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Democracy and Social Choice: A Response to Saari

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1. Introduction

1.1 Majority rule is troublesome if groups choose among more than two options. Suppose Tom, Dick, and Harry must rank A, B, and C. Tom ranks them (A, B, C), Dick (C, A, B), and Harry (B, C, A). Suppose they proceed by taking pairwise majority votes. Yet since A beats B, B beats C, and C beats A, no ranking emerges; instead, we obtain a cycle. This is the *Condorcet paradox*. Majority rule, as sketched here, is *indeterminate*: it does not always deliver a result. Arrow's Impossibility Theorem, in one way of thinking about it, generalizes this phenomenon, isolating those features that imply that majority rule sometimes ascribes intransitive preferences to groups. In light of these results, some argue that majoritarian democracy is conceptually flawed.

My article "Arrow's Theorem, Indeterminacy, and Multiplicity Reconsidered"[1] defends a conception of majoritarian decision making that demonstrates that there *is* a coherent

[1] *Ethics* 111 (2001): 706-34; I refer to that article as "my earlier article." This article is a response to Donald G. Saari (2003), "Capturing the 'Will of the People'," *Ethics* 113 (2003): 333-349. Thanks to Hélène Landemore and Richard Zeckhauser for valuable comments on an earlier draft. I write (A, B, etc.) for rankings, and {A, B, etc.} for sets. Where Saari's notation differs, I substitute mine. I refer to A, B, C as "options." When talking about "rankings," I mean "ordinal rankings," rankings that do not convey any information about options other than to identify other options to which they are preferred To have different names, I refer to the rule introduced here as "majority rule," but argue that the "Condorcet proposal" is what we should mean when talking about majoritarian decision making. While "majority rule" is disqualified by the indeterminacy problem, we do have a coherent account of majoritarian decision making. Note that my discussion, in virtue of being a response, will omit many important issues that arise in this debate; in particular, I ignore considerations about strategic voting altogether, and thus also most considerations that bear on how the two proposed methods do in practice. (By not debating strategic voting, I think I am making a concession to defenders of the Borda count.)

majoritarian decision rule. Since my proposal bears affinities to ideas of the Marquis the Condorcet, I call it the *Condorcet proposal* (often referring to it as "the Proposal"). This proposal solves the indeterminacy problem, as well as the related incoherence problem.[2] I also argue for the *multiplicity* thesis, the claim that in the same situation, different methods may be reasonable. So some people will be "losers" relative to what they would receive had another, equally reasonable method, been adopted. This claim, I argue, does not undermine common justifications of democracy. All this is consistent with skepticism about the range of settings where majoritarian decision making should apply. In particular, I hold the following two claims: (1) Whenever it is reasonable to use the Proposal, it is reasonable to use a competing rule, the Borda count (cf. section 2). (2) There are situations where more information should be aggregated than mere rankings, and there are situations where the decision should be made in non-aggregative ways (say, by fair-division methods).[3] So while majoritarian decision making is conceptually sound, it *never* is the uniquely reasonable decision rule. But again, I do not take that to be detrimental to democracy.

1.2 In his article "Capturing the 'Will of the People'," Donald Saari claims my "main points vary between questionable and wrong" (p 333). Saari is a prolific and insightful contributor to voting

[2] Sometimes Arrow's theorem is taken to pose an *incoherence* problem. For one of its conditions is that every profile of rankings be transformed into a group ranking. (A "profile" of rankings is one ranking for each voter.) One way of stating the theorem is that in the presence of its other conditions, this condition cannot be satisfied (indeterminacy formulation); another way is that this and the other conditions cannot be jointly satisfied (incoherence formulation).

[3] I also develop this skeptical position regarding the range of settings where majoritarian decision making should apply in my article "Arguing for Majority Rule," forthcoming in the *Journal of Political Philosophy*, which responds to Jeremy Waldron's recent defense of majority rule in *The Dignity of Legislation* (Cambridge: Cambridge University Press, 1999).

theory from a mathematical perspective, and a supporter of the Borda count. The disagreement between Saari and myself, I think, is important and complex. What is at issue is what social choice theory teaches about democracy, and how to reason soundly about insights of formal social choice theory. As I understand Saari, our views relate as follows:

> (1) Saari grants that the Condorcet proposal is a coherent, general conception of majoritarian decision making. While showing this was one of the "main points" of my earlier article, Saari does not think of this as a claim in need of much argument.
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> (2) Saari argues that the Proposal, though it may be coherent, is implausible and in particular does not solve the indeterminacy problem.
>
> (3) Saari also argues that the multiplicity thesis fails. The Borda count turns out to be the preferred voting rule, at least among methods that aggregate preferences.
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> (4) In light of (2) and (3), Saari takes critics of majoritarian democracy to be vindicated. Saari argues that majoritarian democracy is flawed *although* majoritarian decision making is sound (since it is implausible), and *because* the multiplicity thesis is false (since the Borda count is the best rule for aggregating rankings). I argue that majoritarian democracy is sound *because* there is a coherent majoritarian method, and that democracy is vindicated *although* the multiplicity thesis holds. The falsity of the multiplicity thesis does not defeat democracy.

In short, Saari argues that the Borda count is the preferred method of aggregating rankings, and solves all problems of social choice mentioned here. Condorcet and Borda themselves, French noblemen in troubled times, debated these matters already in the late 18th century, a golden age of reflection on group rationality. Yet since I merely announce Condorcet a contender whereas Saari pronounces Borda a champion (my position opposing parallel claims about the Proposal as well), our disagreement does not fully re-instantiate "Condorcet vs. Borda."

Fortunately, most terms of the debate are clear. Point (1) captures a shared starting point, the difference consisting in views on how much argument it takes to show that this indeed *is* the starting point. I respond to (2) and (3), and thus also to (4). Addressing (2), section 2 introduces the Condorcet proposal and the Borda count, defending the former against Saari's objections. Addressing (3), section 3 discusses Saari's argument for Borda. Section 4 concludes by assessing where this leaves us regarding justifications of democracy (i.e., addresses point (4)): the multiplicity thesis emerges strengthened, and majoritarian democracy is indeed conceptually sound. Saari's reasoning displays some widespread fallacies, not mathematical fallacies, but fallacies in the reasoning *about* mathematical insights. His arguments fail, but in ways that teach lessons about the philosophy of social choice and the insights social choice theory offers to democratic theory and that thus are instructive beyond the limits of our disagreement.

2. The Condorcet Proposal

2.1 This section defends the Condorcet proposal. I first explain both that proposal and the Borda count and what I claim about them. By that time it will be clear how to approach Saari's objections, and discussing them will occupy us for the remainder of this section.

Suppose we must rank m options in a majoritarian manner. The Proposal looks at all m(m-1)/2 pairs among the options and selects one or more of the m! possible rankings in light of these pairwise votes, regardless of cycles. Those votes are the "data," and we ask which ranking they support best. The Proposal selects rankings *supported by a maximal number of votes in pairwise votes*. For each ranking R, we look at the m(m-1)/2 pairs and count the voters ranking the respective options in the same way as R and thus *support* R. Suppose a group of 48 must rank A, B, and C. 10 people rank them (A, B, C), 12 (A, C, B), 5 (B, A, C), 7 (B, C, A), 3 (C, B, A) and 11 (C, A, B). So we have 3! = 6 rankings and 3(3-1)/2 = 3 pairwise votes. The ranking with highest support is (A, C, B): In A vs. B, 33 people support it (33 people rank A over B), in B vs. C 26 people, and in A vs. C 27. So 86 votes support (A, C, B), compared to 82 for (A, B, C), 64 for (B, A, C), 58 for (B, C, A), 62 for (C, B, A), and 80 for (C, A, B).

One may make three claims about the Concordet proposal: first, it captures what, in ideal theory, we should mean by *majoritarian* decision making; second, it is a *reasonable* method for aggregating rankings; and third, it is the *most or uniquely reasonable* such method. I defend the first and second claim, but reject the third. Saari endorses the first but rejects the second and third claim, insisting instead that the claim parallel to the third is true for Borda (and thus also the claim parallel to the second). Thus I endorse the multiplicity thesis with regard to aggregating rankings, whereas Saari rejects it. Let me begin by arguing for the first claim. Saari grants that claim, but does not think it is worth much argument. However, if we can establish that claim, some important conclusions follow. It follows that Arrow's theorem does not address majoritarian decision making at all. For the Condorcet proposal violates Arrow's Independence condition, which therefore does not hold true for majoritarian decision making. It also follows that, whatever *else* it is, or is not, majoritarian decision making is *coherent*.[4]

[4] What method is adopted in concrete settings must turn on additional political desiderata and practicalities, but none of that undermines what I am arguing here.

My earlier article offers two lines of argument for the claim that the Condorcet proposal is a distinct and distinguished majoritarian rule with. First, I argue that the Proposal has features rendering it such a proposal. Second, I observe that common arguments for majority rule only address cases of two options. I show that several such arguments can be generalized to support the Proposal: anybody who finds them convincing should find the generalizations convincing as well and in support of the Proposal. This is a conditional claim. I do not *thereby* endorse those arguments. It is useful to have two parts of the second line in place. One argument for majority rule (for the case of two options) is

> **Maximization:** Majority rule maximizes the number of people who exercise self-determination. This argument evidently generalizes to whichever property one thinks is expressed in the act of voting or realized by winning an election.

Strictly speaking, Maximization does not generalize. Self-determination is realized in votes, but rankings are not subject to voting, according to the Proposal. Yet the Proposal maximizes the *number of voting acts* expressing self-determination, rather than the *number of people* who express it. For two options, this argument is the original Maximization. The generalization should be convincing to whomever Maximization was convincing. Another argument is

> **Condorcet's Jury Theorem:** Supposes it makes sense to speak of being right or wrong about political decisions. Suppose n agents choose between two options; that each has a probability of $p>½$ of being right; and that their probabilities are independent of each other (i.e., they make up their minds for themselves). Then, as n grows, the probability of a majority's being right approaches 1.

The theorem only applies to two options. However, there is a generalization, which can draw on ideas of the Marquis himself, picking out precisely the rankings with maximal support. That is why I call this proposal the Condorcet proposal to begin with. My conclusion, again, is that the Proposal captures what we should mean by majoritarian decision making.[5]

The second claim is that the Proposal is a reasonable rule for aggregating rankings. If one does not press too much on what makes for "reasonableness" in abstraction from investigating features of voting rules, there is an obvious way of assessing whether majority rule is such a rule, namely by taking arguments for majority rule (and hence their generalizations) at face value (which was not required for claiming that the Proposal is what we mean by majoritarian decision making). This takes us a long way, but rather than exploring this route, let me make a different point that supports our claim. Recall that the same rankings emerge when we apply the Proposal or the generalized Jury Theorem. There is a third approach identifying those rankings. That approach (due to Kemeny) searches for a *compromise* among rankings.[6] Forming their "average" suggests itself. This operation presupposes a notion of distance between any two rankings. Define this distance as the number of pairs with regard to whose ranking they differ. The distance between (A, B, C) and (B, A, C) is 1 since they differ only with regard to {A, B}. A suitable conceptualization for an average of rankings is their *median* relative to this metric, that is, the ranking minimizing the sum over the distances from the rankings. This median is also the result of the maximum likelihood method and the recommendation of the Proposal. Strikingly,

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[5] For this generalization cf. Peyton Young, "Condorcet's Theory of Voting", *American Political Science Review* 82 (1988): 1231-1244. Using the Jury theorem here as only one in two examples has the disadvantage of suggesting that the epistemic framework in social choice is more important than it is: in many situations, after all, it will not make sense to speak of group choices in terms of true and false, and many theorists will deny that it ever does. Yet using the theorem is still useful for our purposes to assess the relative strength to the arguments for Borda and Condorcet, and at any rate, my earlier article emphasizes the limitations of this approach and discusses several other arguments in addition to these two. The reader should keep that in mind.

[6] John Kemeny, "Mathematics Without Numbers", *Daedalus* 88 (1959): 571-591

three methods motivated on different grounds select the same rankings. In particular the fact that the rankings selected by the Condorcet proposal emerge through an intuitively appealing notion of compromise supports the claim that the Proposal is a reasonable aggregation method.

2.2 Before assessing the third claim (that the Condorcet proposal is the uniquely reasonable rule), I introduce the *Borda count*, Saari's preferred method. Again a group ranks m options. First each person ranks the options, assigning 0 to her last-ranked, 1 to her second to the last ranked, etc., until she assigns m-1 to her first ranked. Then a sum over these numbers is formed for each option, which is the *Borda count* of that option. The group ranks the options by decreasing Borda counts. Suppose we have three people {1, 2, 3} and four options {A, B, C, D}. Person 1 ranks them (A, B, C, D), person 2 (B, C, D, A), and person 3 (A, B, D, C). The social ranking is (B, A, C, D): A obtains six points, B seven, C three and D two. Here is an equivalent description of the Borda count: Suppose all votes between any two options are taken. Then for each option, we count the number of elections in which any agent prefers this option to the alternative. If A obtains a Borda count of 23, then in 23 cases some voter, confronted with a pair including A, prefers A. A third characterization is that Borda ranks options by their *average position* across rankings. Like the Proposal, the Borda count uses as "input" all m(m-1)/2 pairwise voting results. But Borda asks about the *support for each of the m options in all rankings*, whereas Condorcet asks about the *support for each of the m! rankings in all pairwise elections*.

What Saari claims about Borda is not symmetrical to what I claim about the Proposal. He claims that the Borda count is the preferred rule for aggregating rankings. My earlier article argues that, whenever we aggregate rankings, Condorcet would not convince Borda, and vice versa. Let me make my argument only with regard to the Jury Theorem and Maximization, and let me only present that bit showing that Borda remains unconvinced by arguments for the Proposal. The other direction is then straightforward. While much remains to be spelled out, this argument shows (a) that the third claim about the Proposal is false (it is not the uniquely reasonable rule), and (b) that a symmetric version of the second claim is true for Borda (it is a reasonable rule as well), whereas (c) a parallel version of the third claim for Borda is also false (i.e., Borda is not the uniquely best aggregation method).

According to the Jury Theorem, rankings selected by the Condorcet proposal bestow the highest likelihood on the election result. Yet, as shown by Peyton Young, the option ranked highest by the Borda count is the *single option* that, if best, bestows highest probability upon the voting results, if the voters' competence p is close to ½. Recall that the Proposal selects *rankings* with highest support, and that Borda ranks *options* in terms of their support. So Borda endeavors to rank *options* in terms of their rightness, and Condorcet to find the right *ranking*. This difference, then, carries over to the epistemic scenario (i.e., to frameworks in which we allow for talk about group choices in terms of "true" and "false"), with Condorcet searching for rankings with maximal likelihood, and Borda ranking options by their maximal likelihood (provided p is close to ½). Borda has his counterpart to the Jury Theorem reflecting this difference and regards arguments drawing on the theorem as non-starters. Now consider Maximization. The Borda count maximizes *agreement among rankings*, not *acts of self-determination*. Having his own idea of what to maximize, Borda fails to be convinced by Maximization. Symmetric arguments show why Condorcet remains unconvinced by parallel arguments that Borda could make.

As far as aggregating rankings is concerned, Condorcet and Borda are on a par: neither has conclusive arguments against the other, and both rules are reasonable. Yet arguments for either are question-begging vis-a-vis methods aggregating other than ordinal rankings, and vis-a-vis methods, like fair division, that make decisions in ways not involving aggregation. Saari seems to concur that the Borda and Condorcet arguments fail against such rules.[7] Our disagreement is about Borda vs. Condorcet. Saari declares the debate in favor of Borda. I claim that he errs when insisting that it can be so decided. The next subsections discuss Saari's objections to the Condorcet proposal. Section 3 explores how he supports the Borda count.

2.3 Saari starts off his argument against the Condorcet proposal as follows (p 337):

> Risse […] argues that 'the majoritarian should select a ranking supported by a maximal number of votes in all pairwise elections.' He then identifies the Kemeny method, which he calls the 'Condorcet proposal,' as uniquely satisfying a specified 'maximal number of votes' condition (…). His argument is not overly compelling. The reason is that the choice of an election procedure which is 'maximal' depends on what is being maximized. Indeed, it is not difficult to design a similar 'maximal number of votes' philosophy to justify just about any reasonable voting method…. Claims of being maximal are standard.

I agree about the value of maximization arguments. I make a related point in my discussion of Borda: claims of maximization *are* standard. The "uncompelling argument" that Saari quotes is part of the *statement* of the Condorcet proposal, and not meant to be an argument.

Yet Saari does have three substantial objections to the Proposal. The first is that since the Proposal does not always deliver a unique solution, it does not solve the indeterminacy problem. However, *non-uniqueness* is different from and does not entail *indeterminacy*. Majority rule (understood as taking a sequence of pairwise votes) is indeterminate because sometimes *no* ranking emerges, not because *more than one* does.[8] By definition, that is what indeterminacy is:

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[7] This becomes clear from the very last sentences of his article, p 349.

[8] One may stipulate that the group is indifferent between rankings obtained by cutting the cycle at some point. Yet doing so introduces an unsatisfactory account of indifference. For two options, we speak of indifference if there is a tie: the group is indifferent because half of them want one thing, and half another. This provides a substantive account of indifference. That is

absence of *any* recommendation offered by a given method. The Condorcet proposal asks which rankings are better supported by pairwise votes than any others. It is *internal* to that method that no *indeterminacy* arises.[9] Alternatively, Saari's objection may target the claims that the Proposal captures majoritarian decision making, or that it is a reasonable rule, rejecting these claims because of the Proposal's non-uniqueness. Either way, it fails. It cannot be a criterion of adequacy for a procedure's being *majoritarian* that it always deliver a unique result. There are ties in pairwise voting, and so there must be room for ties at the general level. In such cases, majoritarian voting *all by itself* fails to make a unique recommendation. In light of the arguments for the thesis that the Condorcet proposal is a distinct and distinguished majoritarian method, there is nothing more to say from a majoritarian standpoint to distinguish among rankings with maximal outcome: other political or practical desiderata should decide the matter, or if there are none, random devices may do so. Similarly, it cannot be a criterion of adequacy for a *reasonable* rule that it always deliver a unique result: there must be room for ties, ties that, if all relevant criteria have been integrated, should be broken by a random mechanism. Either way, the fact that the Proposal does not make a unique recommendation *all by itself* does not pose a problem.

Saari's second objection exploits the fact that the Proposal may display curious discontinuity phenomena. He construes a case where 17,000 voters maximally support rankings

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different from observing that the procedure does not deliver a result and taking *that* to mean that it is indifferent among the rankings obtained by dissolving the source of the indeterminacy (say, by cutting a cycle): no substantive account of indifference is forthcoming in this way. Cyclicity is one way of bringing about indeterminacy, but does not *define* indeterminacy (cf. Saari, p 336).

9 Indeterminacy is always a problem because it leaves the group without any recommendation. Non-uniqueness is not always a problem: maybe what the method tries to accomplish does not entail a unique recommendation. Importantly, the proposal still selects a result if a sequence of pairwise votes leads to a cycle, namely, when the relevant majorities are unequal: not all cases of indeterminacy under sequential pairwise voting become cases of indifference under the proposal.

(A, C, B) and (B, A, C). The addition of one voter might bring about the selection of (A, C, B) as the unique such ranking, whereas the addition of another voter instead triggers the selection of (B, A, C). Saari finds it "difficult to accept that a procedure searches for the 'right ranking' when it certifies radical reversals in the societal outcome – where a candidate drops from top to bottom ranked – with a trivial 'one in 17,001' data change" (p 339). We must ask again whether this objection addresses the claim that the Condorcet proposal is a distinguished majoritarian rule, or that it is a reasonable rule. It is a feature of majoritarian decision making that, in principle (but, plausibly, with *de*creasing likelihood for *in*creasing group size), minor changes may change the outcome. While Saari emphasizes that one candidate can drop from top to bottom by one tiny change, this does not complicate matters. If one acknowledges that it is in the nature of majoritarian decision making that tiny changes may make all the difference, and if one recalls that the Proposal chooses among *rankings*, no *additional* oddity arises in that way. If Saari's objection addresses the claim that the Proposal is a reasonable rule, similar points apply.

Crucially (and this applies to both objections), it is a mistake to dismiss a method by pointing to implications where it allegedly errs without exploring (a) whether what seems counterintuitive looks plausible from a viewpoint that accepts that rule and arguments in its support; and if not, (b) whether arguments for the rule outweigh what seems implausible; and if so (i.e., if (a) is affirmed), (c) whether arguments for that rule show indeed that (in this case) it is a majoritarian rule, or a reasonable rule. Rejecting a method due to counterintuitive implications without processing such an argument is a fallacy we might call (for lack of a more appealing name) the *fallacy of overweighting allegedly counterintuitive consequences.* Saari may not think my arguments for the Condorcet proposal have any force, but he cannot show this by writing as if all that speaks for the Proposal is a maximization claim from its statement.

2.4 Let us discuss Saari's third and most interesting objection. Suppose voters search for a response to an increase of students in a school. One response is to compensate teachers for teaching larger classes; another is to hire more teachers without enlarging classes. Suppose we have 800 voters. 150 (the Deniers) are unsympathetic to the teachers, favoring salary and hiring freezes while enlarging classes. 650 voters wish to help the teachers, but do not both want to increase salaries and hire new teachers. 300 of those (the Raisers) favor raises with increased class sizes over hires, and 350 (the Hirers) favor hires with fixed class size. Voters must choose between A = {no raises} and B = {raises}, and between C = {enlarge classes, no hires} and D = {keep class size, hires}. Given the preferences, A beats B with 500 to 300 votes and C beats D with 450 to 350: the 150 got their way. Saari claims this is the outcome favored by the Condorcet proposal, or a version of it modified to handle this setting (p 339). In virtue of this result, Saari thinks the Proposal fails to *track the right rankings*, as I claim it does. My response, in short, will be that the problems Saari diagnoses arise because he seriously *underdescribes* the situation.

Surely A and C do not capture the "will of the voters," whatever that is. Yet the problem is that the situation is hopelessly *underdescribed:* neither preferences nor voting options are sufficiently specified for the Proposal to apply. There are three questions at stake, and each of the three positions takes stances on all three: (a) Should salaries be raised? (b) Should classes be enlarged? (c) Should teachers be hired? In what I trust is obvious notation, the Deniers' view is (no, yes, no), the Hirers' is (no, no, yes), and the Raisers' is (yes, yes, no). A beats B because Deniers and Hirers join forces against the Raisers, but only because A and B are underdescribed: choosing between A and B is choosing between (no, *blank, blank*) and (yes, *blank, blank*). It is no surprise that the outcome is distorted if the options only partially describe voters' views.

But even if the *options* were fully specified, the Condorcet proposal would not apply. For instance, the Deniers cannot say whether they prefer "hiring teachers while freezing class sizes and salaries" to "enlarging classes with raises but without hires." We must also fully specify the voters' views. The Proposal, placed as it is in ideal theory, requires a *complete description* of the problem that specifies each position in terms of a ranking of the three possible views *and* asks voters about completely specified options. Suppose no other positions are considered: we disregard (say) the view that salaries should be raised, but class sizes should remain fixed, and no teachers should be hired. The Deniers rank (no, yes, no) first, and then split. Suppose 110 Raise-Inclined Deniers rank (yes, yes, no) second, and (no, no, yes) third, and suppose the remaining 40 Hiring-Inclined Deniers rank (no, no, yes) second, and (yes, yes, no) last. Similarly, the Hirers split into 140 Raise-Inclined Hirers ranking (no, no, yes) first, (yes, yes, no) second, and (no, yes, no) last, and 210 Denial-Inclined Hirers ranking (no, no, yes) first, (no, yes, no) second, and (yes, yes, no) third. Finally, the Raisers split into 100 Denial-Inclined Raisers ranking (yes, yes, no) first, (no, yes, no) second, and (no, no, yes) third, and 200 Hiring-Inclined Raisers ranking (yes, yes, no) first, (no, no, yes) second, and (no, yes, no) third. Only now can we use the Proposal. There are six possible rankings and three pairwise votes. The Proposal selects the ranking putting (yes, yes, no) first, (no, no, yes) second, and (no, yes, no) third: the Hiring-Inclined Raisers win. Since most voters want to help the teachers without incurring double expenses, this outcome has a good claim to capturing the "will of the people," as I trust Saari would agree.

If voters with incompletely described views vote on incompletely described outcomes, the Condocet proposal cannot track the right ranking, and we should not expect it to do so. Saari goes on to explore a related scenario in which also A and C as defined above are selected, but by entirely different majorities. He concludes that "we must worry whether procedures based on simple majority votes – in particular the Condorcet proposal – distort outcomes by inheriting and reflecting this loss of information about the voters' wishes" (p 340). In 3.4, we discuss Saari's objection to reliance on pairwise votes in detail, but for now, let us record that *these* distortions here arise due to Saari's set-up, not due to the malfunctioning of the Proposal, or through reliance on pairwise votes. Saari commits a "collectivized" version of an error Jim Joyce calls "the single most common fallacy that people commit in the application of decision theory:"[10] in decision-theoretic contexts, this mistake is the underspecification of outcomes, and in voting scenarios it is the underspecification of voters' views or of options they vote on. To be sure, practical considerations often demand partial descriptions, and it is important to explore what distortions occur depending on what simplifications are imposed. But such scenarios do not threaten claims about ideal theory. Saari's is a wonderful case study in which the Proposal can serve to assess just how far simplified methods (even if practically advisable) are distorting.

3. The Borda Count

3.1 Saari uses an in ingenious method of defending the Borda count. He begins by formulating two seemingly innocuous "neutrality criteria," whose acceptance, however, commits us to Borda. More specifically, Saari defines sets of rankings whose removal from the group should not affect the outcome since those sets constitute a tie:

> To illustrate the basic idea, suppose in a two-person comparison that Sally has forty-five supporters while Bill has forty. A way to determine the will of this group is to combine in pairs a Sally supporter with a Bill supporter. Each of these forty pairs defines a tie; the aggregate tie from the forty pairs is broken in Sally's favor because there are five remaining people who support her. Thus information from the profile identifies Sally as representing the will of these people. A way to extract information about voter

[10] James M. Joyce, *The Foundations of Causal Decision Theory* (Cambridge: Cambridge University Press, 1999), p 52.

> preferences from a profile, then, is to understand which combinations of preferences define ties. (p 342)

This job is done by his neutrality criteria. The first is the Neutral Reversal Requirement (NRR). Call two rankings "opposing" if they rank any two options in reverse order. (Example: (A, B, C) and (C, B, A).) NRR stipulates that voting results remain unchanged when such rankings are removed. If two people disagree about each issue, the group choice should not change if they leave. The second condition is the Neutral Condorcet Requirement (NCR). To explain, we must define Condorcet n-tuples. "To define this configuration with the four candidates A, B, C, and D," Saari explains, "start with any ranking of them, say, (A, B, C, D). Next, move the top-ranked candidate to the bottom to obtain (B, C, D, A). Continue this process to create the four rankings (A, B, C, D), (B, C, D, A), (C, D, A, B), and (D, A, B, C), where, by construction, each candidate is ranked in each position precisely once. With three candidates, the initial ranking (C, B, A) generates the Condorcet triplet (C, B, A), (B, A, C), and (A, C, B)" (p 343). NCR stipulates that group choices remain invariant with regard to the removal of Condorcet n-tuples.

NCR is crucial for Saari. Its acceptance leads to Borda and allows Saari to analyze alleged flaws of other rules. The rationale for NCR is that "[s]ince the construction ranks each candidate in each position precisely once, no candidate has an advantage over any other candidate" (p 343). This argument seems to draw on *fairness to candidates.* Yet it is puzzling how such fairness bears on assessing which voters can be removed without affecting "the will of the people." An argument showing that removing Condorcet n-tuples fails to affect the "will of the people" must turn on that will. Most plausibly, such an argument stresses that no information about the relative standing of candidates in rankings is lost if we remove Condorcet n-tuples.

3.2 NRR captures an important aspect of neutrality. Fortunately, the Condorcet proposal satisfies it.[11] What about NCR? Suppose the Condorcet triplet (C, B, A), (A, C, B), and (B, A, C) is removed. If we look at the situation from the standpoint of disagreements about pairs, we notice that the view that "A is preferred to B" loses one vote; "B is preferred to A" loses two votes; "A is preferred to C" loses one vote, "C is preferred to A" loses one vote, "B is preferred to C" loses one vote, and "C is preferred to B" loses two votes. Three of the six positions lose two votes, and three lose one. For each pair, it is always one view (say, "C is preferred to B") that loses two votes, whereas the opposing view ("B is preferred to C") loses one. This is different for NRR, where each position loses one vote. Thus removing a Condorcet triplet leaves some views about pairs more "damaged" than others. NCR is not neutral with regard to disagreements about pairs.

We have gathered two observations about NCR's neutrality: as far as information about relative standing in rankings is concerned, NCR is neutral, but as far as impact on pairwise disagreements is concerned, it is not. Reflection on pairs leads to a different criterion. Call a set of rankings M "balanced" if for each ranking in M that ranks (any) option A ahead of (any) option B, there is another ranking in M placing B ahead of A. Opposing rankings form balanced sets, but there are balanced sets free from opposing rankings (if there are more than three options). (Example: (A, B, C, D), (C, B, A, D), (A, D, C, B), (D, B, C, A), (B, D, A, C), and (C,

[11] Suppose R is a ranking championed by the proposal, and suppose two opposing rankings are removed from the group. Suppose ranking S, different from R, is chosen by the proposal afterwards. Since S and R are different, there must be a set M of pairs with regard to which they differ, and a set N of pairs with regard to which they agree. For instance, for (A, B, C) and (B, C, A), M consists of {A, B} and {A, C}, and N of {B, C}. For each element of M, R and S lose one vote if two opposing rankings are removed since one of the opposing rankings supports R and one supports S. For each member of N, R and S lose one vote as well: for one of the opposing rankings supports them both on each majority rule involving a member of N. So the difference between the number of votes supporting R and the number supporting S remains the same after the opposing pairs have been removed. Thus the proposal cannot champion S.

D, A, B).) We can now formulate the Neutral Balance Requirement (NBR): NBR stipulates that the group choice remain unchanged if balanced sets are removed. The rationale for this criterion is that removing such sets does not affect the strength of views on the relative standing of pairs.

The Condorcet proposal satisfies NBR, but not NCR, whereas the Borda count satisfies NCR, but not NBR. Clearly, Saari, does not want us to assess NCR from the standpoint of how it affects pairs. So drawing conclusions from that observation may appear question-begging. Yet to the extent that we have touched on this subject in section 2, he has not yet delivered an argument that keeps us from doing so. (We explore what I take to be Saari's main argument to this effect in 3.4 and show that it fails.) So Saari must explain why we should adopt NCR rather than NBR without assuming that criteria in terms of pairs have already been discredited. They have not.

Yet such explanations require commitments regarding the purpose of the aggregation, including statements like "criterion X should be adopted because the purpose of aggregation is such and such," or "X should be adopted because in aggregating rankings we do such and such." Saari may say (for instance) that NCR is persuasive because no information about the relative standing of candidates across rankings vanishes if we remove Condorcet n-tuples. Yet that is convincing only if the purpose of aggregation is to assess such standing and to rank candidates accordingly. That is what the Borda count does. Borda asks about the support for options across rankings. Such claims make NBR looks implausible ("what is the relevance of pairs and hence of "balanced" sets given what we have said the purpose of aggregation is?"), but without some such claims, NBR cannot be dismissed. On the other hand, one may argue that NBR is plausible because it demands that disagreements about pairs be equally damaged by removing rankings. Yet that is plausible only if one thinks the purpose of aggregation is to ask which ranking is most supported by pairwise votes, and then NCR has little pull. Just as NCR led to Borda, so NBR leads to Condorcet, who asks about the support for each ranking in pairwise elections. So by justifying his criteria, Saari must give reasons unavailable to the impartial position from which he means to "assess the data." His attempt to analyze "the data" in an allegedly pre-theoretical manner fails.[12]

3.3 Both Condorcet and Borda capture plausible ideas about group choice, and each is supported by strong arguments. Each has counterintuitive implications from the standpoint of the other, perhaps even some from its own standpoint. Each conforms to neutrality criteria, and has reasons for endorsing those criteria, rather than others. It is fruitless to press on either method to reveal implications that look odd from the standpoint of the other, but (a) do not look implausible from the standpoint of that rule, or (b) do, on balance, not persuade a defender of that rule to abandon her position. This is the fallacy of *overweighting allegedly counterintuitive consequences.* We have now identified a symmetric error, which (again for lack of a catchier name) we may call the fallacy of *overweighting allegedly independently plausible axioms*. This is a fallacy because it is fruitless to show that one rule fails to conform to conditions that look plausible only from the standpoint of the other. Saari seems to commit that error as well. Both fallacies identify some facts about a rule (such as an implication for a given profile, or lack of consistency with some axiom), and declare that those cause devastating problems, without investigating how defenders

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[12] NBR entails NRR, whereas NRR and NCR are independent. One may say this gives Saari an advantage: he has two independent neutrality criteria, whereas the Condorcet proposal draws on only one criterion and a weakened version of it. Yet while NRR and NCR are independent, they are amenable to the Borda count because they both capture the idea that, on average, nothing changes about the ranking of any candidate. So there would not be much merit to saying that the Borda count has "two different criteria" on its side. At any rate, it should matter little whether a decision rule is supported by one or two neutrality criteria.

of that rule may address such claims. Before I suggest what should count as a *successful* objection to a decision method, let me press on Saari's argument a bit more.

That Saari would commit this second error is peculiar. For he insists that his approach avoids precisely that error. Since this is an area where the terms of the debate seem unclear, I elaborate on why I think Saari commits an error that he attributes to others. At the beginning of his article, Saari says that "it is accurate to interpret my [Saari's] comments as questioning whether the traditional approach used by this field, and by Risse, is bankrupt" (p 335). What is this "traditional approach" from which Saari's differs? He tells us in section IV:

> A standard approach is to postulate desirable properties for election procedures and then search for methods which possess them. Stated in another manner, the 'axiomatic approach' specifies measures for election and decision methods and identifies which procedures maximize the measure. This is the approach embraced by Risse. [p 341]

He goes on to sketch his own approach:

> Instead of inventing direct, "maximizing' criteria to suggest what the voters want, a more natural approach is to emphasize the data. By this I mean that we should try to find a way to use the information from the full profile to determine the will of the people.

Next he introduces NRR and NCR. In light of the discussion in my earlier piece[13] and in light of what we have discussed here, Saari's statements are puzzling. I have already commented on the 'maximization' bit. Characteristic of the axiomatic approach is that conditions of adequacy are argued for in advance of exploring individual rules. Once the axioms are in place, a theorem shows that they are consistent, or inconsistent, or characterizes the class of rules satisfying these conditions, and only those are considered "adequate." Yet this is *precisely* the approach that Saari adopts and that I criticize in my earlier paper and here. He identifies NRR and NBR as

[13] Cf. pp 721-23; I discuss there that, if the two lines of argument I sketch in 2.1 above establish that the Condorcet proposal is what we should mean by majoritarian decision making, the fact that the proposal violates Arrow's Independence axiom does not undermine that conclusion.

neutral conditions and shows that only Borda satisfies them. If, instead, Saari means to identify a contrast between "emphasizing the data" (which is what he thinks *he* does) and doing something else (whatever that is, but it is *others* doing it), this fails as well: starting with NRR and NBR instead of NRR and NCR leads to Condorcet. If NRR and NCR "emphasize the data" as opposed to doing something else, then so do NRR and NBR, whatever that "something else" is.

What counts as a successful objection to decision rules? The sketch in 2.3, concluding my response to Saari's second objection, suggests an answer. Suppose we find an apparently counterintuitive implication, or a conflict with an apparently plausible axiom. Then we should check whether this implication or conflict is plausible on the terms adopted by defenders of that rule. If so, opponents must engage arguments for that rule to explore whether the standpoint from which such implications or conflicts appear plausible can itself be supported. If these arguments succeed, the objections fail: they merely spell out consequences of that rule. If these arguments fail, nothing is gained if counterintuitive implications or conflicts with axioms are acceptable to proponents of the rule. The rule has to be withdrawn. If, however, the counterintuitive implication or the conflict is unacceptable to defenders of the rule, we must still explore whether, on balance, arguments for the rule outweigh the objections. This again turns on an investigation of the arguments. Such heavy-handed assessments are, unfortunately, required to obtain insights into group rationality. Saari should not balk at implications whose plausibility depends on arguments he ignores. Intuitions about social choice are too amorphous to allow for quasi-foundationalist isolation of selective features of a method, only to classify them as counterintuitive, and to take that judgment to outweigh, without further ado, what may be said for the rule.

3.4 Yet Saari offers another argument for NCR. This argument is important because it is also an argument against the reliance on pairwise votes.[14] Consider the Condorcet triplet (A, B, C), (C, A, B), and (B, C, A). Those rankings contribute to the selection of the rankings championed by the Condorcet proposal by contributing information about how many individuals have which preference between which options: two individuals support "A over B," one "B over A," two "B over C," one "C over B," two "C over A," and one "A over C." Next consider all profiles generating that distribution of votes over those pairs (which Saari calls "parts" of the Condorcet configuration). One of them is the Condorcet triplet. Another includes (A, B, C) and (C, B, A), while the third voter has cyclical preferences, ranking A over B, B over C, and C over A. Pairwise votes among those people contribute in the same way to the selection of the rankings championed by the Proposal as the Condorcet triplet. Yet in this case, so Saari says, the natural outcome for the group is the cyclical preference structure: (A, B, C) and (C, B, A) cancel out due to NRR, leaving the third person to determine the outcome. The Proposal delivers the wrong result. As Saari informs us, 80% of all profiles that can be constructed in this way (generate the same distribution of votes over pairs) support cyclical social outcomes. As Saari says (p. 347), quoting himself,

> [t]hese numbers capture a sense of Saari's argument (…) that the majority vote statistically interprets the parts of a Condorcet n-tuple as coming from profiles consisting of cyclic voters where the indeterminate cyclic outcome is an appropriate conclusion. Rather than a natural tie, the majority vote introduces a cycle because it is trying to meet the needs of nonexistent cyclic voters! As Saari has argued, the indeterminacy problem and all difficulties where the majority vote and Condorcet proposal distort the wishes of the voters arise only because the majority vote mistakenly interprets the Condorcet configuration as being the contribution of nonexistent votes with cyclic preferences.

[14] Cf. fn. 24, p 719 of my earlier piece; Saari's response is on pp 347/8 of his article.

Let me explain in different words what I think is going on. Suppose three persons must rank A, B, and C. Suppose, again, that two individuals support "A over B," one supports "B over A," two "B over C," one "C over B," two "C over A," and one "A over C." Different remarks apply if we use majority rule introduced in section 1, or the Condorcet proposal. Suppose we use majority rule. Then a cycle emerges, which Saari thinks is appropriate in 80% of all cases leading to these results, but *not* when we have a Condorcet triplet: in *that* case the intuitive outcome is a tie (as Borda delivers it). Yet no tie emerges because "the majority vote mistakenly interprets the Condorcet configuration as being the contribution of nonexistent votes with cyclic preferences." Suppose we use the Condorcet proposal. Saari thinks this example shows that the Proposal is flawed since it uses information about majority votes on pairs that are "parts" of a Condorcet n-tuple. Yet it should disregard such information, since the best explanation for the occurrence of those votes is that they are generated by individuals some of whom have cyclical preferences, which makes it dubious that the outcome should be a ranking (as presupposed by the Proposal). I only discuss Saari's claim about what these phenomena entail for the Proposal.

Saari's argument challenges the applicability of the Proposal *only* if we have reason to think that voters have irrational (non-transitive) preferences. That may happen in two ways. Either (a) we know the preferences and that some are cyclical, or (b) we only know the results of pairwise voting and it is likely that the relevant pairwise results derive from cyclical preferences. As I have pointed out repeatedly, the Proposal captures majoritarian decision making *in ideal theory*. What characterizes such ideal theory for decision rules is, in particular, that we do not worry about availability of information about preferences: (b) never holds. Saari's considerations are worrisome only if (a) holds, that is, only if some voters have intransitive preferences. Nothing has been shown to discredit the claim that, *if* voters are rational, the Proposal should be used. While the Proposal does not *presuppose* transitive preferences, arguments in my earlier piece (cf. section 2 above) show that the Proposal captures majoritarian decision making (and is a reasonable rule) *for rational voters* who also wish to be collectively rational. In ideal theory, we know whether voters have intransitive preferences. That is part of what ideal theory is.

Thus *pace* Saari, his considerations are best understood as a friendly amendment to my claims that the Condorcet proposal captures majoritarian decision making and is a reasonable rule, in ideal theory and restricted to aggregating rankings. His amendment is that those claims *are true only for rational voters*. This is a (very!) friendly amendment because, according to the Proposal, voters assess which rankings are maximally supported by pairwise votes, and there is no point in such a quest if they do not have such rankings themselves. As Saari is aware, if voters had intransitive preferences, we should not assume (as the Proposal does) that one or more of the possible *rankings* must be the group choice on majoritarian grounds. The search for a collectively rational outcome can only be conducted on behalf of rational individuals. It is puzzling why Saari thinks making this explicit constitutes an objection.[15]

Suppose we are not in ideal theory, but instead wonder which method to adopt in a concrete setting. Supposes our concern is to make decisions in a majoritarian manner. Our input is only what voters reveal, and in addition we are concerned that the Condorcet proposal may be impracticable. For such settings, Saari teaches an important lesson, namely, that the Proposal might deliver implausible results if some have cyclical preferences. If there are such preferences,

[15] As I put it in my earlier piece, in ideal theory, Saari's insights discredit the Condorcet proposal "no more than the fact that a decision reached by expected-utility reasoning may also have been reached by drawing lots (if only the right physical conditions hold that make one lot come up rather than another) or by following the Roman technique of observing the flight of birds (if only we have the right kind of weather for their flight to suggest a certain decision) discredits expected-utility theory" (p 719).

cyclical group outcomes should not be by definition excluded as collective outcomes, as the Proposal does. We should worry if, as condition (b) stipulates, voting results are best explained by the presence of irrational voters. There are other ways of dealing with that problem, but one simple way is to ask voters to submit rankings rather than pairwise votes, so that the procedure can extract views on pairs from the rankings. This is legitimate given that Borda *must* ask for rankings, and given that the fact that the Proposal uses only pairwise votes has no methodological or epistemological virtues that would be undermined if we asked voters for rankings. Such questions, however, must be settled in concrete settings. Saari's cases, like his school example in 2.4, tremendously increase our understanding of problems that may arise when the Proposal is put into practice. One source of trouble is that the Condorcet proposal does not presuppose that preferences are transitive, but fails to be intuitively reasonable if they are not. Saari's examples are valuable for providing such insights. But that is their major accomplishment. Again, such insights are friendly amendments, not objections.[16]

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[16] Saari has presented the argument discussed here in several publications, in particular in Saari and Vincent Merlin, "A Geometric Examination of Kemeny's Rule," *Social Choice and Welfare* 17 (2000): 403-438. Consider what Saari and Merlin say, p 421: "These cycles occur because the pariwise vote cannot distinguish the Condorcet profile (of transitive preferences) from ballots cast by irrational voters with cyclic preferences (…). In other words, using the pairwise vote with a Condorcet profile differential has the effect of dismissing, for all practial purposes, the crucial assumption that the voters are rational. Instead (…), the parwise vote treats the Condorcet n-tuple (…) as though the votes are cast by non-existent, irrational voters." To the extent that Saari and Merlin speak about majority rule as defined in section 1, they confirm that that rule should not be taken to capture majoritarian decision making. If they speak about the Condorcet proposal, we must consider that the proposal asks which rankings are best supported by pairwise votes, and we must consider differences between ideal theory and applied settings. The implications of these points are spelled out in 3.4. Cf. also Saari, *Chaotic Elections* (Providence: American Mathematical Society, 2001), chapter 5, in particular section 1.6, and "Mathematical Structure of Voting Paradoxes," *Economic Theory* 15 (2000): 1-53, in particular sections 6 and 8.

4. Conclusion

Let me sum up. The multiplicity thesis emerges strengthened from this discussion. Recall that my claim is that, if we aggregate ordinal rankings, both the Borda count and the Condorcet proposal are reasonable rules. I argued this point in my earlier piece by rehearsing arguments for the Condorcet proposal and by showing that the defender of the Borda count can make a parallel case. Much of section 3 adds to that argument, and section 2 shows that Saari cannot independently discredit the Condorcet proposal. So the fact remains that, in democracy, some people will be "losers" although they would not have been had another, equally reasonable decision rule been adopted. As I argue in my earlier piece, this does not affect common justification of democracy.

What about the claim that majoritarian democracy is "conceptually sound?" What I take that to mean is that majoritarian democracy is coherent, and I think this follows from my arguments for the claim that the Condorcet proposal is what we should mean by majoritarian decision making. Saari does not dispute *that* claim, but still thinks that majoritarian democracy is conceptually flawed. What he means, I gather, and what he takes other critics, like Hardin, to mean, is that majoritarian decision making captured either by majority rule (as introduced in section 1), or by the Condorcet proposal is *implausible* (cf. p 346 of Saari's article). I have argued that it is not, at least as long as we restrict ourselves to aggregating rankings. It becomes implausible under those circumstances under which the Borda count does as well. Vis-a-vis defenders of the Borda count, majoritarian democracy does just fine. While arguing all this, I submitted some ideas about how to reason about group decision making in the first place. I hope these proposals trigger more debate.