Optimal Unemployment Insurance with Variable Skill Levels

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- PRELIMINARY VERSION -

Abstract

I study the consequences of heterogeneity of skills for the design of an optimal unemployment insurance, using a principal-agent set-up with a risk neutral insurer and infinitely lived risk averse agents. Agents are characterised by different productivities. They are employed by firms offering wages that depend both on the agents' individual productivity and the quality of the worker-firm-match. Agents face the risk of losing their job and, if unemployed, they are offered jobs with different match qualities. No search effort by the agent is needed to get offers. Individual productivity declines during unemployment due to depreciation of skills and increases on the job because of learning by doing.

Any insurance offered must take into account the moral hazard problem created by the fact that job offers are private information to the agent. A further complication is due to the unobservability of an agent's productivity.

I find that under an optimal contract, periods of unemployment are characterised by declining benefits. Agents are further punished for long unemployment by reducing expected future utility. A new result obtained from this approach is the observation that under an efficient contract, agents whose productivity is relatively high tend to have a shorter unemployment duration and a higher productivity growth in the future. The mechanism to induce truthful reporting of an agent's productivity is to make his utility depend on the quality of the jobs he accepts.

JEL Classification: C61, D78, D82, D83, J24, J31, J38, J64, J65, J78

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1. Introduction

There has been a great deal of interest in efficient unemployment insurance in the literature. It is a well-established empirical finding that generous unemployment insurance schemes tend to raise the unemployment rate in an economy. High unemployment benefits and long benefit durations increase the attractiveness of being unemployed. In popular theoretical models, this makes workers or unions push for higher wages or reduces the job-search efforts exerted by the unemployed. A good survey of the major developments in the unemployment insurance literature is given in Holmlund (1998).

The model presented here follows a strand of literature that interprets insurance schemes as the solution to principal agent problems.

Shavell and Weiss (1979) were the first to show that if unemployed agents can influence the probability of finding a job, an optimal unemployment insurance pays declining benefits. However, they make the simplifying assumption that once individuals find a job, they are employed forever. The decline of benefits in periods when agents can perform hidden actions seems to be a typical feature of efficient social insurance contracts. Thomas and Worrall (1990) also obtain this result for a model in which the moral hazard problem stems from the fact that the agents' period income is unobservable. They prove that as a consequence, expected discounted utility converges to its infimum with probability one. In the light of this result, Atkeson and Lucas (1993) impose a lower bound on lifetime utility in any period and show that under this additional assumption, efficient insurance leads to a stable cross-sectional distribution that is not fully concentrated at the lower bound. Hopenhayn and Nicolini (1997) explicitly model moral hazard as an unobservable search effort exerted by unemployed agents. They find that incentives to search efficiently should not only be created by decreasing benefits during unemployment, but also by levying higher social security taxes on agents who have been unemployed for a long time. This mechanism to spread the punishment for low search effort over time is required because agents are risk averse. Using a calibrated version of their model, Hopenhayn and Nicolini also show that moving towards this efficient mechanism might result in considerable welfare improvements.

Wang and Williamson (1996) consider a model in which both search effort and job retention effort are required. They find that in this case, optimal unemployment benefits first rise before declining monotonously.

Another approach to the analysis of efficient unemployment insurance arrangements is to embed a labour market with frictions in a general equilibrium model. Although such models are too complicated in general to derive an optimal solution to the insurance problem, they have the virtue of allowing the researcher to compare different policies in a more realistic set-up. Typically, calibrated versions of such models are used to find constrained optima of unemployment insurance mechanisms and to assess welfare effects of alternative policies.

Frederiksson and Holmlund (1999) construct a model of job search abstracting from capital. They consider an unemployment insurance with two benefit levels and show that optimality requires the benefits to decline. In a calibrated version of their model they find significant welfare gains from switching from an optimal one-level to a two level benefit unemployment insurance.

Acemoglu and Shimer (1999) construct a general equilibrium model with search in which highly productive jobs are also riskier. They show analytically that under these circumstances, if agents are risk averse, maximal output is attained only with an unemployment insurance.

Hansen and İmrohoroğlu (1992) study the consequences of moral hazard for optimal replacement ratios in an economy populated by liquidity constrained agents. They find that an optimal insurance has a relatively high benefit level to protect agents from large fluctuations of consumption. However, if moral hazard is introduced, replacement ratios at levels observed in reality may actually make the economy worse off than without any insurance at all.

Using a matching model, Costain (1999) only finds minor improvements of consumption smoothing with unemployment insurance. Also, the importance of moral hazard is relatively small. In the model of precautionary savings discussed in Engen and Gruber (1995), unemployment insurance creates large crowding out effects. Using American micro-data, the authors estimate that an increase of the replacement rate by ten percentage points may reduce asset holdings by more than five percent.

This paper proceeds as follows. Section 2 describes the model economy and the moral hazard and adverse selection problems that any insurer is faced with. In section 3, the first best outcome is derived and compared to a simple insurance contract solving the moral hazard problem. Section 4 discusses a mechanism that solves both the moral hazard and the adverse selection problem. After deriving the dynamic structure of the resulting insurance contract, some distributional issues are mentioned. Section 5 concludes.

2. The model

The economy

Consider an economy populated by a large number of infinitely lived agents. There are also employers (firms), whose only purpose is to offer jobs to agents, paying a wage according to the agents' individual productivity and the quality of the worker-firm-match. Finally, there may be an insurer (e.g. a government agency or some other monopolist "moneylender"), who can offer social insurance contracts to agents. The insurer is assumed only to have information publicly available.

Agents have identical preferences over (positive) consumption c_t and leisure ℓ_t at time $t \geq 0$

$$v_0 = E\sum_{t=0}^{\infty}\beta^t u(c_t, \ell_t), \tag{1.1}$$

where $\beta > 0$ is a discount factor, u is the period utility function, and E is the mathematical expectation operator. ℓ_t depends on the employment status in the current period and can only take on two values, ℓ^e when employed and $\ell^u > \ell^e$ otherwise. Thus, denote by $u_e(c) \equiv u(c, \ell^e)$ and $u_u(c) \equiv u(c, \ell^u)$ the period utility when (un)employed. Both u_e and u_u are assumed to be increasing in consumption, strictly concave, and continuously differentiable. Further, for any level of consumption, both utility and marginal utility of consumption increase in leisure, i.e.

$$u_e(c) \leq u_u(c) \text{ and } u'_e(c) \leq u'_u(c) \quad (\forall c \geq 0). \tag{1.2}$$

Individuals are further characterised by their skill level $q > 0$ that determines their productivity during employment. In period 0, this "human capital stock" is known to have the cross-sectional distribution $F(q)$, but individual skills are private information. Skill levels are subject to change over time. While employed, agents enjoy an increase of q by the factor $1+\delta_e$ per period. This can be thought of as the result of learning by doing or the change to a different job that better fits to the agent's skill profile. During unemployment, however, skills depreciate at the rate δ_u. Immediately after being fired, q decreases by the factor $1-\delta_f$ due to obsolescence of job-specific human capital. [1]

All agents begin their lives without a job in period 0. While unemployed, they have the chance of receiving job offers, which differ in their match quality $m > 0$ drawn from a distribution with cdf. $G(m)$ and density $g(m)$. Whether an agent receives an offer cannot be

[1] Using a similar notion of skill levels, Ljungqvist and Sargent (1997) and Pissarides (1992) assess the consequences of changes of skills during unemployment on level and persistence of unemployment.

observed by the insurer. Jobs with a higher match quality are more attractive to the agent, because they pay higher wages $w = qm$. The probability of being offered a job is p. I assume that m and q are known only to the agent and his potential employer. Thus, although the actual wage paid if a job is accepted is observable, employment only reveals little information about an agent's productivity.

Since both G and p are constant, there is no need for an unemployed agent to exert a search effort. He only has to sit and wait for the right job – one with an acceptable match quality – to come up.

Once an agent has accepted a job, he stays employed until he is fired, which happens at the exogenous rate λ per period.

There exists a capital market paying the risk-free and constant interest rate r. The assumption $R \equiv 1 + r = \beta^{-1}$ simplifies the analysis of the insurance problem and ensures the existence of an interior solution. The parameters of the model are assumed to be such that the expected present value of wage income streams is finite with probability one. This also implies that $v_0 < \infty$.

Insurance

All agents have access to the capital market. If they do not participate in any insurance scheme, they can protect themselves against the fluctuations of labour income by borrowing and saving. However, since any debt must be repaid with probability one, if the marginal utility of consumption tends to infinity as c approaches zero, individuals will choose never to borrow against uncertain future income, because they face a positive probability having difficulties repaying it, leaving them with inefficiently high marginal utilities. The assumption that all agents are equipped with the same small level of wealth a_0 ensures that it is possible to maintain a positive level of consumption under all circumstances. Denote by $\underline{v}(q)$ the expected lifetime utility of an uninsured agent whose initial skill level is q. This is the minimum level of well-being that any insurance contract must offer him to be acceptable. It is clear from the structure of the problem that this outside option is increasing in q.

An unemployment insurance contract constitutes a relationship between the insurer as a risk-neutral principal and a risk-averse agent. The principal also has the same discount factor R^{-1} as the agents. All insurance contracts are offered at date 0. By this time, no information about the actual productivity of individuals has been revealed, so all that is known about it is its cross-sectional probability distribution. I assume that contracts directly rule the consumption of the agents. This simplifies notation as it abstracts from flows such as contributions and benefits. It is also restrictive however, because implies that all these flows can be observed and controlled by the principal. A particularly interesting extension of the model would allow for unobserved saving.

Two complications might prevent the optimal contract offered by the principal from being fully efficient. First, since little is known about the productivity of an agent at time $t = 0$, the principal faces an adverse selection problem if he wants to discriminate between different skill levels. Second, the fact that agents can decline job offers without the principal knowing about it, contracts must provide incentives to overcome this hidden action problem.

In reality, unemployment insurance is usually provided by the state on a non-profit basis. An efficient insurance scheme of this kind maximises some weighted average of the agents' expected lifetime utility subject to a zero profit constraint. The weight attached to a certain period 0 skill level is arbitrary, however. Also, when first initiated, any social security program in democracies must be approved by a majority of the population. This requires that at least fifty percent (or more in the case of a qualified majority) of the agents at least expect a lifetime utility at the level of their outside option.

Before I come to this more complex problem, in the next section I will discuss the first best outcome and a contract that only addresses the moral hazard problem.

3. Moral hazard

The first best outcome

Here and in what follows, a recursive formulation of the problem will be used to characterise the insurance contracts. Denote by $P(q, m, v, s)$ the highest expected profit the principal can make on a contract promising the agent an expected lifetime utility of v, if the agent's current skill level is q, his employment situation is $s \in \{\text{employed}, \text{unemployed}\}$ and the match quality of his current job is m ($m = 0$ during unemployment). As common in the literature, v is used as a state variable. Each period the promise keeping constraint

$$v_t = u(c_t, \ell_t) + \beta E v_{t+1} \qquad \text{(PK)}$$

must hold, meaning that an agent is provided with his promised discounted lifetime utility v_t by giving him some current utility $u(c_t, \ell_t)$ and promising him some "continuation utility" Ev_{t+1} for the future.

A Pareto efficient mechanism is obtained by maximising the principal's profit subject to the requirement to guarantee some promised level of lifetime utility v. To simplify notation, let $P_e(q,m,v) \equiv P(q,m,v,\text{employed})$ and $P_u(q,v) \equiv P(q,0,v,\text{unemployed})$ be the value of the contract to the principal when the agent is employed or unemployed, respectively. The controls of the dynamic programming problem are current consumption, continuation utility, and for unemployed agents a rule determining whether to accept or reject a job.

Conjecture that this job acceptance rule simply requires the agent to take up any job the match quality of which exceeds a threshold level $\underline{m}$. Further, assume for the moment that the continuation utility given to the agent upon accepting a job is independent of the match quality of this job.

The Bellman equations are then

$$P_e(q,m,v) = \max_{c_e, v_{ee}, v_{eu}} \{qm - c_e + R^{-1}[(1-\lambda)P_e(q(1+\delta_e),m,v_{ee}) + \lambda P_u(q(1+\delta_e)(1-\delta_f),v_{eu})]\} \tag{1.3}$$

$$\text{s.t. } v = u_e(c_e) + \beta[(1-\lambda)v_{ee} + \lambda v_{eu}] \tag{PK$_e$}$$

and

$$P_u(q,v) = \max_{c_u, v_{ue}, v_{uu}, \underline{m}} \{-c + R^{-1}[(1-p(1-G(\underline{m})))P_u(q(1-\delta_u),v_{uu}) + p\int_{\underline{m}}^{\infty} P_e(q(1-\delta_u),m,v_{ue})dG(m)]\} \tag{1.4}$$

$$\text{s.t. } v = u_u(c_u) + \beta[(1-p(1-G(\underline{m})))v_{uu} + p(1-G(\underline{m}))v_{ue}]. \tag{PK$_u$}$$

The first order conditions of the maximisation problem are

$[c_e]$ $$-1 - \mu u_e'(c_e) = 0 \tag{1.5}$$

$[c_u]$ $$-1 - \nu u_u'(c_u) = 0 \tag{1.6}$$

$[v_{ee}]$ $$R^{-1}(1-\lambda)\frac{\partial}{\partial v_{ee}}P_e(q(1+\delta_e),m,v_{ee}) - \mu\beta(1-\lambda) = 0 \tag{1.7}$$

$[v_{eu}]$ $$R^{-1}\lambda\frac{\partial}{\partial v_{eu}}P_u(q(1+\delta_e)(1-\delta_f),v_{eu}) - \mu\beta\lambda = 0 \tag{1.8}$$

[v_{uu}] $$R^{-1}(1-p(1-G(\underline{m})))\frac{\partial}{\partial v_{uu}}P_u(q(1-\delta_u),v_{uu})-\nu\beta(1-p(1-G(\underline{m})))=0 \tag{1.9}$$

[v_{ue}] $$R^{-1}p\int_{\underline{m}}^{\infty}\frac{\partial}{\partial v_{ue}}P_e(q(1-\delta_u),m,v_{ue})dG(m)-\nu\beta p(1-G(\underline{m}))=0 \tag{1.10}$$

[$\underline{m}$] $$R^{-1}pg(\underline{m})[P_u(q(1-\delta_u),v_{uu})-P_e(q(1-\delta_u),\underline{m},v_{ue})]-\nu\beta pg(\underline{m})[v_{uu}-v_{ue}]=0, \tag{1.11}$$

where μ and ν are the Lagrange multipliers attached to the promise keeping constraints (PK_e) and (PK_u), respectively. The envelope conditions with respect to v become:

$$\frac{\partial}{\partial v}P_e(q,m,v)=\mu \tag{1.12}$$

$$\frac{\partial}{\partial v}P_u(q,v)=\nu \tag{1.13}$$

The following results can immediately be derived from these equations:

- The marginal utility of consumption is always equal to the inverse of the marginal cost of providing it: $-\frac{1}{\partial/\partial v\, P}=\frac{\partial}{\partial c}u$.
- Both the marginal utility of consumption and the marginal cost of utility to the principal are constant over time and states; By the properties of the period utility function, this means that unemployed agents enjoy a higher utility than those who are working.
- The additional profit from accepting the marginal job $\underline{m}$ must be equal to the reduction of expected utility it causes, valued at the price of utility:

$$P_e(q(1-\delta_u),\underline{m},v_{ue})-P_u(q(1-\delta_u),v_{uu})=\frac{\partial P_u(q,v)}{\partial v}(v_{ue}-v_{uu}) \tag{1.14}$$

  The implied match quality threshold lies above the level $\underline{m}^*$ that would maximise the expected present value of wages, because this would require the expected profit when accepting to be equal to the expected profit when rejecting. However, $\underline{m}=\underline{m}^*$ for $\frac{\partial P_u(q,v)}{\partial v}=0$, which is the case if the marginal utility of consumption is infinite or if $v_{ue}=v_{uu}$, because the period utility does not depend on leisure.

These are rather standard optimality requirements, but they have some interesting implications. It follows from assumption (1.2) that agents not only enjoy a higher utility while unemployed, they also have a higher level of consumption. These consumption levels do not depend on q or m and therefore remain constant over the lifetime of an agent. This does not mean however that the expected lifetime utility does not depend on the individual employment history.

To illustrate this point, assume that the period utility function is of the constant relative risk aversion (CRRA) variety with a coefficient of relative risk aversion $\gamma > 0$.

$$u(c,\ell) = \begin{cases} \sigma \ln c + (1-\sigma)\ln \ell & \text{if } \gamma = 1 \\ \dfrac{(c^{\sigma}\ell^{1-\sigma})^{1-\gamma} - 1}{1-\gamma} & \text{otherwise} \end{cases}, \quad \ell \in \{\ell^e, \ell^u\},\ \sigma \in (0,1)$$

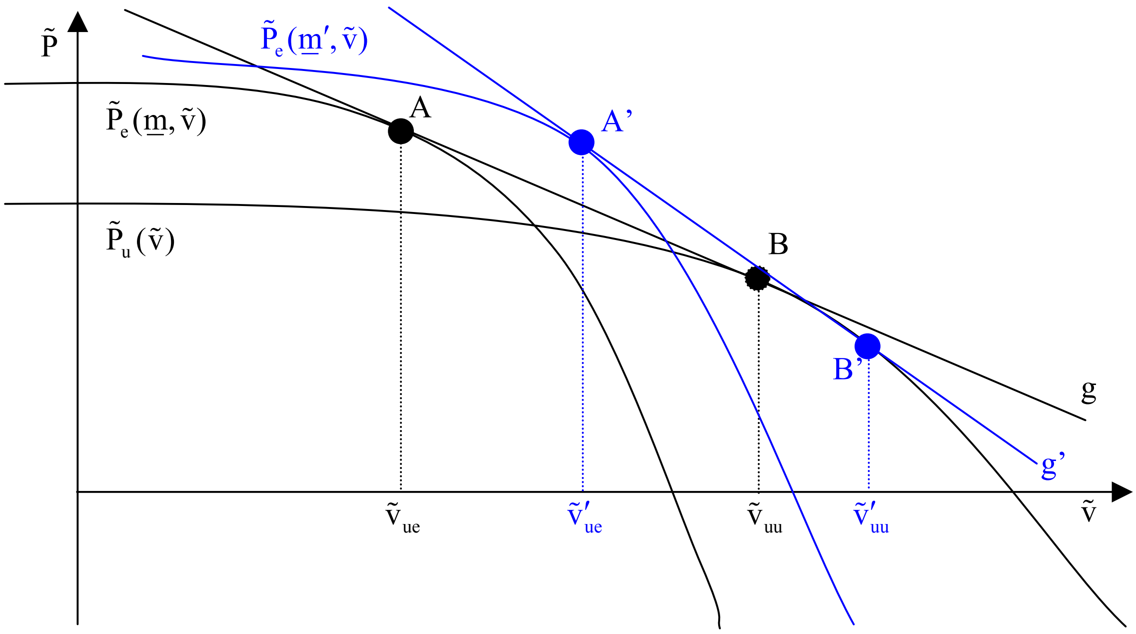

Figure 1

This simplifies the analysis, because q now only affects the scale of the problem: Changing q by a factor $\theta > 0$ while adjusting v appropriately ($v_\theta = u_e(\theta u_e^{-1}(v))$) simply rescales all expected profits and consumption levels of the optimal program, leaving $\underline{m}$ unaffected. Thus, denote by $\tilde{v}$ the adjusted lifetime utility for q normalised to one, and define the corresponding profit functions $\tilde{P}_e(m,\tilde{v}) \equiv P_e(1,m,\tilde{v})$ and $\tilde{P}_u(\tilde{v}) \equiv P_u(1,\tilde{v})$. $\tilde{P}_e$ and $\tilde{P}_u$ can be shown to be bounded above and decreasing and strictly concave in $\tilde{v}$. The parameter m simply shifts $\tilde{P}_e(m,\tilde{v})$ vertically, because it only affects the income stream derived from the current job without having any effect beyond its duration. Also, for any $m \geq \underline{m}^*$, $\tilde{P}_e(m,\tilde{v})$

lies above $\tilde{P}_u(\tilde{v})$ for small $\tilde{v}$, and below it for high $\tilde{v}$. The reason is that for very low $\tilde{v}$ (low promised utility compared with productivity), the cost of consumption to the principal hardly matters, but wage income of $w = qm$ per period does. Conversely, if $\tilde{v}$ is very high, the advantage of being able to provide the agent with utility more cheaply when unemployed clearly outweighs the disadvantage of foregoing some earnings.

The relationship between v_{ue} and v_{uu} implied by (1.14) is sketched for normalised values in Figure 1. Since $\frac{\partial P}{\partial v}$ is constant over time and states, $P_u(q(1-\delta_u), v_{uu})$ and $P_e(q(1-\delta_u), \underline{m}, v_{ue})$ must have the same slope $\frac{\partial}{\partial v}P_u(q, v)$ in points A and B. But by (1.14), this is also the slope of the line g connecting A and B.

Now consider an increase of v to v'. Because $\frac{\partial}{\partial v}P_u(q, v')$ is now bigger in absolute value, $\tilde{P}_u$ must be steeper in the new tangency point of g', $B' = (\tilde{v}'_{uu}, \tilde{P}_u(\tilde{v}'_{uu}))$. Thus, for g' to be also tangent to $\tilde{P}_e(\underline{m}', \tilde{v})$, $\tilde{P}_e(\underline{m}', \tilde{v})$ must lie above $\tilde{P}_e(\underline{m}, \tilde{v})$. This implies that $\underline{m}'$ is greater than $\underline{m}$.

To put it differently, agents who have a higher productivity compared to their expected lifetime utility must also have a lower job match threshold, i.e. they are more likely to accept a job. Because the skill level of agents decreases during unemployment, this implies that an agent who happens to be unemployed relatively long and often during the first periods of his life

- will also have a higher probability of being unemployed in the future, because $\underline{m}$ increases
- will enjoy a higher expected lifetime utility, because both leisure and consumption are higher during unemployment
- will face a lower expected wage income growth, because longer unemployment means more depreciation of human capital.

While agents with a high skill levels are particularly productive workers and therefore have high opportunity costs of unemployment, the comparative advantage of unskilled agents lies in being happy.

The consequences of moral hazard

Before proceeding to the analysis of the full problem, this subsection discusses the effects of introducing moral hazard in isolation. Probably the most obvious reason why the first best solution presented above is not feasible is the fact that unemployed agents enjoy a higher utility than employed agents. This clearly induces an incentive to decline any job offer, with the consequence that production is zero. The only case when the first best solution can be implemented arises if leisure does not affect utility, i.e. $u_u \equiv u_e$. Then, the agent's utility does not depend on his employment situation, and accepting the "right" jobs is a Nash equilibrium. In general however, an agent will only accept a job if this guarantees him a utility at least as high as upon refusal. Thus, the bellman equation for a period of unemployment (1.4) must be amended by an incentive compatibility constraint

$$v_{ue} \geq v_{uu}. \tag{IC}$$

Let ρ be the Lagrange multiplier if (IC). Then, of the first order and envelope conditions (1.5) to (1.13) derived above, only the derivatives with respect to v_{ue} and v_{uu} change.

$$[v_{uu}] \qquad R^{-1}(1-p(1-G(\underline{m})))\frac{\partial}{\partial v_{uu}}P_u(q(1-\delta_u), v_{uu}) - \nu\beta(1-p(1-G(\underline{m}))) - \rho = 0 \tag{1.15}$$

$$[v_{ue}] \qquad R^{-1}p\int_{\underline{m}}^{\infty}\frac{\partial}{\partial v_{ue}}P_e(q(1-\delta_u), m, v_{ue})dG(m) - \nu\beta p(1-G(\underline{m})) + \rho = 0 \tag{1.16}$$

The resulting incentive compatible contract can be characterised as follows:

- The incentive compatibility constraint is always fulfilled with equality, i.e. $v_{ue} = v_{uu}$
- The within-period efficiency condition $-\frac{1}{\partial / \partial v \, P} = \frac{\partial}{\partial c}u$ still holds
- As long as he cannot perform a hidden action, the agent still enjoys full insurance. The marginal utility of consumption only changes *after* a period of unemployment. This implies that the marginal cost of providing utility also changes in this case. However, $\frac{\partial P}{\partial v}$ still follows a Martingale process:

$$\frac{\partial}{\partial v}P_u(q, v) = E \; \frac{\partial}{\partial v'}P(q(1-\delta_u), m', v', s') \tag{1.17}$$

  Next period's values are marked with a prime $'$.
- The condition for the choice of $\underline{m}$ simplifies to

$$P_e(q(1-\delta_u), \underline{m}, v_{ue}) = P_u(q(1-\delta_u), v_{uu}). \tag{1.18}$$

The resulting match threshold however cannot easily be compared to the one obtained in the first best scenario, since the inefficiency introduced by the incentive compatibility constraint increases the cost of providing agents with a given utility, thereby changing the profit functions P_e and P_u.

Again, it is easier to say more about this contract in the special case of CRRA preferences.

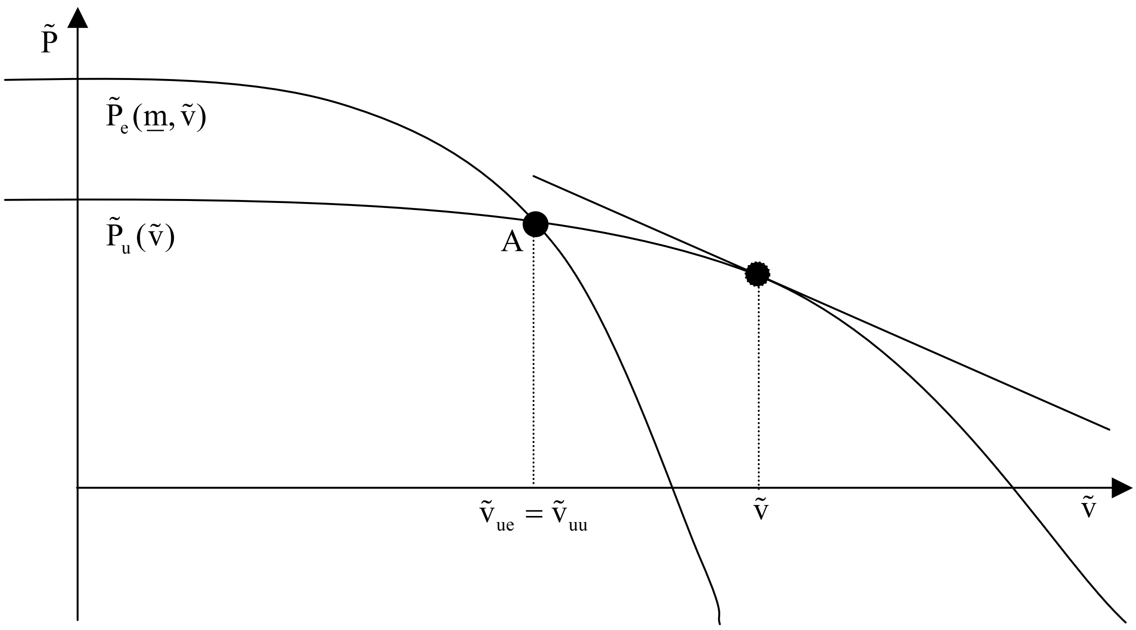

Figure 2

Figure 2 shows the diagram for determining the optimal level of $\underline{m}$ as given by equation (1.18). The promised continuation utility does not depend on whether or not a job is found, $\tilde{v}_{ue} = \tilde{v}_{uu}$, and is the abscissa value of the intersection between $\tilde{P}_e(\underline{m}, \tilde{v})$ and $\tilde{P}_u(\tilde{v})$, point A in figure 2. Since for all m, $P_e(q(1-\delta_u), m, v)$ is steeper in this point than $P_u(q(1-\delta_u), v)$, the Martingale property (1.17) requires that

$$\frac{\partial}{\partial v} P_u(q, v) < \frac{\partial}{\partial v_{uu}} P_u(q(1-\delta_u), v_{uu})$$

or, in terms of adjusted productivity adjusted utilities,

$$\frac{\partial}{\partial \tilde{v}} \tilde{P}_u(\tilde{v}) < (1-\delta_u) \frac{\partial}{\partial \tilde{v}_{uu}} \tilde{P}_u(\tilde{v}_{uu}).$$

By the concavity of P_u, $\tilde{v}_{uu} = \tilde{v}_{ue}$ must therefore be smaller than $\tilde{v}$. But if both "utility per skill unit" and the skill level decline, this must mean that the promised lifetime utility unambiguously decreases after a period of unemployment.

To summarise, both period consumption and continuation utility are constant during periods of employment. Immediately after losing his job, an agent's consumption level increases at the cost of a lower promised future utility. During unemployment, both consumption and continuation utility decline every period. Because agents are unemployed for an infinite number of periods with probability one, both period consumption and continuation utility have a tendency to decline over time.

This result is in line with the findings of Shavell and Weiss (1979), Thomas and Worrall (1990), and Hopenhayn and Nicolini (1997). All these share the common feature that under an optimal contract, promised utility declines after periods in which there is a moral hazard problem. To create the right incentives, agents must be rewarded in the case of a good outcome and punished otherwise. Thus, the principal must offer continuation utilities with different marginal utilities of consumption at different states of the nature in the following period, which is costly because the agent is risk averse. This cost is lower, however, if the expected utility offered is lower. Therefore, it pays for the principal to provide the agent with a slightly higher period utility and save on costs of creating incentives.

The relationship between $\tilde{v}$ and $\underline{m}$ is not as clear as in the first best scenario. Depending on the probability distribution G, even multiple $\underline{m}$ may fulfil the first order conditions. As argued above, the slope of P_u in $\tilde{v}$ must be a weighted average of the slopes of $\tilde{P}_e(\underline{m},\cdot)$ and $\tilde{P}_u$ in $\tilde{v}_{uu}$. Increasing $\tilde{v}$ requires the absolute value of this weighted average to be higher. A higher threshold $\underline{m}$ makes both $\tilde{P}_e(\underline{m},\cdot)$ and $\tilde{P}_u$ steeper at their intersection, but at the same time increases the weight of the flatter slope.

For very large $\tilde{v}$ however, this change of weights effect is small, thus in this case a further increase of promised utility results in a higher threshold level $\underline{m}$. Providing the agent with a high utility is cheaper during unemployment.

If $\tilde{v}$ is very low, the utility advantage of unemployment becomes small compared to the to the wages that can be earned. Therefore for small $\tilde{v}$, the threshold $\underline{m}$ approaches the level that maximises the expected discounted value of wage income, implying a shorter expected unemployment duration and thus a higher expected wage growth. As $\tilde{v}$ declines over time, this case becomes more and more relevant.

The conjectures made about the v_{ue} and the job acceptance rule remain to be justified. The fact that $v_{ue}(m) = v_{ue}$ almost everywhere will come out as a special case from the analysis in the next section. Thus, focus on the conjecture that there is no loss in generality limiting attention to contracts in which jobs are accepted if and only if their match quality m is greater or equal to some threshold $\underline{m}$.

Let $M \subset \mathbb{R}_+$ describe an optimal acceptance rule for some skill level and promised utility:

$$\text{job with match quality } m \text{ must be accepted} \Leftrightarrow m \in M$$

Define M^* by

$$M^* = [\underline{m}, \infty) \text{ and } \int_{M^*} 1dG(m) = \int_{M} 1dG(m),$$

i.e. M^* is the interval with upper bound infinity that has the same probability mass as M. By construction, $E[m \mid m \in M^*] \geq E[m \mid m \in M]$. Now replace M by M^* in the optimal contract. As $v_{ue}(m) = v_{ue}$ with probability one, this does not require any further specification. Neither changes the probability of finding a job, nor is incentive compatibility affected in any way in the moral hazard case. The only thing that changes is the expected revenue from an accepted job, which cannot be lower than under the original contract. Therefore, M^* is optimal.

4. Optimal unemployment insurance

The mechanism just described is feasible if there is no need to discriminate between agents of different skill levels in the first period. Under an insurance scheme that guarantees to everybody the same expected lifetime utility irrespective of individual productivity, no agent has an incentive to misreport his skill level. Reporting a wrong q to the insurer in the initial period simply results in a different range of jobs that an agent can accept at any time. Reporting too low a skill level enables the agent to also accept jobs that provide a worse match than required under the contract, without earning a wage that lies below the minimum value expected by the insurer. Such a strategy, however, cannot affect the expected lifetime utility of the agent, because the continuation utility promised after a period of unemployment neither depends on whether or not a job is accepted, nor on the wage earned on a job. Thus, there is no incentive to cheat.

It may however be impossible or undesirable *not* to differentiate among agents on the basis of their initial skill level. An insurer that cannot force agents into contracts might find it difficult to offer an insurance that guarantees the same utility to everybody who wants to participate while still covering its costs.

In what follows, an incentive compatible insurance mechanism that allows to discriminate between agents based on their initial productivity will be discussed. Then, some basic properties of profit maximising insurance contracts based on this mechanism will be described.

Incorporating adverse selection

In the initial period, the insurer as a principal is confronted with a group of heterogeneous individuals. The only thing known about them is the statistical distribution of their skill levels. Choosing the optimal menu of contracts to be offered involves two choices.

Firstly, the static adverse selection problem must be solved. Since the revelation principle applies here, this basically amounts to offering a contracts for different levels of q that are compatible with truthful reporting. Secondly, the dynamic structure of the contracts must be chosen such that the requirements for truth-telling are met at minimal cost and that the moral hazard problem caused by the unobservability of the job offers is also solved efficiently.

As skill levels q are private information to the agent, insurance contracts can of course only be based on reported values, which will be marked with a circumflex ^. Denote by $v(\hat{q}, q-\hat{q})$ the expected lifetime utility an agent with human capital q realises when reporting $\hat{q}$. For an insurance contract to locally induce truth telling, reporting $\hat{q}=q$ must be locally optimal for the agent. This requires the first order condition

$$\frac{d}{d\hat{q}} v(\hat{q}, q-\hat{q}) = \frac{\partial}{\partial \hat{q}} v(\hat{q}, q-\hat{q}) - \frac{\partial}{\partial (q-\hat{q})} v(\hat{q}, q-\hat{q}) = 0 \qquad (1.19)$$

and the second order condition

$$\frac{d^2}{d\hat{q}^2} v(\hat{q}, q-\hat{q}) \le 0 \qquad (1.20)$$

to hold for $\hat{q}=q$. These conditions can be shown to imply global optimality.[2]

[2] I won't have much more to say about the second order condition, except that I assume it to hold. I have not been able to characterise cases for which this condition is fulfilled. Rewriting it in terms of parameters of the model yields:

$v(\cdot,0)$ is the expected lifetime utility a contract offers to a truth-telling agent. The shape of this function will be determined when solving the adverse selection problem. $v(\hat{q},\cdot)$ is what an agent actually gets when announcing $\hat{q}$, depending on by how much his real skill level deviates from the announced value. (1.19) and (1.20) show how this function must be chosen to induce truth telling. Denote by $\kappa = \frac{\partial}{\partial \hat{q}} v(\hat{q},0)$ the marginal increase of the utility promised to truth-telling agents with the skill level. By (1.19), this must be equal to the marginal effect of lying

$$\kappa = \frac{\partial}{\partial (q-\hat{q})} v(\hat{q}, q-\hat{q}) \Big|_{q=\hat{q}} . \tag{1.21}$$

The optimal incentive compatible contract constructed in the last section yields the same expected utility for any level of q for a given announced level $\hat{q}$. Therefore, implementing constraint (1.21) for any $\kappa \neq 0$ is likely to be costly for the principal. These costs however can be spread over time. If κ is equal to zero, the simple moral hazard case discussed above arises, where the requirement (1.21) that there be no marginal effect of the deviation $q-\hat{q}$ on the expected lifetime utility is automatically implemented though the incentive compatibility constraint, which also requires v to be independent of any decision taken by the agent. The case most plausible here is $\kappa = \frac{\partial}{\partial \hat{q}} v(\hat{q},0) \geq 0$, i.e. agents with a higher period 0 productivity are promised a higher utility. It will however be seen below that $\kappa < 0$ is also a possibility under some circumstances.

To come to a recursive structure again, interpret κ_t as the marginal punishment for lying that still must be implemented from period t on. To make sure that the principal not only threatens to make lying costly without ever attempting to punish untruthful announcements, a transversality condition of the kind

$$\lim_{t\to\infty} \kappa_t q_t = 0 \text{ a.s.} \tag{1.22}$$

$$(v_{ue}(\underline{m}) - v_{uu})\underline{m}g(\underline{m})(1+\underline{m}\frac{g'(\underline{m})}{g(\underline{m})}) + \int_{\underline{m}}^{\infty} v'_{ue}(m)m(1+m\frac{g'(m)}{g(m)})dG(m) \geq 0$$

is required.
Instead of (1.21), I will use the more intuitive (but formally not fully correct) notation

$$\kappa = \frac{d}{dq} E(v \mid q = \hat{q}), \tag{1.23}$$

emphasising the interpretation of κ as the marginal change of the expected utility for a given level v promised to the truth telling agent, if the actual skill level q deviates from the one reported.
This leads to the following Bellman equations for periods of employment and unemployment, respectively:

$$\begin{aligned} P_e(q,m,v,\kappa) = \max_{c_e, v_{ee}, v_{eu}, \kappa_{ee}, \kappa_{eu}} \{ & qm - c_e + R^{-1}[(1-\lambda)P_e(q(1+\delta_e), m, v_{ee}, \kappa_{ee}) + \\ & \lambda P_u(q(1+\delta_e)(1-\delta_f), v_{eu}, \kappa_{eu})]\} \end{aligned} \tag{1.24}$$

$$\text{s.t. } v = u_e(c_e) + \beta[(1-\lambda)v_{ee} + \lambda v_{eu}] \tag{PK$_e$}$$

$$\kappa = \beta[(1-\lambda)(1+\delta_e)\kappa_{ee} + \lambda(1+\delta_e)(1-\delta_u)\kappa_{eu}] \tag{PP$_e$}$$

$$\begin{aligned} P_u(q,v,\kappa) = \max_{c_u, v_{ue}(m), v_{uu}, \kappa_{ue}(m), \kappa_{uu}, \underline{m}} \{ & -c_u + R^{-1}[(1-p(1-G(\underline{m})))P_u(q(1-\delta_u), v_{uu}, \kappa_{uu}) + \\ & p\int_{\underline{m}}^{\infty} P_e(q(1-\delta_u), m, v_{ue}(m), \kappa_{ue}(m))dG(m)]\} \end{aligned} \tag{1.25}$$

$$\text{s.t. } v = u_u(c_u) + \beta[(1-p(1-G(\underline{m})))v_{uu} + p\int_{\underline{m}}^{\infty} v_{ue}(m)dG(m)] \tag{PK$_u$}$$

$$v_{ue}(m) \geq v_{uu} \text{ a.e. on } [\underline{m}, \infty) \tag{IC(m)}$$

$$\begin{aligned} \kappa = \beta[& (1-p(1-G(\underline{m})))(1-\delta_u)\kappa_{uu} + p(1-\delta_u)\int_{\underline{m}}^{\infty} \kappa_{ue}(m)G(m) + \\ & \frac{d}{dq}E(v \mid q = \hat{q})] \end{aligned} \tag{PP$_u$}$$

A few things are different from the simple first best case. First, of course, κ now enters the equations as a new state variable. Further, both v_{ue} and κ_{ue}, i.e. next period's states in the case that an unemployed worker finds a job, are allowed to depend on the match quality m of this job. This is necessary for an efficient implementation of the truth telling mechanism, as will be seen. Finally, the "planned punishment" constraints (PP_e) and (PP_u) have been added, to make sure that the initially planned marginal discrimination with respect to the reported skill level can be enforced. This is basically done by guaranteeing that the required effect κ is equal to the discounted expected value of next period's κ, adjusted by the change of q, also taking into account the discrimination effected in the current period. This is captured by the term $\frac{d}{dq}E(v \mid q=\hat{q})$ in the (PP_u) constraint. This marginal effect of the actual skill level on the continuation utility can, by Leibnitz's rule, be decomposed in two effects: First, an agent with a higher q than reported can also accept worse jobs, thus increasing the probability of employment. Second, his probability distribution over different wages changes.

Before proceeding, $\frac{d}{dq}E(v \mid q=\hat{q})$ must be rewritten in terms of the variables of the model. Using the notation of equation (1.25) and noting the difference between 'effective' values of m and those that the principal erroneously believes to observe for $q \neq \hat{q}$, one gets

$$\begin{aligned} E(v \mid q,\hat{q}) &= (1-p(1-G(\underline{m}^{\text{effective}})))v_{uu} + p\int_{\underline{m}^{\text{effective}}}^{\infty} v_{ue}(m^{\text{observed}})dG(m^{\text{effective}}) \\ &= (1-p(1-G(\underline{m}\frac{\hat{q}}{q})))v_{uu} + p\int_{\underline{m}\frac{\hat{q}}{q}}^{\infty} v_{ue}(m\frac{q}{\hat{q}})dG(m) \end{aligned} .$$

Differentiation of this expression with respect to q, integration by part of the resulting integral, and setting $q=\hat{q}$ yields the result

$$\frac{d}{dq}E(v \mid q=\hat{q}) = -\frac{p}{q}\{\underline{m}g(\underline{m})v_{uu} + \int_{\underline{m}}^{\infty} v_{ue}(m)(1+\frac{g'(m)}{g(m)})dG(m)\} . \qquad (1.26)$$

Now it is straightforward to derive the rather long list of first order and envelope conditions for the Bellman equations (1.24) and (1.25). Attach the Lagrange multipliers μ, ν, ϕ, and ψ to the constraints (PK_e), (PK_u), (PP_e), and (PP_u). (IC(m)) is actually a continuum of constraints that will be multiplied by $\rho(m)$.

[c_e] $$u'_e(c_e) = -1/\mu \qquad (1.27)$$

[c_u] $$u'_u(c_u) = -1/\nu \qquad (1.28)$$

[v_{ee}] $$\frac{\partial}{\partial v_{ee}} P_e(q(1+\delta_e), m, v_{ee}, \kappa_{ee}) = \mu \tag{1.29}$$

[v_{eu}] $$\frac{\partial}{\partial v_{eu}} P_u(q(1+\delta_e)(1-\delta_f), v_{eu}, \kappa_{eu}) = \mu \tag{1.30}$$

[v_{uu}] $$\frac{\partial}{\partial v_{uu}} P_u(q(1-\delta_u), v_{uu}, \kappa_{uu}) = \nu + \frac{1}{1-p(1-G(\underline{m}))}\{R\int_{\underline{m}}^{\infty} \rho(m)dG(m) - \psi p \frac{\underline{m}}{q} g(\underline{m})\} \tag{1.31}$$

[$v_{ue}(m)$] $$\frac{\partial}{\partial v_{ue}(m)} P_e(q(1-\delta_u), m, v_{ue}(m), \kappa_{ue}(m)) = \nu - R\frac{\rho(m)}{p} - \psi\frac{1}{q}(1+m\frac{g'(m)}{g(m)}) \tag{1.32}$$

[κ_{ee}] $$\frac{\partial}{\partial \kappa_{ee}} P_e(q(1+\delta_e), m, v_{ee}, \kappa_{ee}) = \phi(1+\delta_e) \tag{1.33}$$

[κ_{eu}] $$\frac{\partial}{\partial \kappa_{eu}} P_u(q(1+\delta_e)(1-\delta_f), v_{eu}, \kappa_{eu}) = \phi(1+\delta_e)(1-\delta_f) \tag{1.34}$$

[κ_{uu}] $$\frac{\partial}{\partial \kappa_{uu}} P_u(q(1-\delta_u), v_{uu}, \kappa_{uu}) = \psi(1-\delta_u) \tag{1.35}$$

[$\kappa_{ue}(m)$] $$\frac{\partial}{\partial \kappa_{ue}(m)} P_e(q(1-\delta_u), v_{ue}(m), \kappa_{ue}(m)) = \psi(1-\delta_u) \tag{1.36}$$

[$\underline{m}$] $$\begin{aligned} &P_u(q(1-\delta_u), v_{uu}, \kappa_{uu}) - P_e(q(1-\delta_u), \underline{m}, v_{ue}(\underline{m}), \kappa_{ue}(\underline{m})) = \\ &\nu(v_{uu} - v_{ue}(\underline{m})) - \psi\{\frac{1}{q}(1+\underline{m}\frac{g'(\underline{m})}{g(\underline{m})})(v_{uu} - v_{ue}(\underline{m})) - (1-\delta_u)(\kappa_{uu} - \kappa_{ue}(\underline{m}))\} \end{aligned} \tag{1.37}$$

$$\frac{\partial}{\partial v} P_e(q, m, v, \kappa) = \mu \tag{1.38}$$

$$\frac{\partial}{\partial v} P_u(q, v, \kappa) = \nu \tag{1.39}$$

$$\frac{\partial}{\partial \kappa} P_e(q, m, v, \kappa) = \phi \tag{1.40}$$

$$\frac{\partial}{\partial \kappa} P_u(q, v, \kappa) = \psi \tag{1.41}$$

Note that (1.32) and (1.36) each impose a continuum of constraints, one for each $m \geq \underline{m}$. For an optimal solution, they need not hold for any m, but with probability one.

The following properties of the optimal contract can readily be derived from these conditions:

- The within-period efficiency condition

$$-[\frac{\partial}{\partial c} u(c, \ell)]^{-1} = \frac{\partial}{\partial v} P(q, m, v, s, \kappa) \tag{1.42}$$

holds.

- After periods of employment, the marginal cost to the principal of providing the agent with the promised lifetime utility does not change, i.e. the first best intertemporal efficiency condition is fulfilled. If the agents is unemployed, next period's marginal cost of v depends on next period's uncertain state. But even in this case, a Martingale property like (1.17) above still holds:

$$\frac{\partial}{\partial v}P(q,m,v,s,\kappa) = E\ \frac{\partial}{\partial v'}P(q',m',v',s',\kappa') \tag{1.43}$$

- The marginal cost of enforcing truth telling $\frac{\partial P}{\partial \kappa}$ changes exactly in proportion to q, i.e. $\frac{1}{q}\frac{\partial}{\partial \kappa}P(q,m,v,s,\kappa)$ is constant over time and states. This basically means that the marginal cost of inducing truthful reporting of the skill level in terms of the *initial* q does not change.
- The promised lifetime utility v is expected to decline over time. This can be shown as follows. From (1.42) and the Martingale condition (1.43) it follows that

$$[\frac{\partial}{\partial c}u(c,\ell)]^{-1} = E[\frac{\partial}{\partial c'}u(c',\ell')]^{-1}$$

or equivalently

$$\frac{\partial}{\partial c}u(c,\ell) = \frac{1}{E[\frac{\partial}{\partial c'}u(c',\ell')]^{-1}} \leq E\frac{1}{[\frac{\partial}{\partial c'}u(c',\ell')]^{-1}} = E\frac{\partial}{\partial c'}u(c',\ell'),$$

where the inequality follows from Jensen's inequality. Thus, the expected value of the marginal utility of consumption increases over time.
- An interesting new aspect of this contract is the nontrivial shape of $v_{ue}(m)$. Two properties of this function are already clear from the set-up of the problem. First, $v_{ue}(m)$ must be above or equal to v_{uu}, i.e. the expected utility from accepting a job must be at least as high as that from rejecting it to confirm to the incentive compatibility constraint (IC(m)). Second, $v_{ue}(m)$ must be strictly greater than v_{uu} at least for some m with a positive probability mass, if $\kappa \neq 0$. The reason is that otherwise the announcement of the marginal punishment for lying κ would be carried over period by period without ever implementing it. This would violate the transversality condition (1.22).

Looking at the right hand side of equation (1.32), it is clear that the effect of the match quality m of an accepted job on the promised continuation utility is driven by two forces. The first of these is the potentially binding (IC(m)) constraint that is represented by the term $R\frac{\rho(m)}{p}$. The second is the term $\psi\frac{1}{q}(1+m\frac{g'(m)}{g(m)})$, which is more interesting. Consider first the product $\psi\frac{1}{q}$. By the envelope condition (1.41), ψ is the marginal effect of κ on the principal's expected profits. This has above been shown to be proportional to q. Therefore, $\psi\frac{1}{q}$ is constant over time and states of nature. The second factor $(1+m\frac{g'(m)}{g(m)})$ could be interpreted as a measure of information about the agent's type revealed by the observation of the wage corresponding to m.

Consider the plausible case $\kappa > 0$, i.e. the principal wants to guarantee more productive agents a higher lifetime utility. Since the principal's profits are maximised at $\kappa = 0$, at least for values close enough to zero we have $\psi = \frac{\partial}{\partial\kappa}P_u(q, v, \kappa) \leq 0$. Thus, the right hand side of (1.32) increases in $(1+m\frac{g'(m)}{g(m)})$. As the marginal cost of v to the principal increases in v, this implies that $v_{ue}(m)$ decreases as $(1+m\frac{g'(m)}{g(m)})$ gets bigger.

Without specifying the distribution G, not much more can be said about the shape of $v_{ue}(m)$. In the special case when G is a truncated normal, $m\frac{g'(m)}{g(m)}$ is constant and consequently $v_{ue}(m)$ is also constant almost everywhere. A rather typical case seems to be a declining $m\frac{g'(m)}{g(m)}$. If m is distributed log-normal, for example, $(1+m\frac{g'(m)}{g(m)}) = 1-\theta\ln m$ for some positive θ and thus diverges to negative infinity for large m. The resulting $v_{ue}(m)$ would monotonously increase in m, and might be equal to v_{uu} for some low values (see figure 3).

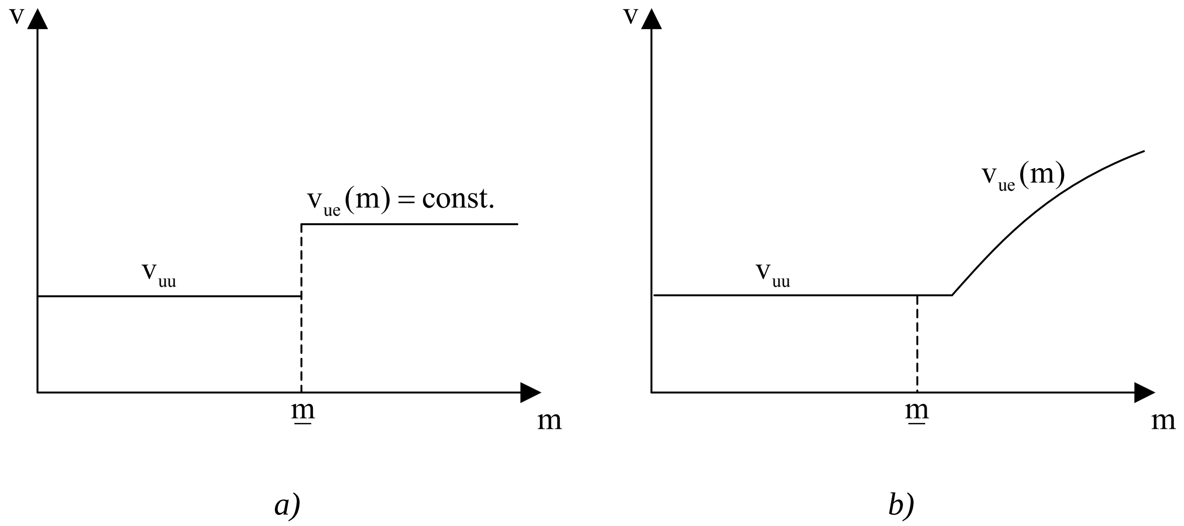

Figure 3. Continuation utility profiles offered to agents depending on the match quality of the job accepted if the distribution of m is: a) a truncated normal, b) log-normal.

The static adverse selection problem

The mechanism just described enables the principal to offer an agent a menu of contracts that yield a lifetime utility $v(q)$ to an agent of skill level q, where v is a continuous and almost everywhere continuously differentiable function.[3]

There are many Pareto-optimal insurance schemes that offer different levels of welfare to different groups. Consequently, some additional assumptions are required that help determine the distributional outcome.

The following analysis is based on the extreme assumption that a monopolistic insurer can offer contracts that are accepted by the agents whenever they yield an expected lifetime utility at least as high as their outside option $\underline{v}(q)$.

After looking at this case of a 'private' insurance, it will briefly be discussed what changes if the insurance is offered in a revenue maximising way by a government that need not provide each agent at least with his outside option, but instead is subject to a median voter constraint.

Finally, a few comments will be given on optimal insurance schemes offered by more benevolent governments that run a non-profit social insurance system and distribute the benefits among all agents.

[3] This formulation allows for a countable number of kinks in v. These may distort the incentives created by the mechanism discussed above and can therefore lead to invalid solutions. However, it is easy to show that such kinks can be smoothed out at an arbitrarily low cost to the principal. In this sense, the maximum profit derived in this section is actually a supremum to the original problem, which may not have a maximum.

Before looking at the profit maximising unemployment insurance scheme, note that an agent's outside option $\underline{v}(q)$ is increasing in its argument. It is also likely to inherit the concavity of the period utility function. As described above, q is distributed with cdf. $F(q)$. Assume $f(q) = F'(q) > 0$ for all $q \in (\underline{q}, \overline{q})$. The principal's objective function can be written as:

$$\max_{v(q),\kappa(q)} \int_{\underline{q}}^{\overline{q}} P_u(q, v(q), \kappa(q)dF(q) \tag{1.44}$$

$$\text{s.t. } v(q) \geq \underline{v}(q) \ (\forall q \in [\underline{q}, \overline{q}]) \tag{IR}$$

$$v'(q) = \kappa(q) \ (\forall q \in [\underline{q}, \overline{q}]) \tag{PP}$$

(IR) is the individual rationality constraint that makes sure an agent only accepts an insurance at least as good as his outside option. The set of constraints (PP) makes sure that truth telling is optimal for agents of each skill level.

This implicitly assumes that it is optimal for the principal to serve every level of q.

The Hamiltonian corresponding to the maximisation problem (1.44) is

$$H = P_u(q, v(q), \kappa(q))f(q) + \pi(q)\kappa(q) + \delta(q)(v(q) - \underline{v}(q)). \tag{1.45}$$

The first order conditions are then:

$$\frac{\partial H}{\partial v(q)} = \frac{\partial}{\partial v(q)} P_u(q, v(q), \kappa(q))f(q) + \delta(q) = -\pi'(q) \tag{1.46}$$

$$\frac{\partial H}{\partial \kappa(q)} = \frac{\partial}{\partial \kappa(q)} P_u(q, v(q), \kappa(q))f(q) + \pi(q) = 0 \tag{1.47}$$

Conjecturing that the individual rationality constraint (IR) is binding for the highest q, the following relationship can be derived from (1.46) and (1.47):

$$\frac{\partial}{\partial \kappa(q)} P_u(q, v(q), \kappa(q))f(q) = \int_{\underline{q}}^{\overline{q}} \frac{\partial}{\partial v(q)} P_u(q, v(q), \kappa(q))f(q) + \delta(q)dq \tag{1.48}$$

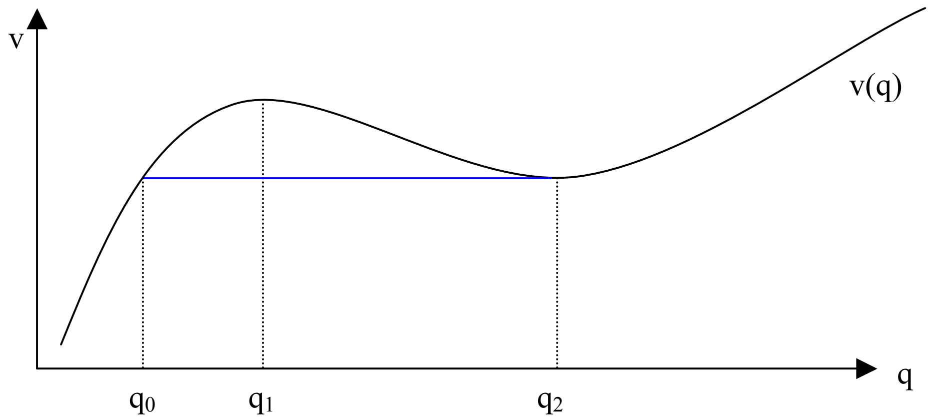

Figure 4

This makes clear that without a binding (IR) constraint, the slope of v is driven by the trade-off between the cost of implementing an high increase of v in q (left hand side of (1.48)) and the cost having to provide a higher lifetime utility for all agents with a skill level below q if the increase of v in q is small (right hand side of (1.48)). These considerations are modified by a potentially binding (IR) constraint.

Although the exact shape of v changes significantly with the parameters of the problem (in particular with the distribution F), it always has to be monotonously increasing. To see this, assume that v is decreasing on $[q_1, q_2]$. Now replace for $q < q_2$ the function $v(q)$ by $\max\{v(q), v(q_2)\}$ as indicated in figure 4. By the monotonicity of $\underline{v}$, this does not cause any problems with the (IR) constraint. Since $\frac{\partial}{\partial \kappa} P_u(q, v, \kappa)$ is minimised at $\kappa = 0$, the principal saves on this kind of cost for all $q \in (q_0, q_2)$. Further, as the promised lifetime utility is also lower on this interval now, the principal's profit unambiguously increases.

Will the principal really offer contracts to all agents?

To address this question, first notice that the principal earns money on each contract for which the (IR) constraint is binding. The reasoning is as follows: The principal offers an incentive compatible contract to the agent that, by the binding (IR) constraint, must have the properties $\left.\frac{\partial}{\partial (q - \hat{q})} v(\hat{q}, q - \hat{q})\right|_{q=\hat{q}} = \kappa$, where κ is the slope of the outside option $\underline{v}$, and $v(q) = \underline{v}(q)$ in this point $q = \hat{q}$. One potential contract that fulfils this requirement is the trivial contract where no payments between the agent and the principal are made. The profit earned on this contract is zero. But the contract actually offered by the principal is the best of all admissible contracts, and thus must yield a nonnegative profit.

Allowing the principal to exclude agents from his insurance would mean to let him design a $v(q)$ that lies below $\underline{v}(q)$ for some skill levels. Yet, the principal never chooses to do so, since for any interval (q_1, q_2) with $v(q) < \underline{v}(q)$ $(\forall q \in (q_1, q_2)$ and $v(q_1) = \underline{v}(q_1)$ or $v(q_2) = \underline{v}(q_2)$, he could earn more when making the (IR) constraint bind on the interval.

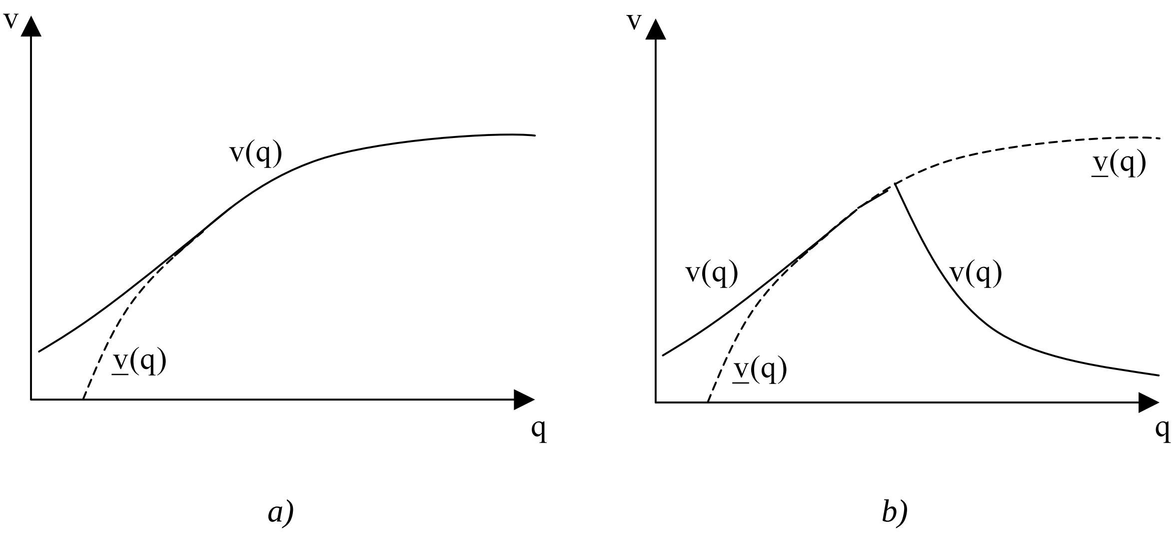

Figure 5. Possible contracts offered by an insurer facing a) an individual rationality constraint for each agent, b) a median-voter constraint only.

After describing an unemployment insurance that could be offered this way by a private, profit-maximising monopolist, I would like to make some brief remarks about how an insurance scheme imposed by a government could differ from that.

Consider first the case of a revenue maximising insurance implemented by a democratic government. To be acceptable to the majority of the voters, it must provide at least fifty percent of the agents with an expected lifetime utility above or equal to the autarky level. This means that the insurance contract described above can be altered such that the (IR) constraint is violated for not more than half of the population. It appears that an optimal contract under such a regime would most likely lower $v(q)$ for high levels of q, because the potential gain from offering them less than their relatively high outside options are large. However, it seems possible to find combinations of the distribution F and costs of enforcing truth-telling $-\frac{\partial P}{\partial \kappa}$ that justify a promised utility below the outside option for pretty much any skill level group.

Figure 5 depicts possible shapes of v for the monopoly insurance and the revenue maximising median voter scheme.
In reality, social security systems run by the government typically only try to cover their cost. But insofar as insurance enhances efficiency in an economy, there is a surplus that must be distributed in some way. Thus, an institution offering insurance on a non-profit basis must have some distributional objective. One potential objective is the maximisation of a weighted average of the agents' lifetime utility.
If equal weight is given to each skill group, an optimal insurance scheme is one that gives equal lifetime utility to everybody in the economy by using the contract of the simple moral hazard case described, if this does not violate the median voter constraint.

5. Conclusion

A principal-agent model of unemployment insurance with moral hazard and adverse selection was described and some aspects of an optimal insurance mechanism were worked out. Although such a model can capture some interesting aspects of the problem, I would like to point out some obvious shortcomings that seem likely to have a strong influence on the results obtained.
As the model only describes the behaviour of workers, any reaction of the demand side of the labour market to changes of workers' behaviour is ignored. Modelling a two-sided search process might have important implications for the design of an insurance mechanism. Another critical assumption is the principal's full control over agents' wage income and consumption. Allowing, for example, for unobserved saving by the agent would introduce another source of moral hazard and is likely to render the mechanism derived ineffective.
Further, by the partial equilibrium nature of the model, any effects of saving on the equilibrium productivity are ignored. Engen and Gruber (1995) however find considerable crowding-out of unemployment insurance due to the decrease of precautionary saving.

In spite of its weaknesses, the model points to some interesting aspects of efficient contracts that real-world insurance systems lack. As argued by Hopenhayn and Nicolini (1997), unemployment insurances should create incentives not only by means of declining benefits during unemployment, but also through increased insurance contributions after a new job is found. Thereby the punishment for inappropriate actions by the agent is spread over several periods, which is more efficient if agents are risk-averse. However, this effect may be quite small in reality, because agents can self-insure against quickly declining replacement rates by saving. It remains to be seen if currently implemented insurance systems that do not use continuously declining replacement rates over time but often adopt two benefit levels (e.g. an unemployment insurance benefit paid for a certain time, social assistance thereafter), are reasonably good approximations of the optimal scheme.

It was shown that under an optimal contract, the requirements to accept a job are less strict for workers who have a very low productivity compared to their promised benefit level. Such individuals can be provided with leisure at low opportunity costs in the form of foregone wage income. This implies that insurances should be relatively lenient towards workers who have claims to high benefits because of high past wages, but are unlikely to find a good job, e.g. because they have human capital specific to a declining industry. It could also be an argument in favour of early retirement schemes.

Current unemployment insurance systems typically exhibit fixed replacement rates rather than fixed absolute benefits, and social insurance taxes are relatively moderate. This suggests that the design of these systems has been dominated by insurance rather than distributional objectives. The mechanism they use to differentiate between individuals of different income prospects is to base both contributions and benefits on current and past wage income. Although an agent’s employment history is an important source of information, especially for young people who have not been able to demonstrate their skills, the insurance could be improved. It was shown that since expected future income is likely to be private information to each individual, an optimal insurance relies on some announced expected lifetime income and implements incentive mechanisms that induce truthful announcements. These mechanisms are based on the quality of the jobs actually held during the working life of a person, rewarding the employee for accepting work that corresponds to the individual skills announced. Such a system would not only insure workers against the risk of losing their job, but also against potential income fluctuations.

It was also shown that a monopolist insurer would offer insurance to all skill groups, promising higher lifetime utility to agents with higher skill levels. In this sense, there is no way an optimal insurance system run by the government could improve the resulting allocation, since it would serve the same group of agents and use the same type of incentive mechanism. The main difference between privately offered insurance and a public scheme would be that the latter could easily implement any distributional objective.

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