Savings, Investment and Capital in a System of General Intertemporal Equilibrium – a Comment on Garegnani

Second version. Comments welcome.

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1. Garegnani's paper as an indirect critique[1]

Is intertemporal general equilibrium concerned by the Cambridge critique of the theory of capital? Many thought, and for a long time, that it was not the case, since there is no aggregate of capital in general equilibrium theory, at least not in a form which would be visible immediately. Others suspected that the problems of capital theory would affect all versions of neoclassical theory, without being able to indicate the consequences for general equilibrium. Burmeister (1980, p. 122) introduced the assumption of regularity, i.e. essentially the postulate that the total change in the values of capital goods employed falls whenever a rise of the rate of interest causes a switch of technique. A variant of this assumption was used by Epstein (1987) to demonstrate the convergence of an intertemporal equilibrium with an infinite horizon towards a steady state in which the rates of return on all assets became equal among themselves and equal to the (variable) rates of time preference of the consumers. It therefore was remarked (see also Burmeister 1980; p. 125, Schefold 1997, chapter 18.1) that the absence of reswitching and reverse capital deepening were necessary conditions in neoclassical theory for reaching a terminal state with a uniform rate of profit after starting from arbitrary initial endowments of capital goods. Not only the relative quantities of capital goods adapt over time (as in the old neoclassical theories where a value of aggregate capital is given and relative quantities of individual capital goods are thought to adapt) but it is characteristic for the consideration of the very long run that even the general level of the production of equipment adapts and distribution depends eventually only on preferences and technology, not on quantities of capital.

The terminal state reached differs from a classical long period position not only with regard to the theory of distribution, but also the state of employment, for classical long period positions are not necessarily full employment equilibria.

However, the destabilising effect of a "perverse" relationship between factor prices and quantities of factors need not only be associated with the path of accumulation towards a "distant" horizon. For the destabilisation happens in a certain period in the process of transition, and it can therefore be analysed by restricting one's attention to a small number of periods around the one where the destabilisation occurs. Schefold (2003) analyses accumulation in a two-period model and compares the stability of two scenarios. In the first, the response to an increase of the labour force is a substitution of technique such that the more labour intensive technique is chosen at a lower wage rate, as conventional theory predicts. Reswitching, in the second scenario, means that the adoption of the more labour intensive technique may be associated with a higher wage rate. A full employment equilibrium still exists, but its stability is in doubt. The analysis of instability presupposes the specification of out-of-equilibrium behaviour, and even *tâtonnement* can take several forms. A general

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instability result cannot be obtained because of the diversity of potential reactions, but it can be shown that reswitching and related problems of capital theory imply less stability than technical changes in which the quantities of factors and factor prices are inversely related as in the neoclassical world.

Such an analysis represents a *direct* critique of modern intertemporal general equilibrium theory. It accepts the methods of the theory and its representation of economic reality, and it shows that neglected problems of technical change question not the existence, but the significance of equilibria by showing that the problems of capital theory surface as problems of stability.

The paper by Garegnani I have been asked to comment upon (Garegnani 2003) follows a different strategy by proposing a critique which I should like to characterise as *indirect*. This indirect critique is based on concepts of aggregate capital which, in a *reformulation* of intertemporal theory, are made to play a causal role in the determination of equilibrium. Once this role of aggregate capital is accepted, it is not surprising to see that unstable equilibria may result since similar phenomena have been known from the debate on the aggregation of capital which is now over forty years old. Garegnani is aware that the proponents of intertemporal general equilibrium theory have consciously moved away from the use of aggregate concepts of capital – indeed, Garegnani (1976) is well known for having clarified the difference between an "old" neoclassical concept of equilibrium based on the datum of an aggregate quantity of capital on the supply side, and a "new" theory in which capital is given as a vector of endowments so that, since these endowments are inherited from the past in arbitrary proportions, different commodities have different own rates of interest which reflect different degrees of scarcity relative to consumption (except in the very long run where only preferences and technology matter, if convergence obtains).

Garegnani (2003) chooses a model with a period of production. There is consumption at the beginning of the period (time 0) and the more is consumed at the beginning, the less remains for production to have consumption at the end of the period (time 1). Moreover, production is constrained by available labour, if demand for future goods is high. Total output at the end of the period (time 1) is consumed. The inputs needed for production, valued at equilibrium prices, represent investment. This is an aggregate which may be derived for each period in any intertemporal equilibrium with production but it is usually not calculated explicitly. On the other hand, goods not consumed immediately, i.e. not consumed at the beginning of the period of production, are left over and can – and in equilibrium will – be invested. They represent saving, valued at equilibrium prices, and, in equilibrium, saving and investment coincide not only in value terms, but also in physical terms. The coincidence usually is not noticed since no causal significance is attributed to these magnitudes in intertemporal theory. But Garegnani introduces an out-of-equilibrium behaviour in which saving and investment become dependent on a rate of interest so that, in this perspective, there emerges a macroeconomic determination of general equilibrium. He thus argues "that contemporary reformulations" of neoclassical theory are erroneously claimed not to rely on any concept of a "quantity of capital"; Garegnani asserts "that the claim is unfounded and that the deficiency of the concept undermine the reformulations no less than they do the traditional versions" (Garegnani 2003, par.1).

Garegnani's critique therefore is indirect in that the reintroduction of a "quantity of capital" in intertemporal general equilibrium is what first must be established to permit a critique of the theory – indeed, the very introduction of the "quantity of capital" is presented as Garegnani's main result. This is stated in his conclusions: "The presence *at some stage* of the theory of a quantity representing aggregates of capital goods is ... as inevitable for the neoclassical determination of prices on the basis of demand and supply decisions of individuals, as is the presence of the quantity of each consumption good ... It is on a single commodity 'capital' and not on individual capital goods that savers' preferences operate, whether in the traditional 'fund' context or in the intertemporal 'flow' context" (Garegnani 2003, par. 27).

Garegnani's reformulation of neoclassical theory implies a considerable modification of intertemporal equilibrium, both in spirit and in form. The indirect nature of the critique leads to three questions:

1. Are the new concepts introduced for the reformulation of intertemporal equilibrium consistent with the theory?
2. Is this reformulation compelling, in the sense of being necessary as the unique way for arriving at a full understanding of the theory?
3. Is the indirect critique likely to be effective, i.e. will it convince neoclassical authors?

It may be useful to state my views on these three questions in preliminary form before I proceed to a more detailed analysis.

1. Garegnani's approach is sophisticated but stands in contrast with the premises of intertemporal theory in the following respects: (a) The reader of either Garegnani's paper or of the summary to be given below will realise that the definition of *separate* aggregates for savings and investment which can be *different* in intertemporal equilibrium is not possible without artificial conventions, in particular without a proportionality of gross out-of-equilibrium demands and endowments or some similar assumption. Garegnani assumes (b) that consumer demand is given in the form of demand functions which are not explicitly derived from utility functions of individual agents. This represents an important deviation from the usual modern representation of intertemporal equilibrium in which the microfoundation of the market process is based on individual decisions of independent agents. There is therefore, from the start, a conceptual difference between the representation of saving as a macroeconomic aggregate in Garegnani and the interplay of the consumption demands and the factor supplies based on the vast multiplicity of individual decisions in the writings of the progenitors of modern intertemporal theory or in textbooks. We shall propose a microeconomic foundation for the disequilibrium behaviour implied by Garegnani and compare it to Neokeynesian and Neowalrasian schemes of rationing; Garegnani's scheme appears as less well founded. The paper is difficult to read because (d) it provides an idiosyncratic synthesis of heterogeneous elements of different theoretical traditions; hence

the multiplicity of notes which are needed to explain why peculiar assumptions are made, and hence the roundabout character of the main text itself. Then there is a consistency problem in the representation of technology (e) in the form of a discrete spectrum of activities while the diagrams suggest that Garegnani really thinks of a model with continuous and differentiable substitution possibilities, for which he possibly has numerical examples in this drawer, to be presented in the future in a more complete exposition. There is a problem of consistency in the fact that Garegnani's savings and investment schedules are drawn as differentiable although the choice among discrete activities will in our interpretation of the system lead to discontinuities in those curves. The savings schedule is being drawn as positive throughout although savings, as defined in his paper, will be negative at low rates of interest. It is a major consistency problem (f) that the choice of techniques, the determination of prices of investment goods and the determination of activity levels are interdependent even outside equilibrium so that there is less freedom in the drawing of the schedules than Garegnani seems to believe.

2. It is obvious that the necessity of the introduction of the aggregates will be contested. Intelligent readers will not limit their defence to pointing out that intertemporal theory has thrived in textbooks and in advanced research for half a century without them. Rather, they will notice that Garegnani attempts here a rollback of what one might call the second neoclassical revolution in which intertemporal equilibrium emerged and became the dominant paradigm, superseding the older versions of neoclassical theory, based on the "quantity of capital". They will say that neoclassicals had their reason to move away from Wicksell, while Garegnani is telling them that Wicksell was right after all, if one wants to be a good neoclassical economist, – but then, he will continue telling them, neoclassical theory is wrong and the good neoclassical economists should see it.

Garegnani's main argument is based on the claim that the preferences of savers do not concern individual capital goods but capital in an abstract form, "a single commodity 'capital' " in Garegnani's phrase quoted above. From the point of view of savers, all capital goods are perfect substitutes and savers are indifferent between them, he argues. An answer to Garegnani might be that the complexity of economic reality requires use of different approaches, at different levels of abstraction. Saving in an uncertain world is the saving of money – unspent income that accumulates in monetary form to satisfy as yet unknown needs. Money may be saved in conditions where prices are not equilibrium prices, indeed in situations in which there are not even uniform prices for the same goods. Intertemporal equilibrium, by contrast, is concerned with the equlilibrium of demand and supply for commodities at different dates. Under the assumptions of this peculiar theory, I buy a refrigerator, to be obtained two months from now, not because I save today and decide in sixty days to purchase, but because I buy the promise of the delivery of the refrigerator today (Debreu 1959, p. 32). It amounts to the same thing if I commit myself to the purchase and pay later, at date *S* in what Debreu (1959, p. 33) called the price system at date *S*, and the producer invests as much as corresponds to the money which I have reserved for the purchase. There is no room for a *separate* decision for consumers to save in this conceptual framework. The producer who invests in the production of the refrigerator has already sold it before he commits his own or borrowed 'capital'; at any rate, the order has been placed. Uncertainty is taken into account only in a very specific form, as we shall see below (Debreu 1959, chapter 7). It is true that the consumer saves in that he does not consume all his income at once, and that the producer invests, but the market for dated goods provides a direct coordination of their decision because the consumer commits his 'savings' to definite acquisitions. Intertemporal theory describes a production to order, somewhat like that in medieval towns where the artisan manufactured a chest according to the specifications of the customer, and most of the price was paid in advance so that the craftsman could buy his materials. Medieval economies suffered from bad harvests and other calamities, but not from business cycles driven by discrepancies between savings and investment decisions.

Garegnani offers a quote from Keynes which illustrates the point: "If saving consisted not merely in abstaining from present consumption but in placing simultaneously a specific order for future consumption, the effect *might* indeed be different. For in that case ... the resources released from preparing for present consumption could be turned over to preparing for the future consumption" (Keynes 1936, 210–211; Garegnani 2003, note 551). Garegnani italicises the "might", in order to indicate that Keynes could have liberated himself further from traditional ways of thinking. A good new theory will be welcome, but here we are concerned with that analysis in which saving is in fact "placing a specific order for future consumption".

Since Garegnani is not so radical as to introduce saving under uncertainty in monetary form in his framework, saving in his account really only is, in equilibrium, a rearrangement of investment goods and, in disequilibrium, this is complemented by unplanned saving as endowments which are neither consumed nor invested because of false prices. Planned saving for an uncertain future is not part of the story. It cannot be denied that intertemporal equilibrium is capable of explaining an intertemporal allocation of resources. In order to demonstrate that a theory of saving must be introduced to understand this equilibrium better, Garegnani gets close to moving in a circle: A different economic tradition must be invoked in order to explain why the intertemporal theory has to be modified, and then this traditional theory emerges from the modification.

3. Neoclassical theorists therefore will feel that they are drawn to follow on a peculiar path with many windings – in order to reach which goal? In order to be told: Your theory is as erroneous as the old theory, because it really *is* the old theory. I fear that this dialectical procedure will not carry conviction for many.

Among Plato's dialogues, the classical models of dialectics, there are examples (e.g. *Gorgias*) showing that it is often helpful in discourse to convince an opponent of the necessity to change his argument by first correcting it, e.g. by saying: Your doctrine looks fine, but it contains a gap; fill the gap as follows and you will realise the error in your doctrine. But if the filling of the gap completely transforms the doctrine, the opponent will feel that it is not his own doctrine anymore which is being discussed, and he remains unconvinced.

In a nutshell: Garegnani says that Debreu cannot emancipate himself from Wicksell, hence Debreu's theory is as erroneous as Wicksell's. We say by contrast: Neoclassical theory is in fact held together by one common idea: to explain distribution in terms of demand and supply for factors of production (instead of analysing the forces which determine the distribution of a surplus). This explanation will run into difficulties whenever factor prices are not inversely related to the quantities of the corresponding factors, and this will happen in particular if the choice of technique involves reswitching and reverse capital deepening, but not all forms of neoclassical theory involve aggregates. There may be similar phenomena involving land and joint production even in the absence of durable capital goods. This multiplicity of phenomena makes it necessary to analyse different formulations of neoclassical theory separately and, where appropriate, to question them by means of *direct* critiques.

2. Garegnani's model

We now turn to the analysis of Garegnani's model. He advises his readers not to introduce a more modern notation too soon, but I need comparability with my own approach, and it is one of the advantages of vector notation to render obvious what happens when the dimension of the commodity space is increased.

Initial endowments, at the beginning of the period of production, are $\bar{\mathbf{q}} \geqq \mathbf{0}$, initial consumption is $\mathbf{c}^0 \geqq \mathbf{0}$, and consumption at the end of the period is $\mathbf{c}^1 \geqq \mathbf{0}$. Activity levels during the period of production are $\mathbf{q}^1$, gross outputs therefore also are $\mathbf{q}^1$ and $\mathbf{q}^1 \geqq \mathbf{c}^1$, and initial endowments are used for consumption or investment:

$$\bar{\mathbf{q}} \geqq \mathbf{c}^0 + \mathbf{q}^1\mathbf{A},$$

where we may replace $\mathbf{q}^1$ by $\mathbf{c}^1$, since both goods are capital goods as well as consumption goods so that production, whatever it is, will be consumed and $\mathbf{q}^1 = \mathbf{c}^1$. So far, the input-output matrix $\mathbf{A}$ is square, with only one process in each industry – the choice of techniques is considered only subsequently and is expressed by means of activity levels $\mathbf{q}^1$; $\mathbf{c}^1 = \mathbf{q}^1\mathbf{B}$ then is output at time $t = 1$, $\mathbf{B}$ being the output matrix for single product processes. The price vectors at the beginning and at the end of the period are $\mathbf{p}^0$ and $\mathbf{p}^1$ respectively and w is the wage paid at the end, all discounted to the present. Therefore

$$\mathbf{p}^1 \leqq \mathbf{A}\mathbf{p}^0 + w\mathbf{l},\ \mathbf{p}^0 \geqq \mathbf{0},\ \mathbf{p}^1 \geqq \mathbf{0},\ w \geqq \mathbf{0},$$

(insert $\mathbf{B}\mathbf{p}^1$, if there is a choice of techniques). The labour market is constrained by the availability of labour L, therefore

$$\mathbf{q}^1\mathbf{l} \leqq L.$$

Goods not used are free,

$$(\overline{\mathbf{q}} - \mathbf{c^0} - \mathbf{q^1 A})\mathbf{p^0} = 0,$$

but this condition is not really necessary since there will be no overproduction, all capital goods also being consumption goods.
We have already seen that the analogous condition

$$(\mathbf{q}^1\mathbf{B} - \mathbf{c}^1)\mathbf{p}^1 = 0$$

similarly is not really needed.
Unprofitable processes are not used:

$$\mathbf{q}^1(\mathbf{Bp}^1 - \mathbf{Ap}^0 - w\mathbf{l}) = 0.$$

This condition will play an important role as soon as alternative methods of production are available in at least one of the industries. Of immediate relevance is the condition

$$w(L - \mathbf{q}^1\mathbf{l}) = 0,$$

for labour may not be fully employed. The wage rate then is zero. The wage is to be interpreted as a surplus wage, and the subsistence for the workers employed is included among the means of production in matrix $\mathbf{A}$.

Garegnani's article is most opaque in his assumptions about consumption. He introduces consumption demand functions $\mathbf{c}^0 = \mathbf{c}^0(\mathbf{p}^0, \mathbf{p}^1, w)$ and $\mathbf{c}^1 = \mathbf{c}^1(\mathbf{p}^0, \mathbf{p}^1, w)$, but the reader is only gradually being told what their properties are; they emerge fully only in the Appendix I, while $\mathbf{c}^0$ and $\mathbf{c}^1$ curiously are not even treated as unknowns the first system presented (E) although it is clear from the Mathematical Note by M. Tucci in Garegnani (2003) that the amounts consumed are the values of "standard Walrasian demand functions" assumed at the equilibrium prices.

The reader presumably is expected to know that consumption demand functions are usually derived in intertemporal general equilibrium theory from the utility functions of consumers, using their intertemporal budget equations. For most of what follows I shall test Garegnani's results by assuming that there is only one consumer, with a utility function $U(\mathbf{c}^0, \mathbf{c}^1)$ which is strictly concave, and the intertemporal budget equation is

$$\overline{\mathbf{q}}\mathbf{p}^0 + wL = \mathbf{c}^0\mathbf{p}^0 + \mathbf{c}^1\mathbf{p}^1;$$

consumption demand functions follow from this. The introduction of utility maximisation under the simplest possible hypothesis will help us to determine in what sense Garegnani's hypotheses are compatible with neoclassical assumptions about 'rational' agents. It is well known that these equations define a unique optimum (where utility is maximised under the condition that the quantity constraints are fulfilled) and a unique equilibrium, equal to the optimum, where the price and the quantity relations are observed and where utility is maximised subject to the budget constraint. It is possible to construct intertemporal equilibria in such a way that they are steady states, either by choosing a suitable utility function (Schefold 1997, chapter 18.2) or a suitable vector of endowments (Schefold 2003).

The equation for the simplest case, leaving aside technical choice and omitting non-negativity conditions for brevity, now are:

$$\overline{\mathbf{q}} = \mathbf{c}^0 + \mathbf{c}^1\mathbf{A}, \quad (1)$$

$$\mathbf{p}^1 = \mathbf{A}\mathbf{p}^0 + w\mathbf{l}, \quad (2)$$

$\mathbf{c}^1\mathbf{l} \le L$, with (3)

$$w(L - \mathbf{c}^1\mathbf{l}) = 0, \quad (3a)$$

$$(\mathbf{c}^0, \mathbf{c}^1) = \arg\max U(\breve{\mathbf{c}}^0, \breve{\mathbf{c}}^1) \quad (4)$$

$$s.t.\ \overline{\mathbf{q}}\mathbf{p}^0 + wL = \breve{\mathbf{c}}^0\mathbf{p}^0 + \breve{\mathbf{c}}^1\mathbf{p}^1. \quad (4a)$$

It is useful to have all equations assembled in one table (table 1) which are necessary to define the variants of this system – variants which will be explained later:

<table>
<tr><td>Garegnani-semiequilibrium</td><td>Clower-semiequilibrium</td></tr>
<tr><td colspan="2">$\mathbf{q}^* = \mathbf{c}^0 + \mathbf{c}^1\mathbf{A}$ (1')
$\mathbf{p}^1 = \mathbf{A}\mathbf{p}^0 + w\mathbf{l}$ (2)
$\mathbf{c}^1\mathbf{l} \le L$ (3)
$w(L - \mathbf{c}^1\mathbf{l}) = 0$ (3a)</td></tr>
<tr><td>$(\mathbf{c}^0, \mathbf{c}^{1*}) = arg\ max\ U(\breve{\mathbf{c}}^0, \breve{\mathbf{c}}^1)$ (4')
$s.t.\ \overline{\mathbf{q}}\mathbf{p}^0 + wL = \breve{\mathbf{c}}^0\mathbf{p}^0 + \breve{\mathbf{c}}^1\mathbf{p}^1$ (4a)
$(\mathbf{c}^{0*}, \mathbf{c}^1) = arg\ max\ U(\mathbf{c}^0, \breve{\mathbf{c}}^1)$ (4")
$s.t.\ \mathbf{c}^1\mathbf{A}\mathbf{p}^0 + wL = \breve{\mathbf{c}}^1\mathbf{p}^1$ (4a")</td><td>$(\mathbf{c}^0, \mathbf{c}^1) = arg\ max\ U(\breve{\mathbf{c}}^0, \breve{\mathbf{c}}^1)$ (4)
$s.t.\ \mathbf{q}^*\mathbf{p}^0 + wL = \breve{\mathbf{c}}^0\mathbf{p}^0 + \breve{\mathbf{c}}^1\mathbf{p}^1$ (4a')</td></tr>
<tr><td colspan="2">$\mathbf{I} = \mathbf{c}^1\mathbf{A}\mathbf{p}^0$ (5a)
$\mathbf{S} = (\overline{\mathbf{q}} - \mathbf{c}^0)\mathbf{p}^0$ (5b)
$\mathbf{q}^* = \xi\overline{\mathbf{q}}$ (6)
$r_2 = p_2^0 / p_2^1 - 1$ (7)</td></tr>
</table>

Table 1: The formulas which define the Garegnani-semiequilibrium GSE and the Clower-semiequilibrium CSE. Full equilibrium is here given by (1'), (2), (3), (3a), (4), (4a) and (6) with $\xi = 1$.

If (1) - (4a) hold, we shall speak of a full equilibrium. (1) must hold with equality since Garegnani assume the goods to be both capital and consumption goods (Garegnani 2003, par. 3) – utility can be increased as long as there are unconsumed amounts of endowments according to (4) with (4a). The (apart from the choice of the numéraire) unique solution to (1) -(4a) will be denoted by $\bar{\mathbf{c}}^0, \bar{\mathbf{c}}^1, \bar{\mathbf{p}}^0, \bar{\mathbf{p}}^1, \bar{\mathbf{w}}$.

One of my aims in confronting Garegnani's interpretation of intertemporal equilibrium with what I take to represent the original intertemporal approach is to identify which elements of Garegnani's critique have to be ascribed to problems of the choice of technique and of the theory of capital and which derive from assumptions about consumption. It is well-known that the problem of multiplicities of equilibria and their stability exists in pure exchange economies. I agree with Garegnani about the importance of the stability problems due to the structure of production, at least in principle, but the different sources of instability have to be kept separate as far as possible. Unfortunately, Garegnani's assumptions about consumption are strewn like strips of paper in a forest where a paper chase has taken place; rather than following the path with all its windings thus indicated, I prefer to look down at the landscape from some convenient vantage point; this is provided by the one-consumer model.

The intertemporal model, as written above, contains interest rates. If a numéraire $\mathbf{s}$ is given, $r_s = (\mathbf{s}\mathbf{p}^0 / \mathbf{s}\mathbf{p}^1) - 1$ is the own rate of interest, and if discounted prices $\mathbf{p}^t$ are converted into undiscounted prices defined by $\bar{\mathbf{p}}^t = \mathbf{p}^t / \mathbf{s}\mathbf{p}^t$, $\bar{w} = w / \mathbf{s}\mathbf{p}^1$, we obtain

$$\bar{\mathbf{p}}^1 = (1 + r_s)\mathbf{A}\bar{\mathbf{p}}^0 + \bar{w}\mathbf{l}.$$

The allocation of goods is determined by relative prices, and it has therefore always been recognised that interest rates, representing specific relative prices, are relevant for the intertemporal allocation of goods. The controversial question is whether aggregates of saving and investment are also relevant for the understanding of this allocation. Neither saving nor investment are autonomous forces in the usual understanding of intertemporal equilibrium. It is certainly possible to calculate for a given allocation how much is being saved by income receivers in each period, it is possible to calculate the value of goods not consumed but allocated for production in each period, and both aggregates must be equal period per period; what is proved in elementary national accounting for actual economies must hold in general equilibrium as well, and Garegnani derives the corresponding equations which are also to be found in macroeconomic textbooks with microeconomic foundations (e.g. Malinvaud 1983, vol. I, p. 52). But do these aggregates play separate causal roles in an intertemporal equilibrium (as opposed to temporal equilibria) with flexible prices (without rigidities, with perfect competition), with rational agents and with perfect foresight? Since saving and investment coincide in full equilibrium, Garegnani tries to show that their discrepancy is essential for understanding if and why there is a tendency to equilibrium.

This is not the direction in which mainstream economics has moved in recent decades. Many controversies about Keynesian macroeconomics were concerned with the problem of finding the appropriate microeconomic foundations for macroeconomics. These debates sprang from the neoclassical belief that, since the intertemporal allocation of resources can be based on individual decisions about present and future consumption in intertemporal equilibrium, aggregate behaviour must be reducible to individual behaviour, and individual acts of saving are not simply indefinite acts of not spending but represent commitments to buy in the future. The obvious objection is to point out that the future is uncertain. The answer of the proponents of intertemporal theory was to make decisions to buy in the future contingent upon future states of nature (Debreu 1959, chapter 7). They thus moved away from the Keynesian concept of uncertainty and introduced a different theory of it, which is coherent, however artificial it may appear to be in the Keynesian perspective. If I save 5.000 Euro for 3 years in order to buy a car for 15.000 provided I shall then still be able to run it, the model represents this decision as the purchase of a car, to be delivered 3 years from now on condition that I pay 5.000 Euro annually and contingent on my health being sufficiently good to warrant the execution of the purchase – if not, i.e. if another 'event' takes place, the contract provides for the corresponding alternative; possibly no purchase at all (Debreu 1959, p. 95).

Innumerable economists have complained about the lack of realism of this representation of the world, by pointing out that the relevant forward markets are lacking and that the future states of nature cannot be enumerated. The analysis of incomplete contracts has become a special discipline. But the idea of intertemporal equilibrium is both daring and consistent. It has roots in the 'old' neoclassical theory, in particular in Böhm-Bawerk. He opposed the idea of deriving interest from the analysis of intertemporal exchange to the notions which attributed interest to the productivity of capital or interpreted it as the price for letting durable capital goods. (Böhm-Bawerk 1921, vol I., chapter VIII). He interpreted uncertainty as risk (Böhm-Bawerk 1921, vol. II, book IV, chapter I).

The discrepancy of saving and investment in Garegnani's model thus is a construct: the aggregate result of disequilibria in markets for goods and factors. The neoclassical economist hopes that equilibrium will be achieved by equilibrating forces operating in all individual markets. But Garegnani's rollback of the intertemporal revolution involves the idea that the aggregates of saving and investment, each considered as a function of the rate of interest, represent the decisive equilibrating forces.

The procedure involves an important deviation from the ordinary model; if saving and investment are causal determinants of the equilibrium, there must be a market for them such that savings and investment functions, dependent on some rate of interest, intersect in equilibrium. Because of Walras' law, it is not sufficient simply to assume that one market is in disequilibrium, since at least one other market then also is in disequilibrium. Garegnani here takes up one of his early ideas: a constraint in the model is relaxed by treating one of the endowments as a variable, and this is made dependent on the equilibrium value of one of the other variables of the model, which can now be varied parametrically. The approach bears a similarity to Clower's (1969) dual decision hypothesis.[2] As a simple example, consider a general equilibrium model

[2] Garegnani (1964/1965) proposed, starting from J.B. Clark, Böhm-Bawerk and Wicksell, to construct a "funzione di domanda", which he summarised as follows: "In breve, alle normale equazioni dell'equilibrio generale, si sottrae l'equazione relativa all' ugualianza

involving labour as e.g. in the model above. Treat the labour endowment L as a variable. The equilibrium equations may be solved (in the case of our model with one consumer uniquely) for each value of the endowment. There will therefore also be an equilibrium value for the real wage for each level of the labour endowment. The schedule of the real wage $w/\mathbf{sp}^1$ associated with each level of the labour endowment L then is an equilibrium trajectory relating potential levels of the labour supply with the real wage rate. But we may also trace the relationship between this real wage and the amount of labour which will be employed in each equilibrium $\mathbf{q}^1\mathbf{l} = L$ and interpret the equilibrium trajectory as the demand for labour $L_D = L_D(w/\mathbf{sp}^1)$; this schedule, confronted with any actual supply L_S as a fixed vertical line, could, at the intersection, represent an equilibrium of demand and supply for labour.

This application of Garegnani's method to the labour market appeals to intuition insofar as it often seems to be the case that a considerable disequilibrium (unemployment or over-employment) is found in the labour market with the wage rate remaining rigid, while all other markets are near equilibrium (there are reasons why wages are more sticky than prices of goods).

Analytically, equations (1) - (4a) of full equilibrium remain unchanged, except in that L is to be replaced by L_D in equations (3), (4a) and also in (3a). The real wage in terms of good i at data t, $\tilde{w} = w/p_i^t$, then is a function of parameter L_D. By inverting this relationship (where that is possible), one obtains $L_D(\tilde{w})$ which may be confronted with a labour supply L_s. A solution to (1) - (4a) with this modification will be called semiequilibrium. To have disequilibrium thus confined to the labour market is a hallmark of Keynesian theory, according to Clower (1969), but whether the analytical structure of the semiequilibrium really contains the explanation of unemployment envisaged by Keynes has remained controversial.

It seems logical that only labour demanded appears in the budget equation (4a), insofar as the unemployed can have purchasing power only as owners of other resources (endowments). Labour thus is 'rationed' in the labour market - only part of the labour force gets employment and demands goods out of wage income. The 'effective' demand for goods is what results from (4) and (4a), with L_D replacing L in the budget equations. Demand would have been higher, if the entire labour force L_s could exert demand at the given wage rate and prices: this is 'notional' demand (formally what results from (4) and (4a) with L_s replacing L). Textbooks, following Bénassy, associate the rationing scheme and the effective demand so defined with the name of Clower (Felderer-Homburg 1987), while another rationing scheme, applicable in other

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trà quantità impiegata e disponibile del capitale, e il grado di liberta acquisito al sistema permette di definire il modo in cui la quantità di capitale impiegata varia al variare del saggio dell' interesse" (Garegnani 1964/1965, p. 25, note 2). It should be noted that this early version of Garegnani's construction refers to the 'old' neoclassical economists who supposed that the endowments of capital goods had adapted so as to permit the formation a uniform rate of profit. I should like to thank P. Garegnani for having mentioned Clower in a discussion of an earlier version of this paper. The similarity (as I interpret it) will be worked out below.

circumstances (e.g. if it is costly to maintain a supply which then turns out to be excessive) is associated with the name of Drèze. Here, effective supply adapts to effective demand (in our example: L_s reduces to L_D – the rate of participation adapts to the state of employment).

The Neowalrasian/Neokeynesian Schools have produced a variety of rationing schemes and of explanations of why and how rationing occurs: the administrative rationing of goods by the State in war time (everybody gets proportionately less goods) is strikingly different from that in the labour market (most people get work, some do not). Garegnani's procedure is equivalent to Clower's as long as effective demand is smaller than notional demand. But if notional demand for labour exceeds the available supply, Clower reduces the effective demand for labour by definition to the supply and the demand for goods emanating from the labour market corresponds to that engendered by full employment (as in Keynes who thought that the traditional theory came into its own at full employment). Garegnani, by contrast, treats excess demand symmetrically to deficient demand.

Disequilibrium (deficient demand or excess demand) is confined to one market in the Clower-Garegnani scheme, as we now show. The budget equation (4a), modified according to Garegnani (L replaced by L_D), combined with (2), yields

$$\overline{\mathbf{q}}\mathbf{p}^0 + wL_D = \mathbf{c}^0\mathbf{p}^0 + \mathbf{c}^1\mathbf{p}^1 = \mathbf{c}^0\mathbf{p}^0 + \mathbf{c}^1\mathbf{A}\mathbf{p}^0 + w\mathbf{c}^1\mathbf{l},$$

hence

$$(\overline{\mathbf{q}} - \mathbf{c}^0 - \mathbf{c}^1\mathbf{A})\mathbf{p}^0 + (L_D - \mathbf{c}^1\mathbf{l})w = 0. \qquad (4b)$$

This relation looks like Walras' law. It holds – as long as prices $\mathbf{p}^1$ depend on $\mathbf{p}^0$ and w according to (2) – not only in full or semiequilibrium. (4b) indicates that a disequilibrium in the labour market $(L_s > L_D = \mathbf{c}^1\mathbf{l})$ is compatible with an equilibrium in the goods markets at time $t = 0$, where we have $(\overline{\mathbf{q}} - \mathbf{c}^0 - \mathbf{c}^1\mathbf{A})\mathbf{p}^0 = 0$, and also at $t = 1$, for $\mathbf{c}^1 = \mathbf{q}^1$, the equality of consumption at $t = 1$ with total production, was for simplicity already expressed in (1) and (3). Walras' law proper, by contrast, with L_s replacing L_D in equation (4b), shows that a disequilibrium in the labour market at a positive wage, coupled with an equilibrium at $t = 1$ because of $\mathbf{q}^1 = \mathbf{c}^1$, implies a second disequilibrium at $t = 0$.

The Neowalrasian/Neokeynesian School – for which Malinvaud (1977) once was a central reference[3] – most often works with definitions of effective demand such that there are connected disequilibria and rationing in several markets, and the nature of the disequilibria and rationing can be different. There is e.g. 'classical unemployment' (wages are too high in the labour market) or 'Keynesian unemployment' (demand for goods produced is too low at high prices because of low purchasing power of government expenditure fixed in monetary terms) and prices are not equilibrium prices because of imperfect

3 For a Postkeynesian critique of Malinvaud see Kahn (1977) and Schefold (1983).

competition or simply because there is no auctioneer and equilibrium prices have not been been found.

Garegnani, like Clower, defines semiequilibria in which the disequilibrium is confined to one market, and it is caused by 'false' prices. If we compare to Malinvaud (1977) and use his terminology, the disequilibria are classical rather than Keynesian. In the present example, they depend on the real wage, not on effective demand for autonomous investment. The investment introduced by Garegnani is not autonomous and depends, as we shall see, on the prices guiding the choice of future consumption. There are here no money, government expenditure or uncertainty, hence no 'Keynesian' unemployment in the sense of Malinvaud.

Equilibrium trajectories such as $L_D(\tilde{w})$ and L_s, compared to ordinary demand and supply curves, could be said to have the advantage of incorporating the total of the reactions of the economy in all other markets for any given state of disequilibrium, whereas ordinary demand and supply curves cannot be constructed without arbitrary assumptions as to the states of the other markets and the prices formed there which are needed to calculate the curves; a disequilibrium in an actual market, accompanied by either a shortage and rationing or by excess supply, characteristically shows spillover effects in other markets. However, normal slopes of demand and supply curves (supply curves rising, demand curves falling) are primarily expected to result from the behaviour of the agents in the market concerned, if the prices formed in other markets can be regarded as given. If an interdependence with other markets is taken into account, the likelihood that slopes will be normal diminishes, and it is no surprise that equilibrium trajectories of the kind constructed by Garegnani are not monotonic functions.

Garegnani uses the equilibrium trajectories to analyse the stability of general equilibrium: the deviation from equilibrium is as it were projected into a single market, and he seems to believe that an instability thus found must be indicative of an instability of the system as a whole. While similar procedures have been used by others, I am not aware that his conclusions regarding stability are a commonly accepted proposition of neoclassical theory, and it is certainly not generally plausible. A mechanical system which moves with many degrees of freedom close to equilibrium may well be stable only if there is an equilibrating mechanism for each degree of freedom and unstable if the disequilibrating forces are constrained so as to engender one large disequilbrium of the variables in one degree of freedom while the others are fixed at the equilibrium values. One may think of a tandem with two men riding it as an analogue. It will be easier to drive if both riders are allowed to move than if one is replaced by a stiff puppet as heavy as a man and the other man has to stabilise for two.

3. Garegnani's semiequilibrium

The construction of equilibrium trajectories, as proposed by Garegnani, is more complicated in the case of the market for savings and investment (which are aggregates of goods, valued at their prices, in his definition) than for the labour market (where labour is a factor, measured in terms of natural units). These magnitudes have to be defined and a rate of interest has to be chosen. But which, since own rates are different? Garegnani's choice amounts to putting $\mathbf{s} = (0,1) = \mathbf{e}_2$ (second unit vector).

Garegnani defines investments I and savings S as follows:

$$I = \mathbf{q}^1\mathbf{A}\mathbf{p}^0 \tag{5a}$$

$$S = (\overline{\mathbf{q}} - \mathbf{c}^0)\mathbf{p}^0 . \tag{5b}$$

Investment is the value of the amount of capital goods needed at activity levels $\mathbf{q}^1$ "today" to produce consumption at activity levels $\mathbf{c}^1 = \mathbf{q}^1$ for "tomorrow", and savings is the amount of endowments not consumed "today". This concept of saving embodies no concept of uncertainty, neither that of the proponents of modern intertemporal general equilibrium theory, nor the concept linked to monetary theory of the Keynesians.

Garegnani thus introduces *two* new variables which are well defined and equal $(I = S)$ for every full equilibrium (1) - (4a); the equality follows from multiplying (1) by $\mathbf{p}^0$. In order to portray a disequilibrium, he relaxes condition (1), but not completely, as follows: the assumptions is that endowments $\overline{\mathbf{q}}$ and the demand for endowments denoted by

$$\mathbf{q}^* = \mathbf{c}^0 + \mathbf{c}^1\mathbf{A} \tag{1'}$$

remain proportional in disequilibrium. Hence there is always a positive multiplier ξ such that

$$\mathbf{q}^* = \xi\overline{\mathbf{q}} .^4 \tag{6}$$

It is then proposed to let the own rate of interest, r_s (Garegnani takes that of the second commodity, hence

$$1 + r_2 = p_2^0 / p_2^1) \tag{7}$$

vary as the independent parameter solving equations (1'), (2), (3), (3a), (4), (4a), (6) and (7); and the unknowns of the system, $\mathbf{c}^0, \mathbf{c}^1, \mathbf{p}^0, \mathbf{p}^1, w, \xi$, together with I and S according to (5a), (5b) become functions of r_s.

This system as it stands, however, is inconsistent except in full equilibrium, with $r_s = \bar{r}_s$, where $\bar{r}_s$ is the unique equilibrium interest rate of the original system (1) - (4a). There are no disequilibrium solutions. This may be seen in various ways.

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4 Garegnani writes this condition in a slightly different form, applicable for $n = 2$ (n is the number of commodities) in formula (7f) in Garegnani (2000, p. 403) and (5.7f) in Garegnani (2003, p. 123); both versions contain the same misprint (a missing fractional line).

One possibility is to start from the observation that we have not yet introduced a rationing scheme which would allow to bypass Walras' law. Equation (3a) implies that the labour market is in equilibrium, either at full employment or at unemployment with $w = 0$. The markets for goods at $t = 1$ are in equilibrium because we assumed $\mathbf{q}^1 = \mathbf{c}^1$. The only possibility which remains would consist in simultaneous disequilibria in the two markets for goods at $t = 0$: the disequilibria would have to be such that the two excess demands were of opposite sign and such that their sum would be equal to zero, in accordance with Walras' law according to which the sum of all excess demands must be equal to zero. But, because of (6), the excess demands must be of the same sign, hence they must be equal to zero; therefore, (1'), (2), ..., (7) admits only of full equilibrium solutions.

It is also possible to verify this assertion by starting from the budget equation (4a). It might be thought that (4a) holds only because I have introduced demand by assuming only one consumer, maximising his utility subject to (4a) while Garegnani uses Walrasian demand functions. But if there are several utility maximising consumers, their individual budget equations must add up to (4a) so that (4a) holds in any case. (4a) implies with (2) and (5a, b)

$$\overline{\mathbf{q}}\mathbf{p}^0 + wL = \mathbf{c}^0\mathbf{p}^0 + \mathbf{c}^1\mathbf{p}^1 = \mathbf{c}^0\mathbf{p}^0 + \mathbf{c}^1\mathbf{A}\mathbf{p}^0 + w\mathbf{c}^1\mathbf{l},$$

hence, using (3a),

$$S - I = w(\mathbf{c}^1\mathbf{l} - L) = 0\,;$$

the labour market equilibrium implies that of saving and investment, and from there one can conclude that there must be full equilibrium as above.

In order to permit a disequilibrium to occur, and to confine it to one market, that for S and I, Garegnani resorts to peculiar assumptions which are explained in par. 9 and note 24 in Garegnani 2003 (par. 9 and note 19 in Garegnani 2000).

These assumptions must (and do) imply a modification of Walras' law and of the budget equation (4a). In order to understand them, it is necessary to understand first that the confinement of the disequilibrium to I and S is only a *façon de parler*: It is obvious from (1a) and (6) that the disequilibrium, if it exists with $I \neq S$, will be one involving both goods at $t = 1$, for $S \neq I$ implies

$$0 \neq S - I = (\overline{\mathbf{q}} - \mathbf{c}^0 - \mathbf{c}\mathbf{A})\mathbf{p}^0 = (\overline{\mathbf{q}} - \mathbf{q}^*)\mathbf{p}^0 = (1-\xi)\overline{\mathbf{q}}\mathbf{p}^0,$$

hence $\xi \neq 1$, which expresses the fact that we have either excess saving ($S > I$ and $\xi < 1$) or excess investment ($S < I, \xi > 1$). The intended disequilibrium of S and I therefore really means that goods at $t = 0$ remain unsold ($S > I,\ \xi < 1$) or are in excess demand ($S < I, \xi > 1$), not because there is saving/dissaving motivated by uncertainties, but because there is a deficient demand/excess demand for goods at $t = 0$, caused by diseqilibrium prices, and this may be caused by demand for present goods $\mathbf{c}^0$ or for investment $\mathbf{c}^1\mathbf{A}$ or both being low or high. Since the labour market is in equilibrium anyway, the isolated diseqilibrium with $S \neq I$ can be permitted only to happen by preventing a spillover to the market for goods at $t = 1$, and here a kind of dual decision hypothesis must be introduced.

The spillover was avoided in the previous example of the labour market assuming that only labour employed was able effectively to demand goods and consequently only L_D entered the budget equation. Here, the problem looks more complicated because the spillover is between periods. We analyse it by looking at the balance of proceeds and expenditure in $t = 0$ and $t = 1$ separately, assuming that the consumer cannot shift purchasing power between periods). In equilibrium, consumption at $t = 0$ can be bought out of the proceeds of selling the endowments, after deducing the cost of investment (wages become available only at $t = 1$). The generalised formula, reflecting the possibility of disequilibrium, is

$$\mathbf{c}^0\mathbf{p}^0 = \bar{\mathbf{q}}\mathbf{p} - \mathbf{c}^1\mathbf{A}\mathbf{p}^0 - \delta^0 .$$

We have deduced δ^0 for the value of unsold endowments (or excess demands for endowments, if $-\delta^0 > 0$) in order to account for effective demand in disequilibrium. Consumption in period one can be bought out of wages (paid at the end of the production period) and gross revenue of the investment goods industry, plus, in disequilibrium, if the value of unsold equipment/excess demand for equipment δ^0 can be realised in $t = 1$ as δ^1 :

$$\mathbf{c}^1\mathbf{p}^1 = wL + \mathbf{c}^1\mathbf{A}\mathbf{p}^0 + \delta^1 .$$

All magnitudes are discounted to $t = 0$ (it is a useful exercise, left to the reader, to follow Malinvaud and to express these relationships in undiscounted prices in order to separate the periods even more clearly). If the intertemporal budget equation holds, it is equal to the sum of these to equations, with $\delta^1 = \delta^0$ and $-\mathbf{c}^1\mathbf{A}\mathbf{p}^0$ and $\mathbf{c}^1\mathbf{A}\mathbf{p}^0$ cancelling each other. Then, if Walras' law holds, purchasing power *is* shifted between periods and the deficient demand/excess demand of time 0 is turned into excess demand/deficient demand at time 1. This must be avoided, if the disequilibrium is to happen at $t = 0$ only, hence effective demand for $\mathbf{c}^1$ is defined by setting $\delta^1 = 0$. There results a new budget equation for purchases in $t = 1$

$$\mathbf{c}^1\mathbf{p}^1 = wL + I ,$$

where I is the investment $\mathbf{c}^1\mathbf{A}\mathbf{p}^0$ in semiequilibrium. This corresponds to equation (5.8c) in Garegnani (2003, p. 162).

Garegnani justifies his rationing scheme by saying: " ... households failing to sell part of their ... resources because of excess savings can hardly exert demand on the commodities of $t = 1$" (Garegnani 2003, par. 9).

We now represent Garegnani's semiequilibrium in full on the assumption that there is only one consumer. We assume that there is a unique solution for each given level $\tilde{r}_s$ of the rate of interest r_s within a finite range between zero and a certain maximum (Garegnani 2003, p. 174); the solutions are denoted by $\tilde{\mathbf{c}}^0, \tilde{\mathbf{c}}^1, \tilde{\mathbf{p}}^0, \tilde{\mathbf{p}}^1, \tilde{w}, \tilde{\xi}, \tilde{S}, \tilde{I}$. The solutions must fulfil (1'), (2), (3), (3a), (5a), (5b), (6), (7), and the equilibrium values of consumption are obtained in simultaneous determination from the following problems of maximisation:

$$(\tilde{\mathbf{c}}^0, \mathbf{c}^{1*}) = arg\,max\,U(\mathbf{c}^0, \mathbf{c}^1) \quad (4)$$

subject to

$$\mathbf{c}^0\mathbf{p}^0 + \mathbf{c}^1\mathbf{p}^1 = \overline{\mathbf{q}}\mathbf{p}^0 + wL \quad (4a)$$

and

$$(\mathbf{c}^{0*}, \tilde{\mathbf{c}}^1) = arg\,max\,U(\tilde{\mathbf{c}}^0, \mathbf{c}^1), \quad (4")$$

subject to

$$\mathbf{c}^1\mathbf{p}^1 = wL + \tilde{\mathbf{c}}^1\mathbf{A}\mathbf{p}^0. \quad (4a")$$

Note that the demand $\tilde{\mathbf{c}}^0$ resulting from (4') and (4a) – which have remained unchanged – is both notional and effective, if $\mathbf{p}^0 = \tilde{\mathbf{p}}^0, \mathbf{p}^1 = \tilde{\mathbf{p}}^1, w = \tilde{w}$. Demand $\tilde{\mathbf{c}}^1$ as resulting from (4"), (4a") then is effective, $\mathbf{c}^{0*} = \tilde{\mathbf{c}}^0$, while $\mathbf{c}^{1*}$ resulting from (4'), (4a) is only notional.

Garegnani's semiequilibrium is not easy to understand in the form in which he presents it, using given demand functions, and it is not easy to analyse in the form into which we have brought it, after the explicit introduction of utility maximisation. The existence proof by Tucci in Garegnani (2000) or (2003) is incomplete in that it is not shown constructively which assumptions about utility and the distribution of wealth among them have to be made in order to justify the series of assumptions about consumption demand functions which are used in the attempted proof. Moreover, Garegnani uses a drastically simplified form of the dual decision hypothesis by defining the effective demand at $t = 1$, $\mathbf{c}^1$, as proportionately reduced from $\mathbf{c}^{1*}$ in his (unnumbered) second but last equation of note 24 in Garegnani (2003). He puts

$$\mathbf{c}^1 = [(wL + I)/(wl + S)]\mathbf{c}^{1*}. \quad (8)$$

This definition leads to the desired macroeconomic adjustment, in that (4a") will be fulfilled. The spillover of the disequilibrium at $t = 0$ to $t = 1$ is avoided at the aggregate level, but I see no reason why $\mathbf{c}^1$ should be proportional to $\mathbf{c}^{1*}$; there remains a microeconomic disequilibrium in $t = 1$, if Garegnani's definition is adopted, which is avoided in (4"), (4a") here.

Turning from formal aspects to the doctrine, we may ask how Garegnani's semiequilibrium compares to the Neowalrasian/Neokeynesian equilibria. The construction is quite original and sophisticated in its own way, but Garegnani's critique can then not be justified as the *reductio ad absurdum* of a pre-existing model. The question therefore must be posed whether this particular rationing scheme is plausible – perhaps more plausible than others.

We here accept the idea of concentrating the disequilibrium phenomena in one market, although it is an open question how the stability results derived on this basis relate to those derived from assumptions more frequently made, such as a simultaneous *tâtonnement*. The crucial question then concerns the rationing scheme. Clower's assumption is convincing, if applied to the labour market, and for a reason advanced long ago: if there are capitalists owning the means of production, they will not employ more than is profitable at the ruling wage rate, which is sticky, and the unemployed cannot demand goods effectively because they cannot employ themselves. Garegnani's rationing scheme leads to an amount of endowments $\overline{\mathbf{q}}-\mathbf{q}^{*}=(1-\xi)\overline{\mathbf{q}}$ remaining unsold (for simplicity we consider only the case $\xi<1$). But this is in contrast which the assumptions that all goods are both capital goods and consumption goods and that decisions to maximise utility can be revised: The utility of the consumer(s) will be increased beyond the level attained in equilibrium, if they consume $\overline{\mathbf{q}}-\mathbf{q}^{*}$; for one consumer the difference is $U(\tilde{\mathbf{c}}^{0}+\overline{\mathbf{q}}-\mathbf{q}^{*},\tilde{\mathbf{c}}^{1})-U(\tilde{\mathbf{c}}^{0},\tilde{\mathbf{c}}^{1})$. As the model stands, the consumer(s) appear(s) to be irrational. Most recent attempts to provide microfoundations for Keynesian macroeconomics (cf. e.g. Malinvaud 1983) do not depart from the assumption of rational consumers who are informed about the amounts of their resources, although they do not know their prices, prior to the establishment of equilibrium. There is no full coordination in a temporary equilibrium, e.g. because of rigid prices, so that not all decisions are compatible *ex ante*. But even in disequilibrium "chaque agent sait qu'il ne saurait échapper à son équation budgétaire ... S'il est raisonnable, cet agent ... retient sont équation budgétaire et ... l'équilibre de ses différents comptes" (Malinvaud, 1983, vol. I, p. 43).

Garegnani himself admits that it "would then seem natural to suppose an 'initial' reaction in the markets" for endowments "more directly affected by the disequilibrium, which would occur *before* adjustments can take place in connected markets" (Garegnani 2003, par. 17).

The inaction of the consumers in the market at $t=0$ is crucial for Garegnani's stabilisation which centres around I and S, each regarded as a function of the rate of interest. An additional assumption is needed to justify the unsold stocks. In order to get on with the discussion, we shall simply suppose that endowments are like capital goods insofar as they can be consumed only after having been sold. As for own consumption or if there is if there is only one consumer: the person as an owner must sell to the same person as a consumer. The consumer, however, cannot buy before the owner has obtained the necessary income. The disequilibrium then persists: A cannot buy from B because B has not bought from A and vice versa. Such a lock-in among different persons is familiar: not being able to buy and not being able to sell coexist in crises. The lock-in is less plausible if the prices are flexible and if credit is avoidable to make the first step, and since wages are for good reasons less flexible than prices, rationing in the labour market is more plausible than rationing in the market for endowments. The argument so far has been confined to the market at $t=0$, a spillover to $t=1$ having been ruled out by Garegnani's dual decision hypothesis. Stabilisation is achieved by means of *tâtonnement* in the market for $I(r_s)$ and $S(r_s)$, and all other markets are thought to adjust during each step of the process.

4. A simplified model

It would now be desirable to provide a precise mathematical analysis of Garegnani's model in order to verify his results, in particular regarding the multiplicity and stability of equilibria and the shapes of the $I-S$-curves in his diagrams. In order to simplify the task, I propose a rationing scheme which is not less plausible than Garegnani's and easier to handle.

We propose to consider the system (1'), (2), (3), (3a), (6),(7) and (4), but to change the budget equation (4a) and to replace it by

$$\mathbf{q}^*\mathbf{p}^0 + wL = \mathbf{c}^0\mathbf{p}^0 + \mathbf{c}^1\mathbf{p}^1. \qquad (4a')$$

The budget consists of wages (which may be zero) and of the value of the *hypothetical* endowments $\mathbf{q}^*$ which are equal to demand and therefore are sold at time 0.

The point is that this rationing scheme is strictly analogous to the most successful rationing scheme we have, that of Garegnani *and* Clower for the labour market. Total endowments $\overline{\mathbf{q}}$ correspond to the labour supply L_s. At false prices, only $\mathbf{q}^* = \mathbf{c}^0 + \mathbf{c}^1\mathbf{A}$ as part of the endowments are demanded, hence no more income than $\mathbf{q}^*\mathbf{p}^0$ is derived from selling the endowments and can be turned into effective demand, hence $\mathbf{q}^*\mathbf{p}^0$ must enter the budget equation in the same way as L_D enters it in the case of the labour market. The reasoning why $\overline{\mathbf{q}} - \mathbf{q}^*$ remain unsold (again, only the case of deficient demand is discussed for brevity) is the same as in Garegnani's construction: a lock-in prevents owners and consumers from revising their purchases and increasing their utility.

With (1) and (4a) left aside, the endowment appears only in the definition of S and in (6). Substituting (6) in the budget equation and in (1a) yields a system which, for every ξ given, is formally equivalent to our original system (1), (2), (3), (3a), (4), (4a), with $\overline{\mathbf{q}}$ replaced by $\xi\overline{\mathbf{q}}$. We conclude that, for every ξ given, there will then exist a unique solution which is an optimum and formally an equilibrium (Schefold 1997). We here regard it as a full equilibrium, if $\xi = 1$, and as a semiequilibrium, if $0 < \xi < \infty$.[5] For, given $\overline{\mathbf{q}}$, ξ will determine $\mathbf{q}^*$, and considering $\mathbf{q}^*$ as a vector of hypothetical endowments, the remaining equations and the inequality determine equilibrium consumption vectors, activity levels and prices such that demand coincides with these hypothetical endowments. Each variable of the model may therefore be considered an equilibrium trajectory in function of parameter ξ, and each trajectory will actually be a well-defined function of ξ for all $\xi > 0$; it may be or may not be monotonic. This will also be true for the rate of interest $r_s = (\mathbf{sp}^0 / \mathbf{sp}^1)$, hence $r_s = r_s(\xi)$.

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5 When it is necessary, we denote the solutions of the semi equilibria as $\hat{\mathbf{c}}^0, \hat{\mathbf{c}}^1, \hat{\mathbf{p}}^0$ etc. in order to distinguish them from Garegnani's solutions $\tilde{\mathbf{c}}^0, \tilde{\mathbf{c}}^1, \tilde{\mathbf{p}}^0$ etc.

The modification of Garegnani's model provides a new basis for an evaluation of his approach. We see that, for each ξ, there is a semiequilibrium with $\mathbf{q}^*$ as (hypothetical) endowments. We call this a Clower-semiequilibrium (CSE), in contrast to Garegnani's semiequilibrium (GSE). In accordance with the earlier definition, we may speak of a full equilibrium, if $\xi = 1$ and if hypothetical endowments or, to remember the analogy with the labour market, endowments demanded are equal to real endowments.

Garegnani does not take ξ but r_s as independent variable which implies that some trajectories may become multivalued even in a one consumer economy, as we shall see. It is economically more interesting to regard the rate of interest as independent, but it is mathematically simpler to start from a variation of ξ, at least in a one-consumer model, since all variables are then uniquely determined for each ξ; once this function $r_s(\xi)$ is obtained, it may be inverted where its derivative exists and does not vanish.

It is clear that

$$I = S$$

now is necessary and (for $\mathbf{p}^0 \neq \mathbf{0}$) also sufficient for a full equilibrium, for $\xi = 1$ implies $\overline{\mathbf{q}} - \mathbf{c}^0 = \mathbf{q}^1\mathbf{A}$, hence $I = S$, and conversely $I = S$ yields

$$\overline{\mathbf{q}}\mathbf{p}^0 = S + \mathbf{c}^0\mathbf{p}^0 = I + \mathbf{c}^0\mathbf{p}^0 = (\mathbf{c}^0 + \mathbf{q}^1\mathbf{A})\mathbf{p}^0 = \mathbf{q}^*\mathbf{p}^0 = \xi\overline{\mathbf{q}}\mathbf{p}^0,$$

therefore $\xi = 1$, if $\mathbf{p}^0$ does not vanish (cf. Garegnani 2003, par.16, for the case $\mathbf{p}^0 = \mathbf{0}$). If the rate of interest r_s is regarded as the exogenous variable, determining $I(r_s)$ and $S(r_s)$, a full equilibrium is characterised by a r_s such that $I = S$. Garegnani provides a description of how his schedules $I(r_s)$ and $S(r_s)$ can be determined (Appendix I), using his demand functions.[6] We summarise our main findings:

EXISTENCE THEOREM

1) The Clower-semiequilibrium (CSE) defined by equations (1'), (2), (3), (3a), (6), (7) and the maximisation (4) with budget equation (4a'), complemented by (5a) and (5b), exists and is unique for every ξ, $0 < \xi < \infty$.

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6 The Appendix in turn is based on a Mathematical Note (really an appendix to the Appendix), by Michele Tucci, where we read e.g. about "border solutions" that they are examined in connection with Assumption

"(iii), Paragraph $[\mathbf{5}]$ of Appendix I, and in Paragraphs $[\mathbf{6}]$, $[\mathbf{10}]$, $[\mathbf{13}]$ and $[\mathbf{14}]$ of the same Appendix, where there will be a continuous set of solutions characterised by $W = 0$."

This quote is a specimen of what I meant when I spoke of a paper chase of assumptions above.

2) Suppose $\mathbf{p}^0 \geq \mathbf{0}$. The Clower-semiequilibrium (CSE) is a unique full equilibrium with $I = S$, if and only if $\xi = 1$.

3) A semiequilibrium according to Garegnani's (GSE) system (1'), (2), (3), (3a), (5a), (5b), (6), (7), (4'), (4a), (4"), (4a") with $\mathbf{p}^0 \geq \mathbf{0}$ is a full equilibrium if and only if $\xi = 1$. The solution coincides with that of the corresponding CSE with $\xi = 1$ and $\tilde{\mathbf{c}}^0 = \hat{\mathbf{c}}^0, \tilde{\mathbf{c}}^1 = \hat{\mathbf{c}}^1, \tilde{\mathbf{p}}^0 = \hat{\mathbf{p}}^0, \tilde{\mathbf{p}}^1 = \hat{\mathbf{p}}^1, \tilde{w} = \hat{w}$.

We only need to prove the third assertion: The necessity of $\xi = 1$ is shown as for the CSE above. CSE and GSE coincide for $\xi = 1$ because notional and speculative demand then coincide.

Having analysed the CSE in the remainder of this section, we shall turn to the GSE subsequently, and we shall show that the essential properties of the GSE emerge from a comparison.

$S = (\overline{\mathbf{q}} - \mathbf{c}^0)\mathbf{p}^0$ will be low – indeed negative – for large ξ, hence $S < I$. Since hypothetical endowments $\mathbf{q}^*$ tend with ξ to infinity, and since $\mathbf{c}^1$ is bounded by the availability of labour, following (3), $\mathbf{c}^0$ becomes infinite so that $S = (\overline{\mathbf{q}} - \mathbf{c}^0)\mathbf{p}^0$ must tend to minus infinity, at least if $\overline{\mathbf{q}}$ is taken as numéraire with $\overline{\mathbf{q}}\mathbf{p}^0 = 1$.

As $\xi \to 1$, one arrives at the (unique) full equilibrium with $I = S$. Suppose that the labour constraint is binding in this full equilibrium. As ξ is reduced below one, hypothetical endowments fall and a point will be reached where full employment of labour ceases to be possible so that the wage is driven to zero. The transition is marked by a level of ξ with full employment, $\mathbf{c}^1\mathbf{l} = L$ and $w = 0$. Further reductions of ξ yield a continuum of unemployment semiequilibria with $w = 0$ (though this unemployment would be classified as "voluntary" in the standard general equilibrium literature). We obviously must have $S > I$ for $0 < \xi < 1$, and r_s will be expected to rise as $\mathbf{q}^*$ tends to zero, but not indefinitely. p_i^0 / p_i^1 must fulfil $\mathbf{p}^1 = \mathbf{A}\mathbf{p}^0$ at $w = 0$. If $\mathbf{p}^0$ happens to be the Frobenius eigenvector $\mathbf{p}^*$ of $\mathbf{A}$, $(1+R)\mathbf{A}\mathbf{p}^* = \mathbf{p}^*$, as in a steady state, we get $p_i^0 / p_i^1 = 1 + R$, R being the maximum rate of profit of $\mathbf{A}$; $i = 1,2$, and $p_1^0 / p_2^0 \to p^*{}_1 / p^*{}_2$ as $\xi \to 0$ implies that both own rates of interest converge to R. If the scarcity relations for small ξ deviate from those characterising the steady state, the own rates converge to other values.

The general picture therefore is this: as ξ falls from "large" values to zero, I, S and r_s are definite functions of ξ, with $S < I$ for $\xi > 1$ and $S > I$ for $\xi < 1$ and $S = I$ for $\xi = 1$. There will be a broad tendency for S and r_s to rise and for I to fall, but not necessarily everywhere. A definite result obtains with our assumptions for $S - I$, in terms of the numéraire $\overline{\mathbf{q}}$, since

$$S - I = (\overline{\mathbf{q}} - \mathbf{c}^0 - \mathbf{c}^1\mathbf{A})\mathbf{p}^0 = (\overline{\mathbf{q}} - \mathbf{q}^*)\mathbf{p}^0 = (1-\xi)\overline{\mathbf{q}}\mathbf{p}^0 = 1 - \xi.$$

The formula confirms $S \to -\infty$ for $\xi \to \infty$. Diagram 1 shows plausible schedules for $S-I, r_s$, w in terms of $\overline{\mathbf{q}}$ as numéraire, in function of ξ, on the assumption that $\xi = 1$ is a full employment equilibrium:

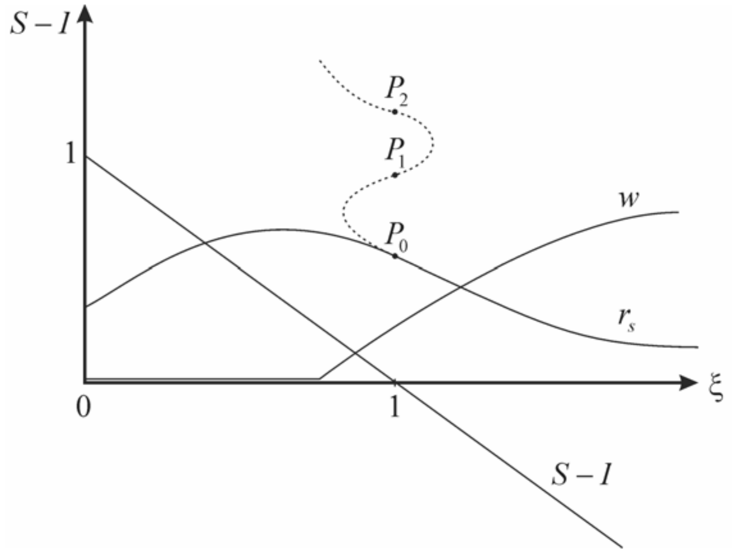

Diagram 1: $S-I, r_s$, w as functions of ξ. Dotted line: additional hypothetical equilibria roughly according to Garegnani (not possible in one consumer model); they lead to equilibria P_1 and P_2, (besides P_0 which is a full equilibrium of the one consumer model).

The underemployment equlibria obtained by diminishing ξ correspond to situations in which lower hypothetical endowments prevent the full employment of available labour. The rate of interest $r_s = (\overline{\mathbf{q}}\mathbf{p}^0 / \overline{\mathbf{q}}\mathbf{p}^1) - 1$ then depends on prices which must fulfil $\mathbf{A}\mathbf{p}^0 = \mathbf{p}^1$; it can therefore not vary much but it can fall to some extent even without a change of technique. The relationship between $S-I$ and r_s which follows from the elimination of ξ is shown in diagram 2:

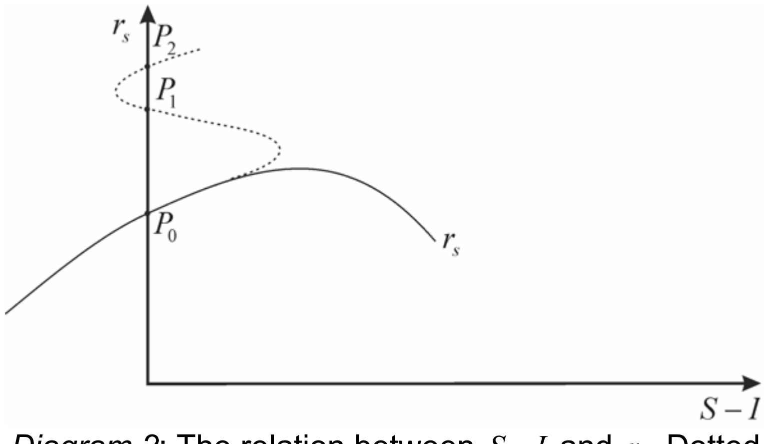

Diagram 2: The relation between $S-I$ and r_s. Dotted lines: extension of the schedule according to Garegnani. P_0: equilibrium with $\mathbf{q}^* = \overline{\mathbf{q}}$, $\xi = 1$. P_1, P_2 additional unstable and stable equilibrium roughly according to Garegnani. Only P_0 is a full employment equilibrium (Garegnani 2003, par.14, Assumption II).

After the elimination of ξ where we have $dr_s(\xi)/d\xi \neq 0$, one obtains schedules $I(r_s)$ and $S(r_s)$ which bear a similarity to the savings and investment functions of traditional theories. Should they therefore be used for the analysis of the stability of intertemporal equilibria? I already have indicated my theoretical doubts, in particular regarding the interpretation of $S(r_s)$ – there is no separate decision to save in the intertemporal model. Moreover, additional difficulties arise if there is a choice of techniques.

Garegnani represents the choice of methods of production as a selection of ”systems of production"; a "system" will include two methods of production, "one for the commodity and one for the other commodity as means of production of the former".[7] Since each commodity is a mean of production for the other, Garegnani compares viable "systems" with $n = 2$ according to the cost of production, asking what the "order of cheapness" at the given level of the own rate of interest will be. He refers for comparison also to Sraffa (1960, chapter XII); there is a "possible coexistence at some r_b" – corresponding to our r_2, equation (7) – "which will then entail the same wage and prices for the given level of r_b" (Garegnani 2003, p. 134 and note 44). He draws continuous (even smooth) schedules for $I(r_b)$ and $S(r_b)$, apparently incorporating such switches. Other forms of technical change, engendered by variations of the rate of interest, are not explained.

But changes of methods in intertemporal theory only presuppose an equality of the costs of the different methods, hence of prices at $t = 1$, not of prices at $t = 0$, contrary to what Garegnani seems to assert in the sentence just quoted, for input prices and output prices need not be proportional in intertemporal theory. Suppose the 'switches' are defined as changes of technique which occur at given levels (points) of ξ or r_2, with different "systems" being profitable on either side of the switch and both systems being equally profitable only at the switchpoint. Then we can prove a paradoxical theorem, still restricting our attention for simplicity to the CSE-case:

DISCONTINUITY THEOREM

Switches of technique are generically associated with discontinuities in the schedules $I(\xi)$, $S(\xi)$, $I(r_s)$, $S(r_s)$ at full employment with positive wage rates.

The reader has briefly been reminded in section 2 of the well-known conditions which must be fulfilled in equilibrium (by extension: in semiequilibrium) in

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7 Garegnani (2003, p. 133). But each commodity is "both a consumption and a (circulating) capital good" (Garegnani 2003, p. 119). Garegnani's surprising asymmetrical treatment of the two commodities in the quote above induces the reader to ask whether he has not something like the Samuelson model in mind, with a consumption good, a capital good and a continuous spectrum of techniques, as used in the discussion of the surrogate production function. But this construction would not be consistent with the assumptions about technology used here, and it would hardly appear as suited for the explanation of general equilibrium, being a half-way house between a general equilibrium model and the production function.

addition to table 1 if there is a discrete set of activities. We restrict the proof to $n=2$. We first consider two techniques $(\bar{\mathbf{A}},\bar{\mathbf{l}})$ and $(\tilde{\mathbf{A}},\tilde{\mathbf{l}})$, with $n=2$, which have the second process, producing the second good, in common and which differ only in the process employed in the first industry: $\bar{\mathbf{a}}_1 \neq \tilde{\mathbf{a}}_1$ *and* $\bar{l}_1 \neq \tilde{l}_1$, but $\bar{\mathbf{a}}_2 = \tilde{\mathbf{a}}_2$, $\bar{l}_2 = \tilde{l}_2$. Denote the solutions to the CSE pertaining to any given ξ, taking each technique in isolation, by $\bar{\mathbf{c}}^0, \bar{\mathbf{c}}^1, \bar{\mathbf{p}}^0, \bar{\mathbf{p}}^1, \bar{w}$ and $\tilde{\mathbf{c}}^0, \tilde{\mathbf{c}}^1, \tilde{\mathbf{p}}^0, \tilde{\mathbf{p}}^1, \tilde{w}$. Now the cases are two: either we have what we shall call a clean switch with $\tilde{\mathbf{p}}^0 = \bar{\mathbf{p}}^0$ and $\tilde{w} = \bar{w} > 0$, and, by definition of the switch, $\tilde{\mathbf{p}}^1 = \bar{\mathbf{p}}^1$. Quantities $\mathbf{c}^0$ and $\mathbf{c}^1$ are continuous functions of prices in the neighbourhood of the switch – otherwise, there is nothing to be proved. Hence demands will also change continuously with ξ, as determined by (4), (4a') and (6), but, generically $\tilde{\mathbf{c}}^1\tilde{\mathbf{l}} \neq \bar{\mathbf{c}}^1\bar{\mathbf{l}}$ because $\tilde{\mathbf{l}}_1 \neq \bar{\mathbf{l}}_1$; if there is full employment on one side of the clean switch, there cannot be full employment on the other side. But this means that there is a discontinuity of wage rates, prices and consequently of the schedules; a clean switch at full employment does not exist. The same reasoning holds if four processes coexist: then the switch is necessarily clean and the four relative prices are determined. But, with $n>1$, it is not necessary for the equality of cost in systems with dated prices to have equality of all prices; suppose that costs are equal because there are two different sets of prices and wage rates at the switchpoint.[8] We then arrive at the other case: the 'dirty' switch: the discontinuous change of prices at ξ generically entails the discontinuity of the I and S schedules. The formal reasoning is the same if r_s is the independent variable.

The reader may be puzzled: We know that relative prices are uniquely determined in the one consumer equilibrium: this rules out the dirty switch. In fact, one expects continuity. But how do techniques change if clean switches cannot exist either in intertemporal equilibrium? The answer will be given in the next section; it differs depending on whether ξ or r_s is the independent variable.

To conclude the present section, we turn to the two functions, $\tilde{r}_s(\xi)$ and $\bar{r}_s(\xi)$ which result from the consideration of the CSE associated with each given ξ, considering each technique in isolation, and we may imagine that both are drawn in a diagram similar to diagram 1, and in each interval of ξ, where one 'system' (e.g. $\tilde{\mathbf{A}},\tilde{\mathbf{l}}$) is used because it is cheaper, the corresponding curve – here $\tilde{r}_s(\xi)$ – shall be drawn in bold; dominance of $\tilde{r}_s(\xi)$ need not imply $\tilde{r}_s(\xi) > \bar{r}_s(\xi)$, however, and the transitions between the 'systems' cannot take the form of 'switches' at isolated levels of ξ. Since it may easily happen that both curves are not monotonic, the inverted 'function' $\xi(r_s)$ will not be single valued. If both, $\tilde{r}_s$ and $\bar{r}_s$, have an ascending and a descending branch as r_s in diagram 1, up to four different levels of ξ may be associated with a given level of r_s such that all four correspond to a CSE.[9] The schedules $I(r_s)$ and $S(r_s)$ then will also

8 Switchpoints on the envelopes of Sraffa's single product systems always represent clean switches.

9 This multiplicity of the CSE in function of r_s is not in contradiction with the unicity of semiequilibrium in function of ξ, nor with the uniqueness of full equilibrium.

have as many values, and since they will be discontinuous as well, the utility of the schedules is in doubt. A similarity with a well-known and important form of analogue in capital theory exists. As one moves down the frontier of wage curves of e.g. different single product Sraffa systems, the rate of profit varies continuously, and so does the value of capital per head along any section of the envelope where one technique dominates, but there will be discontinuous changes of the value of capital per head at switchpoints. The analysis of reswitching and reverse capital deepening resulted in a critique of neoclassical theory. The analysis of the multiplicities of the I and S schedules results in a critique of this (in the present form and application) new tool.

A less negative conclusion may emerge if we restrict our attention to only one technique. We have obtained a general linear relationship between $S-I$ and ξ in diagram 1 and we have related $S-I$ and r_s in diagram 2 which suggests that P_0 is a stable equilibrium: An excess of savings over investment results from too high a rate of interest. The schedules, in our transformed version, seem to allow a stability analysis which combines the simplicity of a partial equilibrium diagram with the complexity of general equilibrium. This is a merit Garegnani may claim for his construction, as long as only one technique is under consideration.

5. Garegnani's semiequilibrium (GSE)

The comparison of the GSE with the CSE starts from the observation that amounts $\overline{q}-q^{*}=(1-\xi)\overline{q}$, in price terms $(1-\xi)\overline{q}p^{0}=S-I$, remain unsold (in excess demand) in both models, if $0<\xi<1(1<\xi<\infty)$, for the available endowments are $\overline{q}$, the endowments demanded $\xi\overline{q}$.[10] A first and important conclusion concerns the status of the I and S schedules as indicators of the deviation of semiequilibrium from equilibrium: it is nowhere necessary to have recourse to the aggregates of S and I in order to analyse either the equilibrium or the semiequilibrium: all the essential relationships follow from the commodity markets themselves. In particular, it is not necessary to use these aggregates to *define* the GSE. It is true that Garegnani (2003) uses I and S to define his rationing scheme in his note 24 to par. 9, but our formulation in equations (4a) and (4a'') show that this is not necessary and less transparent than explicit rationing in terms of the commodity markets. S and I are at best indicators of what happens in the GSE; they do not represent essential causal forces, as we shall argue in more detail below.

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10 $S-I=1-\xi$ with $\overline{\mathbf{q}}\mathbf{p}^{0}\neq 0$ is confirmed also from budget equations of the GSE. Notional demand $\mathbf{c}^{1*}\mathbf{p}^{1}$ in (4') is in semiequilbrium according to (4a) and (4a''))

$$\mathbf{c}^{1*}\mathbf{p}^{1}=\overline{\mathbf{q}}\mathbf{p}^{0}-\mathbf{c}^{0}\mathbf{p}^{0}+wL=S-I+\chi^{1}\mathbf{p}^{1}.$$

This, inserted in (4a), yields in semiequilibrium:

$$\overline{\mathbf{q}}\mathbf{p}^{0}+wL=\mathbf{c}^{0}\mathbf{p}^{0}+\mathbf{c}^{1}\mathbf{p}^{1}+S-I.$$

Replacing $\mathbf{p}^{1}$ according to (2), using (3a), then (1') and (6), completes the proof.

The validity of the formula $S-I=1-\xi$ in the GSE demonstrates that savings again tend to minus infinity with $\xi\to\infty$. Garegnani's Figure 5.3 is drawn as if $I\to S$ for $r_s=-1$ which is wrong with our assumptions.[11]

A numerical example with $n=1$ shall permit a more detailed comparison. There are two methods of production to produce a unit of corn by means of $a_1=1/2$ or $a_2=1/4$ of corn as input and using $l_1=1/2$ or $l_2=1$ as labour. Labour available and the endowment of corn are both unity, and the utility function is $U=\ln c^0+2\ln c^1$. The demand functions follow from $\partial U/\partial c^0=1/c^0=\lambda p^0$, $\partial U/\partial c^1=2/c^1=\lambda p^1$. For λ, the Lagrange multiplier, we get $1/\lambda+2/\lambda=wL+\xi p^0$, hence $c^0=(wL+\xi p^0)/3p^0$, $c^1=(wL+\xi p^0)/(3/2)p^1$, where, in full equilibrium, $\xi=1$.

We show that the full equilibrium with $\xi=1$ here is characterised by a coexistence of both methods. The coexistence fully determines relative prices $p^0/p^1=4/3$, $w/p^1=2/3$; demand then is $c^0=\big((2/3)+(4/3)\big)\big/3(4/3)=1/2$, $c^1=\big((2/3)+(4/3)\big)\big/(3/2)=4/3$. The activity levels with which the two processes are combined are q_1 and q_2. Full employment means $q_1/2+q_2=1$, output c^1 in time 1 is $c^1=q_1+q_2$, hence $q_1=2/3$, $q_2=2/3$. The consistency of the solution follows from the fact that the sum of real consumption and real investment equals the endowment: $c^0+q_1/2+q_2/4=1/2+1/3+1/6=1=\xi$.

All CSE of this example now follow from repeating the calculation for different ξ. Small levels of ξ do not permit full employment and the labour-intensive technique (a_2,l_2) will be chosen, with $q_1=0$, $q_2=c^1$. Since $w=0$, the demand functions yield $c^0=\xi/3$ and $c^0+c^1a_2=\xi$, $c^1=4(\xi-\xi/3)=8\xi/3$. From the demand function $p^0/p^1=(3/2)c^1/\xi=4$. A graphic representation by means of a Fisher diagram shall visualise these two and the remaining CSE.

[11] Garegnani's other assertions about the behaviour of the savings function at low rates of interest are not all confirmed in our version of his model. With U specified e.g. as $U=u(\mathbf{c}^0)+\rho^{-1}u(\mathbf{c}^1)$, where $\rho-1$ is the rate of time preference, the own-rate of interest of commodity $i(i=1,2)$ is $p_i^0/p_i^1=\rho(\partial u/\partial c_i^0)\big/(\partial u/\partial c_i^1)$. With $\mathbf{q}^1=\mathbf{c}^1$ limited by the availability of labour, a large ξ means a large $\mathbf{c}^0$ and a low $r_s>-1$, whichever $\mathbf{s}$ is chosen. But r_s will not necessarily fall below zero, let alone to -1 as in Garegnani's construction, for if $u(\mathbf{c}^t)=\sqrt{c_1^t c_2^t}$ and if it happens that $c_1^t/c_2^t\to 1$ for $\xi\to\infty$; $t=1,2$; one obtains that r_s tends to ρ, whereas $1+r_s$ tends in fact to zero for $u(\mathbf{c}^t)=\sqrt{c_1^t}+\sqrt{c_2^t}$; it suffices to calculate the limits for $1+r_1=(\partial U/\partial c_1^0)\big/(\partial U/\partial c_1^1)$ to see the point.

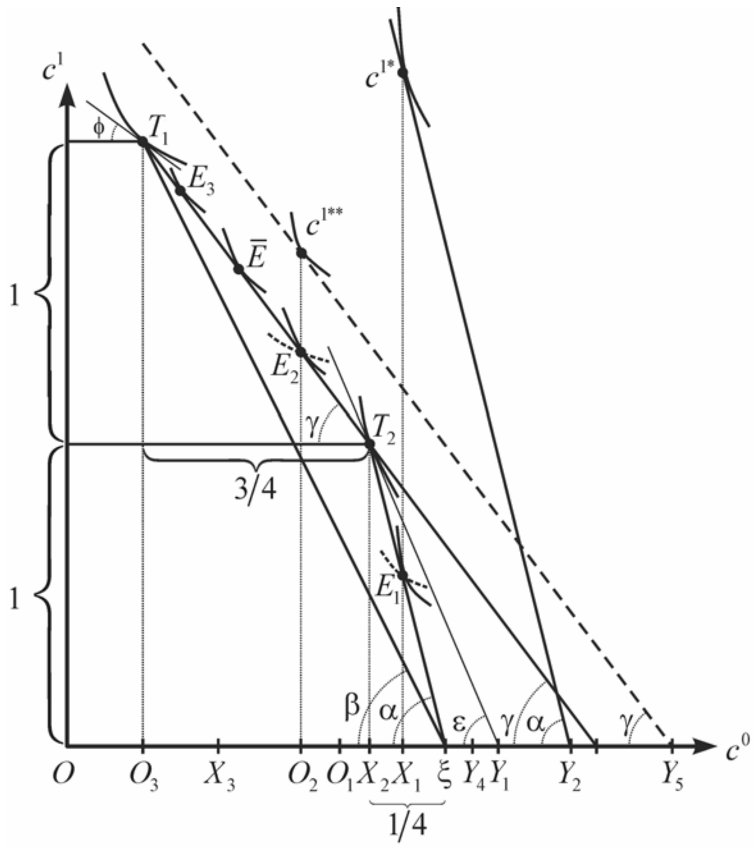

Diagram 3: The technology representing CSE and GSE. The methods are given by $(a_1,l_1)=(1/2,1/2)$, with full employment at T_1, and $(a_2,l_2)=(1/4,1)$, with full employment at T_2, given the hypothetical endowment (GSE: demand). Therefore $tg\alpha=4, tg\beta=2, tg\gamma=4/3$. As ξ increases from zero, the technology frontier $\overline{\xi T_2 T_1}$ of the CSE (GSE: frontier of demand) touches indifference curves at points with $\overline{E}$ (full equilibrium), E_3, T_1 successively. These are also roughly the equilibria of effective demand for GSE. The notional demand for GSE is higher $(\xi<1)$ or lower $(\xi>1)$ than effective demand for CSE (given ξ).

Diagram 3 expresses how the technology frontier for the two techniques, $\overline{T_1T_2\xi}$, moves to the right with rising ξ so that equilibria at higher levels of c^1 are encountered, defined by the point where an indifference curves tangent to the frontier. However, instead of showing how the frontier moves to the right, in a fixed system of coordinates with fixed indifference curves, we keep the technology frontier fixed and let the coordinate system (i.e. its origin) and the indifference curves move to the left in our exposition. [12] The full employment equilibrium is at $\overline{E}$; the origin then is at O_3 and $\xi=1$. Small ξ yield unemployment equilibria of the type E_1, with the origin at O_1 and $c^0=\overline{O_1X_1}$,

[12] ξ and c^1 need not be monotonically related, however, if U is not homothetic, as the reader will easily discover.

$c^1 = \overline{X_1E_1}$. Real investment c^1a_2 is $\overline{X_1\xi}$, using equations (1') and (5a), and real saving is $\overline{O_1Y_2} - \overline{O_1X_1} = \overline{X_1Y_2}$, using equation (5b), if 1 is at Y_2. The rate of interest (unique in the one-commodity-model) is $tg\alpha - 1 = 4 - 1 = 3$, $tg\alpha = -(\partial U/\partial c^0 / \partial U/\partial c^1)$, and $w = 0$.

As ξ increases, full employment is reached at T_2. The origin moves to O_2 and then further left. The budget line is given by $\overline{T_2Y_1}$ and the real wage wL/p^0 equals $\overline{\xi Y_1}$, the budget in terms of p^0 is $\overline{O_2Y_1}$. The rate of interest equals $tg\varepsilon - 1$ and falls from 3 to $tg\gamma - 1 = 4/3 - 1 = 1/3$. Consumption at $t = 0$ increases as the origin O_2 moves to the left, but real investment $\overline{X_2\xi}$ and consumption at $t = 1$, c^1, stay constant as ξ moves to the right.

The limit is reached when the slope of the indifference curves touching the frontier has fallen to $tg\gamma = 4/3$; then, both techniques are combined and q_1 rises from zero, $q_2 = 1 - q_1$ begins to fall, and the GSE moves from T_2 via E_2, full equilibrium at $\overline{E}$ and E_3 to T_1, where $q_2 = 0$, $q_1 = 1$. The rate of interest, prices and the wage rate are constant throughout this movement from T_2 to T_1, but quantities change in function of ξ; one obtains $c^0 = 1/6 + \xi/3$, $c^1 = (4 + 8\xi)/9, 5/8 \leq \xi \leq 7/4$. For full equilibrium, the origin is at O_3 and $\xi = 1$. The last type of equilibrium occurs at T_2, with $r = tg\varphi - 1$ falling from $1/3$ to -1, and w/p^0 tending to infinity; the origin O continues to move to the left as $\xi \to \infty$ and $(\partial U/\partial c^0 / \partial U/\partial c^1) \to 0$.

The understanding of the GSE requires a reinterpretation of the diagram. The technology frontier, given by T_1, T_2 and ξ, becomes the frontier of effective demand. The exact location of the equilibria depends on the point where an indifference curve touches the budget line representing the equation, but the budget for the GSE is $wL + p^0$, not $wL + \xi p^0$ as for the CSE. We first consider GSE of the type E_1, with ξ small and 1 at Y_2, say. We have $w = 0$, because there is unemployment, and the budget equation for speculative demand (4a) reduces to $c^0p^0 + c^1a_2p^0 = p^0$. If the origin again is at O_1, c^0 is O_1X_1 and c^{1*} is found on a line through $Y_2 = 1$ parallel to $\overline{\xi T_2}$, with slope $tg\alpha - 1/\mathbf{a}_2$, as shown in diagram 3. Note that the indifference curve at E_1 now is, except by coincidence, not tangent to the frontier of effective demand (dotted indifference curve). The point E_1 determines effective demand, nevertheless, according to the maximisation (4"), (4a"). For the budget in (4a") equals $\overline{X_1\xi}$ and the slope of the budget line is determined by that of $\overline{\xi T_2}$, according to (4a), but this has been found to equal $-1/a_2$.

A similar determination of the GSE results for equilibria of the type E_2 and E_3, since the prices here result as in the CSE from the combination of two methods so that the wage rate, in particular is given, equal to that of full equilibrium and fixed: $wL/p^0 = \overline{\xi Y_3}$. The budget line for speculative demand is parallel to $\overline{T_1T_2}$ with slope $-4/3$. One then has, taking wL from full equilibrium, to use (8) of section 3 (Garegnani's proportional rationing is here admissible since $n = 1$):

$$\frac{c^{1*}}{c^{1}} = \frac{wL + S}{wL + I}.$$

Now $S/p^0 = 1 - c^0$, $I/p^0 = q_1 a_1 + q_1 a_1$. If $Y_4 = 1$, $S/p^0 = \overline{O_2 Y_4}$, $I/p^0 = \overline{O_2 \xi}$, $(S-I)/p^0 = \overline{\xi Y_4}$, $wL/p^0 = \overline{\xi Y_3}$. Hence $(wL+I)/p^0 = \overline{O_2 Y_3}$ and $wL + S = \overline{O_2 Y_5}$ where the length of $\overline{Y_3 Y_5}$ equals $(S-I)/p^0$. One then finds c^{1*} according to Garegnani's proportional rationing at point c^{1**} in the diagram.

The budget line for $\xi < 1$ is above $\overline{T_1 T_2}$, it passes through T_1 and T_2 for $\xi = 1$ (full equilibrium) and is below $\overline{T_1 T_2}$ for $\xi > 1$ in accordance with the idea of GSE: excess demand in time 1 corresponds to reduced speculative demand at $t = 1$. The constructions for CSE T_1 and T_2 follow the same ideas, but the relation between r and w has to follow from the indifference curves; details must be omitted for reasons of space.

This cumbersome exposition has revealed (I reserve further details on $n > 1$ for possible later controversy): It requires no substantial effort, only patience, to represent semiequilibria of the Clower type (though the variation of L instead of ξ might have been more elegant). The dual decision hypothesis of the GSE, by contrast, is not only analytically but also geometrically more difficult. Prices and quantities vary evidently in a continuous manner in our CSE example with ξ, $0 < \xi < \infty$, hence $S(\xi)$ and $I(\xi)$, being continuous functions of prices and quantities, are also continuous functions of ξ (though not necessarily differentiable at levels of ξ where techniques change. These changes of technique are not 'switches' as in Sraffa (1960), however, in that the transition from one technique to the other is gradual, not sudden. There is a shift from CSE 3 (technique 2 in use) to CSE 1 (technique 1 in use) in our example where activity levels change in opposite directions over an interval $J = (5/8 \leq \xi \leq 7/4)$ of ξ. These are 'switches', by contrast, if r is taken as the independent variable, for the transition takes place at a given rate of interest of 1/3. The other prices are also stationary for $\xi \in J$, but quantities are not, so that $I(r)$ and $S(r)$ change discontinuously, and the same happens for the CSE of type E_4. The discontinuity means that a whole range of CSE is associated with $r = 1/3$ and $r = 3$; the $I - S$-schedules are not functions but correspondences. The switch in the jump from T_2 to T_1 at $r = 1/3$ is compatible with a maintenance of full employment not despite but because of the discontinuity of quantities as functions of r.

Is this a problem for neoclassical theory or for Garegnani's critique? I think both. It is a problem for neoclassical theory when it is asked how changes of the rate of interest (dependent perhaps on monetary factors) influence the choice of technique. But it is also a problem for Garegnani since $I(r)$ and $S(r)$, not being well defined, are not adequate to explain the change.

The Discontinuity Theorem of section 4 thus is explained: there are no switchpoints, if ξ is the independent variable: the intertemporal equilibrium does not change from one 'system' to another 'system' at a point ξ (as in classical long-period analysis) but over an interval. There are switchpoints, if r is the independent variable, but quantities and I and S then change discontinuously.

I should like to remark by the way that our comparison leads to an interesting test for the theories: up to $2n$ methods of production can coexist in the manner of the equilibrium at $\overline{E}$ in an intertemporal model of the type considered here with n commodities. Classical theory predicts that competition will lead to the selection of n methods since the rate of profit has to be uniform. The emergence of dominant techniques in the real world seems to confirm the classical position. However, matters are more complicated. Classical theory also admits more methods in use than commodities produced as a temporary phenomenon; this was discussed in the context of joint production (Schefold 1997, chapter 13) as over-determination and under-determination. The contrast disappears if each model is seen in its proper time frame. The intertemporal model also refers to short run 'market' prices; it admits more than n methods because the rate of profit is not uniform. The turnpike theorems predict that, under certain conditions, prices in an intertemporal model with a 'distant' time horizon converge to states with a unique uniform rate of interest: such states exclude the use of more than one method in the production one commodity with single production (except by a fluke), like the models of classical theory.

Some other aspects of the $I-S$-schedules need discussion. Their use for stability analysis would have to be based on their precise slopes, but the exact proportionality of $\mathbf{q}^*$ and $\overline{\mathbf{q}}$-equation (6) in Garegnani's and in our version is an arbitrary assumption. If one had instead, other things being equal, $q_i^* = \varphi_i(\xi)\overline{q}_i$; $\varphi_i > 0$; $\varphi_i' > 0$; $\varphi_i(1) = 1$; $i = 1,2$; the I and S schedules would be just as good from the point of view of the theory, but their shapes would look different (except for $\xi = 1$). The extrema of $S-I$ and hence the levels of r_s which represented watersheds between one equilibrium and another would shift. This consideration adds to the suspicion that the consideration of equilibrium trajectories is not sufficient to analyse the stability problems deriving from the paradoxes of capital. The proportionality of hypothetical and actual endowments may seem an innocent assumption in a world with only two goods. If their number is large, the condition $\mathbf{q}^* = \xi\overline{\mathbf{q}}$ looks awkward, yet without it (or with some similar condition), the aggregates of investment and savings cannot be defined.

But suppose that we are not interested in the disequilibrium behaviour of the model but only in the equilibria shown in the intersections of the $I-S$-schedules and their multiplicity. Garegnani writes "... regular substitution in consumption has perverse effects on factor demands. Hence the freedom with which we were able to draw the shape of the I schedule ..." (Garegnani 2003, par. 21). The problem here is that parametric variations of ξ or r_s lead not only to changes of technique in response to the variation of distribution, here represented by r_s, as in Sraffa (Sraffa 1960, Part Three), but also to changes in demand and quantities produced and consumed in a general equilibrium in which only the preferences of the consumers, the spectrum of techniques and the relative composition of the endowments are fixed. This interdependence adds to the complications of the 'old' debate about capital theory. For even pure exchange economies can have multiple and unstable equilibria. By contrast, the general equilibrium with only one consumer is unique even in the presence of technologies which allow reswitching, as has been stated above. Hence, if techniques are chosen and demand vectors assumed to construct a certain I-schedule, the S-schedule cannot be drawn independently but is determined, together with the intersections of I and S and the corresponding equilibria. Garegnani would need a novel extension of the Mantel, Sonnenschein and Debreu theorems (Debreu 1983) to show how, to a sequence of techniques, chosen in connexion with to a variation of the rate of interest, there exists (or does not exist) a set of utility functions and a distribution of wealth justifying these choices.

The full equilibrium of diagram 1 is unique, if $\xi = 1$, and also each semiequilibrium is unique as ξ is varied. This is true for the CSE and seems also to be true for the GSE, if there is only one consumer. It is not clear how additional *full* equilibria such as the five equilibria suggested by Garegnani's diagram 5.1 may come about, of which some are regarded as stable, some as unstable according to his disequilibrium analysis, without introducing several consumers. The simplest possible extension of this kind is shown by means of the dotted lines representing hypothetical underemployment equilibria in diagrams 1 and 2 here, with an additional stable and one unstable equilibrium at P_2 and P_1. The decisive question is what role is played by the necessary multiplicity of consumers and what by technology in bringing such a multiplicity of equilibria about. This question cannot be solved by means of Garegnani's approach based on aggregate consumption functions.

He insists that income effects are not relevant (e.g. Garegnani 2003, par. 14) and assumes a parallel movement of the own rates of interest of both commodities which facilitates a comparison with states where a uniform rate of profit is varied. If I have understood Garegnani correctly, the absence of income effect refers to contemporary prices: if the price ratio for contemporary prices moves in one direction, the relative demands for the commodities move in opposite direction. He does not seem to postulate this property for intertemporal prices, however, for a fall of ξ, i.e. a reduced availability[13] of present goods, is not always accompanied by a rise of r_s, and this effect is visible even in the one consumer model of diagram 1 (non-monotonicity of r_s). The deviation from what is possible in a one consumer model is suggested (in analogy with Garegnani's diagrams) in the dotted line of diagram 1: the dotted branch of the r_s-line indicates that a rising rate of interest may be associated with a ξ which rises again after having fallen, so that an underemployment equilibrium is reached, of which the first, P_1, is unstable. What this means in the space of c_i^0 and c_i^1 (supposing that the cases for $i = 1$ and $i = 2$ are symmetrical, in accordance with Garegnani's assumption about the parallel movement of the own rates of interest) is shown in diagram 4, where $\bar{c}_i^1$ indicates the full employment level of consumption at time 1 which must be roughly constant as ξ is lowered (and it is here drawn as exactly constant), because the labour supply is given and fixed.

13 'Availability' in CSE, 'reduced demand' in GSE.

Clearly, the implied slopes for the indifference curves of a consumer in equilibria P_1 and P_2 are impossible in the one consumer world:

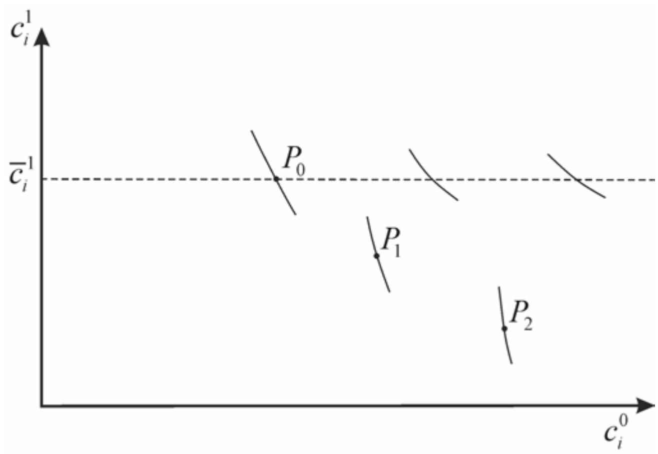

Diagram 4: The full employment level of the consumption of good c_i^1 is indicated by $\bar{c}_i^1$. The level of consumption of c_i^0 falls, with ξ, from right to left (semiequilibria). The full equilibria for $\xi = 1$, suppose to exist according to diagrams 1 and 2, correspond to different levels of c_i^1: P_0 is unique at full employment, P_1 at underemployment, P_2 at possibly still lower underemployment, and the levels of c_i^0 must accordingly be higher, since $\bar{q}$ is the same for all. The implied rising slopes of indifference curves are inconsistent with the map of indifference curves of one consumer.

The task for Garegnani and his followers remains to show how this sequence of equilibria is engendered by the superposition of the indifference maps of several consumers, endowed with different amounts of wealth, and yet to make plausible that the multiplicities and instabilities of equilibria are connected with problems of capital theory rather than with problems typical for exchange economies with many consumers.

6. Preparing for a direct analysis of instabilities caused by reswitching

We now come to an alternative approach, with explicit representation of utility. The method of intertemporal analysis and of the analysis of stability by means of *tâtonnement* are accepted as given. It is well known by virtue of theorems by Mantel, Sonnenschein, Debreu (see e.g. Debreu 1983, chapter 16) that, to essentially any set of continuous excess demand functions for a pure exchange economy, a set of consumers with utility functions and endowments can be found such that the excess demand functions for the economy constructed are, to any given degree of approximation, those of the excess demand functions given initially. But it has also been argued that aggregate demand functions are likely conform to the law of demand (Hildenbrand 1994); instabilities due to problems with the structure of production might be more relevant. Hence the attempts to formulate a direct critique in Schefold (1997, chapter 18; 2000; 2003) which I here wish to develop further in one particular direction by analysing another variant of the *tâtonnement* process. The model is the same as above (one consumer, deciding on consumption using an intertemporal budget equation defined by (1) – or (1') and (6) with $\xi = 1$ – (2), (3), (4), (4a), according to Table 1. But aggregates of saving and investment and rationing schemes are not considered; the rate of interest continues to play a decisive role, however.

The choice of technique is introduced by assuming that there are two alternative processes in the first industry so that we have two techniques

$$\mathbf{A} = \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{pmatrix}, \quad \mathbf{l} = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix},$$
$$\overline{\mathbf{A}} = \begin{pmatrix} \mathbf{a}_0 \\ \mathbf{a}_2 \end{pmatrix}, \quad \bar{\mathbf{l}} = \begin{pmatrix} l_0 \\ l_2 \end{pmatrix}.$$

We analyse the systems first as in Sraffa (1960). We assume that the wage curve of the first technique $(\mathbf{A},\mathbf{l})$ is approximately linear and that the wage curve of the second $(\overline{\mathbf{A}},\bar{\mathbf{l}})$ technique exhibits reswitching as shown in diagram 5.

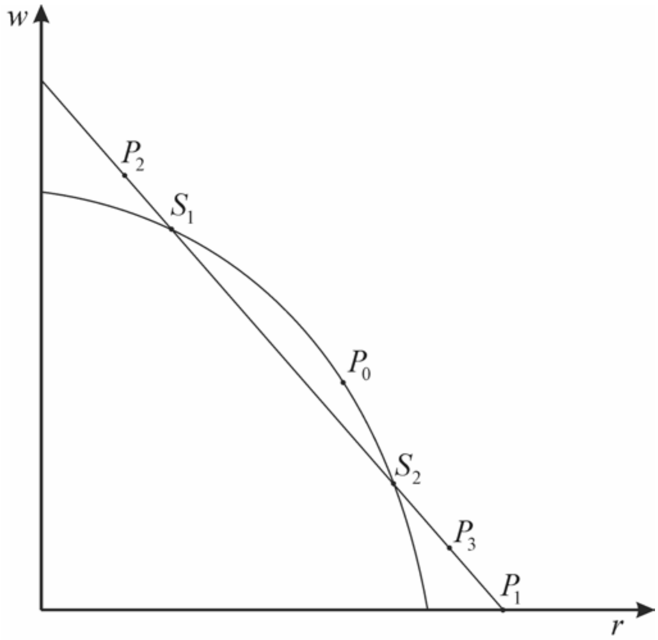

Diagram 5: Two techniques with reswitching.

Reswitching here implies that we have $l_0 > l_1$; technique $(\overline{\mathbf{A}},\bar{\mathbf{l}})$ is used at P_0 and is less capital intensive in a steady state comparison than $(\mathbf{A},\mathbf{l})$ at P_1, P_2 or P_3. Reswitching also implies that we have neither $\mathbf{a}_0 \geqq \mathbf{a}_1$ nor $\mathbf{a}_0 \leqq \mathbf{a}_1$.

The basic idea is very simple. Compare steady states at P_0 and P_1. Suppose that the steady state at P_0 is an intertemporal equilibrium of the type considered above, in that the utility of the consumer is maximal, given the constraints of the endowments and in the labour market where the available labour force happens to be fully employed. Suppose that an auctioneer in a *tâtonnement* announces prices and tests the stability of this state. In so doing, he happens to set the (surplus) wage rate equal to zero and to set prices of endowments such that producers choose technique $(\mathbf{A},\mathbf{l})$, and the economy lands at P_1 where the rate of profit would be equal to R, if a steady state could be obtained immediately. If activity levels do not change much in the transition, employment will be lower, since $l_1 < l_0$. This effect (which we call *technology effect*) confirms the decision of the auctioneer to set wage rate equal to zero, and one begins to wonder whether P_1 is not a second stable equilibrium.

Reswitching is clearly at the root of this technology effect. We here have the characteristic counterintuitive (from the neoclassical point of view) relation between factor prices and quantities. This is best seen by stepping backwards: There is unemployment at P_1. This unemployment can be mended, but, in order to achieve full employment, the wage rate has to be raised, not lowered. Such transitions are modelled as paradoxical equilibria in Schefold (1997) and Schefold (2000).

The paradoxical nature, and the tendency to instability, of equilibria involving reswitching are obvious, but whether the instability prevails depends on utility and consumption. In the present context, there is only one consumer, the optimum is unique, a convergence of a *tâtonnement* process towards a different equilibrium P_1 therefore is ruled out. I now use specific assumptions in order to clarify this point in my comment on Garegnani, because I thus wish to complement his analysis in which demand is not derived from utility explicitly. Other assumptions are made in Schefold (2003) where the emphasis is on showing that equilibria involving reswitching are *relatively less* stable in a general equilibrium with one consumer than equilbria involving technologies which correspond to the neoclassical assumption of a negative correlation between the rate of interest and the intensity of capital.[14]

Mandler (2002) has pointed out that it is useful to concentrate on economies with one consumer, because instabilities due to the differences between utility functions of many consumers as in pure exchange economies are then excluded (cf. also Schefold 1997, p. 482). Prices of the original equilibrium at P_0 are equal to $\mathbf{p}^0, \mathbf{p}^1$, with $\mathbf{p}^1$ proportional to $\mathbf{p}^0$ (initial steady state), the wage rate is w, the numéraire is $\mathbf{s}$, with $\mathbf{sp}^0 = 1$. The auctioneer, to test stability, calls new prices of endowments, $\mathbf{p}^{0*}$, and a new wage rate w^*, with $\mathbf{sp}^{0*} = 1$. The producers (there is perfect competition, with constant returns to scale) report back $\mathbf{p}^{1*}$ which are equal to either $\mathbf{Ap}^{0*} + w^*\mathbf{l}$ or to $\overline{\mathbf{A}}\mathbf{p}^{0*} + \overline{w}^*\mathbf{l}$, depending on which technique is cheaper. Next, the consumer is asked what his demands $\mathbf{c}^{0*}, \mathbf{c}^{1*}$ are, if he maximises his utility under the budget constraint $\overline{\mathbf{q}}\mathbf{p}^{0*} + w^*L = \mathbf{c}^{0*}\mathbf{p}^{0*} + \mathbf{c}^{1*}\mathbf{p}^{1}$. We here assume an ideal auctioneer who is then able

[14] I use steady state analysis, not because of the realism of the assumption but because I want to use what is known about this simple case and because it turns out that the stability problems of intertemporal theory, engendered by the paradoxes of capital, can be approached from that angle, for the auctioneer may be assumed to announce prices such that steady state comparisons result.

to calculate demand $\mathbf{q}^*$ which is equal to $\mathbf{c}^{0*} + \mathbf{c}^{1*}\mathbf{A}$ or to $\mathbf{c}^{0*} + \mathbf{c}^{1*}\overline{\mathbf{A}}$ (for a critique of the assumption about the ideal auctioneer see Schefold, 2003). The next iteration starts with prices $\mathbf{p}^{0**}$. Each component of $\mathbf{p}^{0**}$ will be raised, relatively to that of $\mathbf{p}^{0*}$, if there is excess demand, lowered in case of excess supply and left equal, if the demand for the commodity q_i^* is equal to the endowment q_i. Prices then are normalised so that $\mathbf{sp}^{0**} = 1$. The wage rate w^{**} is adjusted similarly in the labour market. (We need not specify this assumption in greater detail at this point.) The budget for the second round is $\overline{\mathbf{q}}^*\mathbf{p}^{0**} + w^{**}L$. If the subsequent vectors $\mathbf{p}^{0*\ldots*}, \mathbf{p}^{1*\ldots*}$ etc. converge at all, they converge to $\mathbf{p}^0, \mathbf{p}^0$ etc. because of the uniqueness of equilibrium. Total demand for commodities at $t = 0$ then equals $\overline{\mathbf{q}}$.

In our specific case, $w^* = 0$ and $\mathbf{p}^{0*}$ are by assumption such that $\mathbf{A}\mathbf{p}^{0*} \leq \overline{\mathbf{A}}\mathbf{p}^{0*}$ so that the first round of the *tâtonnement* process leads from the technique used at P_0 to the technique used at P_1. With

$$\mathbf{p}^{1*} = \mathbf{A}\mathbf{p}^{0*} \tag{9}$$

determined by the producers and with $\mathbf{c}^{0*}$ and $\mathbf{c}^{1*}$ determined by the consumer, subject to the budget equation $\overline{\mathbf{q}}\mathbf{p}^{0*} = \mathbf{c}^{0*}\mathbf{p}^{0*} + \mathbf{c}^{1*}\mathbf{p}^{1*}$, there results a demand for employment $\mathbf{c}^{1*}\mathbf{l}$, to be compared with labour available L, and a demand for goods $\mathbf{q}^* = \mathbf{c}^{0*} + \mathbf{c}^{1*}\mathbf{A}$, to be compared with endowments $\overline{\mathbf{q}}$.

It is plausible that $\mathbf{p}^{1*}$ will be lower than $\mathbf{p}^1$ in both components, since $w^* = 0$ and $\mathbf{sp}^0 = \mathbf{sp}^{0*}$. This suggests that the own rates of interest $(p_i^{0*}/p_i^{1*}) - 1$ will have risen, compared to $(p_i^0/p_i^1) - 1$. On the other hand, it is possible that $\mathbf{p}^{0*}$ is close to the eigenvector associated with the dominant root of $\mathbf{A}$, and this helps to ensure that $\mathbf{A}$ dominates $\overline{\mathbf{A}}$. If we actually have $(1+R)\mathbf{A}\mathbf{p}^{0*} = \mathbf{p}^{0*}$, it is clear that the own rates of interest of both commodities at P_0 and the own rate of numéraire $\mathbf{s}$ at P_0, r_0, all rise to R. The value of the endowments changes little if the numéraire $\mathbf{s}$ happens to be close in its proportions to $\overline{\mathbf{q}}$, and there is no harm in simplifying our argument by assuming $\mathbf{s} = \overline{\mathbf{q}}$. The budget equation in P_0,

$$b = \overline{\mathbf{q}}\mathbf{p}^0 + wL = \mathbf{c}^0\mathbf{p}^0 + \mathbf{c}^1\mathbf{p}^1,$$

with $\overline{\mathbf{c}}^0, \overline{\mathbf{c}}^1$ as the equilibrium values at P_0, is now replaced by

$$b^* = \overline{\mathbf{q}}\mathbf{p}^{0*} = \overline{\mathbf{q}}\mathbf{p}^0 = \mathbf{c}^0\mathbf{p}^{0*} + \mathbf{c}^1\mathbf{p}^{1*} < b.$$

The budget of the consumer therefore has been reduced by wL and interest rates have increased to R. This means, given 'normal' shapes of the indifference curves, that consumption is postponed, that $\mathbf{c}^{0*}$ is diminished and $\mathbf{c}^{1*}$ increased, in comparison with $\overline{\mathbf{c}}^0$ and $\overline{\mathbf{c}}^1$. To this extent, therefore, employment is likely to rise.

The rise in employment, due to the rise of interest rates, may be called *deferred consumption effect*. It is similar to the consequence of an increase in saving due to a rise in interest rates in the "old" neoclassical theory, but the phenomena are not identical since there is a direct effect on investment and an increased demand for goods in the future. Even in the presence of several consumers, the essential phenomenon in intertemporal equilibrium is an increased demand for future goods, not an increasing unspent income, as we argued above in our critique of Garegnani. This deferred consumption effect is opposed to the technology effect, and the former must dominate the latter eventually in further iterations of the *tâtonnement* process, for that process, if it converges, must converge to the unique equilibrium.

Different conditions may delay this process of convergence. Leaving aside the possibility of cycles, discussed elsewhere, we may first simply note that the two own rates of interest of the two commodities need not move in the same direction, and it is not certain that both c_1^1 and c_2^1 will increase, if the change of relative prices from $\mathbf{p}^0$ to $\mathbf{p}^{0*}$ is large.

Second, the technology effect must predominate if the increase of the rates of interest is small enough because P_0 is close to P_1. In the limit, the present analysis can be started in a steady state in the switchpoint between P_0 and P_1 itself where prices are the same for both techniques. Suppose that $(\overline{\mathbf{A}},\overline{\mathbf{l}})$ is used first[15], that the auctioneer calls prices which deviate from the switchpoint prices marginally so that the producers adopt technique $(\mathbf{A},\mathbf{l})$, the wage rate being marginally lower. Demand $\mathbf{q}^* = \mathbf{c}^{0*} + \mathbf{c}^{1*}\mathbf{A}$ will therefore also only change marginally, but a large, discontinuous change of employment results if l_1 is considerably smaller than l_0. For the change of employment induced by the derived change in consumption is $(c_1^1 l_0 + c_2^1 l_2) - (c_1^{1*} l_1 - c_2^{1*} l_2) > 0$, with $l_0 >> l_1$ and c_i^{1*} only marginally larger than c_i^1. There is therefore unemployment.[16] The auctioneer will have to announce a zero wage rate in the second round and the *tâtonnement* is repeated under conditions corresponding to those encountered at P_1, if p^{0**} is close to the normal prices pertaining to the maximum rate of profit. Later iterations will not involve positive wages as long as the technology effect predominates; only changes of relative endowment prices will break the deadlock.

The process of *tâtonnement* could in principle be accompanied by disequilibria in the markets for both commodities at $t = 0$ and $t = 1$ and in the labour market. However, the disequilibrium at $t = 1$ is avoided if consumption demand c^{1*}

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15 I.e. utility and endowments are such that only $(\overline{\mathbf{A}},\overline{\mathbf{l}})$ is used.

16 This is how the argument underlying the discontinuity theorem may be used to approach the problem of stability. The marginal change of r does here not lead to the discontinuity of quantities encountered in our CSE example of diagram 3 in a transition such as that from $\overline{E}$ to T_1, with r falling from $r = 1/3$ to $1/3 - \varepsilon$, because the auctioneer chooses prices close to the switchpoint prices such that the technique changes, but the demand functions, being continuous, define a continuous change of demand and the discontinuity concerns the state of employment.

translates into investment demand $\mathbf{c}^{1*}\mathbf{A}$ so that the disequilibrium shifts to the market for endowments at $t=0$ where $\overline{q}$ and $q*$ differ. If, moreover, the deferred consumption effect is dominated by the technology effect, we have $c^{1*}l<L$ and $w^{*}=0$ (for this is what we assumed the auctioneer to announce). Hence there is an unemployment equilibrium in the labour market and the disequilibrium is confined to the market for endowments: it follows from Walras' law that the excess demand vector in the market cannot be proportional to the vector of endowments as in the GSE or CSE; the excess demands must here be of opposite sign: $(\overline{\mathbf{q}}-\mathbf{q}^{*})\mathbf{p}^{0*}=0$. This causes the relative prices of endowments to change in the subsequent round so that the economy cannot rest at P_1 with technique $(\mathbf{A},\mathbf{l})$ in the *tâtonnement* process. The main effect in the first round may nevertheless consist in the shift of demand between $t=0$ and $t=1$, and on this we shall concentrate.

The details will have to be worked out elsewhere. In commenting on Garegnani with his consumption demand functions, my main concern is to point to the possibly complementary roles of effects of capital theory and of unusual features of the utility function in bringing about instability or at least in delaying stability. Let us, like Garegnani, assume symmetry in the roles played by the two commodities. Let α_i,α_i^{*} be the share of the budgets b,b^{*} respectively, spent on commodity i at $t=0$ and $t=1$, and suppose $\alpha_i^{*}b^{*}/p_i^{0*}<\alpha_i b/p_i^{0}$, which may be assumed since $(\mathbf{A},\mathbf{l})$ may have been chosen so that relative prices change little and $b^{*}<b$. Diagram 6 shows the equilibrium at P_0 and possible outcomes of the first round of the *tâtonnement* process in P_1 in the space of commodity i; $i=1,2$; at the beginning and at the end of the period, therefore $\overline{c}_i^{\,0}$ and $\overline{c}_i^{\,1}$ in Q_0, corresponding to P_0, and c_i^{0*} and c_i^{1*} in Q_1, corresponding to P_1. This is the solution where it is assumed that the own rate of interest of commodity i has risen from r_0 at Q_0 to a higher value, say R, at Q_1. If Q_0 is on indifference curve I_1 and if Q_1 is on indifference curve I_2 and if I_1 and I_2 are homothetic, the deferred consumption effect follows which dominates the technology effect eventually, if the *tâtonnement* process converges. Note that Q_0 is in the budget of the consumer at Q_1, since the application of the *tâtonnement* prices to the equilibrium value of consumption at P_0 yields, using (1) and (9):

$$\begin{aligned}\overline{\mathbf{c}}^{0}\mathbf{p}^{0*}+\overline{\mathbf{c}}^{1}\mathbf{p}^{1*}&=(\overline{\mathbf{q}}^{0}-\overline{\mathbf{c}}^{1}\overline{\mathbf{A}})\mathbf{p}^{0*}+\overline{\mathbf{c}}^{1}\mathbf{A}\mathbf{p}^{0*}\\&=b^{*}+\overline{c}_1^{1}(\mathbf{a}_1-\mathbf{a}_0)\mathbf{p}^{0*}<b^{*};\end{aligned}$$

$\mathbf{a}_1\mathbf{p}^{0*}<\mathbf{a}_0\mathbf{p}^{0*}$ is the condition for $(\mathbf{A},\mathbf{l})$ to be more profitable than $(\overline{\mathbf{A}},\overline{\mathbf{l}})$ at P_1 (violation of Burmeister's regularity condition mentioned in the beginning). Utility is higher in the first round of *tâtonnement* at Q_1 than at Q_0, in spite of the diminution of the budget from b to b^{*}, because the future price falls (the interest rate rises) and because the technique in use is cheaper ($\mathbf{a}_1\mathbf{p}^{0*}<\mathbf{a}_0\mathbf{p}^{0*}$).

But it is also possible that consumption demand in the first round of *tâtonnement* will correspond to Q_1^{*} on indifference curve I_2^{*} which is such that the higher rate of interest leads to a lower demand for the future at the new budget which has been diminished by the cost of the surplus wage. Although the rate of intertemporal substitution has risen, future consumption is lower according to a kind of intertemporal Giffen effect. This negative deferred consumption effect reinforces the technology effect and the auctioneer sees no reason to raise the wage as long as it lasts. As we know, the situation must get corrected in subsequent iterations sooner or later, if convergence obtains. But, instead of convergence, one might get a cycle.

The intertemporal Giffen effect or income effect considered here reinforces the technology effect due to reswitching. The destabilisation is in part due to rates of time preference which increase with accumulation (the slope of I_2^* is higher than that of I_1 at the intersection with the 45°-line). Hayek observed (Hayek 1941, p. 228) that the rates of time preference of the consumers in an intertemporal equilibrium with a distant time horizon must rise with the levels of income and consumption.[17] I.e. the indifference curves as shown in diagram 6 must exhibit steeper slopes along the 45°-line as one moves to the upper right, for if the converse were true, richer consumers would accumulate more (in the sense of shifting their expenses to definite future goods), and an ever greater concentration of wealth would result. But is it not often the case that the rich have low rates of time preference (if their habits are to be described in neoclassical terminology)? The unrealistic condition which favours a stable distribution of wealth here favours the instability of the equilibrium itself, in conjunction with reswitching and the technology effect.

This analysis of stability is incomplete insofar as we have only looked at special cases, without constructing the time-paths of all iterations. The emphasis here is on the comparison between the destabilising influence of reswitching at switchpoint S_2, in diagram 4 and an ordinary change of technique such as at switchpoint S_1. To observe the difference, compare a change from P_2 to P_0 with one from P_0 to P_3 in diagram 4. If an equilibrium at P_2 is given and the auctioneer tests it by announcing prices p^{0*} and a wage rate w^* pertaining to P_0 (where P_2 may be thought to be 'close' to P_0, with S_1 in-between), the deferred consumption effect and the technology effect operate in the same direction: the demand by the consumer implies an increased employment; since full-employment already reigned at P_2, the auctioneer must raise w^{**} and the reaction is in the right direction. If there is an equilibrium at P_0, by contrast, and the wage is lowered to P_3, across switchpoint S_2, the deferred consumption effect can by a calculation, analogous to the one presented above for the transition across switchpoint S_2 to point P_1, be shown to be opposed to the technology effect. With a small variation of w^*, the deferred consumption effect is small, but the technology effect is large. There therefore likely results unemployment, w^{**} will be lower than w^* and the auctioneer will approach P_1 in several steps, while P_1 was reached at once by assumption in the analysis above. It is therefore the nature of the switch which is at the root of the instability.

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17 For a summary of Epstein's more modern formulation see Schefold (1997, p. 430).

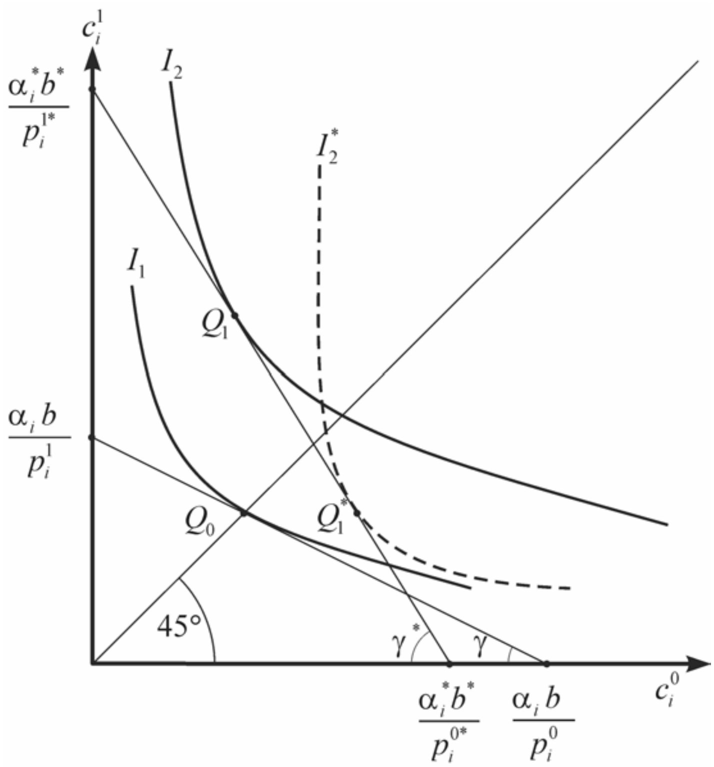

Diagram 6: Intertemporal Giffen effect; $\mathrm{tg}\,\gamma$, $\mathrm{tg}\,\gamma^*$: slopes corresponding to interest rates.

7. Conclusion

Garegnani's indirect critique is an interesting challenge for neoclassical theorists, but I doubt that they will take it up, since his approach to the theory of saving is at odds with the conception of intertemporal equilibrium. There is no room for saving as unspent income without a definite commitment to acquire future goods – if necessary, contingent on the state of nature, with uncertainty as in Debreu (1959). Saving in a world with Keynesian uncertainty is a monetary phenomenon; uncertainty may be a sufficient motive to save in a disequilibrium where not even prices are uniform. Hence, the aggregation of capital to make savers indifferent between capital goods is not necessary for the process of saving to take place.

The introduction of saving and investment in the general equilibrium model, as proposed by Garegnani, involves the construction of disequilibrium states where all markets are in an equilibrium dependent on the rate of interest, and only saving and investment in price terms on the one hand, and the vector of endowments and a proportionate vector of demand for endowments on the other are in disequilibrium. Garegnani's rationing in the market for endowments is economically less convincing than rationing in the labour market which he only mentions. There is probably something arbitrary about all models of disequilibrium, but this one is very peculiar in that all the markets clear in such a way that precisely goods available as endowments, which might be objects of own consumption, remain unsold.

The macroeconomic equality of saving and investment characterises full equilibrium among the GSE. This creates the impression that the aggregates rule the roost in general equilibrium after all as in the "old" neoclassical theory. Will it not be a natural reaction for advanced neoclassicals to insist on the intertemporal method, explained in terms of microeconomic decisions, and to criticise Böhm-Bawerk for the inconsistency of trying to combine an intertemporal approach, in which interest results from intertemporal exchange, with the datum of aggregate capital or aggregate saving?

Garegnani insists that the old method was fruitful because it regarded the composition of endowments as an unknown and only an aggregate of capital as the datum, and he adds, convincingly, that the classical analysis of long-period positions was fruitful because it was assumed that the composition of stocks would adapt, and that this long-period position was not necessarily a steady state. I share Garegnani's conviction that this classical analysis still has much to offer, on the basis of a combination of the classical approach to value and prices and of theories of distribution and employment which do not reduce the division of the surplus to the pricing of factors. However, if Garegnani thinks that neoclassical economists should follow his introduction of savings and investment schedules in intertemporal equilibrium because that would improve the theory (albeit eventually flawed), I doubt that many will share his evaluation. If he wants them to accept his analysis of saving and investment because he believes that it is inevitable in intertemporal equilibrium, he is wrong. We have shown that not only the equations defining full equilibrium but also the semiequilibrium of the Garegnani and the Clower type can be defined without introducing the aggregate concepts. Garegnani uses I and S to define his semiequilibrium, but it is clear from our construction and its summary presentation in table 1 that the definitions of I and S (equation 5c and 5b) are not required to define the GSE or to understand its functioning.

Garegnani argues that intertemporal substitution is inherently more difficult to achieve than substitution within a given period and that the aggregates are required to understand the difficulty. He compares the capital intensities of two systems and argues that changes of distribution (in this case the wage rate) may lead to the paradox known from steady state comparisons: The rise of the wage rate "may well result in the less capital-intensive method ... becoming more profitable" (Garegnani 2003, p. 136). Unfortunately, the analysis at this point reduces to a steady state comparison and neglects the specifity of intertemporal equilibrium and the Discontinuity Theorem. If associated changes of ξ and r_s engender this transition, with its discontinuity regarding $I(r_s)$ and $S(r_s)$: where is the problem? For the transition leads from one semiequilibrium to another? An answer is attempted in his final section ("Some conclusions”, Garegnani 2003, p. 137) where he surprises the reader by considering not a semiequilibrium but a disequilibrium with excess saving in $t = 0$ and excess demand in $t = 1$, neglecting the adjustment of proportional rationing (Garegnani 2003, note 49, p. 165). Prices of goods at $t = 1$ will rise relative to those at $t = 0$, apparently in a process of reconcentrating. Garegnani interprets it s a fall of interest rates which (he now supposes *ad hoc*) neither lead to reduced consumption not to a reduction of excess savings in $t = 0$ but to investment in a still earlier period, introduced *ad h*oc as $t = -1$. Now we are at the heart of the matter, because reverse capital deepening may come in: if the intensity of capital does not fall, the excess future demand is not corrected through increased production and the economy moves away from the equilibrium to which it was close; in Garegnani's exposition towards another (Garegnani 2003, par. 24).

The thesis that reverse capital deepening leads to instability is supported by the present paper (section 6), but Garegnani and I disagree about the method to show it. The reader here again encounters the problem of discontinuities in these transitions. A precise analysis would have to consider changes in intertemporal and contemporary relative prices, as we saw in section 6, and hence it would have to bypass the aggegrates of I and S. Moreover, if the *ad hoc* assumption is removed (according to which the savings and the consumption decisions for $t = 0$ are not revised in view of the excess demand in $t = 1$), intertemporal utility maximisation will re-establish the parallelism between intertemporal substitution and substitution within one period. This parallelism exists in intertemporal theory because of its formal structure, though not in reality because of true uncertainty.

In fact, it is formally possible to make assumptions such that the opposite of Garegnani's conclusion follows. If the disequilibrium consists in excess demand for c_1^0 and contemporary deficient demand for c_2^0, obstacles to a price adjustment may make it necessary to invest more in the production of commodity 1 and less in that of commodity 2 in $t = -1$. Conversely, Garegnani's intertemporal adjustment can be achieved with a reallocation affecting only $t = 0$ and $t = 1$, not $t = -1$, if there is flexibility. Most economists would probably agree with Garegnani that the intertemporal adjustment is more difficult in reality than the contemporary adjustment, but the reasons are not reflected in general equilibrium theory, as was argued in section 1 above. The proponents of general equilibrium theory do not use the $I - S$-analysis, we have now found that it leads to formal problems, and the real reasons why economists are interested in the process of saving and investments are associated with monetary economics which have never successfully been united with general equilibrium theory.

Still, we can grant the assumptions made in the paper and ask what we learn about the multiplicity and instability of equilibria. Then we get to the problem that it has not been shown how the respective influences of changes in technology and of the preferences of consumers lead to the I and S-schedules drawn by Garegnani. Multiple equilibria are not possible if there is only one consumer. It is therefore not clear to what extent the multiplicities and instabilities of equilibrium in Garegnani's construction may be ascribed to paradoxes of capital theory or to the well-known multiplicities and instabilities in exchange economies.

We can test the power of Garegnani's indirect critique by re-interpreting the stability analysis of section 6 above as a transition from one CSE to another.

Suppose we are again in S_2 (diagram 5),using technique $(\overline{\mathbf{A}},\bar{\mathbf{l}})$ only, with endowments which happen to be such that this is a steady state, all own rates of interest being equal to the rate of profit at S_2, and technique $(\mathbf{A},\mathbf{l})$ is equally profitable at S_2. This is a full equilibrium with $\xi=1$, and we assume that a small rise of an own rate of interest, looking at technique $(\overline{\mathbf{A}},\bar{\mathbf{l}})$ in isolation, corresponds to a small fall of ξ below 1 as in diagram 1. By virtue of the welfare theorems, the quantity solutions to the semiequilibria in function of ξ are given by (using the notation of section 6 and omitting non-negativity conditions):

$$Max\, U\; s.t.\, \mathbf{c}^0+\mathbf{q}^1\mathbf{B}\le\xi\overline{\mathbf{q}},\ \mathbf{q}^1\mathbf{m}\le L,\ \mathbf{c}^1\le\mathbf{q}^1\mathbf{C},$$

where $\mathbf{B}=\begin{pmatrix}\mathbf{a}_o\\ \mathbf{a}_1\\ \mathbf{a}_2\end{pmatrix}$, $\mathbf{C}=\begin{pmatrix}1&0\\1&0\\0&1\end{pmatrix}$, $\mathbf{m}=\begin{pmatrix}l_0\\ l_1\\ l_2\end{pmatrix}$.

$(\overline{\mathbf{A}},\bar{\mathbf{l}})$ is used by assumption in $\xi=1$ and coexists with $(\mathbf{A},\mathbf{l})$, but a small reduction of ξ, associated with a small rise of r, would lead to the adoption of $(\mathbf{A},\mathbf{l})$, if the uniformity of the rate of profit, assumed for $\xi=1$, held also for $\xi-\varepsilon$. By an argument analogous to that used in the proof of the Discontinuity Theorem, this transition – which is possible for the auctioneer as shown in section 6 – is impossible as a change of the CSE with ξ, for if prices and the wage rate changed continuously, $\mathbf{c}^0$ and $\mathbf{c}^1$ would also change continuously. But employment would fall with the adoption of $(\mathbf{A},\mathbf{l})$, hence the wage would fall to zero, and this would contradict the continuity assumption. The alternative is clear: the fall of ξ leads to the combined use of $(\mathbf{A},\mathbf{l})$ and $(\overline{\mathbf{A}},\bar{\mathbf{l}})$, prices and activity levels change continuously, employment is maintained and the constellation is the analogue for $n=2$ of the transition from T_1 (with $r=1/3$) to E_3 in diagram 3.

This demonstrates that the instability due to reswitching (or reserved capital deepening) is not to be analysed by means of the analysis of semiequilibria but by means *of tâtonnement*. We would have $S-I=1-\xi$ in the example under consideration for $\xi\ge1$. Garegnani regards not ξ but r_2, the own rate of interest of the second commodity, as the independent variable. S and I would show a discontinuity as 'functions' of r_2 in the interval of transition corresponding to $\xi=1$. The discontinuity of $I(r_2)$ and $S(r_2)$ as such would not be indicative of the reswitch and of an instability, however, since the same discontinuity was encountered for $n=1$; hence it is not clear how Garegnani's aggregates I and S, derived from a shifting semiequilibrium, might help to analyse the influence of the 'paradoxes' of capital theory on the stability of full equilibrium.

I have tried to show by means of the example of section 6 that a more direct critique of intertemporal equilibrium is possible. It demonstrates the destabilising influence of reswitching which can, however, be dominated by the stabilising influences of the preferences of one consumer. This is a direct critique, based on the usual assumptions of intertemporal theory. It uses the *tâtonnement* process which, even if Walras originally may have had another conception of it

(Garegnani 2003, par. 13), has become the standard tool to analyse stability. The results obtained so far are limited. The assertion that reswitching and reverse capital deepening are relevant causes for the instability of intertemporal equilibrium is not a result to be announced but a hypothesis, supported by preliminary results, which leads to a programme for future research.

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