PHYSNET

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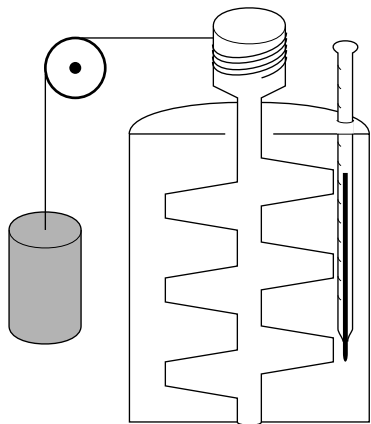
THE FIRST LAW OF Thermodynamics

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by

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Title: **The First Law of Thermodynamics**

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Input Skills:

1. Vocabulary: temperature, equation of state, ideal gas constant, ideal gas, mole, (MISN-0-157); work (MISN-0-20).
2. Express temperatures using both the Celsius and Kelvin scales (MISN-0-157).
3. Solve two algebraic equations in two unknowns (MISN-0-401).
4. Evaluate $\int_a^b x^k \, dx$ for arbitrary k (MISN-0-1).

Output Skills (Knowledge):

K1. Vocabulary: adiabatic, calorie, cycle, heat, heat reservoir, heat capacity, internal energy, isothermal, P-V diagram.

K2. State the expression for the internal energy of an ideal gas.

K3. State the first law of thermodynamics.

Output Skills (Problem Solving):

S1. Compute the work done during isothermal and adiabatic expansions (or compressions) of a given amount of an ideal gas.

S2. Apply the first law of thermodynamics to compute the heat transferred in processes involving ideal gases. Consider processes in which volume, pressure and temperature may change at the same time.

Post-Options:

1. "Entropy" (MISN-0-160).
2. "Energy Distribution Functions: the Boltzmann Distribution" (MISN-0-159).

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1. Examples of Energy Dissipation

A medium size car weighing 4000 lbs and moving with a speed of 55 mph carries a large amount of kinetic energy. This energy is roughly equal to the energy needed for an average person to climb a 2000 ft high mountain, which is equivalent to 2 hrs of hard work; and yet, with a medium hard pressure on the brake pedal, the car can be stopped in 20 seconds. After the brakes are cooled off there is no trace of the kinetic energy left. There are many instances of the disappearance of kinetic energy: a stone dropped on the ground; a collision of trains; gems polished in a grinder; a man trying to warm himself up in cold weather, beating his arms against his body. In some of the cases, the energy loss seems to be accompanied by a heating of the surroundings. Careful examination would show that some heat is produced in all cases. Is the heat a form of energy? Is energy lost? What is heat? We shall discuss these questions now.

2. Internal Energy

2a. Disordered Nature of Molecular Motion. Consider the motion of a stone thrown into the air. The molecules of the air are in a constant motion and so are the molecules from which the stone is built. However, there is a difference between the two kinds of motion. The air molecules are very fast, as fast as 500 m/s, and they move randomly in all directions. Because of this randomness, it is difficult to observe their motion. Only when a very small region containing not too many molecules is observed can the motion be detected. The molecules of the stone move all together and therefore their motion is quite noticeable even though their speed is small; on the order of, say, 40 m/s. Due to collisions with the air molecules, the stone will experience a force called "air resistance." It is quite interesting to consider the effect of these collisions on the speeds of the air molecules.
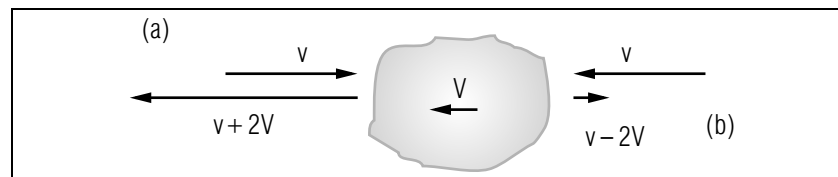
Figure 1. A molecule, velocity $\vec{v}$, hits a stone that has velocity $\vec{V}$. Two cases are shown: (a) the molecule approaches from the left; (b) the molecule approaches from the right. In each case the upper line shows the molecule's velocity before the collision, the lower line after it.

2b. Mechanical Energy into Molecular Motion Energy. During the head-on collision of a molecule, of speed v, with a stone, of speed V, the speed of the molecule will increase by $2V$ (see Fig. 1). A molecule chasing the stone from behind and then colliding will decrease its speed by $2V$. The change in the molecule's kinetic energy in the head-on collision will be

$$\Delta E_k = \frac{1}{2}m(v + 2V)^2 - \frac{1}{2}mv^2 = +2mvV \ ,$$

where we have neglected the small term $2mV^2$ (we assume $V \ll v$). Similarly, for a collision from behind, we have:

$$\Delta E_k = \frac{1}{2}m(v - 2V)^2 - \frac{1}{2}mv^2 = -2mvV \ ,$$

The first impression is that the average kinetic energy of the air molecules should remain constant. However, it takes more time to chase the stone from behind than to meet it head-on, therefore the number of head-on collisions in which energy is gained by the air molecules is larger. Consequently, the average kinetic energy of the air molecules should increase. This is not surprising. We assume that only elastic collisions take place and that the total kinetic energy must be conserved. Because the stone is slowing down, its kinetic energy must be transferring to the disordered motion of the air molecules.

2c. Temperature and Molecular Motion. Although the motion of the molecules of a gas is difficult to observe, a change in their average kinetic energy is easily detected by monitoring the temperature of the gas. For many gases this average kinetic energy is, to a good approximation, proportional to their temperature. In liquids and solids the random motion of molecules is more complicated because they are close together and are interacting with each other most of the time. In spite of the

complicated nature of these internal motions their total average energy (kinetic plus interactions) can be entirely determined by measuring the temperature and volume of a solid or liquid.

2d. Internal Energy. This total average energy of internal molecular motion is called "internal energy" and should always be taken into account when considering the conservation of energy. For an ideal gas, there is no potential energy since there are no interactions. Furthermore there is no "structure" to the gas molecules, so the kinetic energy is simply $mv^2/2$ for each molecule. The internal energy of an ideal gas of N molecules can be expressed in terms of the average kinetic energy of the molecules[1]

$$U = NE_{\text{k,ave}} = \frac{1}{2} N m v^2_{\text{RMS}}, \tag{1}$$

or in terms of temperature:

$$U = \frac{3}{2} N k T = \frac{3}{2} n R T. \tag{2}$$

For a non-ideal gas, the internal energy is usually expressed as:

$$U = C_v T, \tag{3}$$

where C_v is called the "heat capacity" of the gas. It is a quantity which is proportional to the amount of gas in the system.

3. Heat and Its Mechanical Equivalent

3a. Intensive Character of Internal Energy. It has been explained in the previous section that the internal energy is not related to the global motion of a system but to its temperature. In everyday language, we would say that a body with increased internal energy has higher temperature or is hotter.

as. I. nternal Energy Transfer[]3bHeat The transfer of internal energy can take place directly (without exchange of mechanical energy) when two bodies are in contact. The internal energy will be transferred from the higher temperature body to the lower temperature one. Again we would say that there was flow of heat. It took a long time to recognize the nature of heat and its relationship with energy and work. In textbooks, heat is explained as a "form of energy" or a "form of energy transfer." The second definition of heat, as an energy transfer, is more precise.

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[1]For the connection between average kinetic energy, RMS speed and temperature, see "Temperature and Pressure of an Ideal Gas: The Equation of State" (MISN-0-157).
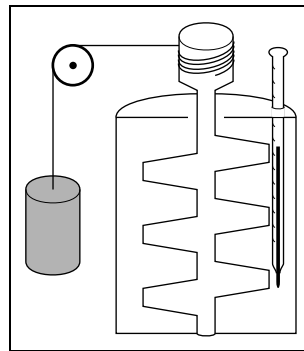
Figure 2. Apparatus used to measure the mechanical equivalent of heat. The weight is let drop, thereby forcing the shaft to turn and the paddles to stir the liquid.

3c. Heat Unit: Calories. The unit of heat is the "calorie." It is defined as the amount of heat needed to increase the temperature of one gram of water from $14.5\,°C$ to $15.5\,°C$. It is a good approximation to assume that one calorie will increase the temperature of one gram of water by $1\,°C$ at any initial temperature in the liquid phase.

3d. Heat's Mechanical Equivalent. In the case of the stone thrown into the air, we noticed that the mechanical energy of the stone was changed into the internal energy of the air and that the temperature of the air would be changed by the heat flow. One should be able to determine precisely the amount of heat produced when a definite amount of mechanical energy is converted into internal energy. The experiment with the stone flying in the air is not well suited for this purpose. The kinetic energy of the stone could be determined by measuring its mass and by controlling its initial velocity. However, the amount of heat produced would be very difficult to measure because of the large volume of air involved. The first careful measurement of the heat equivalent of mechanical energy was made by James Joule (1818-1889). In his experiment, a falling weight pulled a string which rotated on a shaft with attached paddles immersed in a water container (see Fig. 2). The lost mechanical energy is known from the distance the weight falls and from the weight's mass. The gain in heat can be determined from the mass of water and the increase in its temperature. The results obtained by Joule were quite accurate in his time. The value accepted today is:

$$1\,\text{cal} = 4.184\,\text{joules}.$$

In whatever manner the mechanical energy E is changed into heat—by stirring, rubbing, friction or during an inelastic collision—the amount of

heat transferred, Q, is always found to be:

$$4.184 \times Q[\text{in calories}] = E[\text{in joules}].$$

4. Mechanical Work of Gases

4a. Change of Internal Energy into Work. The internal energy of a system can be increased at the expense of mechanical energy as was demonstrated in Joule's experiment. The direct transfer of internal energy from one body to another is possible when the two bodies have different temperatures and are in contact with each other. Is it possible to obtain mechanical energy from the internal energy of a gas or some other substance?

4b. Work Done by a Gas: The Steam Engine. Indeed, such a process can take place in machines which produce mechanical work while burning some fuel. Let us consider the work stroke of a steam engine. At the beginning of the stroke the steam is contained in a small part of the cylinder and its pressure and temperature are high. When the stroke is finished, the volume is larger and the pressure and temperature are smaller. For example, in Fig. 3 the force F on the piston is

$$F = PA, \tag{4}$$

where P is the pressure of the gas and A the surface area of the piston exposed to the gas. If the displacement ΔS of the piston is small, the
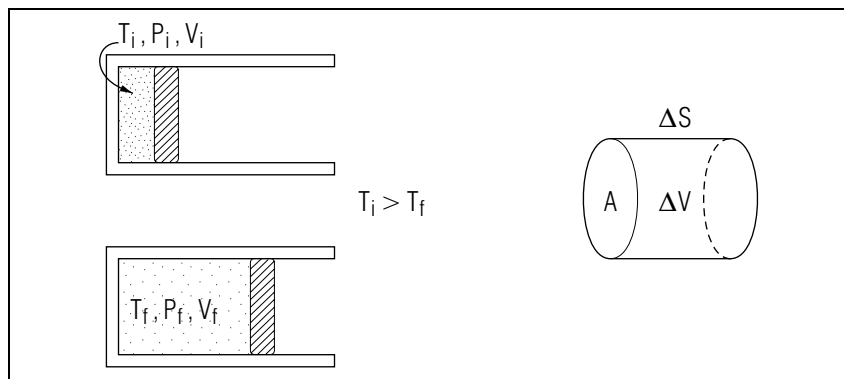
Figure 3. Changes in state of the gas in an engine cylinder during the piston stroke.
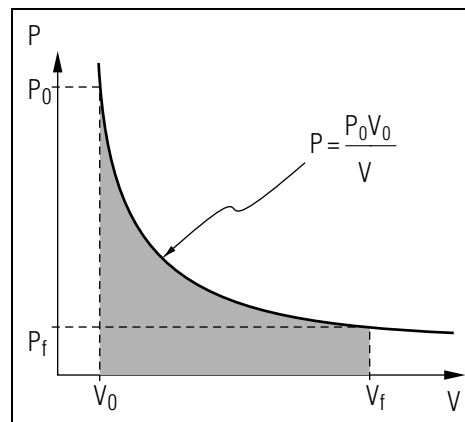
Figure 4. $P - V$ diagram of an isothermal expansion. Note that pressure decreases as volume increases while temperature is held constant.

pressure can be assumed to be approximately constant, and the work done on the piston by the gas is:

$$W = F\Delta S = PA\Delta S = P\Delta V, \tag{5}$$

where ΔV is the change in the volume of the gas during expansion. For large displacements where P is not constant over the change in volume, one should integrate over variable pressure with the result:

$$W = \int_{V_0}^{V_f} P \, dV. \tag{6}$$

This formula can be used if we know the pressure of the gas as a function of its volume, either by mathematical formula or by a graphical representation known as a $P - V$ diagram.[2]

4c. Work in an Isothermal Process. There are many ways in which gas pressure may vary during an expansion. Let us imagine that the expanding mass of gas is in thermal contact with a large body which has a constant temperature. Such a body is often called a "heat reservoir." Because heat is allowed to flow freely between the gas and the heat reservoir, the temperature of the gas will remain constant. An expansion during which the temperature is constant is called an "isothermal" expansion and if the system is closed, i.e. the amount of gas remains fixed, the pressure will be inversely proportional to the volume:

$$PV = nRT = \text{ constant } = P_0V_0,$$

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[2]If the $P - V$ diagram is given, the work is equal to the area under the curve. See the relevant problems in this module's *Problem Supplement*.
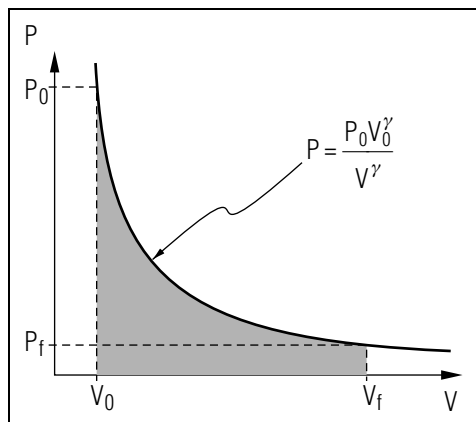
Figure 5. $P - V$ diagram of an adiabatic process. Note that pressure decreases as volume increases and energy is held constant.

so

$$P = \frac{P_0 V_0}{V} , \qquad (7)$$

where P_0 and V_0 are initial values of the pressure and volume. The work done during a isothermal expansion can be calculated now using Eq. (6):

$$W = \int_{V_0}^{V_f} P \, dV = \int_{V_0}^{V_f} \frac{P_0 V_0}{V} \, dV = P_0 V_0 \ell n \left(\frac{V_f}{V_0} \right) . \qquad (8)$$

We have assumed here that the gas expanded from the initial volume V_0 to the final volume V_f and that the initial pressure was P_0. The isothermal expansion is illustrated graphically in the $P - V$ diagram of Fig. 4. The shaded area is the integral of P with respect to V and thus represents the work done by the gas during the expansion.

4d. Work in a Adiabatic Process. When the cylinder and piston are good thermal insulators and/or the expansion is fast, the heat flow to the expanding gas can be neglected. A process in which the gas does not receive or produce any heat is called an "adiabatic" process. The pressure in the adiabatic process depends on the volume in a more complicated manner:

$$PV^\gamma = \text{ constant } = P_0 V_0^\gamma ,$$

so,

$$P = \frac{P_0 V_0^\gamma}{V^\gamma} , \qquad (9)$$

where P_0 and V_0 are the initial values of the pressure and volume and γ is a constant. The constant γ is equal to approximately 1.4 for all gases

with molecules composed of two atoms. This type of expansion is also illustrated in a $P - V$ diagram (see Fig. 5).

4e. Negative Value for Work Done *on* a Gas. Let us notice that the work as defined in Eq. (6) is the work done *by* a gas. If the gas is compressed, however, work is done *on* the gas by its surroundings. Equation (6) still holds, but note that its value will be negative. In general, a negative value of the integral shows that work is done *on* the system, rather than *by* the system.

5. The First Law of Thermodynamics

5a. Conserving Mechanical + Internal Energy. We have learned about a new form of energy (internal energy) and about different ways in which mechanical and internal energy can be exchanged. We are ready to introduce the "first law of thermodynamics," which is nothing more than the conservation law for these two kinds of energy.

5b. Energy Conservation in an Adiabatic Process. Consider first a well insulated cylinder, with a well insulated piston, containing a compressed gas that can only expand or contract adiabatically (see Fig. 6). If the gas is allowed to expand, the piston may carry a load and mechanical work will be performed. This work is done entirely by the gas in the cylinder, which is thermally insulated from its surroundings. Conservation of energy requires that the work W done by the gas be equal to the loss of its internal energy U:

$$W = -\Delta U , \qquad (10)$$

where the minus sign means that the internal energy is lost when the gas expands and does work.

5c. Energy Conservation in an Isothermal Process. Let us now consider an isothermal expansion of the gas. In this case the cylinder is made from a good thermal conductor and is in contact with a heat reservoir. During the expansion the temperature of the gas will remain constant and equal to the temperature of the heat reservoir. Therefore its internal energy will remain constant too. This means that the work of the expanding gas must come from some other source than the gas itself. The only other source of energy is the heat reservoir. The energy will flow in the form of heat from the heat reservoir to the gas. The work W done by the gas must be equal to the heat Q absorbed by it:
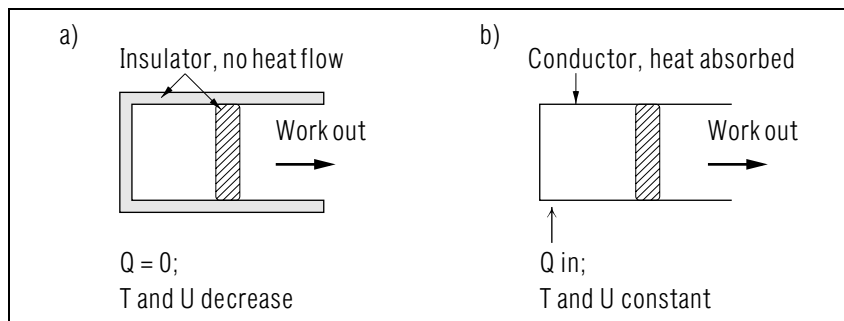
$$W = Q . \qquad (11)$$

Figure 6. A piston moving in a cylinder: (a) an adiabatic expansion; (b) an isothermal expansion.

5d. Energy Conservation in a General Process. In a real life process usually both heat and internal energy are converted into work. Such would be the case with a cylinder which is neither a perfect thermal conductor nor a perfect insulator. The total balance of work, heat and internal energy is:

$$W = Q - \Delta U .$$

Traditionally, this equation is written as

$$\Delta U = Q - W , \tag{12}$$

and is known as the first law of thermodynamics. It says that the internal energy gained by a system must be equal to the heat absorbed by the system minus work done by the system.

5e. Generalization to Other Substances. We arrived at Eq. (12) by assuming that energy is conserved and considering all possible ways in which it can be exchanged. We should not be surprised to find that Eq. (12) is valid, not only for an expanding gas, but also for all processes in which work is produced, heat is exchanged and internal energy is gained or lost. As with the simpler law of conservation for mechanical energy, the first law of thermodynamics cannot be proved or derived. It was discovered as a result of many observations and experiments. No violation of the law has ever been observed.

Acknowledgments

Glossary

- **adiabatic**: an adjective describing a process in which a system does not experience any heat transfer. This means that the system is thermally insulated from its environment.

- **calorie**: a unit of heat energy; the amount of internal energy transfer necessary to raise the temperature of one gram of water from 14.5 °C to 15.5 °C; abbreviated "cal."

- **cycle**: a series of changes of state in a system resulting in a return to the initial state.

- **heat**: a form of energy transfer involving the internal energy of a system and mechanical work done on or by the system. The unit of heat is the calorie.

- **heat reservoir**: a large system with enough constituents so that transfers of heat to or from the reservoir do not significantly change its temperature.

- **heat capacity**: a quantity used to express the internal energy of a system as a function of temperature.

- **internal energy**: the total average energy of internal motion of a system. At thermal equilibrium the internal energy can be expressed as a function of temperature.

- **isothermal**: an adjective describing a process in which a system undergoing a change in state does not experience a change in temperature. For an ideal gas under closed conditions this implies that the product of pressure and volume is constant.

- $P - V$ **diagram**: a two-dimensional graph of the pressure of a system as a function of volume. The work done on or by the system can be represented as the area under the $P(V)$ curve between the initial and final volumes.

PROBLEM SUPPLEMENT

$$
\begin{aligned}
1\,\text{atm} &= 1.013 \times 10^5\,\text{N/m}^2 = 1.013 \times 10^5\,\text{J/m}^3 \\
1\,\text{liter} &= 1000\,\text{cm}^3 = 10^{-3}\,\text{m}^3 \\
1\,\text{N} &= 4.45\,\text{lb} \\
1\,\text{watt} &= 1\,\text{joule/sec} \\
1\,\text{mi/hr} &= 0.447\,\text{m/s} \\
1\,\text{cal} &= 4.184\,\text{J} \\
T/\,^\circ\text{C} &= \frac{5}{9}\,[(T/\,^\circ\text{F}) - 32] \\
T/K &= T/\,^\circ\text{C} + 273
\end{aligned}
$$

Note: T denotes temperature

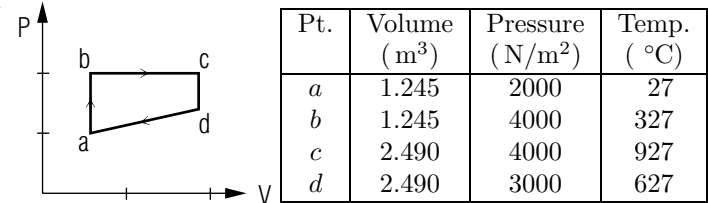Note: Problem 7 also occurs in this module's *Model Exam.*

1. A closed vessel contains $32\,\text{g}$ of oxygen gas at $1.00\,\text{atm}$ pressure. The gas is heated with its volume kept constant. It is found that $5.0\,\text{calories}$ of heat are needed to raise the temperature of the oxygen by $1.00\,^\circ\text{C}$.

   Compute how much heat is needed to raise the temperature of $1.00\,\text{liter}$ of oxygen, initially at $1.00\,\text{atm}$ pressure, from $0.0\,^\circ\text{C}$ to $1.00 \times 10^2\,^\circ\text{C}$ with its volume remaining constant? *Help: [S-1]*

2. The internal energy of the oxygen gas in Problem 1 changes according to the formula $\Delta U = C_v \Delta T$. Compute the heat capacity, C_v:

   a. for $32\,\text{g}$ of oxygen

   b. for $1.0\,\text{liter}$ of oxygen at a pressure of $1.0\,\text{atm}$ and a temperature of $0.0\,^\circ\text{C}$ (STP).

3. Compute the heat produced when a car weighing 4.0×10^3 lbs brakes and its velocity decreases from $55\,\text{mph}$ to $2.0 \times 10^1\,\text{mph}$, assuming all of the lost mechanical energy is converted to heat. *Help: [S-6]*

4. A gas vessel with a piston is heated up and is allowed to expand in such a way that its pressure remains constant and equal to $2.0\,\text{atm}$. The initial volume of gas is $6.0\,\text{liters}$ and the final volume is $10.0\,\text{liters}$. Find the work done by the gas. *Help: [S-2]*

5. An ideal gas with volume $5.0\,\text{liters}$ and pressure $2.5\,\text{atm}$ undergoes an adiabatic expansion. The final volume is $7.5\,\text{liters}$. Find the work done by the gas during the expansion. *Help: [S-3]*

6. One mole of an ideal gas goes through the cycle shown here in this $P-V$ diagram:
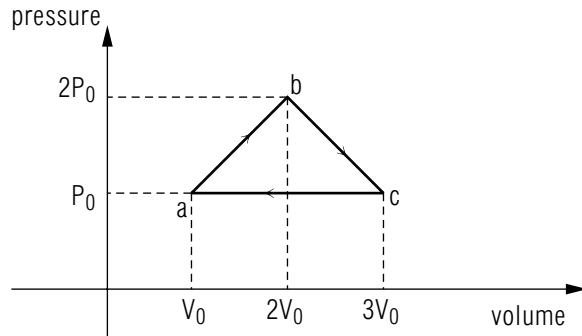
a. Use the equation of state of an ideal gas, $PV = nRT$, and the following data to find the temperature of the gas at points a, b, and c. The answers we give follow from this "ideal gas" assumption.
$P_0 = 1.00 \times 10^6 \, \text{N/m}^2$
$V_0 = 1.00 \times 10^{-3} \, \text{m}^3$
$R = 8.314 \, \text{J/(mole K)}$

b. What is the internal energy (in joules) of the gas at points a, b, and c? *Help: [S-4]*

c. Calculate the work done by the gas as it goes from a to b; b to c; c to a. *Help: [S-8]*

d. Calculate ΔU for each step: a to b; b to c; c to a.

e. Calculate Q for each step: a to b; b to c; c to a.

f. Calculate ΔU, W and Q for one complete cycle. Where does the energy come from for the work output?

g. If this apparatus is to be used as an engine, how rapidly should it complete each cycle for it to have a power output of 3.0 HP? (1 HP = 746 watts)

7.

Pt.	Volume (m^3)	Pressure (N/m^2)	Temp. ($^\circ$C)
a	1.245	2000	27
b	1.245	4000	327
c	2.490	4000	927
d	2.490	3000	627

A gas goes through the cycle illustrated above. The starting point is at "a." The values of the volume, pressure and temperature of the gas during the cycle are shown in the table. The heat capacity of the gas is $C_V = 5 \, \text{cal/K}$.

a. Calculate the internal energy for each of the states a, b, c and d. *Help: [S-7]*

b. Calculate the net work done by the gas during one complete cycle. *Help: [S-7]*

c. How much heat is absorbed or liberated by the gas during the part of the cycle from $a \to b$? *Help: [S-7]* Repeat for the parts of the cycle from $b \to c$, $c \to d$, $d \to a$. *Help: [S-7]*

d. Where does the energy come from for the work done during the complete cycle? Substantiate your answer quantitatively.

e. If the system going through this cyclic process is to be used as an engine, how rapidly should it complete this cycle for its output to be 30 kilowatts?

Brief Answers:

1. $Q = 22.3 \, \text{cal}$

2. Heat capacity:

   a. $5 \, \text{cal}/^\circ\text{C}$

   b. $0.223 \, \text{cal}/^\circ\text{C}$

3. $113.6 \, \text{Kcal}$

4. $808 \, \text{J}$

5. $W = 474 \, \text{J}$ *Help: [S-5]*

6. An ideal gas cycle:

   a. $T_a = 120\,\text{K}$; $T_b = 481\,\text{K}$; $T_c = 361\,\text{K}$

   b. $U_a = 1.50 \times 10^3\,\text{J}$ *Help: [S-4]*
      $U_b = 6.00 \times 10^3\,\text{J}$; $U_c = 4.50 \times 10^3\,\text{J}$

   c. $W_{ab} = 1.50 \times 10^3\,\text{J}$ *Help: [S-6]*
      $W_{bc} = 1.50 \times 10^3\,\text{J}$; $W_{ca} = -2.00 \times 10^3\,\text{J}$; negative, compression

   d. $\Delta U_{ab} = 4.50 \times 10^3\,\text{J}$; $\Delta U_{bc} = -1.50 \times 10^3\,\text{J}$; $\Delta U_{ca} = -3.00 \times 10^3\,\text{J}$

   e. a to b: $Q_{ab} = \Delta U_{ab} + W_{ab} = 6.00 \times 10^3\,\text{J}$
      b to c: $Q_{bc} = 0$
      c to a: $Q_{ca} = -5.00 \times 10^3\,\text{J}$

   f. $\Delta U = 0$ for one cycle, since the gas returns to its original condition. Summing the results of part (d) will verify this.
      Using part (c) gives W = 1000 J.
      Part (e) gives Q = 1000 J.
      All the work output energy comes from heat input energy.

   g. $P = W/t$ so that $t = W/P$.
      Using 1000 J as the work for one cycle, P as 3.0 HP = 2238 W gives t = 0.45 sec, the time for one cycle.

7. A gas cycle:

   a. point a, 1500 cal; point b, 3000 cal; point c, 6000 cal; point d, 4500 cal

   b. 1868 joules

   c. $a \to b$, 1500 cal absorbed; $b \to c$, 4190 cal absorbed; $c \to d$, 1500 cal liberated; $d \to a$, 3744 cal liberated

   d. Net heat absorbed $= 1500\,\text{cal} + 4190\,\text{cal} - 1500\,\text{cal} - 3744\,\text{cal} = 446\,\text{cal} = 1868\,\text{J} = $ net work done

   e. 16.1 cycles per second

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS, Problem 1)*

Note that there are two different quantities of oxygen in this problem, heated by different amounts: first 32 grams heated 1 degree and then 1 liter heated 100 degrees.

The molecular weight of oxygen is 32, so 32 g of oxygen is one mole of oxygen. To find out how many moles of oxygen are in 1 liter at 1 atm pressure, (initially at 0 °C) use the equation of state for an ideal gas, $PV = nRT$, or recall that at STP (0 °C, 1 atm) a mole of ideal gas occupies a volume of 22.4 liters.

S-2 *(from PS, Problem 4)*

Since the pressure is constant, the integral for work reduces to the following simple form:

$$W = \int_{V_i}^{V_f} P\,dV = P \int_{V_i}^{V_f} dV = P(V_f - V_i).$$

On the $P - V$ diagram, below, the shaded area represents this integral and is easy to compute. UnframedEpsFigurem158gr09
Use: area = width × height = $(2\,\text{atm})(4\,\ell) = 8\,\text{atm}\,\ell$. Using the appropriate conversion factors this can be converted to joules.

S-3 *(from PS, Problem 5)*

Here is the $P - V$ diagram for this adiabatic expansion:

pressure (atm)

$PV^\gamma = (2.5\,\text{atm})(5.0\,\text{liter})^{1.4}$
$= \text{constant}$

5.0   7.5   volume (liters)
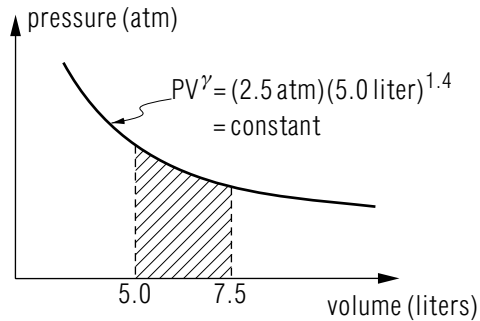
The work done by the expanding gas is still represented by the area under the curve, but there is no simple geometrical formula for computing it. You must use the integral definition of work and integrate $P = P_0 V_0^\gamma / V^\gamma$ with respect to V from $V = 5.0$ liters to $V = 7.5$ liters. *Help: [S-5]*

S-4 *(from Problem 6b)*

The equation for the internal energy of the ideal gas is given, in terms of the gas's temperature, in the text part of this module.

S-5 *(from [S-3])*

integral $= \dfrac{p_0 V_0^\gamma}{-\gamma + 1}\left(V_f^{-\gamma+1} - V_0^{-\gamma+1}\right) = 4.676\,\text{atm}\,\ell$

S-6 *(from PS, Problem 3)*

Advice: first convert to MKS and find: $m = 1805\,\text{kg}$, $v_i = 24.59\,\text{m/s}$, $v_f = 8.94\,\text{m/s}$.

S-7 *(from PS, Problem 7)*

Successfully work and thoroughly understand Problems 1-6, for which help is provided, before attempting this problem.

S-8 *(from PS, Problem 6c)*

You could compute the work done on/by the gas for each segment of the cycle by finding the mathematical equation expressing pressure as a function of volume and integrating between the appropriate limits. For example from a to b the pressure varies linearly with volume so the equation of a straight line is used:

$$P = \alpha V + \beta$$

where α is the slope of the line and β is the intercept. These two parameters are determined by evaluating the expression at two points where the pressure is known, [obviously at point $a(V_0 P_0)$ and point $b(2V_0, 2P_0)$] and solving the two resulting expressions for α and β:

$$P_0 = \alpha V_0 + \beta\,; \qquad \alpha = \frac{P_0}{V_0} = 1.00 \times 10^9\,\text{N m}^{-5}$$

hence:

$$2P_0 = 2\alpha V_0 + \beta \qquad \beta = 0$$

Thus $P = (P_0/V_0)V$ and this may be integrated from a to b. Similarly the pressure from b to c is $P = -(P_0/V_0)V + 4P_0$, and the pressure from c to a is simply P_0. If this seems too complicated you may use simple geometry to calculate the relevant areas under each $P(V)$ curve. For example from a to b, the area under the curve is shown below:
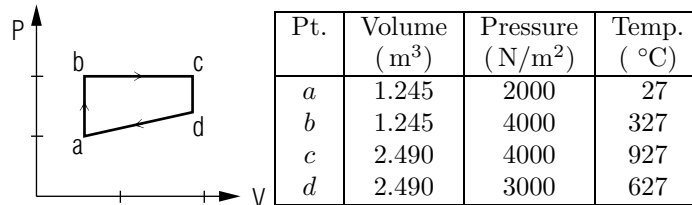

pressure

$2P_0$ b

P_0

P_0 a   c

V_0 P_0

V_0 $2V_0$ $3V_0$ volume
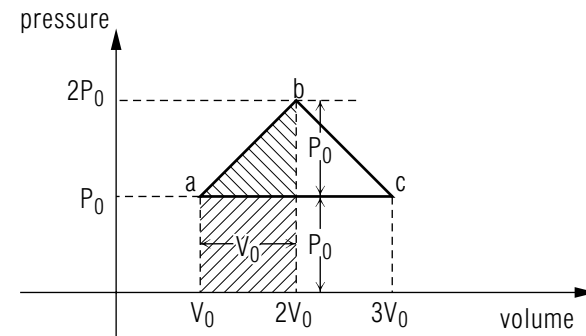
The work done by the gas in expanding from V_0 to $2V_0$ is simply the area of the shaded triangle and the shaded rectangle:

$$\text{Area} = \frac{1}{2}(\text{base}) \times (\text{height}) + (\text{width}) \times (\text{height})$$

$$= \frac{1}{2}(V_0)(P_0) + V_0 P_0 = \frac{3}{2}P_0 V_0$$

MODEL EXAM

1. See Output Skills K1-K3 in this module's *ID Sheet*.

2.

Pt.	Volume (m^3)	Pressure (N/m^2)	Temp. $(°C)$
a	1.245	2000	27
b	1.245	4000	327
c	2.490	4000	927
d	2.490	3000	627

A gas goes through the cycle illustrated above. The starting point is at "a." The values of the volume, pressure and temperature of the gas during the cycle are shown in the table. The heat capacity of the gas is $C_V = 5\,cal/K$.

a. Calculate the internal energy for each of the states a, b, c and d.

b. Calculate the net work done by the gas during one complete cycle.

c. How much heat is absorbed or liberated by the gas during the part of the cycle from $a \rightarrow b$? Repeat for the parts of the cycle from $b \rightarrow c$, $c \rightarrow d$, $d \rightarrow a$.

d. Where does the energy come from for the work done during the complete cycle? Substantiate your answer quantitatively.

e. If the system going through this cyclic process is to be used as an engine, how rapidly should it complete this cycle for its output to be 30 kilowatts?

Brief Answers:

1. See this module's *text*.

2. See this module's *Problem Supplement*, problem 7.