14.02 Principles of Macroeconomics, Spring 2000
2 pages

Problem Set 5
Solutions

Question 1.
a) False. They both contribute in approximately the same amount, which is consistent with investment being more volatile because consumption accounts for a larger fraction of total output.
b) True. They both equal capital depreciation allowance.
c) False. The J-curve predicts that net exports (which equals the *value* of exports minus the *value* of imports) should fall in the short run. The quantity of exports and imports should start moving (although slowly) towards their long-run values.
d) False. If the Fed raises interest rates, uncovered interest parity requires a *depreciating* dollar which, with relatively unchanged expected future exchange rates, implies an *appreciation* of the dollar now.
Question 2.
a) Taking the derivative of the profit function with respect to capital and equating to zero we obtain:

$$K_t = N\left(\frac{\alpha A_t}{r+\delta}\right)^{\frac{1}{1-\alpha}}$$
$$\frac{K_t}{Y_t} = \frac{\alpha}{r+\delta}$$

The capital/output ratio is decreasing in r and δ because the higher the cost of capital, the less capital firms will want to use.
b) Gross investment:

$$I_t = K_t - (1-\delta)K_{t-1} = \delta K_t$$

Net investment:

$$I_t^{net} = K_t - K_{t-1} = 0$$

c)

$$Y_1 = Y_2 = Y_3 = 8N$$
$$Y_4 = Y_5 = Y_6 = 12.62N$$

$$K_1 = K_2 = K_3 = 32N$$
$$K_4 = K_5 = K_6 = 50.48N$$

$$
\begin{aligned}
I_1 &= I_2 = I_3 = 3.2N \\
I_4 &= 21.68N \\
I_5 &= I_6 = 5.05N
\end{aligned}
$$

d) In a small open economy, investors and consumers can save and borrow abroad. As a result, the cost of capital is given by the international interest rate.

Question 3.

a) In steady state $Y = \bar{Y}$. Then $i = 0$. Then $p = m_0$.

b) Since prices are continuous, the initial $i = p - m = -(m_1 - m_0)$. Then the initial $Y = \bar{Y} + (m_1 - m_0)$.

c) While $Y > \bar{Y}$, $\dot{p} = \pi$. Then

$$
p(t) = \begin{cases} m_0 & \text{if } t < 0 \\ m_0 + \pi t & \text{if } t \in \left[0, \frac{m_1 - m_0}{\pi}\right] \\ m_1 & \text{if } t > \frac{m_1 - m_0}{\pi} \end{cases}
$$

Then

$$
i(t) = \begin{cases} 0 & \text{if } t < 0 \\ -(m_1 - m_0) + \pi t & \text{if } t \in \left[0, \frac{m_1 - m_0}{\pi}\right] \\ 0 & \text{if } t > \frac{m_1 - m_0}{\pi} \end{cases}
$$

Then

$$
Y(t) = \begin{cases} \bar{Y} & \text{if } t < 0 \\ \bar{Y} + (m_1 - m_0) - \pi t & \text{if } t \in \left[0, \frac{m_1 - m_0}{\pi}\right] \\ \bar{Y} & \text{if } t > \frac{m_1 - m_0}{\pi} \end{cases}
$$

d) From uncovered interest parity we know that $\dot{e}(t) = i(t)$. Integrating, we obtain

$$
e(t) = -(m_1 - m_0)t + \frac{\pi}{2}t^2 + C
$$

for $t \in \left[0, \frac{m_1 - m_0}{\pi}\right]$, where C is a constant of integration. From purchasing power parity and continuity at $t = \frac{m_1 - m_0}{\pi}$ we know that $e\left(\frac{m_1 - m_0}{\pi}\right) = m_1$ which allows us to solve for C. Then

$$
e(t) = \begin{cases} m_0 & \text{if } t < 0 \\ m_1 + \frac{(m_1 - m_0)^2}{2\pi} - (m_1 - m_0)t + \frac{\pi}{2}t^2 & \text{if } t \in \left[0, \frac{m_1 - m_0}{\pi}\right] \\ m_1 & \text{if } t > \frac{m_1 - m_0}{\pi} \end{cases}
$$

The exchange rate depreciates (discontinuously) at the time of the unexpected monetary expansion. It depreciates to a higher level than its new steady state value (i.e. it overshoots) because the currency must appreciate during the adjustment process in order to compensate for the low interest rates.