s} &= \frac{2}{3}\\
s &= 0.6
\end{aligned}
$$

d) If $\varepsilon = 0$, nothing is changed.

If $\varepsilon = 0.4$, we should use capital per effective units of labor: $k_t = \dfrac{K_t}{H_t N}$.

Net investment:

$$K_{t+1} - K_t = sK_t^{0.6} (H_t N)^{0.4} - \delta K_t$$

Divide this by $H_{t+1}N$:

$$\begin{aligned}
\frac{K_{t+1}}{H_{t+1}N} - \frac{K_t}{H_{t+1}N} &= \frac{sK_t^{0.6} (H_t N)^{0.4}}{H_{t+1}N} - \delta \frac{K_t}{H_{t+1}N} \\
k_{t+1} - \frac{K_t}{1.03 \cdot H_t N} &= \frac{1}{1.03} \left(\frac{sK_t^{0.6} (H_t N)^{0.4}}{H_t N} - \delta \frac{K_t}{H_t N} \right) \\
k_{t+1} - \frac{1}{1.03} k_t &= \frac{1}{1.03} \left(s k_t^{0.6} - \delta k_t \right) \\
s k_t^{0.6} - \delta k_t &= 1.03 k_{t+1} - k_t
\end{aligned}$$

In steady state $k_{t+1} = k_t = k^* \Rightarrow$

$$\begin{aligned}
s (k^*)^{0.6} - \delta k^* &= 0.03 k^* \\
s (k^*)^{0.6} &= (\delta + 0.03) k^* \\
k^* &= \left[\frac{s}{(\delta + 0.03)} \right]^{\frac{1}{0.4}} \\
&= \left[\frac{0.15}{(0.02 + 0.03)} \right]^{2.5} = 15.588
\end{aligned}$$

3.True-False.

a) False. In the absence of nominal rigidities, it is best to reduce the rate of growth of money stock sharply in the beginning to gain credibility; in the final stages it should be accelerated a little bit.

b) False. The Lucas critique states that coefficients of the Phillips curve depend on the policy, in particular, whether there is reform or not.

c) False. The key property responsible for this is the decreasing returns to capital in the production function.

d) False. Nominal rigidities argument implies that although people have no illusions and predict anything perfectly, adjustment is slowed down because contracts are written sometime in advance and they are overlapping.