

A St. Mary's Project: The Aerodynamics of Golf Ball Flight

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## **I. Preface:**

I was introduced to golf at a fairly young age of 12. At such a young age, golf can be intimidating to learn but it quickly becomes a work in progress. As I got older and throughout high school I was an avid baseball player, so the golf swing, which is very similar to the baseball swing, came natural to me. But after playing golf for 9 years I realize that hitting a few buckets of golf balls at the local driving range and watching the professionals play on Sunday afternoon can only take you so far. It quickly becomes evident that golf is as mentally challenging as it is physically challenging. The golfer has to be able to predict the outcome of a shot before it happens. This requires a player to do things such as manipulate the plane of their swing, change the strength of their swing, and select the proper club. Each of these changes will affect the outcome of the shot. Also, conditions such as wind and rain are factors the player must adjust to as well. With all of these variables, I could not resist researching the golf shot.

The golf shot can be broken into three sections: the player's swing, collision between club and ball, and finally golf ball flight. Each section could be studied in depth but, after talking with my advisor, Dr. Charles Adler, we decided to restrict ourselves to a limited part of the problem. We choose to focus on golf ball flight. Specifically, we choose to study the forces acting on a ball when in flight. The Drag Force and a phenomenon called the Magnus Effect are analyzed and modeled.

## II. Introduction

We have examined the Drag Force and Magnus effect acting on a golf ball using a phenomenological model developed by Dr. Robert Adair and others to model the aerodynamic forces on a baseball.[1] We decided that these models, modified to the smaller size and higher spin rate of the golf ball, can be used to successfully predict the golf ball trajectory. In our model we can predict the trajectory of a golf ball given its initial launch angle, velocity, and spin rate. From this we hope to gain insight on how these variables affect a golfer's choice of club, choice of ball, and their affect on golf ball design.

## III. Introductory Fluid Dynamics

Air, although it can not be seen by the naked eye, is a fluid that follows the principles of fluid dynamics. Air forms around an object just as if the object were submerged in water. Similarly, when an object moves through air it causes air to flow about the object very much like water flowing past a pole.

Newton's third law states: *For every action there is an equal and opposite reaction.* An example is a book sitting on a desk. The book's weight exerts a force on the table and the table exerts an equal and opposite force to hold it up. Similarly, when an object in motion produces a force on the air molecules, the air molecules react with an equal and opposite force on the ball. This Drag force will decelerate the moving object. Similarly, if an object forces air molecules downward, air molecules will react with an upward force causing the object to rise. To better understand this situation we will consider an airplane wing.

Air that flows over an airplane wing is diverted downward. Newton's first law states: *a body at rest will remain at rest unless subjected to an external force*. Given Newton's first law, air that is initially at rest is diverted downward by a wing because a force acted on the air. This is the action. Newton's third law states that there must be an equal and opposite force on the wing. This is the reaction. Lift on an airplane is the reaction force due to air being diverted downward.[2]

But, how does the wing divert air down? When a moving fluid comes into contact with a surface it will follow that surface. The tendency of fluids to follow a surface is the *Coanda effect*. This effect is largely due to the viscosity of air. Viscosity is the resistance of a material to change shape and form. Viscosity also produces the attraction between a fluid and surface. Specifically, when air flows over a surface the relative velocity between the surface and closest air molecules is zero. The viscosity produces a force of friction high enough to hold the air molecules in place. As you get further away from the surface the molecules are still affected by the viscosity, but are able to move. Eventually the distance between air molecules and the surface is large enough that the viscosity does not produce any friction to slow air molecule movement. These molecules move freely.[3] The distance from the surface to the point at which friction can be neglected is called the boundary layer.[4]

Boundary layers exist on the wings of a plane but also in other situations when the fluid is moving slowly. For example, dust particles on glass are close enough to the surface to be in the boundary layer. As a result, when water is poured over glass the dust is not washed away. Instead, the viscous forces of the boundary layer keep the dust

particles from being washed away. In general, boundary layers change width depending on the type of surface, fluid, and speed of flow.

#### **IV. The Dimpled Golf Ball and Drag**

The modern golf ball is changing annually. Companies such as *Titleist* and *Callaway* have patented dimple designs promising less drag and further range. *Titleist* praises its new 392 multi-dimple high coverage icosahedral design.[5] *Callaway* introduced a revolutionary design that does not have dimples but raised, interconnecting and rounded ridges.[6] Every new design tries to do the same thing. They hope to reduce drag and increase distance.

Intuition would lead most people to think that a smooth golf ball would travel further than one with dimples. After all, there is less surface friction with a smooth ball than with a dimpled one. But, contrary to intuition, a dimpled ball has been shown to travel more than 4 times further than a smooth ball.[7] The *Navier-Stokes* partial differential equations that usually give insight into the flow of incompressible fluids like air have not been solved. The exact way dimples affect the boundary layer is not clear. There are, however, qualitative theories supported by properties of physics. Vincent Mallette explained the situation as follows, “when the boundary layer ‘fits like a glove’ it slows down rapidly and separates quickly. But turbulence provides coupling to the ‘outside’ air stream and enables the boundary layer to continue receiving momentum from the outside air.”[8] Theodore Jorgensen, in his second edition of *The Physics of Golf* offers a more in-depth explanation. Refer to figure 1.

Jorgensen explains air flow over a smooth sphere:

“The air flowing from A to B outside the boundary layer is going from a high pressure region to a low pressure region and we may look upon this pressure difference as helping to increase the air velocity. However the air in flowing from B to C moves from a low pressure region to a high pressure regions and loses velocity in going against this pressure difference. When the viscous effect in the boundary layer becomes large enough so that the air near the surface of the ball is stopped before it reaches C, turbulent motion takes the place of the streamline flow.”

For a dimpled ball,

“The dimpled surface makes the boundary layer turbulent; it stirs the air up a bit. Instead of stalling near B, as in the previous example, the rapidly moving air carries the turbulent boundary layer along with it, helping it to extend further along the surface of the ball from the low pressure region at B toward the higher pressure region at C.”[9]

When a dimpled ball causes the boundary layer to extend further along the ball the resulting wake of air is smaller than with the smooth ball. See Figure 2. The smaller the wake, the lower the pressure and the easier it is for air to flow around the ball.

The wake is smaller but it still is a source of high pressure. This high pressure is a resistive force. The force resisting air flow from the low pressure region B to the high pressure region C is the balls drag. The equation of for drag is proportional to the Drag coefficient,  $C_d$ . The Drag coefficient is a dimensionless constant of order 1 which is a function of the Reynolds number,  $Re$ . For a sphere traveling through air,

$$Re = (\rho v D) / \eta \quad (1)$$

In this equation, the sphere has diameter  $D$  and is moving with a velocity  $v$ . Air has density  $\rho$  and kinematic viscosity  $\eta$ . The equation for drag is as follows:

$$F_d = (.5) C_d \rho_{air} A v^2 \quad (2)$$

where  $A$  is the projected cross sectional area of the ball.

For a sphere,

$$A = \pi r^2 \tag{3}$$

where  $r$  is the radius of the ball. Note that  $A$  is not the total surface area, but the area of a circle of the same radius as the sphere.

One thing to consider is how the drag force is proportional to velocity squared. As an object moves faster the drag acting on it increases. We wonder whether the drag force will have a large effect on the golf ball. For more details on the Reynolds number and Drag coefficient see section XIV, the Technical Appendix.

## **V. The Magnus Force**

The second major force acting on the golf ball results from its spin. Spinning, however, does not cause the wake to increase or decrease. Instead, spinning causes the wake to change shape.

Like the affects of dimples, the change in shape due to spinning is directly related to the boundary layer. The spinning motion imparts a spin onto the boundary layer due to air's viscosity. This spin then affects where the boundary layer releases from the golf ball. On the side of the ball which is spinning into the wind a force is being imparted against the flow of the boundary layer. This causes early separation and a high pressure wake. On the side of the ball which is spinning away from the wind a force is imparted on the boundary that assists its flow around the ball. This causes late separation and a



low pressure wake. Taking both changes into account, this means the wake is shifted toward the side moving against the wind.[10] See figure 3.

The effects of the wake shift can be explained in two ways. The first is with Newton's laws. There is a net force, due to the defection of air, acting on air molecules that results in an equal and opposite force on the ball. The reaction force can be separated into components by setting up a coordinate system with the x axis parallel to ball trajectory. The x component will be resistive against its flow in the x direction. The y component, perpendicular to motion, will move the ball toward the side spinning away from the wind. Another way of thinking of this same situation is to imagine a ball rolling down a hill. It may encounter bumps or steep drops. The ball will naturally take the path of least resistance. A golf ball in flight follows the same principle. It moves toward the side of least resistance-the low pressure wake.

Due to the backspin of a well hit golf ball, the Magnus force provides lift and increases range. In addition to lift, the Magnus force is also the reason for bad golf shots like the slice and hook. Therefore, plotting the trajectory without taking the Magnus force into account would be very unrealistic.

For a ball spinning with angular velocity  $\omega$  and moving with velocity  $v$ , a phenomenological model of the Magnus force is,

$$F_M = \kappa v \omega \quad (4)$$

such that  $\kappa$  is a constant,

$$\kappa = (2/3) \Pi r^3 \rho_{air} \quad (5)$$

Here  $r$  is the radius of the ball and  $\rho_{air}$  is the density of air. This model of the Magnus force and its legitimacy is discussed further by Dr. Robert Adair in *The Physics of Baseball*. In our model, we believe it is a good approximation to the truth.

## VI. Modeling Golf Ball Flight

The United States Golf Association has set standards on golf ball size and weight. The golf ball can not weigh more than 1.620 ounces (45.93 grams). The minimum diameter is 1.680 inches (42.67 mm). Throughout the research conducted in this paper we use 45.25 grams for weight and 43 mm for diameter.[11]

From a typical launch velocity of a golf ball, around 60 m/s, we can calculate a typical Reynolds number of  $Re = 150,000$ . Using this Reynolds number, we considered a chart for a smooth sphere to get a rough estimate for the Drag coefficient. The smooth sphere with Reynolds number of 150,000 has a drag coefficient of (.5). But, as we know the Drag force on the dimpled golf ball is very different. For a golf ball's typical value of  $Re$ , there are good estimates for  $C_d$  ranging from (.25) to (.30). For the research we chose (.3).[12]

Our program begins with inputting initial conditions. Some are initial velocity, angular velocity, and launch angle. The value for each varies depending on numerous things. Conditions such as swing speed, swing plane, and choice of ball will change initial velocity. The choice of club will vary the angular velocity and launch angle.

Whether a ball is hit from thick grass or short grass changes the angular velocity as well. Changing any of these values will have a direct effect on the trajectory. For example, a player can hit a low punch shot with low angular velocity. On the other hand, they could also hit a lofting sand wedge with high angular velocity. Both shots will have different initial conditions but each shot could result in the same range of 145 yards.

Understanding initial conditions and how they are produced is important to consider.

## **VII. Starting the Problem: The Falling Golf Ball**

To model the trajectory of a golf ball one must be able to write out the equations of motion using Newton's second law. This will give a set of two coupled differential equations for  $x(t)$  and  $y(t)$ .

To begin with, however, we would like to start with an easier problem. We consider the fall of a non-spinning golf ball under the influence of gravity and a drag force pointing up on it. See Figure 4. Terminal velocity,  $v_t$ , will occur when the drag force is equal but opposite of gravity,  $mg$ ,

$$(.5) C_d A \rho v_t^2 = mg \quad (6)$$

and,

$$(.5) C_d A \rho = mg / v_t^2 \quad (7)$$

Generally then,

$$\begin{aligned} F_d &= (.5) C_d A \rho v^2 \\ &= mg (v / v_t)^2 \end{aligned} \quad (8)$$

The net force acting on the ball is,

$$F_{net} = m dv/dt = mg - mg (v/v_0)^2 \quad (9)$$

solving for  $dv/dt$ , the equation of motion for a non-spinning, falling golf ball under the influence of drag is,

$$dv/dt = g(1 - (v/v_0)^2) \quad (10)$$

also,

$$dy/dt = v \quad (11)$$

After writing the equation of motion for the falling golf ball we decided to use Euler's Forward Method to solve it.

Euler's Forward method is very appealing because it can be used in Excel which enables an easy plot of trajectory. A general form of Euler's forward method follows below:

using the method to solve for velocity at time  $t_1 = t_0 + \Delta t$ ,

$$v_x(t_1) = v_x(t_0 + \Delta t) \approx v_x(t_0) + (dv_x(t_0)/dt)\Delta t \quad (12)$$

$$v_y(t_1) = v_y(t_0 + \Delta t) \approx v_y(t_0) + (dv_y(t_0)/dt)\Delta t \quad (13)$$

solving for the components of position,

$$y(t_1) = y(t_0 + \Delta t) \approx y(t_0) + v_y(t_0)\Delta t \quad (14)$$

$$x(t_1) = x(t_0 + \Delta t) \approx x(t_0) + v_x(t_0)\Delta t \quad (15)$$

Notice that the method increments a solution through  $\Delta t$  while only using information from the beginning of the interval; the values at time  $t_0$ . This causes some error, but if the step is very small than error is greatly reduced. Eric W. Weisstein stated, “the accuracy is actually not too bad and the stability turns out to be reasonable as long as the so-called Courant-Friedrich-Lévy condition is fulfilled. This condition states that, given a space discretization, a time step bigger than some computable quantity should not be taken.”[13] Effectively, this means that a small enough time interval between calculations will result in minimal error. To give perspective, in the two dimensional program without Drag or the Magnus force a ball with initial velocity of 40 m/s and launch angle of 20 degrees had a range of 106 meters. The theoretical Range equation, which does not take into account Drag and the Magnus force, predicted 105.1 meters.

To solve the equation of motion for the falling golf ball we solve Euler’s Forward method in Excel. Initial values for  $v(t)$  and  $y(t)$  are zero. The ball is “dropped,” and using Euler’s Forward method Excel calculates  $v(t)$  and  $y(t)$  every (.1) seconds. A graph was then made plotting velocity versus position. See Figure 5. The program verified that velocity continues to increase until it reaches its terminal velocity. The value calculated for  $v_t$  is 41.2 m/s. If the force of drag is not taken into account the velocity of the ball continues to increase indefinitely.

## VIII. Two Dimensions with Drag

The next equation of motion we consider is a non-spinning golf ball moving in two dimensions under the influence of drag. Refer to Figure 6. Continuing from equation 8, we have:

$$\begin{aligned} F_{dx} &= (-)mg(v/v_t)^2 \cos\theta \\ &= (-)mg(v/v_t)^2 (v_x/v) \\ &= (-)mg(vv_x/v_t^2) \end{aligned} \tag{16}$$

similarly,

$$F_{dy} = (-)mg(vv_y/v_t^2) \tag{17}$$

it follows from Newton's Second Law,

$$dv_x/dt = (-)g(vv_x/v_t^2) \tag{18}$$

$$dv_y/dt = (-)g(1 + vv_y/v_t^2) \tag{19}$$

To solve equations 18 and 19 the same approach as with the falling golf ball is used. A new program is made in Excel and by using Euler's Forward method,  $v_x$ ,  $v_y$ ,  $x(t)$ , and  $y(t)$  are calculated every .001 seconds. The program verified that drag has a major affect on ball trajectory. In Figure 7, the range of the ball including the drag force is 29.1 meters less than the range without drag.

## IX. Magnus Force included

The final situation considered in this project is when there is backspin on the ball. We are still only in two dimensions so the ball is rotating in the x-y plane only. Recall equation 3,  $F_M = \kappa v \omega$ . With  $\kappa$  and  $\omega$  constant the components of the Magnus force,  $F_{Mx}$  and  $F_{My}$ , are dependent on the change in components of velocity,  $v \sin \theta$  and  $v \cos \theta$ . It follows that:

$$F_{Mx} = (-)\kappa \omega v \sin \theta \quad (20)$$

$$F_{My} = \kappa \omega v \cos \theta \quad (21)$$

By, again using Newton's second law we can solve equation 20 and 21 and obtain expressions for  $dv_x/dt$  and  $dv_y/dt$  which result from the Magnus force. But we also have the drag force to consider. We start from equations 18 and 19 and add the Magnus effect. Adding the effects of Drag and the Magnus effect, it follows:

$$dv_x/dt_{net} = (-)g(v v_x / v_t^2) + ((-)\kappa \omega v \sin \theta / m) \quad (22)$$

$$dv_y/dt_{net} = (-)g(1 + v v_y / v_t^2) + (\kappa \omega v \cos \theta / m) \quad (23)$$

We now have the set of two coupled differential equations for  $x(t)$  and  $y(t)$  that includes Drag and the Magnus force. The next step is to solve the equations. Initial conditions, constants, and equations 22 and 23 were entered in to a new Excel program. Again, Euler's Forward Method is used with a time step of .001. The values for  $v_x$ ,  $v_y$ ,  $x(t)$ , and

$y(t)$  were calculated and then plotted. The program allows for variation of spin rate, launch angle, and initial velocity. See Figure 8.

The Magnus force was found to greatly affect the trajectory of the ball. The program with out the Magnus force was compared to this one which takes both Drag and the Magnus force into account. In each program the same initial conditions were used. The Magnus force produces lift on the ball. This lift is shown to carry the ball an additional 29.9 meters. See figures 7 and 9. In some situations though, too much lift can decrease range. In Figure 10, spin rates of 800 and 900 radians per second produce the furthest range. With the same initial conditions, a spin rate of 1000 radians per second is too fast of an angular velocity. The range decreases by approximately 4 meters. One can conclude that given initial conditions there will be an angular velocity that results in the furthest range. There will always be a higher and lower angular velocity that results in less range than the optimal one. It is also good to note that the Magnus Force can distort golf ball trajectory into an almost impossible form. If one produces an angular velocity on the order of 1500 radians per second and the launch angle is pretty high, trajectory is over powered by the Magnus affect. See Figure 11. In this situation, the Magnus Force has a y component that is larger than gravity. The ball actually rises to a steeper angle than its initial launch angle.

## **X. Verifying the Program: Comparison with a baseball**

To verify the programs legitimacy we consider a baseball. A comparison is made between the results of our final program including the Magnus force and Adair's results from *The Physics of Baseball*.<sup>[14]</sup> The same ball parameters, drag coefficient, value for  $\kappa$ , weight, rotation speed, and terminal velocity as in Figure 2.4 of *The Physics of*



*Baseball* were used in the program. The range predicted by the program was 347 ft. Bob Adair's results were very close at 350 ft.

## **XI. Applications to Club Choice**

Imagine a golfer that has 160 yards to the green but there is a tree in his/her way. The option of hitting the ball in high and landing it on the green is less likely because of the tree. The golfer must hit a lower shot. To do so, selection of a less angled club is chosen. This will produce a lower launch angle and less angular velocity. To get the ball close to the hole the golfer has to anticipate what will happen when the ball hits the ground. If the tree was not in the way and the golfer could hit a shot with higher angular velocity, the ball would hit the green softly without a large bounce forward. In the case of hitting under the tree, however, the lower angular velocity will not help stop the ball. Instead, it lands on the green and moves forward a significant amount. The golfer must adjust the range they hit the ball. Ideally in this case, the golfer should hit the ball short and roll it up on the green.

Another situation to consider is hitting out of thick grass-the rough. The average golfer knows it is tough to consistently hit shots the same range from the rough. The difficulty arises from the fact that it is difficult to reproduce the same collision of ball and club. This produces various angular velocities. So, for example, a golfer that is used to hitting a pitching wedge 120 yards must adjust when in the rough. A pitching wedge hit firmly in the fairway produces angular velocities on the ball upward of 1000 to 1300 radians per second. These angular velocities are actually higher than the spin rate that produces the furthest range. When hitting out of the rough spin rates slow down because of bad collisions of ball and club. Sometimes rates slow down to the optimal one. As a

result, pitching wedge shots out of the rough with slower spin rates can actually fly further than when hit from the fairway. This is contrary to intuition. It is important to note that some golfers that produce less than perfect impact may not hit any of their clubs from the fairway to produce angular velocities higher than the optimal one. This would mean that all their shots from the rough would have spin rates slower than the optimal one and be shorter than when hit from the fairway. Similarly, golfers with good impact(perhaps the pros) will hit shots from the fairway that induce spin rates higher than the optimal one with numerous clubs. These golfers hit balls further in the rough with multiple clubs.

Many times you see golfers licking their fingers and testing for wind. How could a small 5 mph wind change the trajectory that much? The reason is this: hitting into the wind causes the effects of the Magnus Force to increase. The boundary layer is extended further around one side and forced to separate even earlier on the other side. See Figure 12. Therefore, air is forced downward at a steeper angle than it would naturally be. A larger lift force is produced and the range changes. To hit a correct shot into the wind golfers should use a club with less of an angle. If they usually hit a 7 iron 150 yards then they should use a 6 iron to hit 150 yards into the wind. On the other hand, hitting with the wind has a reverse affect. The boundary layer on one side of the ball is not extended as far as usual. The boundary layer on the other side does not separate as early as usual. This results in less lift and again the range changes. Golfers should hit higher lofted clubs when hitting with the wind.

Understanding how the Drag force and Magnus force acts on the golf ball gives the golfer insight to make decisions like those just mentioned. In many cases the golfer

with this knowledge will be able to predict their shots better. Golfers should not expect to carry a lap-top and enter parameters into the program and make decisions that way. Instead, with the insight gained they should have some rules of thumb.

## **XII. Applications to Ball Design**

As mentioned before, ball manufacturers are constantly updating their ball designs. The two main characteristics to consider are dimple design and the inflexibility of balls on impact. The dimple design is important because it can reduce drag. Manufacturers that produce a better dimple design can promise people further distance. This promise will keep the average golfer buying their dimple design for years. The inflexibility of the ball is the second consideration. Balls are made to be hard or soft. Hard balls make for a more elastic collision so less energy is lost and distance is increased. A more elastic collision also means the balls do not compress as much on impact. This results in less surface of the ball on the face of the club. The force of friction decreases and the angular velocity will be low. A low angular velocity is characteristic of low lift and balls will bounce harder on the green. On the contrary, softer balls will have higher angular velocities to produce both increased lift and a soft touch on the green. Manufacturers have a lot to gain from this because they can design balls that target various golfer profiles. For example, the “older golfer” may need a harder ball to get the distance they are used to.

Golfers, like manufacturers, need to consider the changes in ball design as well. If they are on a par 3, 120 yard hole then distance is not a problem. Instead they need the shot to be close to the hole. Using a softer ball will allow for less bounce on the green and the potential for a closer shot. Another situation is when a golfer is playing a

long par 5. If a golfer wants to reach the green in two shots it is easier to do so when using the hard ball.

### **XIII. Conclusion**

This project has explained the fundamental forces acting on a golf ball while in flight and then modeled them in an Excel program. Drag decreases the range of flight significantly. Dimple patterns, then, become very important and should be researched further. Particularly, the introduction of *Callaway's* raised, interconnecting and rounded ridges is the beginning of a new era of dimple design. The Magnus force has been shown to significantly change trajectory. It allows the ball to travel further but in some cases too high of an angular velocity decrease range. Gaining knowledge about Drag and the Magnus effect can benefit both golfers and golf manufacturers. The knowledge enables the golfer to predict outcomes of golf shots in various conditions. It enables manufacturers to target desired characteristics for their products.

The more that is known about the aerodynamics of golf the more advanced the game of golf will become. Golfers and their equipment will continue to get better making for an only more interesting game of golf in the future.

## XIV. Technical Appendix

The Reynolds number,  $Re$ , is a dimensionless ratio between the viscous forces and inertial forces acting on an object moving through a fluid. The inertia of a fluid will keep it moving steadily in the face of the retarding viscous forces of other fluids. The inertial force is represented with Newton's second law such that mass is replaced by density times area (across the flow) times length (with the flow), and acceleration is replaced by velocity over time. The equation for the inertial force is,

$$F_i = (\rho l S v) / t \quad (24)$$

The viscous forces are proportional to the object and speed of flow,

$$F_v = (\mu S v) / l \quad (25)$$

The result then,

$$Re = F_i / F_v = (\rho v l) / \eta \quad (26)$$

In our case, the characteristic of length,  $l$ , is the diameter of the golf ball. [15]

Given a fluid, the ratio of size and speed can be manipulated. A golf ball of a small radius and high speed can have the same Reynolds number as a basketball at lower speeds. Changing the Reynolds number results in a change in flow over the object. At low Reynolds numbers flow is laminar and orderly. As the Reynolds numbers increase, flow becomes increasingly disordered and turbulence can set in. [16]

In our study, a golf ball traveling at velocities of 60 m/s results in a Reynolds number of 150, 000. This is termed as a “high” number and the flow around the ball is turbulent. As a result there is high pressure behind the ball and drag becomes a factor.

This drag is termed pressure drag. Another type of drag is “skin friction.” This is a direct effect due to the fluid’s viscosity. The skin friction is minimal due to high Reynolds numbers. [17]

The drag coefficient comes from dividing Drag per unit area by pressure. This results in a dimensionless constant that varies only with the Reynolds number. The drag coefficient is,

$$C_d = 2D/S / \rho v^2 \quad (27)$$

By only varying with the Reynolds number, the drag coefficient is particularly effective when describing drag of a particular shape. Given the equations for drag coefficient and the Reynolds number for a given shape one can calculate the Drag force for any size, speed, and fluid.[18]

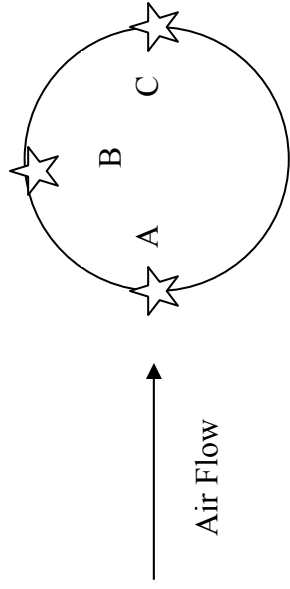
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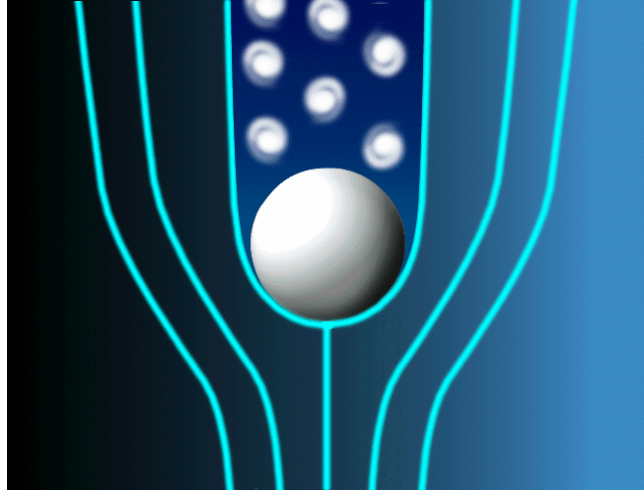
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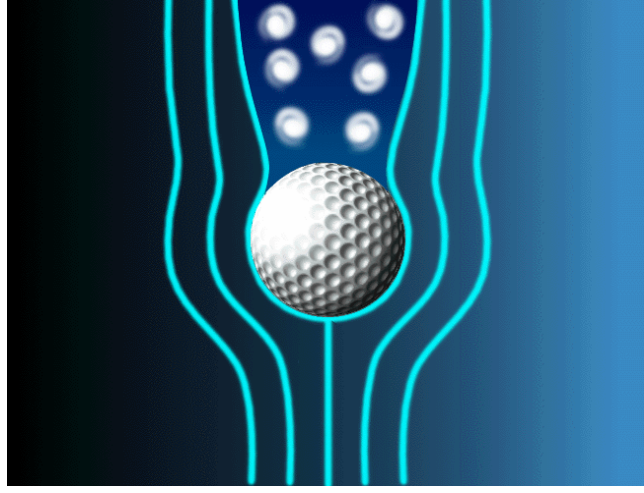
**Figure 1. Air Flow**



**Figure 2. Air Flow: Dimpled vs. Smooth**

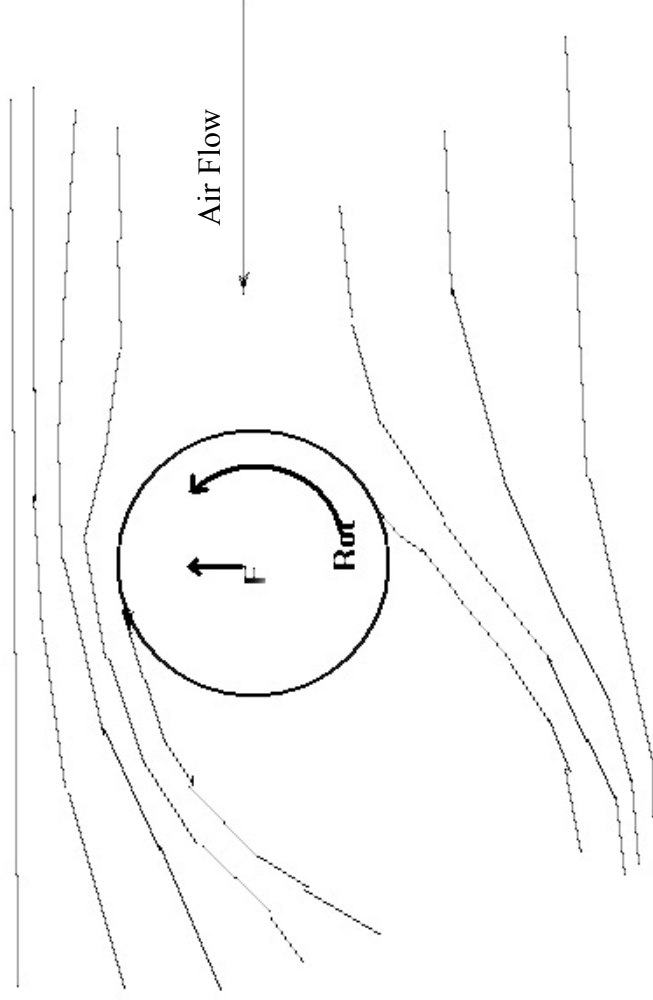


**Smooth Ball-early boundary layer release**

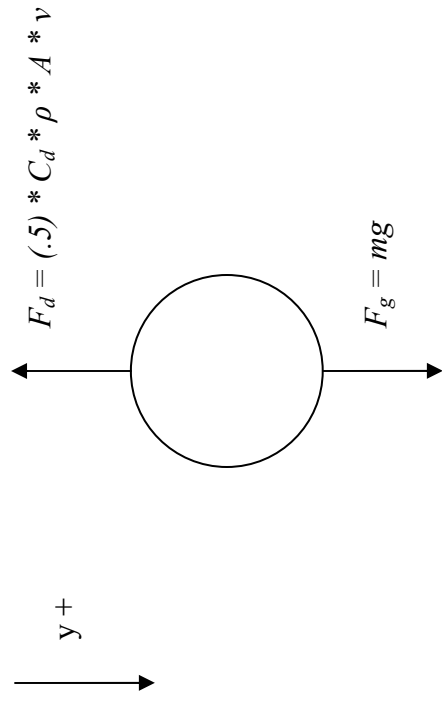


**Dimpled Ball-late boundary layer release**

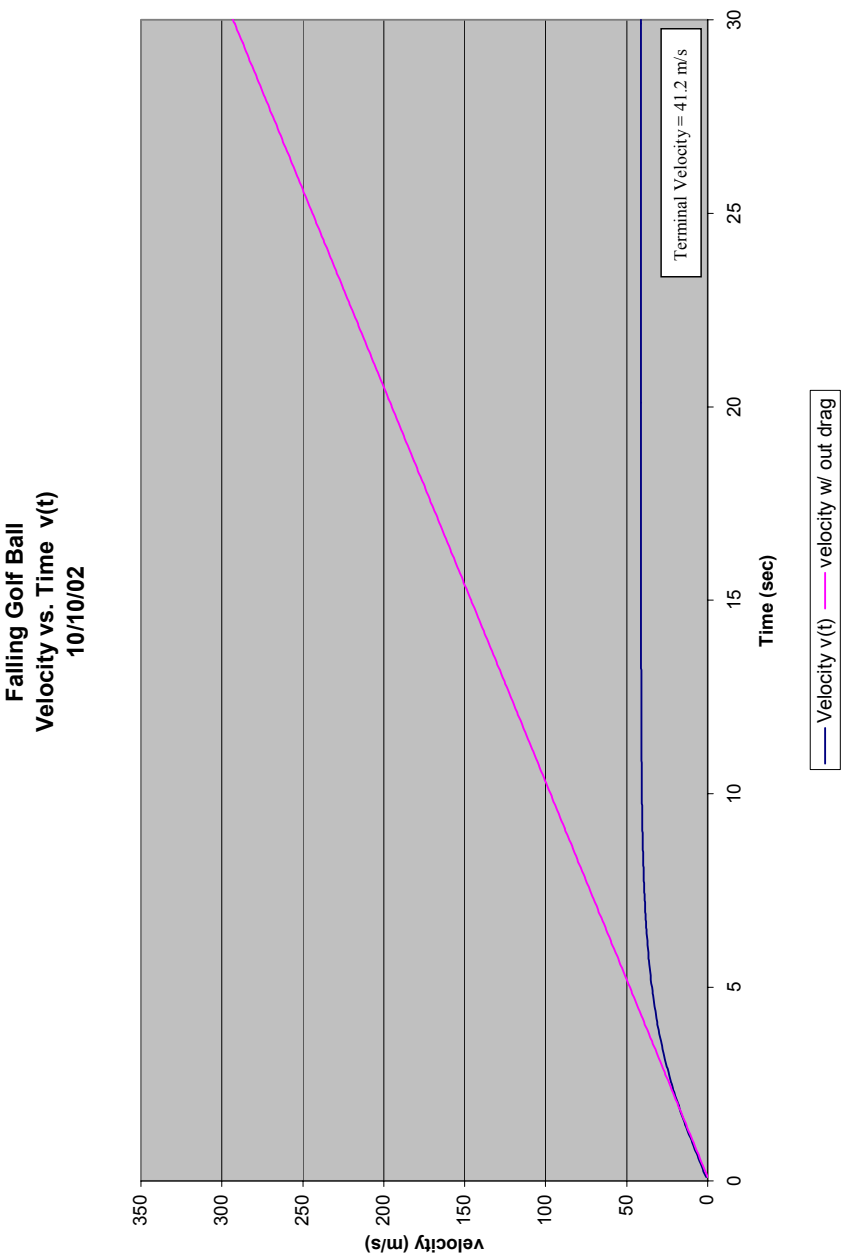
**Figure 3. Magnus Effect (Air flow from right to left)**



**Figure 4. Non-Spinning, Falling Golf Ball with Drag**



**Figure 5: Falling Golf Ball**



**Figure 6. Non-spinning, Two dimensional flight with Drag**

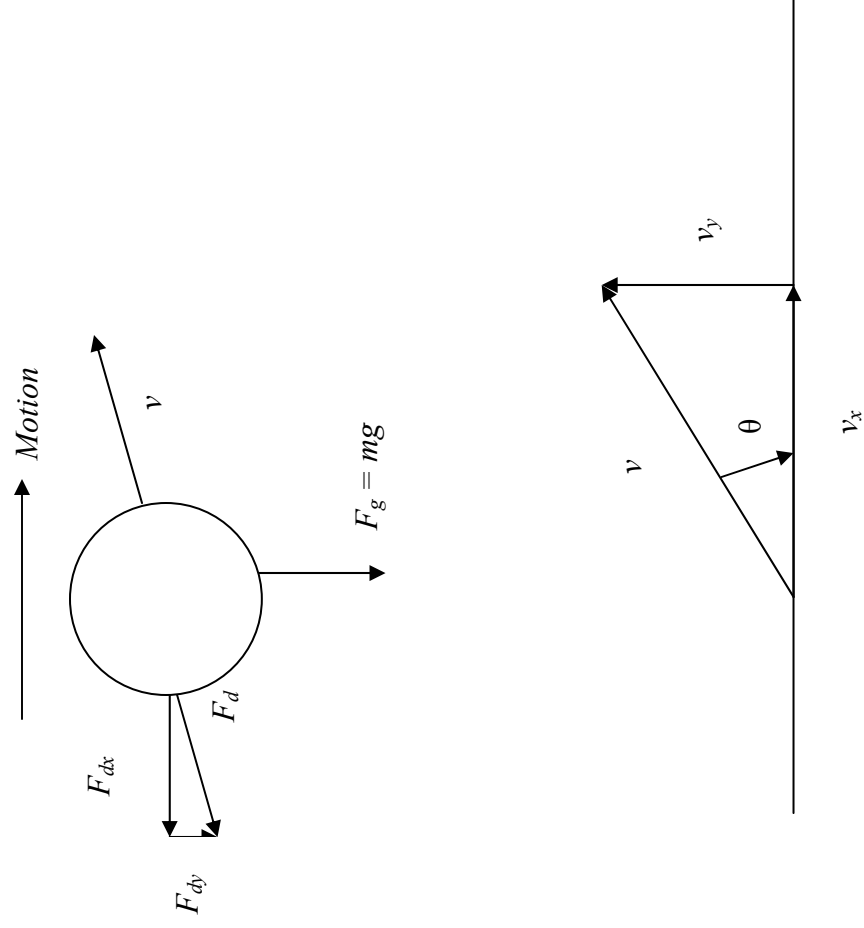
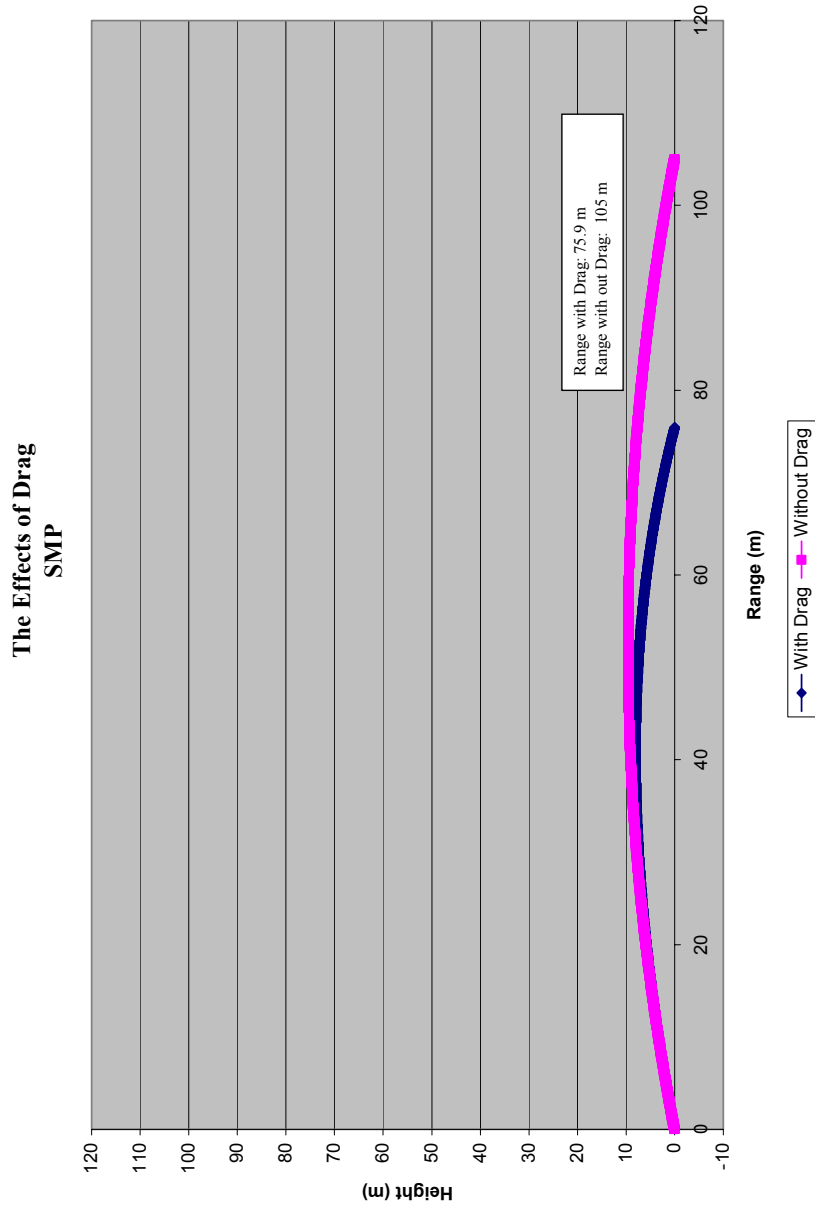


Figure 7: The Effects of Drag, 40 m/s, 20 deg



**Figure 8: Sample of Program**

| t     | Mass  | Vterm | $\Delta t$ | v(t0)<br>m/s | Cos 20 deg<br>( R ) | Sin 20 deg<br>( R ) | Rotation<br>speed<br>(rad/sec) | K         | g<br>m/s^2 | Velocity<br>Angle | Vx(t) m/s   | Vy(t) m/s   | V(t) m/s    | x(t) m     | y(t)      |
|-------|-------|-------|------------|--------------|---------------------|---------------------|--------------------------------|-----------|------------|-------------------|-------------|-------------|-------------|------------|-----------|
|       |       |       |            |              |                     |                     |                                |           |            |                   |             |             |             |            |           |
| 0     | 0.045 | 40.3  | 0.001      | 50           | 0.819152044         | 0.573576436         | 500                            | 0.0000123 | 9.8        |                   | 40.94523716 | 28.67017437 | 49.98491117 | 0.0409576  | 0         |
| 0.001 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610865413       | 40.92896915 | 28.65732282 | 49.96421386 | 0.08190284 | 0.0286788 |
| 0.002 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610841467       | 40.91271292 | 28.64447651 | 49.94352924 | 0.12283181 | 0.057349  |
| 0.003 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610817452       | 40.89646844 | 28.63163542 | 49.92285727 | 0.16374452 | 0.0860063 |
| 0.004 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610793368       | 40.88023572 | 28.61879956 | 49.90219795 | 0.20464099 | 0.1146508 |
| 0.005 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610769215       | 40.86401474 | 28.60596891 | 49.88155128 | 0.24552123 | 0.1432824 |
| 0.006 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610744993       | 40.84780549 | 28.59314347 | 49.86091723 | 0.28638524 | 0.1719012 |
| 0.007 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610720701       | 40.83160796 | 28.58032324 | 49.84029579 | 0.32723305 | 0.2005072 |
| 0.008 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610696339       | 40.81542213 | 28.5675082  | 49.81968695 | 0.36806465 | 0.2291003 |
| 0.009 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610671908       | 40.79924799 | 28.55469836 | 49.79909071 | 0.40888008 | 0.2576807 |
| 0.01  |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610647408       | 40.78308554 | 28.54189371 | 49.77850704 | 0.44967932 | 0.2862482 |
| 0.011 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610622839       | 40.76693477 | 28.52909425 | 49.75793594 | 0.49046241 | 0.3148029 |
| 0.012 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.6105982         | 40.75079565 | 28.51629996 | 49.73737739 | 0.53122934 | 0.3433448 |
| 0.013 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610573491       | 40.73466818 | 28.50351085 | 49.71683138 | 0.57198014 | 0.3718739 |
| 0.014 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610548713       | 40.71855235 | 28.4907269  | 49.6962979  | 0.61271481 | 0.4003902 |
| 0.015 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610523865       | 40.70244815 | 28.47794812 | 49.67577694 | 0.65343336 | 0.4288937 |
| 0.016 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610498948       | 40.68635557 | 28.46517449 | 49.65526848 | 0.69413581 | 0.4573844 |
| 0.017 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610473961       | 40.67027459 | 28.45240602 | 49.63477252 | 0.73482216 | 0.4858623 |
| 0.018 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610448905       | 40.6542052  | 28.43964269 | 49.61428904 | 0.77549244 | 0.5143275 |
| 0.019 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610423779       | 40.63814739 | 28.42688451 | 49.59381802 | 0.81614664 | 0.5427799 |
| 0.02  |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610398583       | 40.62210116 | 28.41413146 | 49.57335947 | 0.85678479 | 0.5712196 |
| 0.021 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610373318       | 40.60606648 | 28.40138354 | 49.55291335 | 0.89740689 | 0.5996465 |
| 0.022 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610347983       | 40.59004335 | 28.38864075 | 49.53247968 | 0.93801296 | 0.6280606 |
| 0.023 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610322578       | 40.57403176 | 28.37590308 | 49.51205842 | 0.978603   | 0.656462  |
| 0.024 |       |       |            |              |                     |                     |                                |           | 9.8        | 0.610297103       | 40.5580317  | 28.36317052 | 49.49164957 | 1.01917703 | 0.6848506 |
|       |       |       |            |              |                     |                     |                                |           |            |                   |             |             |             |            | 0.7132265 |

\* The Program continues downward.



**Figure 9: Addition of the Magnus Force, 40 m/s, 20 deg**

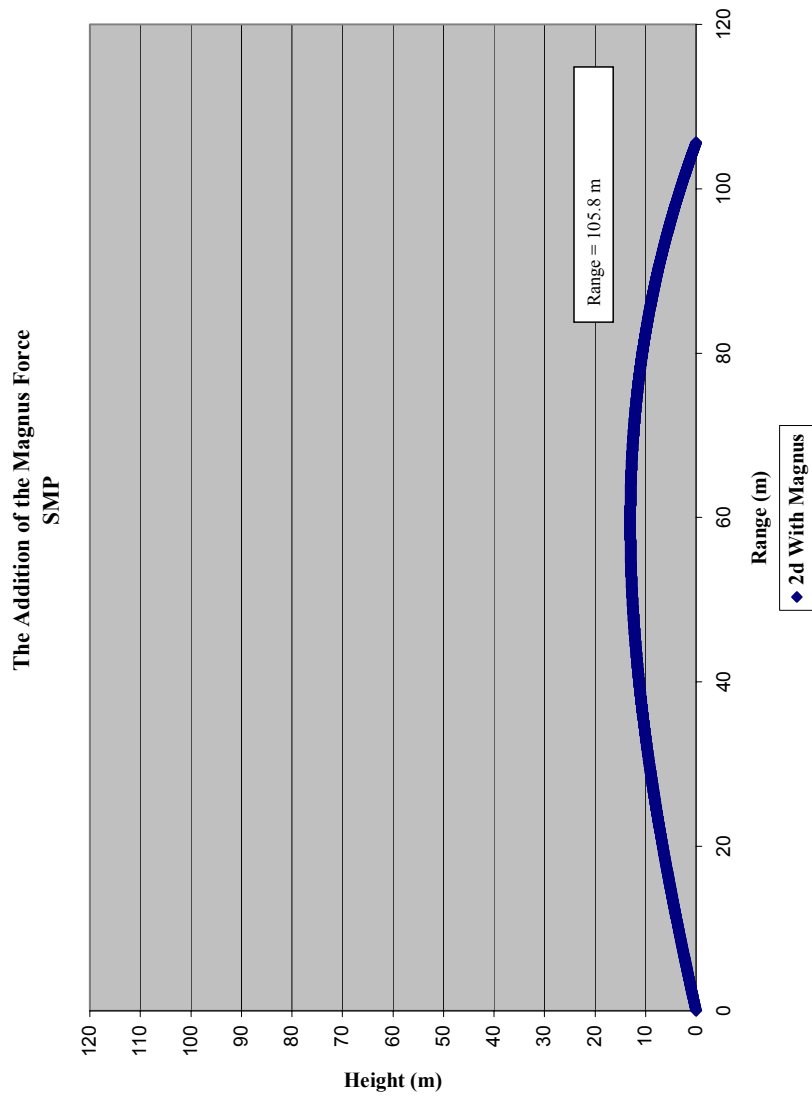
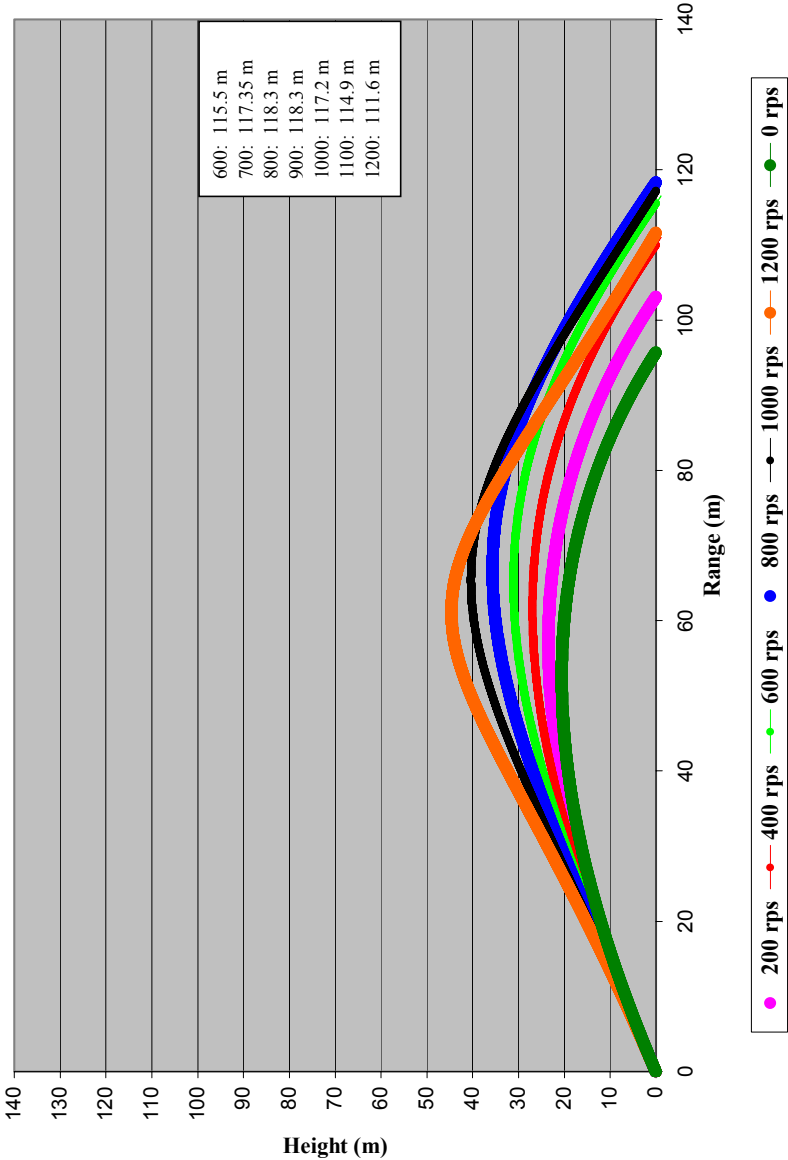
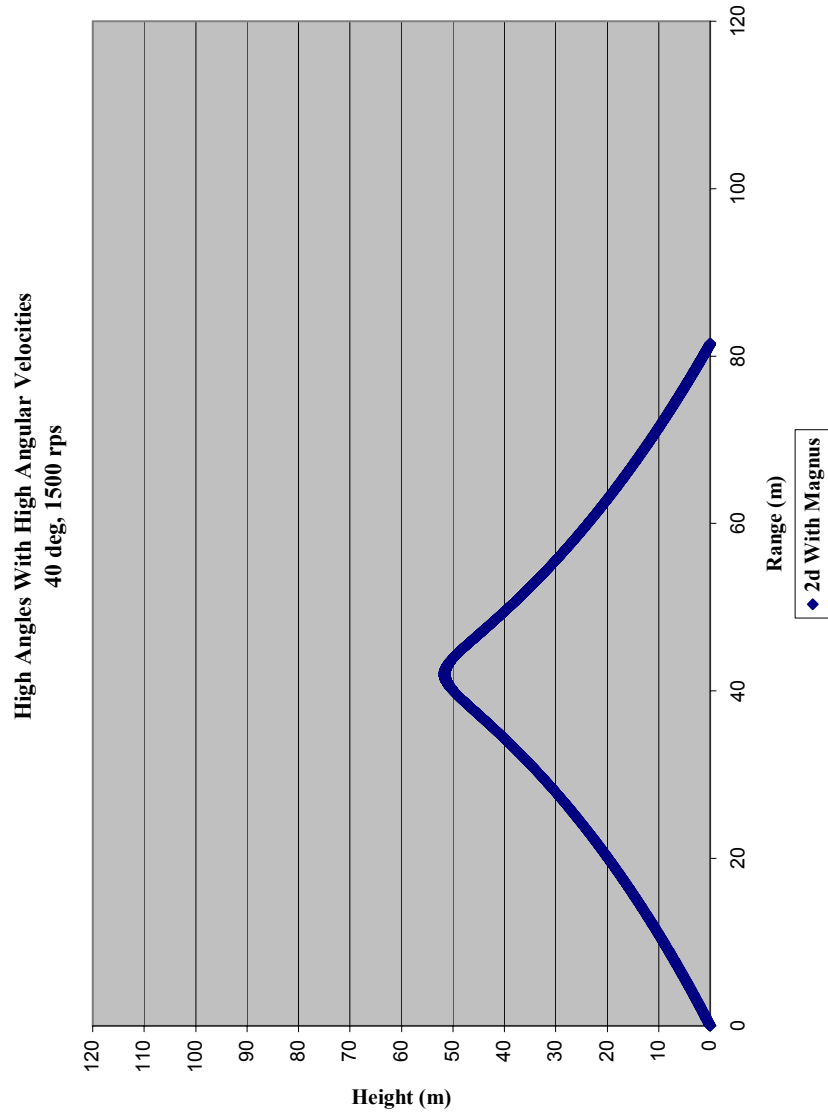


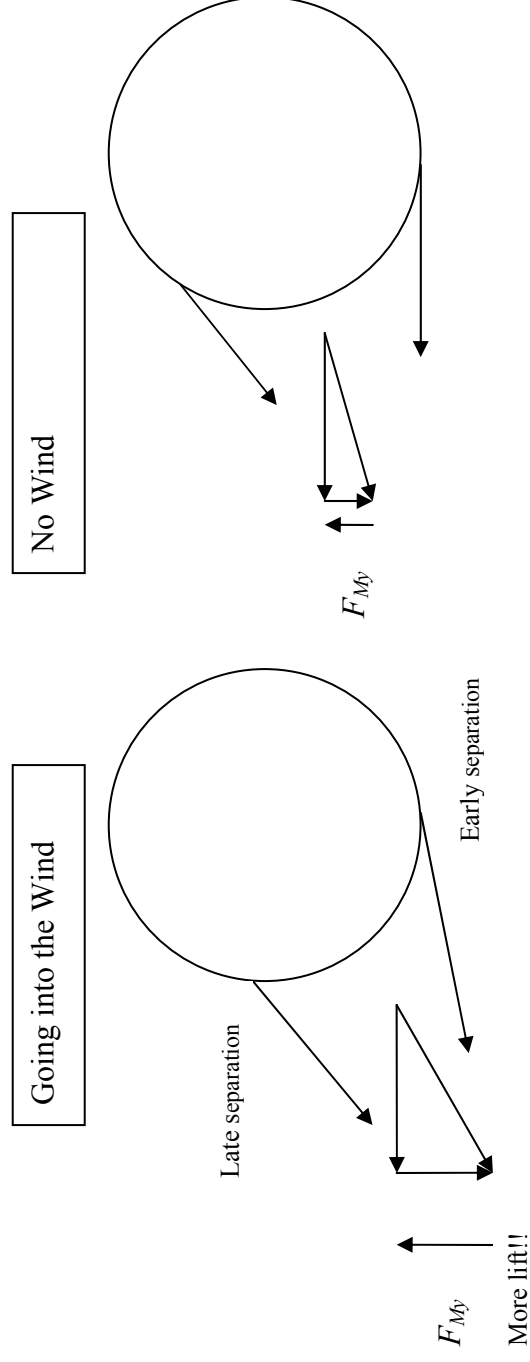
Figure 10: Effects of Magnus Effect on Range: 40 m/s, 35 deg



**Figure 11: Distorted Trajectories Due to the Magnus Force**



**Figure 12: Wind Changes Boundary Layer separation**



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