PROBLEM SET FIVE SOLUTIONS

Problem 1

a) We will use the hint provided and solve first for the second period. We will denote variables in the second period with primes and since there is no third period the behavioral equations reduce to a simpler form given by:

Consumption: $C' = C_0 + \frac{1}{2}(Y' - T')$

Investment: $I' = I_0 + \frac{1}{10}\frac{Y'}{r'}$

$G' = 80 \qquad T' = 80$

$\frac{M'^d}{P'} = \frac{1}{10}\frac{Y'}{r'}$

To solve we must find equilibrium both in the goods and financial markets (i.e. the IS and LM relations). The second one is very easy since it amounts to equate real money demand with real money supply (and we are told that $\frac{M'}{P'} = 40$). This gives a downward sloping LM curve in the (Y', r') space. To get the IS relation we proceed as usual by starting with the identity

$$Y' = C' + I' + G'$$

We could rewrite this relation by using the behavioral equations and then get an upward sloping IS curve. Instead of doing this, we will directly replace the interest rate r' from the LM relation and get an equation with just the output as an unknown (notice how the fact that in the IS relation the ratio $\frac{Y'}{r'}$ appears exactly as it is given by the LM relation will make substitutioin straightforward). This gives us,

$$Y' = C_0 + I_0 + \frac{1}{2}(Y' - T') + \frac{M'}{P'} + G'$$

Which gives output as a function of exogenous variables,

$$Y' = 2[C_0 + I_0 - \frac{1}{2}T' + \frac{M'}{P'} + G'] = 2[20 - 40 + 40 + 80] = 200 \qquad (1)$$

Then the LM relation gives us inmediately that $r' = 0.5$. Some people said that this was too high an interest rate (nobody complained that output was too low ;) But in the set up nothing is telling what are the units of measure. Maybe output is measured in billions of dollars, and interest rates are measured in percentage points (i.e. $r' = 0.5$ means 0.5%, which if something is low).

We now attack the first period equilibrium replacing the second period variables by the values we just found. This gives us,

Consumption: $C = C_0 + \frac{1}{2}(Y - T) + \frac{1}{12}\frac{200-80}{0.5}$

Investment: $I = I_0 + \frac{1}{10}\frac{Y}{r} + \frac{1}{40}\frac{200}{0.5}$

$G = 50 \qquad T = 80$

$\frac{M^d}{P} = \frac{1}{10}\frac{Y}{r}$

We do the same operations that we did before, having the same LM relation (since real money supply is still $\frac{M}{P} = 40$). Substituting in the IS relation now gives us,

$$Y = C_0 + I_0 + \frac{1}{2}(Y - T) + \frac{1}{12}\frac{200-80}{0.5} + \frac{M}{P} + \frac{1}{40}\frac{200}{0.5} + G$$

$$Y = 2[C_0+I_0-\frac{1}{2}T+\frac{1}{12}\frac{200-80}{0.5}+\frac{M}{P}+\frac{1}{40}\frac{200}{0.5}+G] = 2[20-40+20+40+10+50] = 200 \tag{2}$$

Since output is the same in both periods, the interest rate will be the same, i.e. $r = 0.5$ (remember that real money supply is the same in both periods).

b) This point is very simple and it amounts to replace $T = T' = 40$ in the above equations 1 and 2. This gives,

$$Y' = 2[20 - 20 + 40 + 80] = 240 \qquad r' = \frac{240}{400} = 0.6$$

$$Y = 2[C_0+I_0-\frac{1}{2}T+\frac{1}{12}\frac{240-40}{0.6}+\frac{M}{P}+\frac{1}{40}\frac{240}{0.6}+G] = 2[20-20+27.7+40+10+50] = 255.5$$

$$r = \frac{255.5}{400} = 0.638$$

c) This is a little trickier since we now know that $Y = Y' = 200$, and we want to find out what should be the money supply to achieve this (and therefore what the interest rate will be). We proceed first from equation 1 but replacing $Y' = 200$ and find out what M' should be,

$$200 = 2[C_0 + I_0 - \frac{1}{2}T' + \frac{M'}{P'} + G'] = 2[20 - 20 + M' + 80]$$

$$M' = 100 - 80 = 20 \qquad r' = \frac{200}{200} = 1$$

Now we replace the values of $Y' = 200$, $M' = 20$, and $r' = 1$ into equation 2 and equating $Y = 200$ we solve again for M.

$$200 = 2[C_0+I_0-\frac{1}{2}T+\frac{1}{12}\frac{200-40}{1}+\frac{M}{P}+\frac{1}{40}\frac{200}{1}+G] = 2[20-20+13.3+M+5+50]$$

$$M = 100 - 68.3 = 31.6 \qquad r = \frac{200}{316} = 0.631$$

d) We know that stock prices, and therefore the stock market index is a positive function of future and current dividends and a negative function of future and present interest rates (because we have to discount those future cash flows to get the present value). Therefore in case b) there are two effects on the stock market, first output is increasing both today and tomorrow, therefore dividends are increasing (since dividends are correlated with output), but also the interest rate is increasing. Thus in this case the stock market can go either up or down and it will depend on which effect dominates. But in case c) there is only an increase in the interest rate, therefore unambiguously the stock market will go down. One can think that the US economy today is working at full employment therefore if Congress where to vote a decrease in taxes (with the argument that there is a budget surplus), the most likely reaction by the Fed would be to contract money supply (or expand it at a lower rate for a while), to prevent this excess demand to create inflationary preassures. This will lead to a decrease in the stock market.

Problem 2

This problem is basically about turning the interest parity relation inside out, upside down, etc. Let's start by remembering what we mean by interest parity.

$$i_t = i_t^* + \frac{E_{t+1}^e - E_t}{E_t}$$

Where i stands for domestic nominal interest rate, i^* for foreign nominal interest rate, and E for nominal exchange rate. The interest parity relation tells us that there is a link between domestic and foreign interest rates and the expected nominal depreciation. This relation comes from an arbitrage argument in the foreign exchange market.

a) Looking at everything from the perspective of country M, if $E_t = 10$, in three years we should have that

$$0.21 = 0.15 + \frac{E_{t+3}^e - 10}{10} \qquad E_{t+3}^e = 10.6$$

Therefore the nominal exchange rate must depreciate at a rate of 2% per year.

b) Now we have to use the real version of interest parity, that relates real interest rates at home and abroad with the real expected depreciation

$$r_t = r_t^* + \frac{\epsilon_{t+1}^e - \epsilon_t}{\epsilon_t}$$

Where $\epsilon = \frac{EP^*}{P}$ is the real exchange rate and we assume that initially is equal to 1. Then to find the real expected depreciation we solve

$$0.21 - (0.10 + 0.5 + 0.5) = 0.15 - 3(0.03) + \frac{\epsilon_{t+3}^e - 1}{1} \qquad \epsilon_{t+3}^e = 0.95$$

What we find is that M's real exchange rate appreciates by 5% during the three years. This tells us that despite an expected nominal appreciation, to say whether a country's trade balance position will improve in the future we need to correct the nominal appreciation by expected inflation at home and abroad to see what happens with the expected real exchange rate.

c) We know that this Mexican bond was paying only at maturity (let's say an amount X), and now we are told that its price was \$ 100. This means that $X = 100(1.07)^3 \approx 100(1.21) = 121\$$. Now that we peg the exchange rate and this peg is credible, then the nominal rates in Mexico have to decrease to the level of the United States, therefore $i = 5\%$. This means that the new price of the bond has to be given by the discounted value of X at this new rate, i.e.

$$P = \frac{121}{1.05^3} \approx \frac{121}{1.15} \approx 105$$

d) Now there is an expected devaluation of 10% (average of possible outcomes during the year), which will force the government to raise interest rates if it wants to maintain the peg. To get the magnitude of this increase we use, once again, the interest parity relation,

$$i = 0.05 + 0.10 = 0.15$$

Thus domestic rates have to be increased to 15% just to compensate investors from the risk of a devaluation. This tells us something about the large costs of defending a peg when markets expect a devaluation (and why this expectations might easily be self-fulfilled).