PROBLEM SET FOUR SOLUTIONS

Problem 1.

a. PDV of your lifetime wealth.

Note to graders: please be kind if exponents are off by only one year

Labor market income:
$\$50,000[1+1.05/1.03+\ldots+(1.05/1.03)^{44}]$
$= \$50,000(1-1.019417^{45})/(1-1.019417)$
$= \$50,000(73.23911)$
$= \$3,543,156$

The exponent is the first line is 44 because there must be 45 terms in the series.

Social security income:
$\$75,000[1+1/1.03+\ldots+1/(1.03)^{19}]/(1.03)^{45}$
$= \$75,000[(1-0.970874^{20})/(1-0.970874)]/(1.03)^{45}$
$= \$303,915$

The exponent in the first line is 19 because there must be 20 terms in the series, while the exponent in the denominator is 45 as the stream of Social security benefits is discounted over 45 years.

Lifetime wealth:
W = \$3,847,071

b. Smooth consumption

$C^* = W/65 = \$3,847,071/65 = \$59,185$

c. Borrowing and liquidity constraints

Saving

$S_{20} = Y_{20} – C_{20} = Y_{20} – C^* = \$50,000 - \$59,185 = -\$9,185$
You must borrow this amount in the first year to perfectly smooth consumption over your expected lifetime.

When can you smooth

Note $W_{20} = Y_{20} + W_{21}/(1.03)$ so $W_{21} = (1.03)*(W_{20}-Y_{20})$.
More generally $W_{t+k} = (1.03)*(W_{t+k-1}-Y_{t+k-1})$

Liquidity constraints in the first five years require $C_{t+k} = Y_{t+k}$, so the above sequence captures the evolution of lifetime wealth subject to our consumption plan (which is just the path of income over the next five years). The series for wealth is generated in the second column of the table below. The last column is the smooth consumption consistent with this wealth, simply constructed as $W_{t+k}/(65\text{-}k)$. This is simply the level of consumption consistent with remaining wealth. Note smooth consumption is less than income in each of the first five years, so we will not be able to begin to smooth consumption until after our credit history is long enough for us to borrow at age 25.

Time	**Y_t**	**W_t**	**c^*_t**
20	$ 50,000.00	$ 3,847,071.80	$ 59,185.72
21	$ 52,500.00	$ 3,910,983.95	$ 61,109.12
22	$ 55,125.00	$ 3,974,238.47	$ 63,083.15
23	$ 57,881.25	$ 4,036,686.87	$ 65,107.85
24	$ 60,775.31	$ 4,098,169.79	$ 67,183.11
25	$ 63,814.08	$ 4,158,516.31	$ 69,308.61

Note to graders: again be kind here, as I'm sure many people got lost in doing some complicated math.

d. Tax cuts

A tax cut of $10,000 will be smoothed over the next 65 years of your life so consumption today will rise by $10,000/65 = $154.

The effectiveness of one-time changes in taxes on output is diminished when agents make consumption decisions based on lifetime wealth.

e. Tax cuts under liquidity constraints

If you are liquidity constrained, you will be more likely to consume a large fraction of your one-time tax cut instead of spread it evenly over your lifetime. This implies that the presence of liquidity constraints implies that even if agents are forward-looking in their consumption decisions, one-time changes in the stance of fiscal policy can have significant effects on output.

Problem 2.

a.

$$\Pi = K^b(AN)^{1-b} - (r+\delta)K - wN$$

foc(K):

$$bK^{b-1}(AN)^{1-b} - (r+\delta) = 0$$
$$bK^b(AN)^{1-b}/K = (r+\delta)$$
$$bY/K = (r+\delta)$$
$$K^* = bY/(r+\delta) = \theta_2 Y \text{ where } \theta_2 = b/(r+\delta)$$

$Y^* = [bY^*/(r+\delta)]^b(AN)^{1-b}$
$Y^* = AN[b/(r+\delta)]^{b/(1-b)} = AN*\theta_1$ where $\theta_1 = [b/(r+\delta)]^{b/(1-b)}$

b.
$I^{rep}_t = \delta K_{t-1} = \delta K^*_{t-1} = \delta bY_{t-1}/(r+\delta)$
$I^{net}_t = \Delta K^*_t = b\Delta Y_t/(r+\delta)$

$I_t = I^{rep}_t + I^{net}_t = \delta bY_{t-1}/(r+\delta)+b\Delta Y_t/(r+\delta)$

c.

$Y^* = 1*100*[0.6/(0.05+0.10)]^{0.6/(1-0.6)} = 800$
$K^* = 0.6*800/(0.05+0.10) = 3200$

d.

Note that the following is true after taking natural logs and time derivatives on your equations from part a.

$g_y^* = g_N+g_A+g_{\theta 1}$
$g_k^* = g_y^*+g_{\theta 2}$

Divide your investment equation by the capital stock from the previous period as follows:

$(I_t/K_{t-1}) = \delta+\Delta K^*_t/K_{t-1} = \delta+\Delta K^*_t/K^*_{t-1} = \delta+g_k^*$

Note finally $g_N = g_{\theta 1} = g_{\theta 2} = 0$. We have $g_y^* = 3\%$ and $(I_t/K_{t-1}) = 13\%$.

e.

From the above it follows that $g_y^* = 5\%$ while $(I_t/K_{t-1}) = 15\%$. These are permanent changes in the growth rate of output and investment as the change in technological progress is permanent.

f.

Now we need to take into account $g_{\theta 1} = (\theta_{1,t+1}-\theta_{1,t})/\theta_{1,t}$ and $g_{\theta 2} = (\theta_{2,t+1}-\theta_{2,t})/\theta_{2,t}$.

We have $\theta_{1,t} = [0.6/(0.05+0.10)]^{0.6/(1-0.6)} = 8$ and $\theta_{1,t+1} = [0.6/(0.055+0.10)]^{0.6/(1-0.6)} = 7.6$.
We have $\theta_{2,t} = 0.6/(0.05+0.10) = 4$ and $\theta_{2,t} = 0.6/(0.055+0.10) = 3.87$

These results imply $g_{\theta 1} = -4.7\%$ and $g_{\theta 2} = -3.2\%$, which together imply that $g_{y,t} = 3\%-3.2\% = -0.2\%$ and $(I_t/K_{t-1}) = 13\%-7.9\% = 5.1\%$. These changes in the growth rate of output and investment rate are temporary as there is a one-time permanent change in interest rates, so as there are no changes in the future there are no more changes in θ_1 or θ_2.

Note to graders: it is sufficient to say that the growth rate of output and investment rate increase, and that these increases are temporary if students note the reason (one-time changes in the theta's).

g.

Firms will on average have more than the optimal amount of capital if they follow this policy. The policy simply tells them to compare the current capital stock with the optimal capital stock. If the capital stock is less than optimal, invest until you reach the optimal level. If the capital stock is more than optimal, irreversibility constrains you to do nothing. So part of the time you have optimal capital and part of the time you have more than optimal, so on average you have too much.

A better policy for the firm to follow would be when capital is less than optimal capital, invest more slowly (for example, invest only one-half of the way to the level of optimal capital). Firms are still taking advantage of time being good, but reduce their downside risk if times turn bad again in the future.

h.

Irreversibility in investment implies that firms will be reluctant to invest when times become good, especially if it looks like good times are temporary. In part e, the change in a permanent one, so firms would likely behave in a manner similar to your answer in the case without irreversibility. This might be less so if it takes times for firms to realize the change in technological progress is permanent. In part f the change is an increase in the interest rate, which reduces optimal capital and temporarily reduces output growth, and investment rates. The firm will react less to this interest rate shock than in part e simply because of the irreversibility constraint.

Note to graders: for the second part, I'm sure not many people realized that the shock was negative, so it is fine to talk about a positive shock. Answer in this case as follows:

A decrease in interest rates increases optimal capital and temporarily increases output growth and the investment rate. Shocks to the interest rate are easily reversed by the central bank, so this may be perceived as more of a temporary shock, in which case firms would react by less than they did in the frictionless case discussed in part e above.